Chapter Contents

JEE (MAINS) JANUARAY 2020 TEST PAPERS SOLUTIONS

JEE (MAIN) SEPTEMBER 2020 TEST PAPERS SOLUTIONS

IMPORTANT NOTES

SET # 01

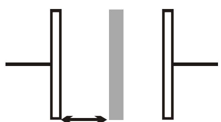
PHYSICS

1. NTA Ans. (1)

Sol. As K is variable we take a plate element of Area A and thickness dx at distance x

Capacitance of element

$$dC=\frac{(A)K(1+\alpha x)\varepsilon_0}{dx}$$



Now all such elements are is series so equivalent capacitance

$$\frac{1}{C}=\int\frac{1}{dC}=\int_0^d\frac{dx}{AK\varepsilon_0(1+\alpha x)}$$

$$\frac{1}{C}=\frac{1}{\alpha AK\varepsilon_0}\ln\left(\frac{1+\alpha d}{1}\right)$$

$$=\frac{1}{C}=\frac{1}{\alpha AK\varepsilon_0}\left(\alpha d-\frac{(\alpha d)^2}{2}+\frac{(\alpha d)^3}{3}+....\right)$$

$$\Rightarrow\frac{1}{C}=\frac{\alpha d}{\alpha AK\varepsilon_0}\left(1-\frac{\alpha d}{2}+\frac{(\alpha d)^2}{3}+....\right)$$

$$\frac{1}{C}=\frac{d}{AK\varepsilon_0}\left(1-\frac{\alpha d}{2}\right)$$

$$C=\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$$

2. NTA Ans. (3)

Sol. Time period of revolution of electron in n^{th} orbit

$$T=\frac{2\pi r}{V}=\frac{2\pi a_0\left(\frac{n^2}{Z}\right)}{V_0\left(\frac{Z}{n}\right)}$$

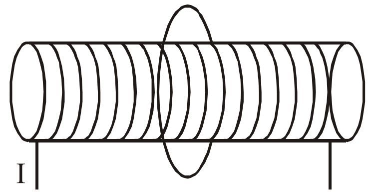
$$\Rightarrow\ T\propto\frac{n^3}{Z^2}$$

$$\frac{T_2}{T_1}=\frac{(2)^3}{(1)^3}=8\ \Rightarrow\ T_2=8\times1.6\times10^{-16}$$

Now frequency $f_2=\frac{1}{T_2}=\frac{10^{16}}{8\times1.6}\approx 7.8\times10^{14}$ Hz.

3. NTA Ans. (1)

Sol.



Magnetic flux (ϕ) through ring is $\phi=\pi(R)^2.B$

$\phi=(\pi R^2)(\mu_0 nI)=(\pi R^2\mu_0 nI_0)(t-t^2)$

Induced e.m.f. of $V_R=\frac{-d\phi}{dt}$

$=\left(\pi R^2\mu_0 nI_0\right)(2t-1)$

and induced current $I_R=\frac{\pi R^2\mu_0 nI_0(2t-1)}{R_R}$

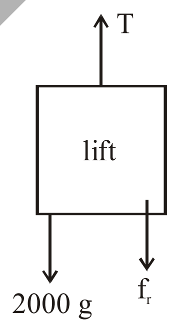
($R_R\to$ Resistance of Ring)

Clearly V_R and I_R are zero at $t=\frac{1}{2}=0.5$ sec.

and their sign also changes at t = 0.5 sec.

4. NTA Ans. (3)

Sol.



Let elevator is moving upward with constant speed V.

Tension in cable

$T=2000\ g+f_r=2000+4000$

T = 24000 N

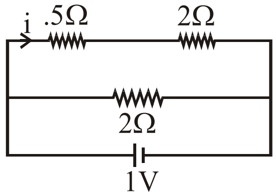
Power P = TV

$\Rightarrow\ 60\times746=(24000)\ V$

$V=\frac{60\times746}{24000}=1.865\approx1.9$ m/s.

5. NTA Ans. (2)

Sol. Equivalent resistance of upper branch of circuit
R = 2.5 Ω



Voltage across upper branch = 1 V

$$\Rightarrow \; i = \frac{1}{2.5} = .4 \text{ A}$$

$\Rightarrow I_1 = 0.2$ A

6. NTA Ans. (1)

Sol. $$w = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{P_1V_1 - P_2V_2}{0.4}$$

$$= \frac{100 - \frac{100}{4.6555} \times 3}{0.4} = 88.90 \cdot$$

7. NTA Ans. (3)

Sol. $$mgh = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}mr^2 \times \frac{v^2}{r^2} = \frac{3}{4}mv^2$$

$$u = \sqrt{\frac{4}{3}gh}$$

$$\omega = \frac{v}{r}$$

8. NTA Ans. (2)

9. NTA Ans. (1)

Sol. $$m = \frac{LD}{f_e \times f_0} = \frac{150 \times 250}{f_e \times 25} = 375$$

$f_e = 20$ mm.

10. NTA Ans. (2)

Sol. $$m\frac{l^2}{12} + m\frac{l^2}{16} = mk^2$$

$$\frac{7l^2}{48} = k^2$$

11. NTA Ans. (1)

Sol. $\vec{E} \times \vec{B} = \vec{C} = -\hat{i}$

where $\vec{B}$ is along $\hat{j}$

$$\frac{E}{B} = C$$

$E = 3 \times 10^{-8} \times 3 \times 10^{8} = 9$ V/m.

12. NTA Ans. (1)

13. NTA Ans. (3)

Sol. $$v = \sqrt{\frac{T}{\mu}}$$

$$90 = \sqrt{\frac{\frac{YA}{l}\Delta l}{\frac{m}{l}}} = \sqrt{\frac{16 \times 10^{11} \times 10^{-6} \times \Delta l}{6 \times 10^{-3}}}$$

$$= \frac{8100 \times 3}{8} \times 10^{-8} = \Delta l$$

14. NTA Ans. (4)

Sol. $$\sin\theta = \frac{2\lambda}{\omega}$$

$$\sin 60° = \frac{2\lambda}{\omega}$$

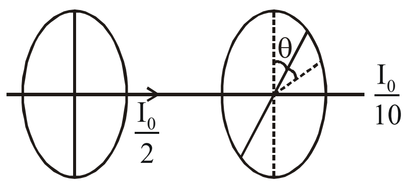
$$\sin\theta_1 = \frac{\lambda}{\omega} = \frac{\sqrt{3}}{4}$$

$\theta_1 = 25°$

15. NTA Ans. (1)

Sol. $$\frac{I_0}{10} = I = \frac{I_0}{2} \times \cos^2\theta$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$



$\theta = 63.44°$

angle rotated = 90 − 63.44° = 26.56°

Closest is 1.

16. NTA Ans. (2)

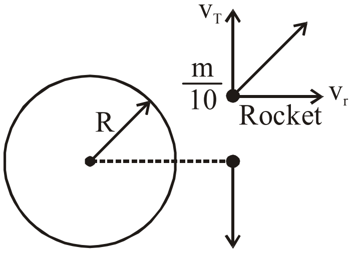
Sol. Applying energy conservation

$K_i + U_i = K_f + U_f$

$$\frac{1}{2}mu^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2 - \frac{GMm}{2R}$$

$$v = \sqrt{u^2 - \frac{GM}{R}} \qquad \text{....(i)}$$

By momentum conservation, we have



$$\frac{m}{10}v_T = \frac{9m}{10}\sqrt{\frac{GM}{2R}} \qquad \text{....(ii)}$$

& $\frac{m}{10}v_r = mv$

$$\Rightarrow \frac{m}{10}v_r = m\sqrt{u^2 - \frac{GM}{R}} \qquad \text{....(iii)}$$

Kinetic energy of rocket

$$= \frac{1}{2}m\left(v_T^2 + v_r^2\right)$$

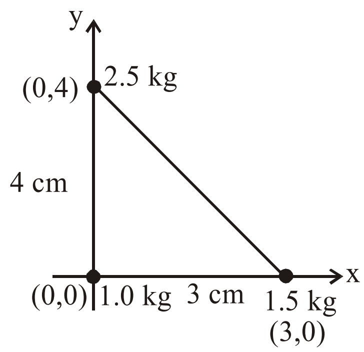
$$= \frac{m}{20}\left(81\frac{GM}{2R} + 100u^2 - 100\frac{GM}{R}\right)$$

$$= \frac{m}{20}\left(100u^2 - \frac{119GM}{2R}\right)$$

$$= 5m\left(u^2 - \frac{119GM}{200R}\right).$$

17. NTA Ans. (2)

Sol.



Let 1 kg as origin and x-y axis as shown

$$x_{cm} = \frac{1(0)+1.5(3)+2.5(0)}{5} = 0.9 \text{ cm}$$

$$y_{cm} = \frac{1(0)+1.5(0)+2.5(4)}{5} = 2 \text{ cm}$$

18. NTA Ans. (2)

Sol. $C_{P_{eq}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2}$

$$C_{V_{eq}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

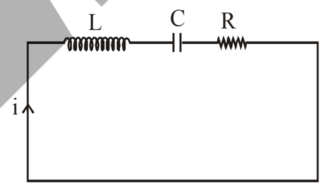
$$\gamma_{eq} = \frac{C_{P_{eq}}}{C_{V_{eq}}} = \frac{2\times\frac{5R}{2} + 3\times\frac{8R}{2}}{2\times\frac{3R}{2} + 3\times\frac{6R}{2}}$$

$$= \frac{5+12}{3+9} = \frac{17}{12} \simeq 1.42$$

Correct Answer : 2

19. NTA Ans. (1)

Sol.



By kVL

$$-L\frac{di}{dt} - \frac{q}{C} - iR = 0$$

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q + R\frac{dq}{dt} = 0$$

for damped oscillator

net force = –kx – bv = ma

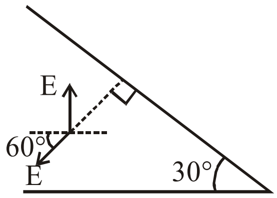
$$\frac{md^2x}{dt^2} + kx + \frac{bdx}{dt} = 0$$

by comparing ; Equivalence is

$$L \to m \; ; \; C \to \frac{1}{K} \; ; \; R \to b.$$

20. NTA Ans. (2)

Sol. Electric field due to each sheet is uniform and equal to $E = \dfrac{\sigma}{2\varepsilon_0}$



Now net electric field between plates

$$\vec{E}_{net} = E\cos 60°(-\hat{x}) + (E - E\sin 60°)(\hat{y})$$

$$= \frac{\sigma}{2\varepsilon_0}\left[-\frac{\hat{x}}{2} + \left(1 - \frac{\sqrt{3}}{2}\right)\hat{y}\right].$$

21. NTA Ans. (10)

Sol. Mechanical energy conservation between A & P

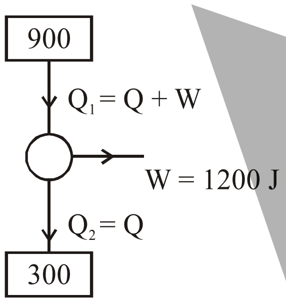
$U_1 + \cancel{K_1} = K_2 + U_2$

$mg \times 2 = mg \times 1 + K_2$

$K_2 = mg \times 1 = 10$ J.

22. NTA Ans. (600)

Sol.



for carnot engine

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q + 1200}{Q} = \frac{900}{300}$$

$Q + 1200 = 3Q$

$Q = 600$ J.

23. NTA Ans. (10)

Sol. Power incident $P = I \times A$

n = no. of photons incident/second

$nE_{ph} = IA$

$$n = \frac{IA}{E_{ph}}$$

$$n = \frac{IA}{\left(\frac{hc}{\lambda}\right)} = \frac{6.4 \times 10^{-5} \times 1}{\frac{1240}{310} \times 1.6 \times 10^{-19}}$$

$n = 10^{+14}$ per second

Since efficiency $= 10^{-3}$

no. of electrons emitted $= 10^{+11}$ per second.

$x = 11$.

24. NTA Ans. (60)

Sol. $\gamma = \alpha_x + \alpha_y + \alpha_z$

$= 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$

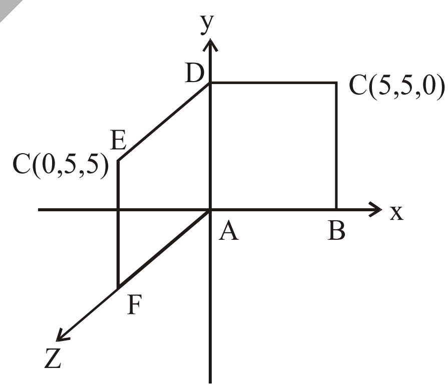
$= (50 + 5 + 5) \times 10^{-6}$

$\gamma = 60 \times 10^{-6}$

$C = 60$.

25. NTA Ans. (175)

Sol.



$\vec{A}_{ABCD} = 25\hat{k}$

$\vec{A}_{ADEF} = 25\hat{i}$

$\vec{A}_{net} = 25\hat{i} + 25\hat{k}$

$\vec{B} = 3\hat{i} + 4\hat{k}$

$\phi = \vec{B}.\vec{A}$

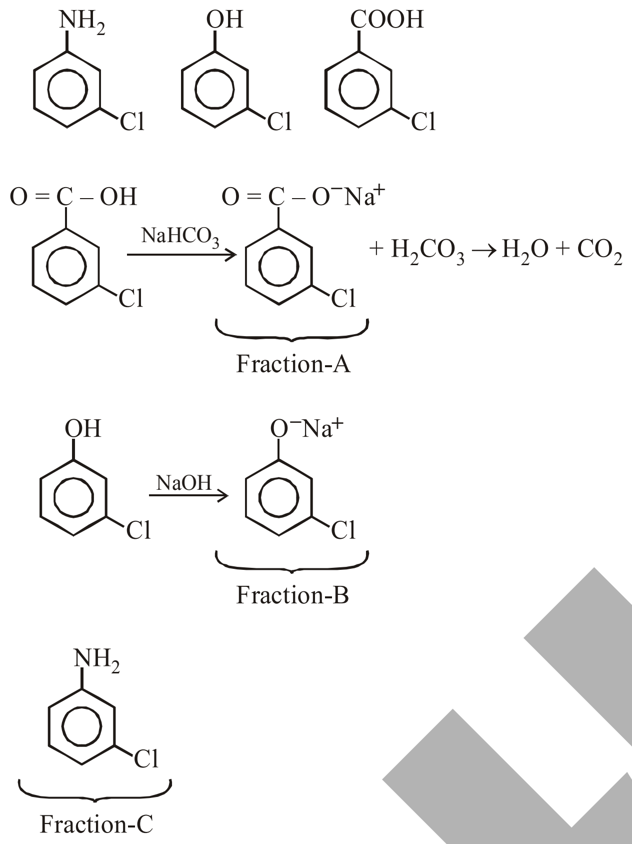
$= 25 \times 3 + 25 \times 4$

$\phi = 175\ W_b.$

CHEMISTRY

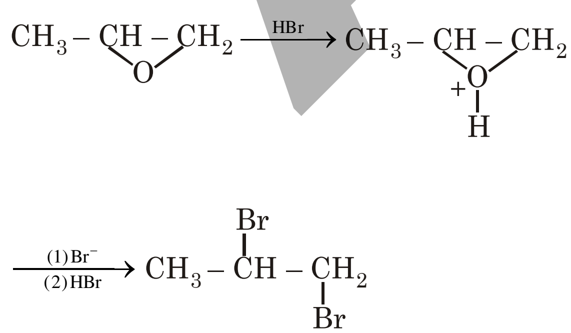
1. NTA Ans. (3)

Sol.



2. NTA Ans. (4)

Sol.

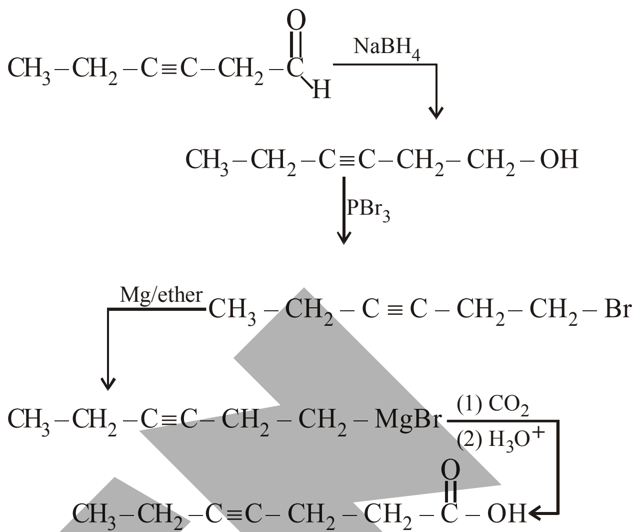


3. NTA Ans. (3)

Sol. Option(3) is according to Gaylussac's law of volume combination.

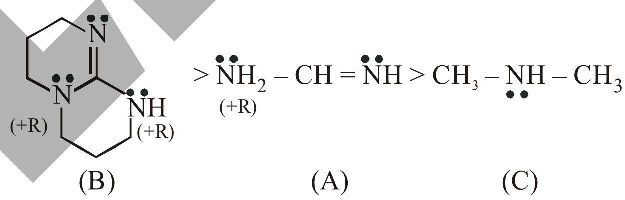
4. NTA Ans. (3)

Sol.



5. NTA Ans. (3)

Sol. Base strength order



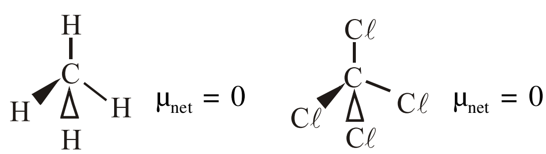
pk_b order (C > A > B)

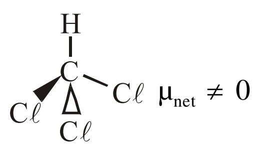
6. NTA Ans. (3)

Sol. Atomic radius of Ag and Au is nearly same due to lanthanide contraction.

7. NTA Ans. (1)

Sol.





8. NTA Ans. (1)

Sol.

	E^0	ΔG^0
$Cu^{2\oplus} + 2e^{\oplus} \longrightarrow Cu$	0.34	= – 2F (0.34)
$Cu^{\oplus} + e^{\oplus} \longrightarrow Cu$	0.522	= –F (0.522)

$Cu^{2\oplus} + e^{\oplus} \longrightarrow Cu^{+}$

$\Delta G^0 = -2F\ (0.34) - (-F(0.522)) = -F\ (0.68 - 0.522) = -F\ (0.158)$

$$E^0 = \frac{-F(0.158)}{-F} = 0.158V$$

9. NTA Ans. (4)

Sol. (a) Zeolite method removes only cations (Ca^{2+} and Mg^{2+} ion) present in hard water

$2NaZ + M^{2+}(aq) \rightarrow MZ_2(s) + 2Na^{+}(aq)$
(M→Mg, Ca)

(b) Synthetic resin method removes cations (Ca^{2+} and Mg^{2+} ion) and anions (like Cl^-, HCO_3^-, SO_4^{2-} etc.)

(i) $2RNa(s) + M^{2+}(aq) \rightarrow R_2M(s) + 2Na^{+}(aq)$
(Cation exchange resin) (M→Mg, Ca)

(ii) $RNH^{+}_3\,OH^{-}(s) + X^{-}(aq) \rightarrow RNH_3^{+}X^{-}(s) + OH^{-}(aq)$
(Anion exchange resin) ($X^- = Cl^-, HCO_3^-, SO_4^{2-}$ etc)

10. NTA Ans. (4)

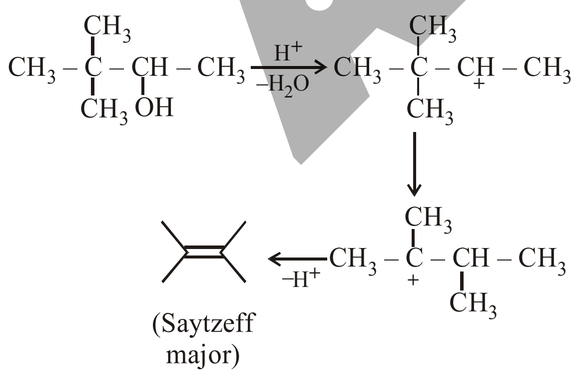
Sol. Order is

ion – ion > ion – dipole > dipole – dipole

11. NTA Ans. (1)

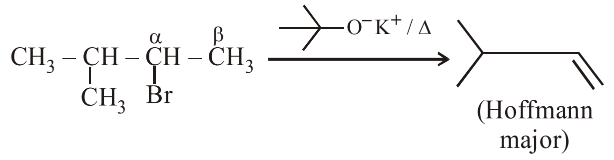
Sol.

(a)

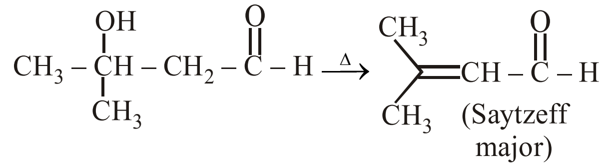


(b) $CH_3 - \underset{\substack{|\\CH_3}}{CH} - \underset{\substack{|\\Br}}{CH} - CH_3 \xrightarrow[\Delta]{alc.KOH} CH_3 - \underset{\substack{|\\CH_3}}{C} = CH - CH_3$
(Saytzeff major)

(c)



(d)



$(CH_3)_3O^-K^+$ is incorrect representation of potassium tert-butoxide $[(CH_3)_3CO^-K^+]$.

So it is possible that it can be given as **Bonus**

12. NTA Ans. (2)

Sol. Wrought iron is purest from of commercial iron.

13. NTA Ans. (3)

Sol. The vapour pressure of mixture (= 600 mm Hg) is greater than the individual vapour pressure of its constituents (Vapour pressure of CS_2 = 512 mm Hg, acetone = 344 mm Hg). Hence, the solution formed shows positive deviation from Raoult's law.

$\Rightarrow (1)\ \Delta_{Sol}H > 0$, (2) Raoult's law is not obeyed

(3) $\Delta_{sol.}$ Volume > 0

(4) CS_2 and Acetone are less attracted to each ether than to themselves.

14. NTA Ans. (1)

Sol. Order of electron gain enthalpy (magnitude) is Cl > F > Br > I

15. NTA Ans. (2)

Sol. No. of orbitals = $n^2 = 5^2 = 25$

For n = 5, no. of orbitals = n^2 = 25

Total number of orbitals is equal to no. of electrons having $m_s = \frac{1}{2}$

16. NTA Ans. (1)

Sol. (i) Riboflavin $\longrightarrow$ (c) Cheilosis

(ii) Thiamine $\longrightarrow$ (a) Beriberi

(iii) Pyridoxin $\longrightarrow$ (d) Convulsions

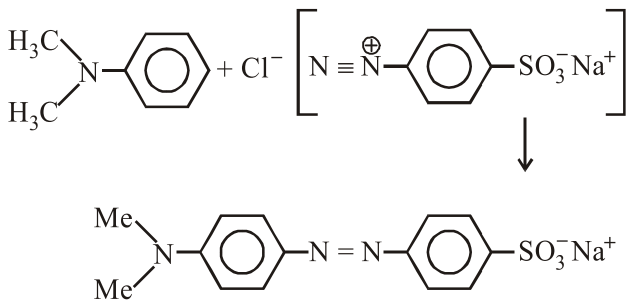
(iv) Ascorbic acid $\longrightarrow$ (b) Scurvy

17. NTA Ans. (4)

Sol. In complex $[Ni(CO)_4]$ decrease in Ni–C bond length and increase in C–O bond length as well as it's magnetic property is explained by MOT.

18. NTA Ans. (1)

Sol.



(Methyl orange)

It is an acid base indicator

19. NTA Ans. (4)

20. NTA Ans. (3)

Sol. Potasisum has an oxidation of +1 (only) in combined state.

21. NTA Ans. (–2.70 to -2.71)

Sol. $A(\ell) \longrightarrow 2B(g)$

$\Delta U = 2.1$ Kcal , $\Delta S = 20$ cal K^{-1} at 300 K

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta G = \Delta H - T\Delta S$

$\Delta G = \Delta U + \Delta n_g RT - T\Delta S$

$$= 2.1 + \frac{2\times2\times300}{1000} - \frac{300\times20}{1000}$$

(R = 2 cal K^{-1} mol^{-1})

= 2.1 + 1.2 – 6 = – 2.70 Kcal/mol

22. NTA Ans. (23 to 23.03)

Sol. All nuclear decays follow first order kinetics

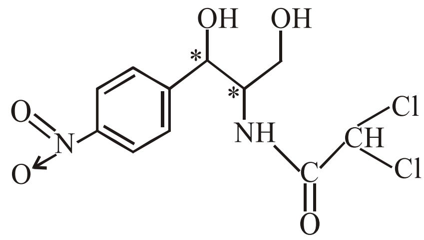
$$t = \frac{1}{k}\ell n\frac{[A_0]}{[A]}$$

$$= \frac{(t_{1/2})}{0.693}\times 2.303 \ \log_{10}10 \ = 10 \times 2.303 \times 1$$

= 23.03 years

23. NTA Ans. (2)

Sol.



Chloramphenicol

24. NTA Ans. (10.60)

Sol. 4 gm of NaOH in 100 L sol. $\Rightarrow 10^{-3}$ M sol.

9.8 gm of H_2SO_4 in 100 L sol. $\Rightarrow 10^{-3}$ M sol.

Mixture : 40L of 10^{-3} M NaOH and 10 L of 10^{-3} M H_2SO_4 sol.

Final Conc. of OH^-

$$= \frac{10^{-3}(40\times1-10\times1\times2)}{40+10} = 6\times10^{-4}\,M$$

pOH = – log (6 $\times 10^{-4}$)

= 4 – log 6 = 4 – 0.60 = 3.40

pH = 14 – 3.40 = 10.60

25. NTA Ans. (1.66 to 1.67)

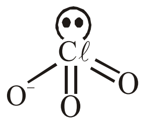
Sol. $3Cl_2 + 6NaOH \rightarrow 5NaCl + NaClO_3 + 3H_2O$

(X) (X)

$NaCl + AgNO_3 \rightarrow AgCl \downarrow + NaNO_3$

(X)

ClO_3^-



Bond order of Cl–O Bond = $1 + \frac{2}{3} = \frac{5}{3}$

= 1.66 or 1.67

MATHEMATICS

1. NTA Ans. (2)

Sol. $g(x) = x^2 + x - 1$

$g(f(x)) = 4x^2 - 10x + 5$

$= (2x - 2)^2 + (2 - 2x) - 1$

$= (2 - 2x)^2 + (2 - 2x) - 1$

$\Rightarrow f(x) = 2 - 2x$

$$f\left(\frac{5}{4}\right) = \frac{-1}{2}$$

2. NTA Ans. (2)

Sol. $\operatorname{Re}\left(\frac{z-1}{2z+i}\right)=1$

Put z = x + iy

$\operatorname{Re}\left(\frac{(x+iy)-1}{2(x+iy)+i}\right)=1$

$\operatorname{Re}\left(\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right)\left(\frac{2x-i(2y+1)}{2x-i(2y+1)}\right)\right)=1$

$\Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$

$x^2 + y^2 + x + \frac{3}{2}y+\frac{1}{2}=0$

$\Rightarrow$ locus is a circle whose

Centre is $\left(-\frac{1}{2},-\frac{3}{4}\right)$ and radius $\frac{\sqrt{5}}{4}$

$\Rightarrow$ diameter = $\frac{\sqrt{5}}{2}$

3. NTA Ans. (3)

Sol. Let the A.P is

a – 2d, a – d, a, a + d, a + 2d

$\because$ sum = 25 $\Rightarrow$ a = 5

Product = 2520

$(25 - 4d^2)(25 - d^2) = 504$

$4d^4 - 125d^2 + 121 = 0$

$\Rightarrow d^2 = 1, \frac{121}{4}$

$\Rightarrow d=\pm1,\pm\frac{11}{2}$

d = $\pm$ 1 is rejected because none of the term can be $\frac{-1}{2}$.

$\Rightarrow d = \pm\frac{11}{2}$

$\Rightarrow$ AP will be $-6, -\frac{1}{2}, 5, \frac{21}{2}, 16$

Largest term is 16.

4. NTA Ans. (1)

Sol. $y(\alpha)=\sqrt{2\frac{(\tan\alpha+\cot\alpha)}{1+\tan^2\alpha}+\frac{1}{\sin^2\alpha}}$, $\alpha\in\left(\frac{3\pi}{4},\pi\right)$

$=\frac{|\sin\alpha+\cos\alpha|}{|\sin\alpha|}=\frac{-(\sin\alpha+\cos\alpha)}{\sin\alpha}$

$= -1 - \cot\alpha$

$y'(\alpha) = \operatorname{cosec}^2\alpha$

$y'\left(\frac{5\pi}{6}\right)=4$

5. NTA Ans. (1)

Sol. $x^2 + x + 1 = 0$

$\alpha = \omega$

$\alpha^2 = \omega^2$

$A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}$

$A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$

$\Rightarrow A^4 = A^2 . A^2 = I_3$

$A^{31} = A^{28}. A^3 = A^3.$

6. NTA Ans. (3)

Sol. y = mx + 4 is tangent to $y^2 = 4x$

$\Rightarrow m = \frac{1}{4}$

$y=\frac{1}{4}x+4$ is tangent to $x^2 = 2by$

$\Rightarrow x^2-\frac{b}{2}x-8b=0$

$\Rightarrow$ D = 0

$b^2 + 128b = 0$

$\Rightarrow$ b = – 128, 0

$b \neq 0 \Rightarrow b = -128$

7. NTA Ans. (3)

Sol. Given $2ae = 6 \Rightarrow \boxed{ae = 3}$ ......(1)

and $\frac{2a}{e} = 12 \Rightarrow \boxed{a = 6e}$ ....(2)

from (1) and (2)

$6e^2 = 3 \Rightarrow \boxed{e = \frac{1}{\sqrt{2}}}$

$\Rightarrow \boxed{a = 3\sqrt{2}}$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 18\left(1 - \frac{1}{2}\right) = 9$

Length of L.R $= \frac{2(9)}{3\sqrt{2}} = 3\sqrt{2}$

8. NTA Ans. (3)

Sol.

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

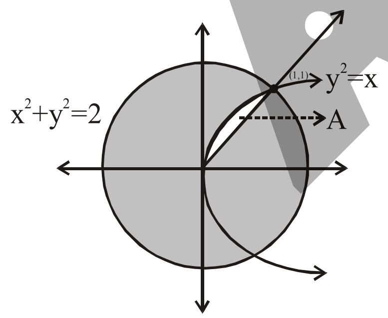
Expected value $= \sum XP(k)$

$-\frac{1}{32} - \frac{12}{32} - \frac{11}{32} + \frac{15}{32} + \frac{8}{32} + \frac{5}{32}$

$= \frac{28 - 24}{32} = \frac{4}{32} = \frac{1}{8}$

9. NTA Ans. (2)

Sol.



$A = \int_0^1 \left(\sqrt{x} - x\right) dx$

$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{1}{6}$

Required Area : $\pi r^2 - \frac{1}{6} = \frac{1}{6}(12\pi - 1)$

10. NTA Ans. (3)

Sol. $x^k + y^k = a^k$ $(a, k > 0)$

$kx^{k-1} + ky^{k-1}\frac{dy}{dx} = 0$

$\frac{dy}{dx} + \left(\frac{x}{y}\right)^{k-1} = 0 \Rightarrow k - 1 = -\frac{1}{3} \Rightarrow k = 2/3$

11. NTA Ans. (4)

Sol. $e^y\frac{dy}{dx} - e^y = e^x$, Let $e^y = t$

$\Rightarrow e^y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - t = e^x$

I.F. $= e^{\int -dx} = e^{-x}$

$t e^{-x} = x + c \Rightarrow e^{y-x} = x + c$

$y(0) = 0 \Rightarrow c = 1$

$e^{y-x} = x + 1 \Rightarrow y(1) = 1 + \log_e 2$

12. NTA Ans. (1)

Sol. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 is

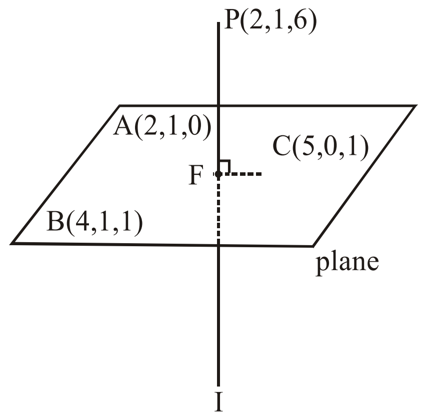
${}^5C_1 \times \frac{6!}{2!}$

13. NTA Ans. (1)

Sol. Plane passing through : (2, 1, 0), (4, 1, 1) and (5, 0, 1)

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow x + y - 2z = 3$



Let I and F are respectively image and foot of perpendicular of point P in the plane.

eq^n of line PI $\frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\lambda(\text{say})$

Let I (λ + 2 , λ + 1, –2λ + 6)

$$\Rightarrow F\left(2+\frac{\lambda}{2},1+\frac{\lambda}{2},-\lambda+6\right)$$

F lies in the plane

$$\Rightarrow 2+\frac{\lambda}{2}+1+\frac{\lambda}{2}+2\lambda-12-3=0$$

$\Rightarrow \lambda = 4$

$\Rightarrow$ I (6, 5, –2)

14. NTA Ans. (4)

ALLEN Ans. (Bonus)

Sol. $\vec{a}=\lambda\left(\hat{b}+\hat{c}\right)=\lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}+\frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$$\vec{a}=\frac{\lambda}{3\sqrt{2}}\left(4\hat{i}+2\hat{j}+4\hat{k}\right)\Rightarrow\frac{\lambda}{3\sqrt{2}}(4\hat{i}+2\hat{j}+4\hat{k})$$

$$=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$$

$\Rightarrow \alpha = 4$ and $\beta = 4$

So, $\vec{a}=4\hat{i}+2\hat{j}+4\hat{k}$

None of the given options is correct

15. NTA Ans. (1)

ALLEN Ans. (1 or 3)

Sol. f(x + 1) = f (a + b – x)

$$I=\frac{1}{(a+b)}\int_a^b x\left(f(x)+f(x+1)\right)dx \quad .....(1)$$

$$I=\frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(x+1)+f(x)\right)dx \quad ...(2)$$

from (1) and (2)

$$2I=\int_a^b\left(f(x)+f(x+1)\right)dx$$

$$2I=\int_a^b f\left(a+b-x\right)dx+\int_a^b f(x+1)dx$$

$$2I=2\int_a^b f(x+1)dx\Rightarrow I=\int_a^b f(x+1)dx$$

$$=\int_{a+1}^{b+1} f(x)dx$$

OR

$$I=\frac{1}{(a+b)}\int_a^b x\left(f(x)+f(x+1)\right)dx \quad ...(1)$$

$$=\frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(a+b-x)+f(a+b+1-x)\right)dx$$

$$I=\frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(x+1)+f(x)\right)dx \quad ...(2)$$

equation (1) + (2)

$$2I=\frac{1}{(a+b)}\int_a^b (a+b)\left(f(x+1)+f(x)\right)dx$$

$$I=\frac{1}{2}\left[\int_a^b f(x+1)dx+\int_a^b f(x)dx\right]$$

$$=\frac{1}{2}\left[\int_a^b f(a+b+1-x)dx+\int_a^b f(x)dx\right]$$

$$=\frac{1}{2}\left[\int_a^b f(x)dx+\int_a^b f(x)dx\right]$$

$$I=\int_a^b f(x)dx$$

Let x = T + 1

$$=\int_{a-1}^{b-1} f(T+1)dT$$

$$I=\int_{a-1}^{b-1} f(x+1)dx$$

16. NTA Ans. (2)

Sol. Using LMVT in [–7, –1]

$$\frac{f(-1)-f(-7)}{-1-(-7)}\le 2$$

f(–1) – f (–7) $\le$ 12

$\Rightarrow$ f (–1) $\le$ 9 ....(1)

Using LMVT in [–7, 0]

$$\frac{f(0)-f(-7)}{0-(-7)}\le 2$$

f(0) – f (–7) $\le$ 14

f(0) $\le$ 11 ....(2)

from (1) and (2)

f(0) + f (–1) $\le$ 20

17. NTA Ans. (4)

Sol. For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0, \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$\Rightarrow$ (3b – 2a) (c –a) – (b – a) (4c – 2a) = 0

$\Rightarrow$ 2ac = bc + ab

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Hence $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

18. NTA Ans. (2)

Sol. $\tan\alpha + \tan\beta = \frac{\lambda\sqrt{2}}{k+1}$

$\tan\alpha . \tan\beta = \frac{k-1}{k+1}$

$$\tan(\alpha + \beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1 - \frac{k-1}{k+1}} = \frac{\lambda\sqrt{2}}{2} = \frac{\lambda}{\sqrt{2}}$$

$$\Rightarrow \frac{\lambda^2}{2} = 50 \Rightarrow \lambda = 10 \,\&\, -10$$

19. NTA Ans. (3)

Sol. $(p \to q) \wedge (q \to \sim p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$

$\equiv \sim p \vee (q \wedge \sim q)$

$\equiv \sim p \vee C \equiv \sim p$

20. NTA Ans. (3)

Sol. $1 + 49 + 49^2 + \ldots + 49^{12}$

$$= (49)^{126} - 1 = (49^{63} + 1)\,\frac{(49^{63} - 1)}{(48)}$$

So greatest value of k = 63

21. NTA Ans. (36)

Sol. $$\lim_{x\to 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}} \Rightarrow \lim_{x\to 2} \frac{3^{2x} - 12.3^x + 27}{3^{x/2} - 3}$$

$$= \lim_{x\to 2} \frac{(3^x - 9)(3^x - 3)}{(3^{x/2} - 3)}$$

$$= \lim_{x\to 2} \frac{(3^{x/2} + 3)(3^{x/2} - 3)(3^x - 3)}{(3^{x/2} - 3)}$$

= 36

22. NTA Ans. (18)

Sol. Variance of first 'n' natural numbers = $\frac{n^2 - 1}{12} = 10$

$\Rightarrow$ n = 11

and variance of first 'm' even natural numbers

$$= 4\left(\frac{m^2 - 1}{12}\right) \Rightarrow \frac{m^2 - 1}{3} = 16 \Rightarrow m = 7$$

m + n = 18

23. NTA Ans. (30)

Sol. Let $(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$

$= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots + a_{4n}x^{4n}$

So,

$a_0 + a_1 + a_2 + \ldots + a_{4n} = 2n + 1$ ...(1)

$a_0 - a_1 + a_2 - a_3 \ldots + a_{4n} = 2n + 1$ ...(2)

$\Rightarrow a_0 + a_2 + a_4 + \ldots + a_{4n} = 2n + 1$

$\Rightarrow 2n + 1 = 61$ $\qquad \Rightarrow n = 30$

24. NTA Ans. (5)

Sol. P is centroid of the triangle ABC

$$\Rightarrow P \equiv \left(\frac{17}{6}, \frac{8}{3}\right)$$

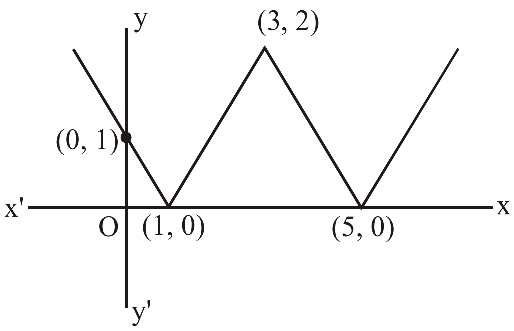
$\Rightarrow$ PQ = 5

25. NTA Ans. (3)

Sol. f(x) = |2 – |x – 3||

f is not differentiable at

x = 1, 3, 5



$$\Rightarrow \sum_{x \in s} f(f(x)) = f(f(1)) + f(f(3)) + f(f(5))$$

= f(0) + f(2) + f(0)

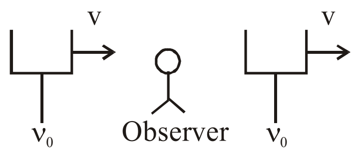
= 1 + 1 + 1 = 3

SET # 02

PHYSICS

1. NTA Ans. (4)

Sol.



$$\nu_1 = \left(\frac{c}{c-v}\right)\nu_0$$

$$\nu_2 = \left(\frac{c}{c+v}\right)\nu_0$$

beat frequency = $\nu_1 - \nu_2$

$$= c\nu_0 \left(\frac{1}{c-v} - \frac{1}{c+v}\right)$$

$$= c\nu_0\left(\frac{c+v-c+v}{c^2-v^2}\right) = \frac{2c\nu_0^2 v}{c^2-v^2}$$

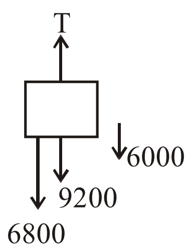
$$\approx \frac{2c\nu_0 v}{c^2} = \frac{2\nu_0 v}{c} = 2$$

$$\Rightarrow \frac{2\times1400\times v}{350} = 2$$

$$\Rightarrow v = \frac{1}{4}\ \text{m/s}$$

2. NTA Ans. (3)

Sol.



elevater moving with constant speed hence

T = 6800 + 9200 + 6000

T = 22000 N

power = T·v = 22000 × 3

= 66000 W

3. NTA Ans. (4)

Sol. $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$$500 = 700 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

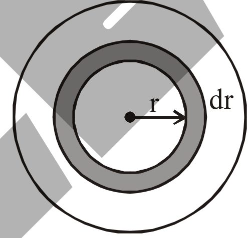
$$0.7 \approx \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$\left(\frac{1}{2}\right)^{1/2} \approx \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$\frac{30}{T_{1/2}} \approx \frac{1}{2} \Rightarrow T_{1/2} = 60$$

4. NTA Ans. (1)

Sol.



$dI = dmr^2$

$dI = \sigma 2\pi r\ dr\ r^2$

$dI = 2\pi(A + Br)\ r^3\ dr$

$$\int dI = 2\pi\int_0^a \left(Ar^3 + Br^4\right)dr$$

$$I = 2\pi a^4 \left(\frac{A}{4} + \frac{B9}{5}\right)$$

5. NTA Ans. (3)

Sol. $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

$$\vec{E} = E_0\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\cos\pi$$

$$= -E_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

as $\vec{E}\times\vec{B} = \vec{c}$

$$+E_0\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\times \vec{B} = c\hat{k}$$

$$\Rightarrow \vec{B} = -\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\frac{E_0}{c}$$

$$\vec{F} = q\left(-E_0\frac{(\hat{i}+\hat{j})}{\sqrt{2}} - \frac{v_0\hat{k}}{c}\times(\hat{i}-\hat{j})E_0\right)$$

since $\frac{v_0}{c} << 1$

$\Rightarrow$ F is antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

6. NTA Ans. (3)

Sol. $(2V_0)^2 = v_0^2 + v_x^2$

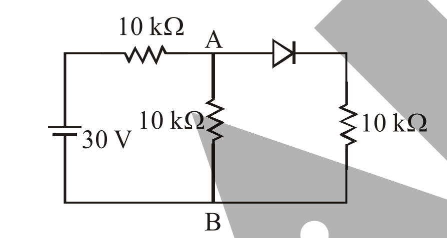
$v_x = \sqrt{3}\, v_0$

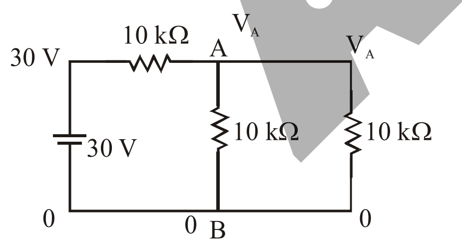
$\sqrt{3}\, v_0 = 0 + \frac{qE_0}{m}t$

$t = \frac{\sqrt{3}v_0 m}{qE_0}$

7. NTA Ans. (2)

Sol.





$$\frac{30-V_A}{10} + \frac{0-V_A}{10} + \frac{0-V_A}{10} = 0$$

$3 = \frac{3V_A}{10}$

$V_A = 10$ V

8. NTA Ans. (1)

Sol. Magnetic energy stored per unit volume is

$\frac{B^2}{2\mu_0}$

Dimension is $ML^{-1}T^{-2}$

9. NTA Ans. (4)

Sol. $220\,I = P = 15 \times 45 + 15 \times 100 + 15 \times 10 + 2 \times 10^3$

$I = \frac{4325}{220} = 19.66$

$I \simeq 20$ A

10. NTA Ans. (2)

ALLEN Ans. (1)

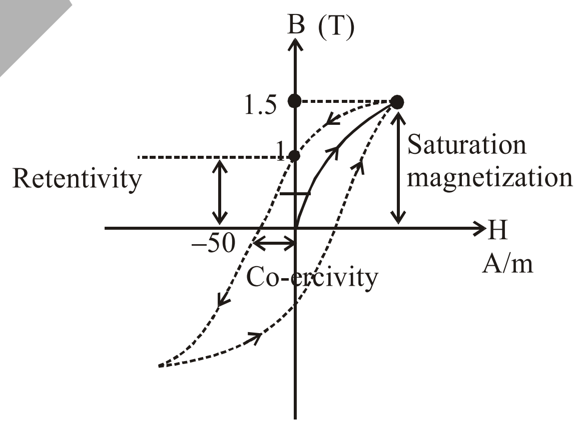
Sol. $i = i_0\,(1 - e^{-Rt/L})$

$\frac{i_0}{i} = \frac{1}{1-e^{-2\times10^4}}$

$\frac{i_0}{i} \simeq 1$

11. NTA Ans. (2)

Sol.



Retentivity = 1.0 T

Co-ercivity = 50 A/m

Saturation = 1.5 T

12. NTA Ans. (2)

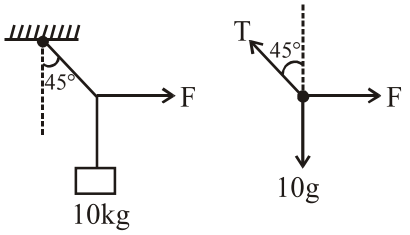
Sol. Fringe width, $\beta = \frac{D\lambda}{d} = \frac{1.5\times589\times10^{-9}}{0.15\times10^{-3}}$

$= 5.9 \times 10^{-3}$ m

$= 5.9$ mm

13. NTA Ans. (1)

Sol.



For equilibrium,

$T \sin 45° = F$ ....(1)

and $T \cos 45° = 10g$ ....(2)

equation (1)/(2)

we get $F = 10g$

$= 100$ N

14. NTA Ans. (3)

Sol. Using $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ....(1)$$

and $\frac{1}{f_1} = \left(\frac{1.5}{1.42} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ....(2)

equation (1)/(2),

we get $\frac{f_1}{f} = \frac{0.5}{0.056}$

$= 8.93 \approx 9$

15. NTA Ans. (4)

Sol. Flux $\phi = \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos\omega t$

$|\text{Induced emf}| = |e| = \left|\frac{d\phi}{dt}\right| = |BA\omega \sin \omega t|$

$|e|$ will be maximum at $\omega t = \frac{\pi}{2}$

$$\left(\frac{2\pi}{T}\right)t = \frac{\pi}{2}$$

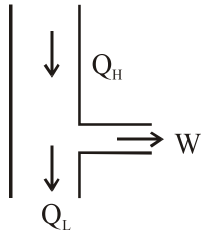
$$\left(\frac{2\pi}{10}\right)t = \frac{\pi}{2} \Rightarrow t = 2.5 \text{ sec}$$

$|e|$ will be minimum at $\omega t = \pi$

$$\left(\frac{2\pi}{10}\right)t = \pi \Rightarrow t = 5 \text{ sec}$$

16. NTA Ans. (3)

Sol.



$\frac{Q_H}{Q_L} = \frac{T_1}{T}$ and $W = Q_H - Q_L$ ....(1)

$\frac{Q_L}{Q'_L} = \frac{T}{T_2}$ and $W = Q_L - Q'_L$ ....(2)

17. NTA Ans. (4)

Sol. $W = 196 - m\omega^2 R$

18. NTA Ans. (1)

Sol. $t \propto \frac{V}{\sqrt{T}}$ ....(1)

$TV^{\gamma-1} = \text{constant}$ ....(2)

$\therefore t \propto V^{\frac{\gamma+1}{2}}$

19. NTA Ans. (4)

Sol. $A_1 v_1 = A_2 v_2$

$$\frac{v_{min}}{v_{max}} = \frac{A_{min}}{A_{max}}$$

$$\frac{v_{min}}{v_{max}} = \left(\frac{4.8}{6.4}\right)^2$$

$$\frac{v_{min}}{v_{max}} = \frac{9}{16}$$

20. NTA Ans. (2)

Sol. $\frac{\lambda_{electron}}{\lambda_{photon}} = ?$

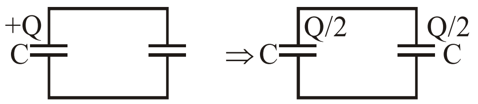
$E = \frac{hc}{\lambda_{photon}}$ ....(1)

$\lambda_{electron} = \frac{h}{\sqrt{2mE}}$ ....(2)

from (1) and (2)

$$\frac{\lambda_{electron}}{\lambda_{photon}} = \frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$$

21. NTA Ans. (6)

Sol. 

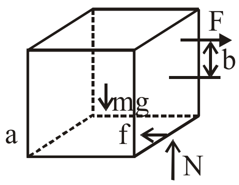
$$\Delta Q_L = \frac{Q^2}{2C} - \left[\frac{(Q/2)^2}{2C} \times 2\right] = \frac{Q^2}{4C}$$

$$= \frac{1}{4} CV^2$$

$$= \frac{1}{4} \times 60 \times 10^{-12} \times 4 \times 10^2$$

$= 6nJ$

22. NTA Ans. (75)

Sol. 

$F = \mu mg$ ...(1)

$F\left(b + \frac{a}{2}\right) = mg\ \frac{a}{2}$ ...(2)

$$\mu mg \left(b + \frac{a}{2}\right) = mg \times \frac{a}{2}$$

$$\left(b + \frac{a}{2}\right) \mu = \frac{a}{2}$$

$$0.4 = \mu = \frac{a}{2b + a}$$

$0.8b + 0.4a = a$

$0.8\ b = 0.6\ a$

$$\frac{b}{a} = \frac{3}{4}$$

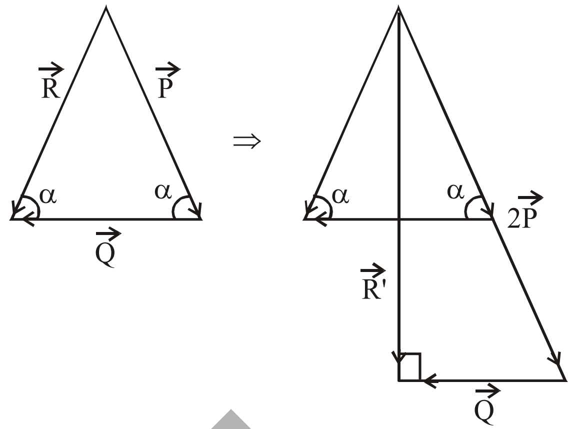
23. NTA Ans. (12)

Sol. $r = R\left(\frac{x - x'}{x'}\right)$

$$= 10 \times \frac{60}{500}$$

$= 12$

24. NTA Ans. (90)

Sol. 

Hence angle 90°

25. NTA Ans. (40)

Sol. $M \times 540 + M + 60 = 200 \times 80 + 200 \times 1 \times (40 - 0)$

$\Rightarrow M = 40$

CHEMISTRY

1. NTA Ans. (3)

Sol. (i) Electron affinity of second period p-block element is less than third period p-block element due to small size of second period p-block element.

E.A. order : F < Cl

(ii) Down the group electron affinity decreases due to size increases.

EA. order : S > Se

Li > Na

2. NTA Ans. (1)

Sol. (i) $N_2 + O_2 \xrightarrow{2000\ K} 2NO$ (Redox reaction)

during the reaction, oxidation of nitrogen take place from 0 to 2 and reduction of oxygen take place from 0 to –2. It means this reaction is redox reaction.

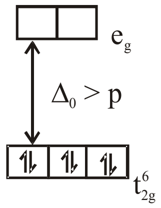
(ii) $3O_2 \xrightarrow{h\nu} 2O_3$ (Non – redox reaction)

(iii) $H_2SO_4 + 2NaOH \rightarrow Na_2SO_4 + 2H_2O$ (neutralization reaction)

(iv) $[Co(H_2O)_6]Cl_3 + 3AgNO_3 \rightarrow 3AgCl\downarrow + [Co(H_2O)_6](NO_3)_3$ (White ppt.)

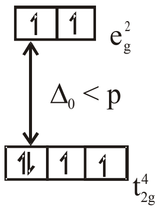
3. NTA Ans. (4)

Sol. (a) Co^{+3} (with strong field ligands)



diamagnetic

(b) If $\Delta_0 < p$;



(c) Splitting power of ethylenediamine (en) is greater than fluoride (F^-) ligand therefore more energy absorbed by $[Co(en)_3]^{3+}$ as compared to $[CoF_6]^{3-}$.

So wave length of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$

(d) $\Delta_t = \frac{4}{9}\Delta_0$

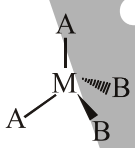
so if Δ_0 = 18,000 cm^{-1}

$$\Delta_t = \frac{4}{9} \times 18000 = 8000 \text{ cm}^{-1}$$

Statement (a) and (d) are incorrect.

4. NTA Ans. (1)

Sol. (a) If the complex MA_2B_2 is sp^3 hybridised then the shape of this complex is tetrahedral this structure is opticaly inactive due to the presence of plane of symmetry.



Optical isomes = 0

(b) If the complex MA_2B_2 is dsp^2 hybridised then the shape of this complex is square planar.

(Cis-isomer) (Trans-isomer)

Both isomers are optically inactive due to the presence of plane of symmetry.

Optical isomers = 0

5. NTA Ans. (1)

Sol. $6NaOH + 3Cl_2 \longrightarrow NaClO_3 + 5NaCl + 3H_2O$

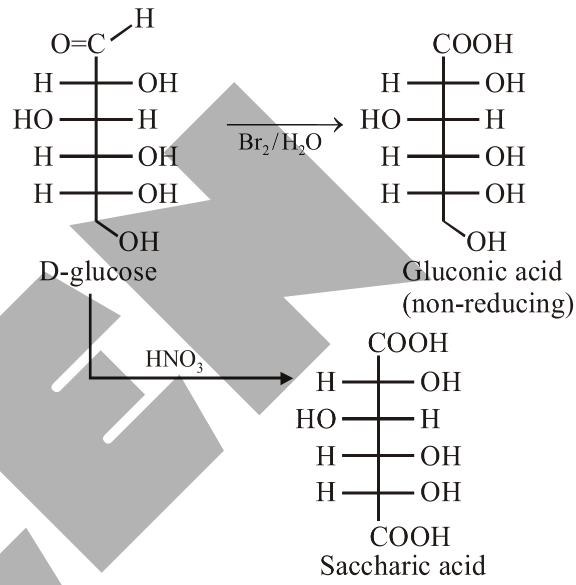
(hot and conc.) (A) side product

$2Ca(OH)_2 + 2Cl_2 \longrightarrow Ca(OCl)_2 + CaCl_2 + 2H_2O$

dry (B) side product

6. NTA Ans. (2)

Sol.



7. NTA Ans. (1)

Sol. According to MOT (If z is internuclear axis)

The configuration of

$$CN^- : \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2 = \pi_{2p_y}^2, \sigma_{2p_z}^2$$

$$\text{Bond order} = \frac{1}{2}(10-4)$$

$$= 3$$

CN^- is diamagnetic due to absence of unpaired electron

8. NTA Ans. (1)

Sol. Option (1) is incorrect.

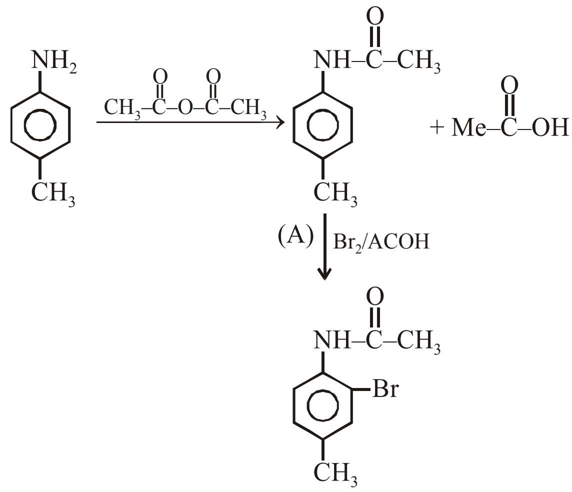
According to Kohlrausch's law correct expression is

$$\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaI} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{KI}$$

The other statements are correct.

9. NTA Ans. (1)

Sol.



10. NTA Ans. (3)

Sol. The pure solvent solution will try to maintain higher vapour pressure in the sealed container and in return the solvent vapour molecules will condense in the solution of non-volatile solute as it maintains an equilibrium with lower vapour pressure. (Lowering of vapour pressure is observed when a non volatile solute is mixed in a volatile solvent)

This will eventually lead to increase in the volume of solution and decrease in the volume of solvent.

11. NTA Ans. (2)

Sol. (A) Benzanilide $\rightarrow$ $Ph{-}NH{-}\overset{O}{\overset{\|}{C}}{-}Ph$ (μ = 2.71 D)

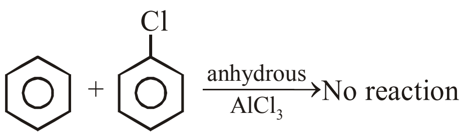
(B) Aniline $\rightarrow$ $Ph{-}NH_2$ (μ = 1.59 D)

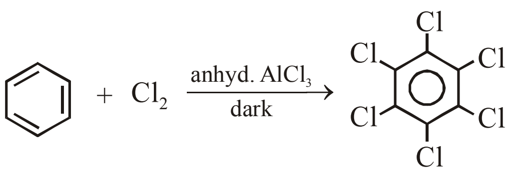
(C) Acetophenone $\rightarrow$ $Ph{-}\overset{O}{\overset{\|}{C}}{-}CH_3$ (μ = 3.05 D)

Dipole moment : C > A > B

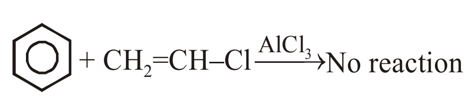
Hence the sequence of obtained compounds is (C), (A) and (B)

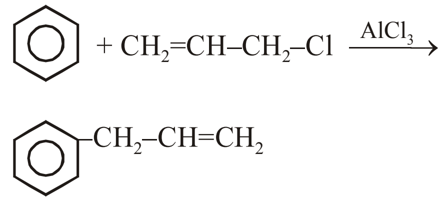
12. NTA Ans. (2)

Sol. (a) 

(b) 

(electrophilic substitution)

(c) 

(d) 

13. NTA Ans. (4)

Sol. $V_{mp}\left(=\sqrt{\frac{2RT}{M}}\right) < V_{av}\left(=\sqrt{\frac{8RT}{\pi M}}\right) < V_{rms}\left(=\sqrt{\frac{3RT}{M}}\right)$

14. NTA Ans. (3)

Sol. (a) $H_2O_2 \rightarrow 2H_2O + O_2$

(b) $KClO_3 \xrightarrow[MnO_2]{\Delta} KCl + \frac{3}{2}O_2$

$Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2 + \frac{1}{2}O_2$

$NaNO_3 \xrightarrow{\Delta} NaNO_2 + \frac{1}{2}O_2$

(c) 2-ethylanthraquinol $\underset{H_2/Pd}{\overset{O_2(air)}{\rightleftharpoons}}$ 2-ethylanthraquinone + H_2O_2

(d) $2H_3BO_3 + 2NaOH + 2H_2O_2 \longrightarrow Na_2[B_2(O_2)(OH)_4] + 4H_2O$

All statements are correct

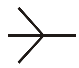
15. NTA Ans. (3)

Sol.

$CH_3{-}CH_2{-}CH_2{-}Br + Z^{\ominus} \xrightarrow{K_s} CH_3{-}CH_2{-}CH_2{-}Z$

$CH_3{-}CH_2{-}CH_2{-}Br + Z^{\ominus} \xrightarrow{K_e} CH_3{-}CH{=}CH_2 + HZ + Br^-$

(A) $CH_3{-}CH_2{-}O^- = Z^{\ominus} \longrightarrow$

(B)  $O^{\ominus} = Z^{\ominus}$

(B) with more steric crowding forms elimination product compared to substitution.

$K_e(B) > K_e(A)$

$\mu_B = \frac{K_s(B)}{K_e(A)} < \mu_A = \frac{K_s(A)}{K_e(A)}$

16. NTA Ans. (2)

Sol. Liquation method is used when the melting point of metal is less compare to the melting point of the associated impurity.

17. NTA Ans. (1)

Sol. (C) > (B) > (A)

18. NTA Ans. (3)

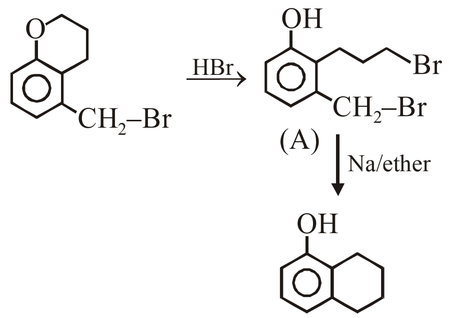
Sol. $NH_2CONH_2 + 2NaOH \rightarrow Na_2CO_3 + 2NH_3$

10 mmoles 20mmoles

Hence, NH_3 will require 20 meq.

19. NTA Ans. (4)

Sol.



20. NTA Ans. (4)

Sol. $K_{eq} = \dfrac{k_f}{k_b} = \dfrac{[N_2][H_2O]^2}{[H_2]^2[NO]^2}$

At equilibrium $r_f = r_b$

$$k_f \underset{[\text{Given}]}{[H_2][NO]^2} = k_b \frac{[N_2][H_2O]^2}{[H_2]}$$

Hence, rate expression for reverse reaction.

$$= k_b \frac{[N_2][H_2O]^2}{[H_2]}$$

21. NTA Ans. (18.00 to 18.00)

Sol. $4NaCl + K_2Cr_2O_7 + 6H_2SO_4$

$\downarrow$

$2CrO_2Cl_2 + 4NaHSO_4 + 2KHSO_4 + 3H_2O$

(A)

$CrO_2Cl_2 + 4\,NaOH \longrightarrow Na_2CrO_4 + 2NaCl + 2H_2O$

(B)

$Na_2CrO_4 + 2H_2SO_4 + 2H_2O_2$

$\downarrow$

$CrO_5 + 2NaHSO_4 + 3H_2O$

(C)

$A = CrO_2Cl_2$

$B = Na_2CrO_4$

$C = CrO_5$

Total number of atom in A + B + C = 18

22. NTA Ans. (5.22 to 5.24)

Sol. 3gm Acetic Acid + 250 ml 0.1 M HCl + Water

$\longrightarrow$ made to 500 ml solution.

$\Rightarrow$ 500 ml solution has 25 meq of HCl

50 meq of CH_3COOH

$\therefore$ 20ml solution has 1 meq of HCl

2 meq of CH_3COOH

We have added 2.5 meq. of NaOH $\left(5M, \dfrac{1}{2}\,ml\right)$

Finally, NaOH & HCl are completely consumed and we are left with 0.5 meq of CH_3COOH

and 1.5 meq of CH_3COONa

$$pH = pKa + \log\frac{1.5}{0.5}$$

$= 4.75 + \log 3 = 4.75 + 0.4771$

$= 5.2271$

23. NTA Ans. (–192.00 to –85.00)

Sol. $2C(graphite) + 3H_2(g) \longrightarrow C_2H_6(g)$

$\Delta_f H\,(C_2H_6) = 2\Delta H_{comb}\,(C_{graphite}) + 3\,\Delta H_{comb}(H_2)$
$- \Delta H_{comb}\,(C_2H_6)$

$= -(286\times 2) - (393.5\times 3) - (-1560)$

$= -572 - 1180.5 + 1560 = -192.5$ kJ/mole

24. NTA Ans. (0.36 to 0.38)

Sol. 1 L solution requires 30 m.mol HCl

250 ml sol. will require 7.5 m.mol HCl

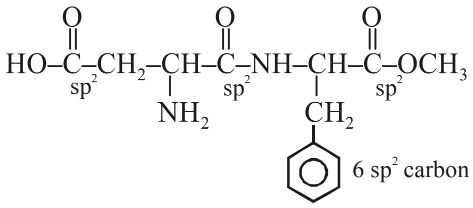
or 3.75 m.mol H_2SO_4

$\Rightarrow \dfrac{3.75\times 98}{1000}$ gm H_2SO_4

$= 0.3675$ gm H_2SO_4

25. NTA Ans. (9.00)

Sol.



no. of sp^2–carbon $\rightarrow$ 9

MATHEMATICS

1. NTA Ans. (2)

Sol. Put $x = \sin\theta$, $y = \sin\alpha$

$$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$$

$\Rightarrow$ $\sin\alpha \cdot \cos\theta + \cos\alpha \cdot \sin\theta = k$

$\Rightarrow$ $\sin(\alpha + \theta) = k$

$\Rightarrow$ $\alpha + \theta = \sin^{-1}k$

$\Rightarrow$ $\sin^{-1}x + \sin^{-1}y = \sin^{-1}k$

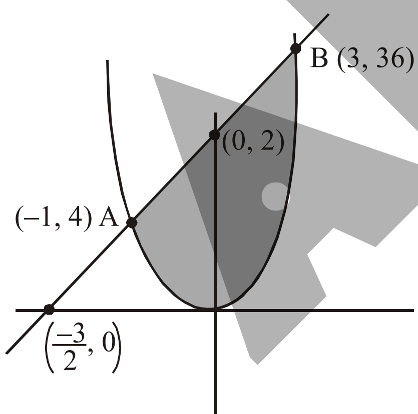
$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}} \times \frac{dy}{dx} = 0$$

at $x = \frac{1}{2}$, $y = \frac{-1}{4}$

$$\frac{dy}{dx} = \frac{-\sqrt{5}}{2}$$

2. NTA Ans. (4)

Sol. $4x^2 - y \le 0$ and $8x - y + 12 \ge 0$



On solving $y = 4x^2$

and $y = 8x + 12$

We get A (–1, 4) & B(3, 36)

Required area = area of the shaded region

$$= \int_{-1}^{3} (8x + 12 - 4x^2)dx = \frac{128}{3}$$

3. NTA Ans. (1)

Sol. $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}) + 2(\vec{b}\cdot\vec{c}) + 2(\vec{c}\cdot\vec{a}) = 0$$

$$\lambda = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = \frac{-3}{2}$$

$$\vec{d} = \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}$$

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\Rightarrow \vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$$

$$\Rightarrow \vec{d} = 3(\vec{a}\times\vec{b})$$

4. NTA Ans. (1)

Sol. Sum of the 40 terms of

3 + 4 + 8 + 9 + 13 + 14 + 18 + 19…

= (3 + 8 + 13 +…upto 20 term)

+ [4 + 9 + 15 +… upto 20 terms]

= 10 [{6 + 19 × 5} + {8 + 19 × 5}]

= 10 × 204 = 20 × 102

5. NTA Ans. (4)

Sol. $f(0) = 11$

$f(1) = 16$

$$\frac{f(1) - f(0)}{1 - 0} = 3c^2 - 8c + 8$$

$\Rightarrow$ $3c^2 - 8c + 8 = 5$

$\Rightarrow$ $3c^2 - 8c + 3 = 0$

$c \in [0, 1]$ $\Rightarrow c = \frac{4 - \sqrt{7}}{3}$

6. NTA Ans. (4)

Sol. $2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0$

$3\sin^2\theta - 5\sin\theta + 2 = 0$

$\sin\theta = \frac{1}{2}$, 2 (Rejected)

$$\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6} \frac{1 + \cos 6\theta}{2} d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$$

7. NTA Ans. (3)

Sol. $6 \times {}^{35}C_r = (k^2 - 3)\, {}^{36}C_{r+1}$

$k^2 - 3 > 0$ $\Rightarrow k^2 > 3$

$$k^2 - 3 = \frac{6 \times {}^{35}C_r}{{}^{36}C_{r+1}} = \frac{r+1}{6}$$

Possible values of r for integral values of k, are

r = 5, 35

number of ordered pairs are 4

(5, 2), (5, –2), (35, 3), (35, –3)

8. NTA Ans. (4)

Sol. $b_{ij} = (3)^{(i+j-2)} a_{ij}$

$$B = \begin{bmatrix} a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2 a_{31} & 3^2 a_{32} & 3^2 a_{33} \end{bmatrix}$$

$$\Rightarrow |B| = 3 \times 3^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2 a_{31} & 3^2 a_{32} & 3^2 a_{33} \end{vmatrix}$$

$= 3^6|A|$

$$\Rightarrow |A| = \frac{81}{27 \times 27} = \frac{1}{9}$$

9. NTA Ans. (1)

Sol. $a_1 + a_2 = 4$

$r^2a_1 + r^2a_2 = 16$

$\Rightarrow r^2 = 4 \Rightarrow r = -2$ as $a_1 < 0$

and $a_1 + a_2 = 4$

$a_1 + a_1(-2) = 4 \Rightarrow a_1 = -4$

$$4\lambda = (-4)\left(\frac{(-2)^9 - 1}{-2-1}\right) = (-4) \times \frac{513}{3}$$

$\Rightarrow \lambda = -171$

10. NTA Ans. (2)

Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$

$(A \subseteq B) \wedge (B \subseteq D) \longrightarrow (A \subseteq C)$

Contrapositive is

$\sim(A \subseteq C) \longrightarrow \sim(A \subseteq B) \vee \sim(B \subseteq D)$

$A \not\subseteq C \to (A \not\subseteq B) \vee (B \not\subseteq D)$

11. NTA Ans. (2)

Sol. $3x + 4y = 12\sqrt{12}$ is tangent to $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$

$c^2 = m^2a^2 + b^2$

$\Rightarrow a^2 = 16$

$$e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

Distance between focii = 2ae = $2\sqrt{7}$

12. NTA Ans. (3)

Sol.
$$4\alpha\left[\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right] = 5$$

$$\Rightarrow 4\alpha\left(\left[\frac{e^{\alpha x}}{\alpha}\right]_{-1}^{0} + \left[\frac{e^{-\alpha x}}{-\alpha}\right]_{0}^{2}\right) = 5$$

$\Rightarrow 4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

Let $e^{-\alpha} = t$, $4t^2 + 4t - 3 = 0$, $t = \frac{1}{2}, \frac{-3}{2}$ (Rejected)

$e^{-\alpha} = \frac{1}{2}$; $\alpha = \ell n2$

13. NTA Ans. (2)

Sol. Coefficient of x^7 is

$${}^{10}C_7 + {}^9C_6 + {}^8C_5 + \ldots + {}^4C_1 + {}^3C_0$$

$$\underbrace{{}^4C_0 + {}^4C_1}_{{}^5C_1} + {}^5C_2 + \ldots + {}^{10}C_7 = {}^{11}C_7 = 330$$

14. NTA Ans. (4)

Sol. $\alpha + \beta = 1$, $\alpha\beta = -1$

$P_k = \alpha^k + \beta^k$

$\alpha^2 - \alpha - 1 = 0$

$\Rightarrow \alpha^k - \alpha^{k-1} - \alpha^{k-2} = 0$

& $\beta^k - \beta^{k-1} - \beta^{k-2} = 0$

$\Rightarrow P_k = P_{k-1} + P_{k-2}$

$P_1 = \alpha + \beta = 1$

$P_2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 + 2 = 3$

$P_3 = 4$

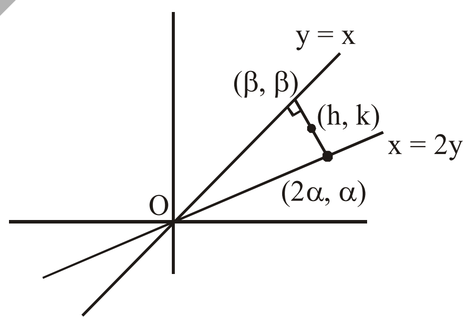
$P_4 = 7$

$P_5 = 11$

15. NTA Ans. (3)

Sol. $\frac{\alpha - \beta}{2\alpha - \beta} = -1$

$3\alpha = 2\beta$



$$h = \frac{2\alpha + \beta}{2}$$

$$2h = \frac{7\alpha}{2}$$

$$k = \frac{\alpha + \beta}{2} \qquad 2k = \frac{5\alpha}{2}$$

$\frac{h}{k} = \frac{7}{5}$; $5x = 7y$

16. NTA Ans. (3)

Sol. $\frac{3 + i\sin\theta}{4 - i\cos\theta}$ is a real number

$\Rightarrow 3\cos\theta + 4\sin\theta = 0 \quad \Rightarrow \tan\theta = \frac{-3}{4}$

argument of $\sin\theta + i\cos\theta = \pi - \tan^{-1}\frac{4}{3}$

17. NTA Ans. (3)

Sol. $(y^2 - x)\frac{dy}{dx} = 1 \qquad \Rightarrow \frac{dx}{dy} + x = y^2$

I.F. $= e^{\int dy} = e^y$

Solution is given by

$xe^y = \int y^2 e^y dy + C \Rightarrow xe^y = (y^2 - 2y + 2)e^y + C$

$x = 0, y = 1,$ gives $C = -e$

If $y = 0$, then $x = 2 - e$

18. NTA Ans. (2)

Sol. $\lim\limits_{x\to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$

$\Rightarrow f(x) = 2x^3 + ax^4 + bx^5$

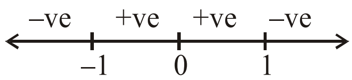
$f'(x) = 6x^2 + 4ax^3 + 5bx^4$

$f'(1) = 0, f'(-1) = 0$

$a = 0, b = \frac{-6}{5} \quad \Rightarrow f(x) = 2x^3 - \frac{6}{5}x^5$

$f'(x) = 6x^2 - 6x^4 = 6x^2(1 - x)(1 + x)$

Sign scheme for $f'(x)$



Minima at $x = -1$ Maxima at $x = 1$

19. NTA Ans. (3)

Sol. Probability that at most 2 machines are out of service

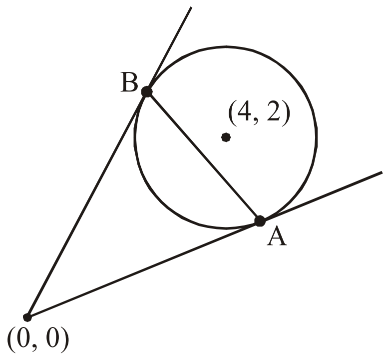
$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_2\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$

$= \left(\frac{3}{4}\right)^4 \times \frac{17}{8} \qquad \Rightarrow k = \frac{17}{8}$

20. NTA Ans. (4)

Sol. $R = \sqrt{16 + 4 - 16} = 2$

$L = \sqrt{S_1} = 4$



AB(Chord of contact) $= \frac{2LR}{\sqrt{L^2 + R^2}} = \frac{8}{\sqrt{5}}$

$(AB)^2 = \frac{64}{5}$

21. NTA Ans. (13.00)

Sol. System has intfinitely many solution

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \qquad \Rightarrow \lambda = 1$

$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 1 \end{vmatrix} = 0$

$\mu = 14 \qquad \mu - \lambda^2 = 13$

22. NTA Ans. (5.00)

Sol. $k = \lim\limits_{x\to 0}\left(\frac{\ell n(1+3x)}{x} - \frac{\ell n(1-2x)}{x}\right)$

$k = 3 + 2 = 5$

23. NTA Ans. (54.00)

Sol. $\frac{3+7+9+12+13+20+x+y}{8} = 10$

$x + y = 16$

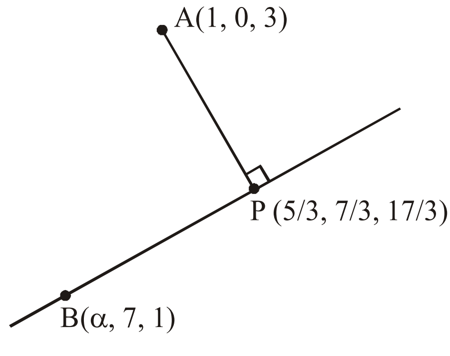
$\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = 25$

$3^2 + 7^2 + 9^2 + 12^2 + 13^2 + 20^2 + x^2 + y^2 = 1000$

$x^2 + y^2 = 148 \qquad xy = 54$

24. NTA Ans. (4.00)

Sol.



D.R. of BP $= \left< \frac{5}{3} - \alpha, \frac{7}{3} - 7, \frac{17}{3} - 1 \right>$

D.R. of AP $= \left< \frac{5}{3} - 1, \frac{7}{3} - 0, \frac{17}{3} - 3 \right>$

BP $\perp^r$ AP

$\Rightarrow \alpha = 4$

25. NTA Ans. (29.00)

Sol. $n(A) = 25$

$n(B) = 7$

$n(A \cap B) = 3$

$n(A \cup B) = 25 + 7 - 3 = 29$

SET # 03

PHYSICS

1. NTA Ans. (4)

Sol. In case of minimum density of liqued, sphere will be floating while completely submerged

So mg = B

$$m = \int_0^R \rho(4\pi r^2 dr) = B$$

$$= \rho_0 \int_0^R \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr = \frac{4}{3}\pi R^3 \rho_\ell g$$

On Solving

$$\rho_\ell = \frac{2\rho_0}{5}$$

2. NTA Ans. (3)

Sol. $\lambda_B = 2\lambda_A$

$$\Rightarrow \frac{h}{\sqrt{2T_B m}} = \frac{2h}{\sqrt{2T_A m}}$$

$T_A = 4T_B$ ....(i)

and $T_B = (T_A - 1.5)$ eV ....(ii)

from (i) and (ii)

$3T_B$ 1.5 eV $\Rightarrow T_B = 0.5$ eV

$T_B = 0.5$ eV $= 4.5$ eV $-\phi_B$

$\phi = 4$eV

3. NTA Ans. (4)

Sol. $5 = \lambda\ell$

where λ is potential gradient & L is total length of wire.

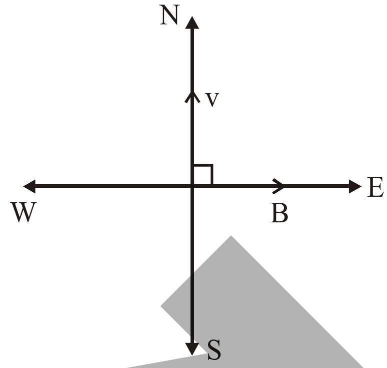
$$5 = \frac{\Delta V}{L}\ell$$

$$\Delta V = \frac{5 \times L}{\ell} = 5 \times \frac{12}{10} = 6V = 60 \text{ mA} \times R$$

R = 100Ω

4. NTA Ans. (4)

Sol. $a = \dfrac{qvB}{m}$



$$B = \frac{ma}{qv} = \frac{ma\sqrt{m}}{\sqrt{2k}}$$

$$= \frac{m^{3/2}a}{e\sqrt{2k}} = \frac{(1.6 \times 10^{-27})^{3/2} \times 10^{12}}{1.6 \times 10^{-19}\sqrt{2 \times 1 \times 10^6 \times 1.6 \times 10^{-19}}}$$

= 0.71 mT

5. NTA Ans. (4)

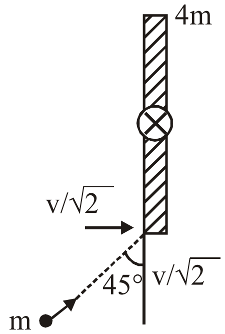
Sol. Mean free time $= \dfrac{\text{Mean free path}}{\text{Average speed}}$

$$= \frac{\frac{1}{\sqrt{2}\pi D^2 n}}{\sqrt{\frac{8RT}{\pi M_w}}}$$

$$t \propto \frac{1}{\sqrt{T}}$$

6. NTA Ans. (2)

Sol.



Let angular velocity of the system after collision be ω.

By conservation of angular momentum about the hinge :

$$m\left(\frac{v}{\sqrt{2}}\right)\left(\frac{\ell}{2}\right)=\left[\frac{4m\ell^2}{12}+\frac{m\ell^2}{4}\right]\omega$$

On solving

$$\omega=\frac{3\sqrt{2}}{7}\left(\frac{v}{\ell}\right)$$

7. NTA Ans. (Bonus)

Sol. $v_0 = h^x c^y G^z A^w$

$$\frac{ML^2T^{-2}}{AT}=(ML^2T^{-1})^x(LT^{-1})^y(M^{-1}L^3T^{-2})^z A^w$$

$\Rightarrow w = -1$

$(x - z = 1)$

$2x + y + 3x = 2$

$- x - y - 2z = -3$

$2x = 0$

$x = 0$

$z = -1$

$2 \times 0 + y + 3x - 1 = 2$

$y = 5$

$\Rightarrow v_0 = h^0c^5G^{-1}A^{-1}$

So Bonus

8. NTA Ans. (4)

Sol. Gravitational field on the surface of a solid sphere $I_g=\frac{GM}{R^2}$

By the graph

$$\frac{GM_1}{(1)^2}=2$$

and $$\frac{GM_2}{(2)^2}=3$$

On solving

$$\frac{M_1}{M_2}=\frac{1}{6}$$

9. NTA Ans. (4)

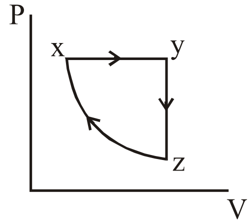
Sol. $|\vec{E}|$ should be constant on the surface and the surface should be equipotential.

10. NTA Ans. (4)

Sol. $x \to y \Rightarrow$ Isobaric

$y \to z \Rightarrow$ Isochoric

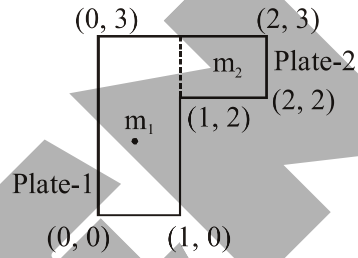
$z \to x \Rightarrow$ Isothermal



11. NTA Ans. (4)

Sol. $m_1 = 3kg$

$m_2 = 1kg$



Mass of plate-1 is assumed to be concentrated at (0.5, 1.5)

Mass of plate-2 is assumed to be concentrated at (1.5, 2.5).

$$x_{cm}=\frac{m_1x_1+m_2x_2}{m_1+m_2}=\frac{3\times0.5+1\times1.5}{4}=0.75$$

$$y_{cm}=\frac{m_1y_1+m_2y_2}{m_1+m_2}=\frac{3\times1.5+1\times2.5}{4}=1.75$$

12. NTA Ans. (4)

Sol. $L = f_0 + f_e = 60$ cm

$$M=\frac{f_0}{f_e}=5$$

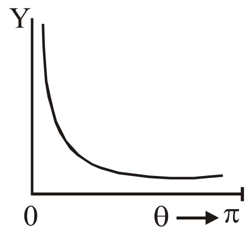
$\Rightarrow f_0 = 5f_e$

$\therefore 6f_e = 60$ cm

$f_e = 10$ cm

13. NTA Ans. (3)

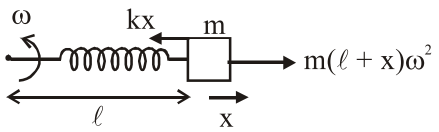
Sol.



$$Y\propto\frac{1}{\left(\sin\frac{\theta}{2}\right)^4}$$

14. NTA Ans. (2)

Sol.



$kx = m\ell\omega^2 + mx\omega^2$

$$x = \frac{m\ell\omega^2}{k - m\omega^2}$$

15. NTA Ans. (4)

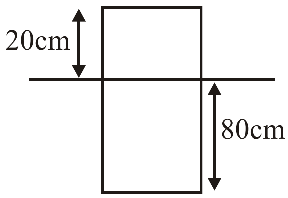
Sol. $\sin\theta_C = \frac{1}{\mu} = \frac{1}{\sqrt{3 \times 4/3}}$

$\theta_C = 30°$

16. NTA Ans. (1)

Sol. $m = \rho_0 A\ (80)$ ....(i)

$m = \rho A\ (79)$ ....(ii)



17. NTA Ans. (3)

Sol.

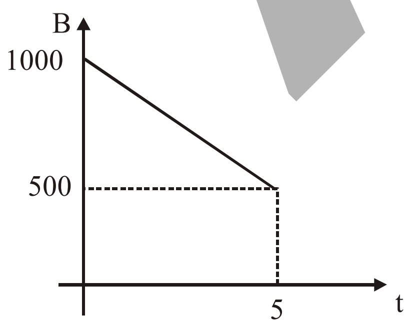
A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

18. NTA Ans. (4)

Sol. $E_x = \frac{K(4q)}{R^2}\cos 30° + \frac{K(2q)}{R^2}\cos 30° + \frac{K(2q)}{R^2}\cos 30°$

19. NTA Ans. (3)

Sol.



$$\frac{dB}{dt} = 100$$

$A = 16 \times 4 - 4 \times 2 = 56\ cm^2$

$$\varepsilon = \frac{dB}{dt}A = 100 \times 10^{-4} \times 56 \times 10^{-4}$$

20. NTA Ans. (3)

Sol. $C_1 + C_2 = 10$ ....(i)

$$\frac{1}{2}C_2V^2 = 4 \times \frac{1}{2}C_1V^2$$

$\therefore\ C_2 = 4C_1$ ....(ii)

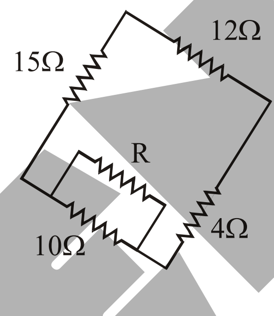
$\therefore\ C_1 = 2\ \&\ C_2 = 8$

For series combination

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = 1.6$$

21. NTA Ans. (10.00)

Sol.



Let the resistance to be connected is R.

For balanced wheatstone bridge,

$$15 \times 4 = 12 \times \frac{10R}{10 + R}$$

$\Rightarrow R = 10\Omega$

22. NTA Ans. (60.00)

Sol. Using Lens-Maker's formula :

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f} = (1.5 - 1)\left(\frac{1}{30} - 0\right)$$

f = 60 cm

23. NTA Ans. (1.00)

Sol. By conservation of linear momentum :

$$(0.1)(3\hat{i}) + (0.1)(5\hat{j}) = (0.1)(4)(\hat{i} + \hat{j}) + (0.1)\vec{v}$$

$$\Rightarrow\ \vec{v} = -\hat{i} + \hat{j}$$

$\therefore$ Speed of B after collision $|\vec{v}| = \sqrt{2}$

Now, kinetic energy = $\frac{1}{2}mV^2 = \frac{1}{2}(0.1)(2) = \frac{1}{10}$

$\therefore\ x = 1$

24. NTA Ans. (580.00)

Sol. $x = 10 + 8t - 3t^2$

$v_x = 8 - 6t$

$(v_x)_{t=1} = 2\hat{i}$

$y = 5 - 8t^3$

$v_y = -24t^2$

$(v_y)_{t=1} = -24\hat{j}$

Now

$$\sqrt{v} = \sqrt{(24)^2 + (2)^2} = \sqrt{580}$$

$\therefore\ v = 580\ m^2/s^2$

25. NTA Ans. (106.00 to 107.20)

Sol. $v_s = \sqrt{\dfrac{\gamma P}{\rho}}$

$$\frac{v_{gas}}{v_{air}} = \sqrt{\frac{\rho_{air}}{\rho_{gas}}} \qquad \Rightarrow \frac{v_{gas}}{300} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow v_{gas} = \frac{300}{\sqrt{2}}$$

$$\therefore\ v_{gas} = 150\sqrt{2}$$

Now $n_2 - n_1 = \dfrac{v_{gas}}{2\ell} = \dfrac{150\sqrt{2}}{2(1)} = 75\sqrt{2}$

$\Rightarrow\ \Delta n = 106.06$ Hz

CHEMISTRY

1. NTA Ans. (3)

Sol. Liquid which have less difference in boiling point can be isolated by fractional distillation and liquid with less boiling point will be isolated first.

2. NTA Ans. (1)

Sol. Electronic configuration of Na = [Ne] $3s^1$

Mg = [Ne] $3s^2$

Al = [Ne] $3s^2 3p^1$

Si = [Ne] $3s^2 3p^2$

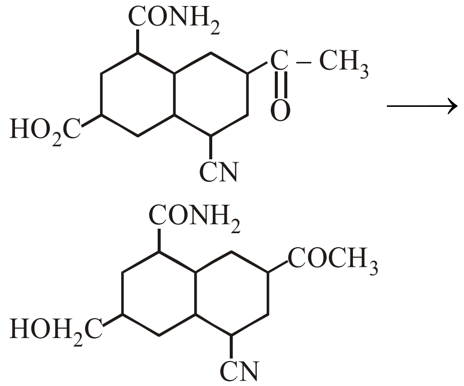
So order of first ionisation energy is

$$\underset{496}{Na} < \underset{737}{Mg} > \underset{577}{Al} < \underset{786}{Si}\ \text{kj/mol}$$

Na < Al < Mg < Si (IE_1 order)

3. NTA Ans. (4)

Sol.



Most suitable reagent for given conversion is B_2H_6 (electrophilic reducing agent)

4. NTA Ans. (1)

Sol. Electronic configuration of

	$_{25}Mn$	$_{26}Fe$	$_{27}Co$	$_{28}Ni$
M =	$[Ar]3d^5 4s^2$	$[Ar]3d^6 4s^2$	$[Ar]3d^7 4s^2$	$[Ar]3d^8 4s^2$
M^{2+} =	$[Ar]3d^5 4s^0$	$[Ar]3d^6 4s^0$	$[Ar]3d^7 4s^0$	$[Ar]3d^8 4s^0$

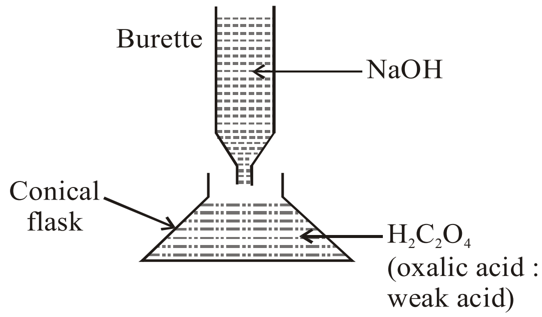
So third ionisation energy is minimum for Fe.

5. NTA Ans. (3)

Sol. Ethyl acetate $\left(H_3C\text{–}\underset{\underset{O}{\|}}{C}\text{–}O\text{–}CH_2\text{–}CH_3\right)$ is polar molecule. Hence there will be dipole-dipole attraction and london dispersion forces are present.

6. NTA Ans. (4)

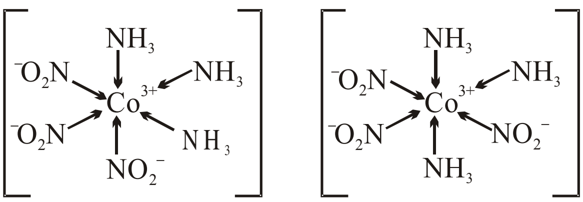
Sol.



7. NTA Ans. (3)

Sol. $[Ma_3b_3]$ type complex shows fac and mer isomerism.

$[Co(NH_3)_3(NO_2)_3]$



fac-isomer mer-isomer

8. NTA Ans. (3)

Sol. From the graph & dimensions salt is : XY_2

$[X] = 1 \times 10^{-3}M$

$[Y] = 2 \times 10^{-3}M$

$$XY_2(s) \rightleftharpoons X^{2+}_{(aq.)} + 2Y^{-}_{(aq.)}$$

$ksp = [X^{2+}]\,[Y^-]^2$

$= (10^{-3})\,(2 \times 10^{-3})^2$

$= 4 \times 10^{-9}\ M^3$

9. NTA Ans. (3)

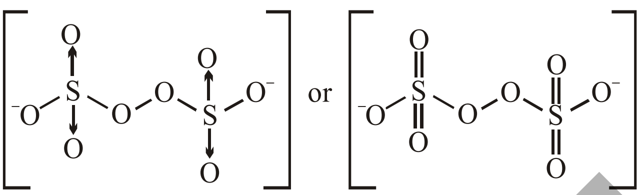
Sol. Reactivity D > B > C > A

Carbocation formed from D is most stable

Carbocation formed from A is least stable

10. NTA Ans. (3)

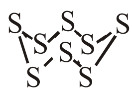
Sol. $\mathbf{S_2O_8^{2-}}$ **:**



8 bonds are present between sulphur and oxygen. (It is best answer in given options)

Rhombic sulphur :

$(\mathbf{S_8})$



8 bonds are present between sulphur and sulphur atoms.

11. NTA Ans. (4)

Sol. $K = Ae^{\frac{-E_a}{RT}}$

$$K' = Ae^{\frac{-E_a'}{RT}} = 10^6 K$$

$$Ae^{\frac{-E'}{RT}} = 10^6 \times Ae^{\frac{-E_a}{RT}}$$

$$\frac{-E_a'}{RT} = \frac{-E_a}{RT} + \ln 10^6$$

$$E_a' = E_a - RT\ \ln 10^6$$

$$E_a' - E_a = -\ RT\ \ln 10^6$$

$= -\ 6RT \times 2.303$

12. NTA Ans. (2)

Sol. Glucose gives negative test with Schiff reagent

13. NTA Ans. (3)

Sol. Order of B.P. is : Z > Y > X

Order of vapour pressure : Z < Y < X

order of intermolecular interaction : Z > Y > X.

14. NTA Ans. (4)

Sol. CO_2, H_2O, CFCs and O_3 are green house gases.

15. NTA Ans. (4)

Sol. Since, $Fe(OH)_3$ is positively charged sol, hence, anionic charge will flocculate

As per Hardy Schulze rules coagulation power of anion follows the order :

$Fe(CN)_6^{3-} > CrO_4^{2-} > Cl^- = Br^- = NO_3^-$

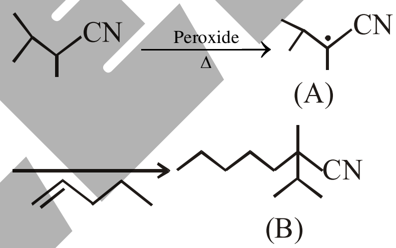
Higher the coagulation power lower will be its flocculation value

therefore order will be :

$Fe(CN)_6^{3-} < CrO_4^{2-} < Cl^- = Br^- = NO_3^-$

16. NTA Ans. (1)

Sol.



17. NTA Ans. (2)

Sol. For balmer : $n_1 = 2,\ n_2 = 3, 4, 5, \ldots \infty$

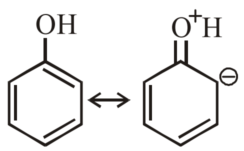
$$\bar{\nu} = \frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda_{longest}} = R_H\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

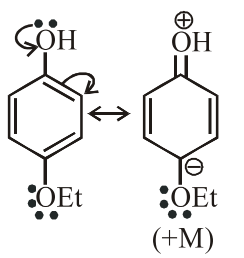
Ans.(2)

18. NTA Ans. (2)

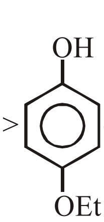
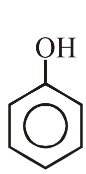
Sol. $H_3C - OH$ (100% single bond)



C–OH bond has partial double bond character

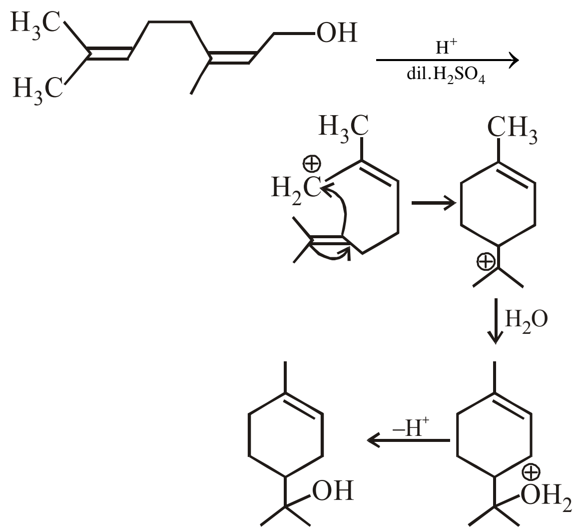


(C–OH bond has some double bond character but double bond character is less)

Ans. CH_3OH >  (p-ethoxyphenol) > 

19. NTA Ans. (2)

Sol.



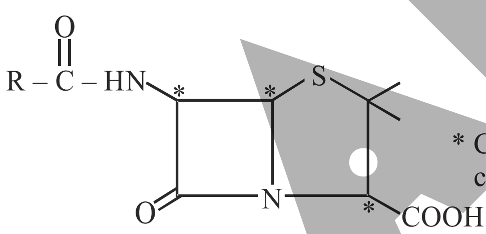
Terpineol

20. NTA Ans. (4)

Sol. $\underset{\text{Gypsum}}{CaSO_4.2H_2O} \xrightarrow{393K} \underset{\text{Plaster of paris}}{CaSO_4.\frac{1}{2}H_2O} + \frac{3}{2}H_2O$

21. NTA Ans. (3.00)

Sol. The structure of penicillin is



* Chiral center = 3

22. NTA Ans. (48.00)

Sol. Area enclosed under P V curve = 48

= 48 Joule

23. NTA Ans. (26.80 to 27.00)

Sol. Number of moles of Cl^- precipitated in $[Co(NH_3)_6]Cl_3$ is equal to number of moles of $AgNO_3$ used.

$$\frac{0.3}{267.46} \times 3 = \frac{0.125 \times V}{1000}$$

where V is volume of $AgNO_3$ (in mL)

V = 26.92 mL

24. NTA Ans. (-0.93 to -0.94)

Sol. $2H_2O(l) \rightarrow O_2(g) + 4H^+ + 4e^-$; $E^0_{red.} = 1.23V$

From nernst equation

$$E_{cell} = E^0_{cell} - \frac{RT}{nF}\ln Q$$

at 1 bar & 298 K

$$\frac{2.303RT}{F} = 0.059$$

pH = 5 $\Rightarrow [H^+] = 10^{-5}$ M

$E^\circ_{oxidation} = -1.23$ volt

$$E_{cell} = -1.23 - \frac{0.059}{4}\log[H^+]^4$$

$$E_{cell} = -1.23 - \frac{0.059}{4}\log(10^{-5})^4$$

$= -1.23 + 0.059 \times 5 = -0.935$ V

25. NTA Ans. (4.95 to 4.97)

Sol. $FeSO_4.7H_2O$ (M = 277.85)

$$\text{ppm} = \frac{\text{wt. of Fe}}{\text{wt. of wheat}} \times 10^6$$

let the wt. of salt be = w gm

$$\text{moles} = \frac{w}{277.85}$$

$$\text{wt. of Fe} = \left(\frac{W}{277.85} \times 55.85\right)\text{gm}$$

$$10 = \frac{\frac{W}{277.85} \times 55.85}{10^5} \times 10^6$$

$$W = \frac{277.85}{55.85} = 4.97$$

MATHEMATICS

1. NTA Ans. (4)

Sol. Any normal to the ellipse is

$$\frac{x\sec\theta}{\sqrt{2}} - y\,\mathrm{cosec}\,\theta = -\frac{1}{2}$$

$$\Rightarrow \frac{x}{\left(\frac{-\cos\theta}{\sqrt{2}}\right)} + \frac{y}{\left(\frac{\sin\theta}{2}\right)} = 1$$

$$\Rightarrow \frac{\cos\theta}{\sqrt{2}} = \frac{1}{3\sqrt{2}} \text{ and } \frac{\sin\theta}{2} = \beta$$

$$\Rightarrow \beta = \frac{\sqrt{2}}{3}$$

2. NTA Ans. (2)

Sol. $f(x)=\dfrac{2\left(2^{x}+2^{-x}\right)+\left(3^{x}+3^{-x}\right)}{2}\geq 3$

(A.M $\geq$ G.M)

3. NTA Ans. (1)

Sol. $\left\|\begin{matrix}1 & 1 & \lambda\\ 1 & 1 & 3\\ 2 & 1 & 1\end{matrix}\right\|=1\Rightarrow\lambda=2,4$

Now, $\cos\theta=\dfrac{\vec{u}.\vec{w}}{|\vec{u}||\vec{w}|}=\dfrac{5}{\sqrt{6}\sqrt{6}}\text{or}\dfrac{7}{\sqrt{6}\sqrt{18}}=\dfrac{5}{6}\text{or}\dfrac{7}{6\sqrt{3}}$

4. NTA Ans. (4)

Sol. $a={}^{19}C_{10}$, $b={}^{20}C_{10}$ and $c={}^{21}C_{10}$

$\Rightarrow a={}^{19}C_{9}$, $b=2({}^{19}C_{9})$ and $c=\dfrac{21}{11}\left({}^{20}C_{10}\right)$

$\Rightarrow b=2a$ and $c=\dfrac{21}{11}b=\dfrac{42a}{11}$

$\Rightarrow a:b:c=a:2a:\dfrac{42a}{11}=11:22:42$

5. NTA Ans. (1)
ALLEN Ans. (Bonus)

Sol. Let $\tan^{-1}x=\theta$, $\theta\in\left(-\dfrac{\pi}{2},-\dfrac{\pi}{4}\right)\cup\left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$

$f(x)=(\sin\theta+\cos\theta)^{2}-1=\sin2\theta=\dfrac{2x}{1+x^{2}}$

Now, $\dfrac{dy}{dx}=\dfrac{1}{2}\dfrac{d}{dx}\sin^{-1}\left(\dfrac{2x}{1+x^{2}}\right)$

$=-\dfrac{1}{1+x^{2}},|x|>1$

Since, we can integrate only in the continuous interval. So we have to take integral in two cases separtely namely for $x<-1$ and for $x>1$.

$\Rightarrow y=\begin{cases}-\tan^{-1}x+c_{1} & ;\quad x>1\\ -\tan^{-1}x+c_{2} & ;\quad x<-1\end{cases}$

so, $c_{1}=\dfrac{\pi}{2}$ as $y\left(\sqrt{3}\right)=\dfrac{\pi}{6}$

But we cannot find c_2 as we do not have any other additional information for $x<-1$. So, all of the given options may be correct as c_2 is unknown so, it should be bonus.

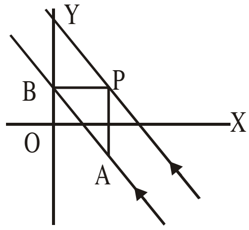
6. NTA Ans. (4)

Sol. Required limit $=e^{\lim\limits_{x\to0}\left(\frac{3x^{2}+2}{7x^{2}+2}-1\right)\frac{1}{x^{2}}}$

$=e^{\lim\limits_{x\to0}\left(\frac{-4}{7x^{2}+2}\right)}=\dfrac{1}{e^{2}}$

7. NTA Ans. (3)

Sol. $\overleftrightarrow{AB}:3x+y-2=0$



Also, $\dfrac{1}{2}\times\sqrt{10}\times h=5$

$\Rightarrow h=\sqrt{10}$

$\Rightarrow\dfrac{|4\lambda-2|}{\sqrt{10}}=\sqrt{10}\Rightarrow\lambda=3,-2$

8. NTA Ans. (1)

Sol. $20p-q=10$ ...(i)

and $2|p|=1\Rightarrow p=\pm\dfrac{1}{2}$ ...(ii)

so, $p=-\dfrac{1}{2}$ and $q=-20$

9. NTA Ans. (2)
ALLEN Ans. (Bonus)

Sol. $\dfrac{dy}{dx}=-\dfrac{\sqrt{1-y^{2}}}{\sqrt{1-x^{2}}}$ so, $\dfrac{dy}{\sqrt{1-y^{2}}}+\dfrac{dx}{\sqrt{1-x^{2}}}=0$

Integrating, $\sin^{-1}x+\sin^{-1}y=c$

so, $\dfrac{\pi}{6}+\dfrac{\pi}{3}=c$

Hence, $\sin^{-1}x+\sin^{-1}y=\dfrac{\pi}{2}$

Put $x=-\dfrac{1}{\sqrt{2}},\sin^{-1}y=\dfrac{3\pi}{4}$ (Not possible)

10. NTA Ans. (4)

Sol. Assuming z is a root of the given equation,

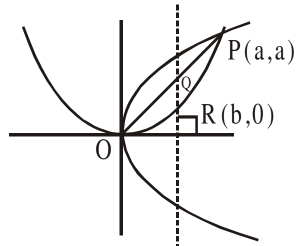
$z=\dfrac{-b\pm i\sqrt{180-b^{2}}}{2}$

so, $\left(1-\dfrac{b}{2}\right)^{2}+\dfrac{180-b^{2}}{4}=40$

$\Rightarrow -4b+184=160\Rightarrow b=6$

11. NTA Ans. (1)

Sol.



$$\int_0^b\left(\sqrt{ax}-\frac{x^2}{a}\right)dx=\frac{1}{2}\times\frac{16\left(\frac{a}{4}\right)\left(\frac{a}{4}\right)}{3}$$

$$\Rightarrow\left[\frac{2\sqrt{a}}{3}x^{3/2}-\frac{x^3}{3a}\right]_0^b=\frac{a^2}{6}$$

$$\Rightarrow\frac{2\sqrt{a}}{3}b^{3/2}-\frac{b^3}{3a}=\frac{a^2}{6} \qquad ...(i)$$

Also, $\frac{1}{2}\times b^2=\frac{1}{2}\Rightarrow b=1$

so, $\frac{2\sqrt{a}}{3}-\frac{1}{3a}=\frac{a^2}{6}\Rightarrow a^3-4a^{3/2}+2=0$

$\Rightarrow a^6 + 4a^3 + 4 = 16a^3 \Rightarrow a^6 - 12a^3 + 4 = 0$

12. NTA Ans. (4)

Sol. (1) $P\wedge(P\vee Q)\equiv P$

(2) $P\vee(P\wedge Q)\equiv P$

(3) $Q\rightarrow(P\wedge(P\rightarrow Q))$

$\equiv Q\rightarrow(P\wedge(\sim P\vee Q))\equiv Q\rightarrow(P\wedge Q)$

$\equiv(\sim Q)\vee(P\wedge Q)\equiv(P\vee(\sim Q))$

(4) $(P\wedge(P\rightarrow Q))\rightarrow Q$

$\equiv(P\wedge(\sim P\vee Q))\rightarrow Q\equiv(P\wedge Q)\rightarrow Q$

$\equiv((\sim P)\vee(\sim Q))\vee Q\equiv(\sim P)\vee t\equiv t$

13. NTA Ans. (2)

Sol. $A(0,-1)\overset{1}{—}P(h,k)\overset{2}{—}Q(2t,t^2)$

$\Rightarrow 3h = 2t$ and $3k = t^2 - 2$

$\Rightarrow 3y=\left(\frac{3x}{2}\right)^2-2\Rightarrow 12y=9x^2-8$

14. NTA Ans. (2)

Sol. $\frac{9+\alpha}{21}=\frac{16+\alpha}{28}\Rightarrow\alpha=12$

Also, $f'(x)=\frac{7x}{x^2+12}\times\frac{x^2-12}{7x^2}=\frac{x^2-12}{x(x^2+12)}$

Hence, $c=2\sqrt{3}$

Now, $f''(c)=\frac{1}{12}$

15. NTA Ans. (4)

Sol. $2\times$ (ii) $-$ $2\times$ (i) $-$ (iii) : -

$0 = 2\mu - 2 - \delta$

$\Rightarrow \delta = 2(\mu - 1)$

16. NTA Ans. (3)

Sol. (1) $P(A/B)=P(A)=\frac{1}{3}$

(2) $P(A/(A\cup B))=\frac{P(A\cap(A\cup B))}{P(A\cup B)}=\frac{P(A)}{P(A\cup B)}$

$$=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}-\frac{1}{18}}=\frac{3}{4}$$

(3) $P(A/B')=P(A)=\frac{1}{3}$

(4) $P(A'/B')=P(A')=\frac{2}{3}$

17. NTA Ans. (3)

Sol. $f(x)=y=\frac{8^{4x}-1}{8^{4x}+1}=1-\frac{2}{8^{4x}+1}$

so, $8^{4x}+1=\frac{2}{1-y}\Rightarrow 8^{4x}=\frac{1+y}{1-y}$

$\Rightarrow x=\ell n\left(\frac{1+y}{1-y}\right)\times\frac{1}{4\ell n8}=f^{-1}(y)$

Hence, $f^{-1}(x)=\frac{1}{4}\log_8 e\ell n\left(\frac{1+x}{1-x}\right)$

18. NTA Ans. (1)

Sol. $\int\frac{\cos x\,dx}{\sin^3x(1+\sin^6x)^{2/3}}=\frac{-6}{-6}\int\frac{\cos x\,dx}{\sin^7x\left(\frac{1}{\sin^6x}+1\right)^{2/3}}$

$$=-\frac{1}{6}\times3\left(\frac{1}{\sin^6x}+1\right)^{\frac{1}{3}}+c$$

$$=-\frac{1}{2}\frac{(1+\sin^6x)^{\frac{1}{3}}}{\sin^2x}+c$$

Hence, $\lambda = 3$ and $f(x)=-\frac{1}{2\sin^2x}$

so, $\lambda f\left(\frac{\pi}{3}\right)=-2$

REMARK : Technically, this question should be marked as bonus. Because $f(x)$ and λ cannot be found uniquely.
For example, another such $f(x)$ and λ can be

$-\dfrac{\left(1+\sin^6 x\right)^{\frac{1}{6}}}{2\sin^2 x}$ and 6 respectively.

19. NTA Ans. (2)

Sol. Shortest distance $=\dfrac{\left\|\begin{vmatrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}\right\|}{\sqrt{11\times 29-49}}=\dfrac{270}{\sqrt{270}}$

$=\sqrt{270}=3\sqrt{30}$

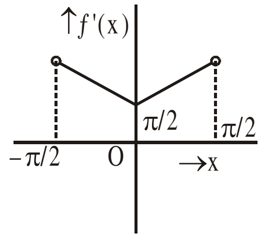
20. NTA Ans. (1)

Sol. $f(x)$ is an odd function.
Now, if $x \geq 0$, then $f(x) = x\cos^{-1}(-\sin x)$

$=x\left(\dfrac{\pi}{2}-\sin^{-1}\left(-\sin x\right)\right)=x\left(\dfrac{\pi}{2}+x\right)$

Hence, $f\left(x\right)=\begin{cases} x\left(\dfrac{\pi}{2}+x\right) & ; \quad x\in\left[0,\dfrac{\pi}{2}\right] \\ x\left(\dfrac{\pi}{2}-x\right) & ; \quad x\in\left[-\dfrac{\pi}{2},0\right) \end{cases}$

so, $f'\left(x\right)=\begin{cases} \dfrac{\pi}{2}+2x & ; \quad x\in\left[0,\dfrac{\pi}{2}\right) \\ \dfrac{\pi}{2}-2x & ; \quad x\in\left(-\dfrac{\pi}{2},0\right) \end{cases}$



21. NTA Ans. (672.00)

Sol. trace $(AA^T) = \Sigma a_{ij}^2 = 3$

Hence, number of such matrices
$= {}^9C_3 \times 2^3 = 672.00$

22. NTA Ans. (8.00)

Sol. $D \geq 0 \Rightarrow (a-10)^2 - 4\times 2\times\left(\dfrac{33}{2}-2a\right)\geq 0$

$\Rightarrow a^2 - 4a - 32 \geq 0$

$\Rightarrow a \in (-\infty, 4] \cup [8,\infty)$

23. NTA Ans. (4.00)

Sol. Let $P(\alpha,\beta)$

so, $\beta^2 - 3\alpha^2 + \beta + 10 = 0$ ...(i)

Now, $2yy' - 6x + y' = 0$

$\Rightarrow m=\dfrac{6\alpha}{2\beta+1}$ ....(ii)

Also, $\dfrac{\beta-\frac{3}{2}}{\alpha}=-\dfrac{1}{m}$

$\Rightarrow \dfrac{2\beta-3}{2\alpha}=-\dfrac{\left(2\beta+1\right)}{6\alpha}$ (from (ii))

$\Rightarrow \beta = 1 \Rightarrow \alpha^2 = 4$ (from (1))

Hence, $|m| = \dfrac{12}{3} = 4.00$

24. NTA Ans. (1540.00)

Sol. $\displaystyle\sum_{k=1}^{20}\frac{k\left(k+1\right)}{2}=\frac{1}{2}\sum_{k=1}^{20}\frac{k\left(k+1\right)\left(k+2\right)-\left(k-1\right)k\left(k+1\right)}{3}$

$=\dfrac{1}{6}\times 20\times 21\times 22 = 1540.00$

25. NTA Ans. (490.00)

ALLEN Ans. (13 or 490)

Sol. The question does not mention that whether same coloured marbles are distinct or identical. So, assuming they are distinct our required answer $= {}^{12}C_4 - {}^5C_4 = 490$

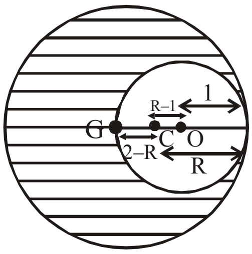
And, if same coloured marbles are identical then required answer $= (2 + 3 + 4 + 4) = 13$

SET # 04

PHYSICS

1. NTA Ans. (3)

Sol.



By concept of COM

$m_1R_1 = m_2R_2$

Remaining mass × (2–R) = cavity mass × (R–1)

$$\left(\frac{4}{3}\pi R^3\rho - \frac{4}{3}\pi 1^3\rho\right)(2-R) = \frac{4}{3}\pi 1^3\rho\times(R-1)$$

$(R^3 - 1)(2 - R) = R - 1$

$(R^2 + R + 1)(2 - R) = 1$

2. NTA Ans. (4)

Sol. $I = I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$

$$\frac{I}{I_0} = \cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\frac{I}{I_0} = \cos^2\left(\frac{\frac{2\pi}{\lambda}\times\frac{\lambda}{8}}{2}\right)$$

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{8}\right)$$

$$\frac{I}{I_0} = 0.853$$

3. NTA Ans. (2)

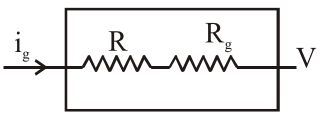
Sol. $E = \vec{B}\times\vec{V}$

$= (5\times10^{-8}\hat{j})\times(3\times10^{8}\hat{k})$

$= 15\,\hat{i}\ \mathrm{V/m}$

4. NTA Ans. (1)

Sol. $i_g = 1$ mA , $R_g = 100\ \Omega$



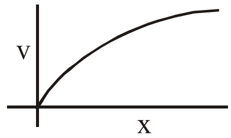
$V = i_g(R + R_g)$

$10 = 1\times10^{-3}(R + 100)$

$R = 9.9\ k\Omega$

5. NTA Ans. (3)

Sol. $v^2 = u^2 + 2as$



$$v^2 = 0 + 2\left(\frac{qE}{m}\right)x$$

$$v^2 = \frac{2qE}{m}x$$

6. NTA Ans. (2)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$g = \frac{4\pi^2\ell}{T^2}$$

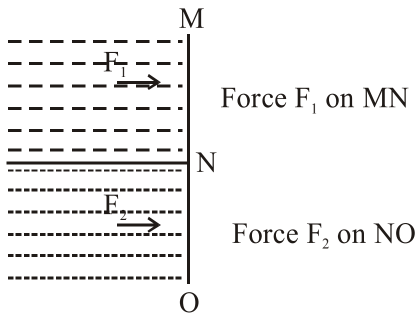
$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$$

$$= \frac{0.1}{25} + \frac{2\times1}{50}$$

$$\frac{\Delta g}{g} = 4.4\%$$

7. NTA Ans. (1)

Sol.



$$F_1 = \frac{\rho gh}{2}\times A$$

$$F_2 = \left(\rho gh + \frac{2\rho gh}{2}\right)A$$

$$\frac{F_1}{F_2} = \frac{1}{4}$$

8. NTA Ans. (2)

Sol. Velocity of transverse wave $V \propto \sqrt{T}$

$$V \to \frac{V}{2} \Rightarrow T \to T' = \frac{T}{4}$$

$$T' = \frac{2.06\times10^4}{4} = 5.15\times10^3 N$$

9. NTA Ans. (4)

Sol. $i = i_0\,(1 - e^{-Rt/L}) = i_0(1-e^{-t/T_C})$

$$q = \int_0^{T_C} i\,dt$$

$$= \int_0^{T_C} \frac{\varepsilon}{R}\left(1-e^{-t/T_C}\right)$$

$$= \frac{\varepsilon}{R}\left(t - \frac{e^{-t/T_C}}{-1/T_C}\right)\Bigg|_0^{T_C}$$

$$= \frac{e}{R}\left(T_C - T_C e^{-1}\right) - \frac{e}{R}(0+T_C)$$

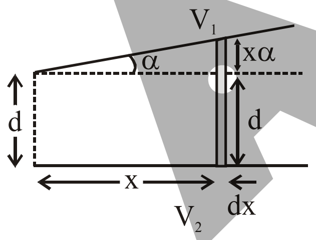
$$q = \frac{e}{R}\times T_C\ e^{-1}$$

$$= \frac{\varepsilon}{R}\times\frac{L}{R}\ \frac{1}{e}$$

$$= \frac{\varepsilon L}{eR^2}$$

10. NTA Ans. (4)

Sol. Assume small element dx at a distance x from left end



Capacitance for small element dx is

$$dC = \frac{\varepsilon_0 a\,dx}{d + x\alpha}$$

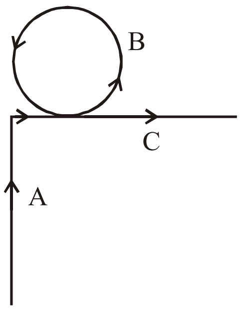
$$C = \int_0^a \frac{\varepsilon_0 a\,dx}{d + x\alpha}$$

$$= \frac{\varepsilon_0 a}{\alpha}\ln\left(\frac{1+\alpha a}{d}\right)\Bigg|_0^a \qquad \left(\ln(1+x) \approx x - \frac{x^2}{2}\right)$$

$$= \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$$

11. NTA Ans. (3)

Sol. We say we have 3 parts (A, B, C)



$$B = B_A + B_B + B_C$$

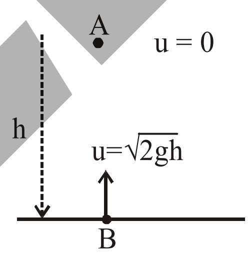
$$= \frac{\mu_0 I}{4\pi R}(\sin 90° - \sin 45°)\otimes + \frac{\mu_0 I}{2R}\odot + \frac{\mu_0 I}{4\pi R}(\sin 45° + \sin 90°)\odot$$

$$= \frac{\mu_0 I}{2\pi R}(\sin 45° + \pi)$$

$$= \frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$$

12. NTA Ans. (4)

Sol.

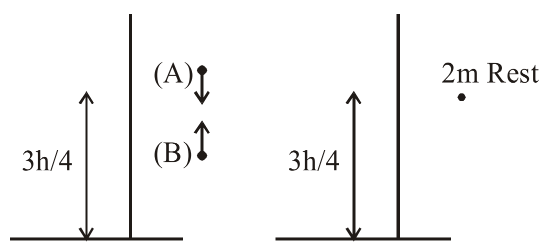


Particles will collide after time $t_0 = \frac{h}{\sqrt{2gh}}$

at collision, $v_A = gt_0$ $\qquad v_B = u_B - gt_0$

$\Rightarrow v_A = -v_B$

Before collision $\qquad$ After collision



Time taken by combined mass to reach the ground

$$\text{time} = \sqrt{\frac{2\times 3h/4}{g}} = \sqrt{\frac{3h}{2g}}$$

13. NTA Ans. (3)

Sol. Refrigerator cycle is :

$$\eta = \frac{W}{Q_+} = \frac{W}{W + Q_-}$$

$$\frac{1}{10} = \frac{10}{10 + Q_-}$$

$Q_- = 90$ J

Heat absorbed from the reservoir at lower temperature is 90 J

14. NTA Ans. (2)

Sol. $$\frac{C_P}{C_V}\text{mix} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

$$\frac{C_P}{C_V}\text{mix} = \frac{n \times \left(\frac{5R}{2}\right) + 2n\left(\frac{7R}{2}\right)}{n \times \frac{3R}{2} + 2n\left(\frac{5R}{2}\right)}$$

$$\frac{C_P}{C_V} = \frac{19}{13}$$

15. NTA Ans. (3)

Sol. By de-Broglie hypothesis

$$\lambda = \frac{h}{mv}$$

$$\lambda_0 = \frac{h}{m\sqrt{2}v_0} \qquad \text{........(1)}$$

$$\lambda' = \frac{h}{\sqrt{v_0^2 + v_0^2 + \left(\frac{eE_0 t}{m}\right)^2}}$$

$$= \frac{h}{m\sqrt{2v_0^2 + \frac{e^2E_0^2t^2}{m^2}}} \qquad \text{........(2)}$$

By (1) and (2)

$$\lambda' = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{2m^2v_0^2}}}$$

16. NTA Ans. (1)

Sol. m = 0.5 kg, v = 5 cm/s

KE in rolling = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

$= 8.75 \times 10^{-4}$ J

17. NTA Ans. (4)

Sol. $E_1 = \frac{KQ_1}{R_1^2}$ $\qquad E_2 = \frac{KQ_2}{R_2^2}$

Given,

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}$$

$$\frac{\frac{KQ_1}{R_1^2}}{\frac{KQ_2}{R_2^2}} = \frac{R_1}{R_2} \Rightarrow \boxed{\frac{Q_1}{Q_2} = \frac{R_1^3}{R_2^3}}$$

$$\frac{V_1}{V_2} = \frac{KQ_1/R_1}{KQ_2/R_2} = \frac{R_1^2}{R_2^2}$$

18. NTA Ans. (1)

Sol. $\vec{r}(t) = \cos\omega\hat{i} + \sin\omega t\,\hat{j}$

On diff. we get

$\vec{v} = -\omega\sin\omega t\ \hat{i} + \omega\cos\omega t\ \hat{j}$

$\vec{a} = -\omega^2\vec{r}$

$\vec{v}\,.\,\vec{r} = 0$

19. NTA Ans. (2)

Sol. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

At focus m = ∞ $\qquad$ x = f

At centre m = –1 $\qquad$ x = 2f

20. NTA Ans. (2)

Sol. Y = $\overline{\overline{AB}.A}$

$= \overline{\overline{AB}} + \overline{A}$

= 0 + 0

= 0

21. NTA Ans. (50)

Sol. According to table and applying law of calorimetry

$1T_1 + 2T_2 = (1 + 2)60°$ ....(1)

$= 180$

$1T_2 + 2T_3 = (1 + 2)30°$ ....(2)

$= 90$

$2T_1 + 1T_3 = (1 + 2)60$

$= 180$ ....(3)

Adding (1) + (2) + (3)

$3\ (T_1 + T_2 + T_3) = 450$

$T_1 + T_2 + T_3 = 150°$

Hence,

$T_1 + T_2 + T_3 = (1 + 1 + 1)\theta$

$150 = 3\theta$

$\theta = 50°C$

22. NTA Ans. (8 or 2888)

Sol. Time to travel 81 m is t sec.

Time to travel 100 m is $t + \frac{1}{2}$ sec.

$81 = \frac{1}{2}\times a\times t^2 \qquad \Rightarrow t = 9\sqrt{\frac{2}{a}}$

$100 = \frac{1}{2}\times a\times\left(t+\frac{1}{2}\right)^2 \qquad \Rightarrow t + \frac{1}{2} = 10\sqrt{\frac{2}{a}}$

$$9\sqrt{\frac{2}{a}}+\frac{1}{2}=10\sqrt{\frac{2}{a}}$$

$$\frac{1}{2}=\sqrt{\frac{2}{a}}$$

$$\boxed{a=8\ m/s^2}$$

23. NTA Ans. (486)

Sol. For Balmer series,

$$\frac{1}{\lambda}=R_H\left(\frac{1}{2^2}-\frac{1}{n_2^2}\right)$$

$$\frac{\lambda_2}{\lambda_1}=\frac{\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}{\left(\frac{1}{2^2}-\frac{1}{4^2}\right)}$$

$$\frac{\lambda_2}{6561}=\frac{5/36}{3/16}$$

$$\lambda_2 = \frac{20}{27}\times 6561$$

$\lambda_2 = 4860$ Å $= 486$ nm

24. NTA Ans. (16)

Sol. $U_1 + K_1 = U_2 + K_2$

$$-\frac{GM_em}{10R}+\frac{1}{2}mv_0^2=-\frac{GM_em}{R}+\frac{1}{2}mv^2$$

$$+\frac{9}{10}\times\frac{GM_em}{R}+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$\frac{9}{10}\times\frac{1}{2}M\times v_e^2+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$v^2 = \frac{9}{10}v_e^2+v_0^2$$

$$= \frac{9}{10}\times(11.2)^2+(12)^2$$

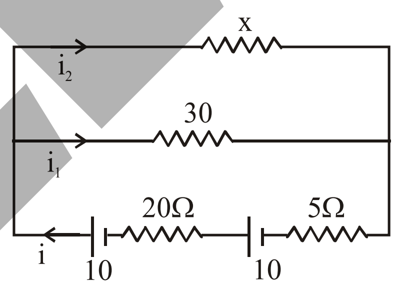
$v^2 = 112.896 + 144$

$v = 16.027$

$v = 16$ km/s

25. NTA Ans. (30)

Sol.



$E_1 = E - ir \qquad\qquad E_2 = E - ir$

$= 10 - i20 = 0 \qquad\qquad = 10 - 0.5 \times 5$

$i = 0.5$ A $\qquad\qquad = 7.5$ V

$$E_{net} = E_1 + E_2 = 7.5V$$

$$i = i_1 + i_2$$

$$0.5 = \frac{7.5}{x}+\frac{7.5}{30}$$

$x = 30\ \Omega$

CHEMISTRY

1. NTA Ans. (1)

Sol. In blast furnace (metallugy of iron) involved reactions are

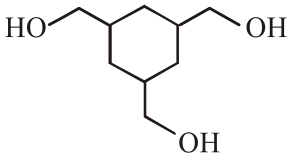
(a) $CaO + SiO_2 \longrightarrow CaSiO_3$

(b) $3Fe_2O_3 + CO \longrightarrow 2Fe_3O_4 + CO_2$

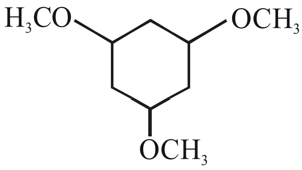
2. NTA Ans. (1)

Sol. Alcohol has more boiling point than ether (due to hydrogen bonding).

So,



has more boiling point than



3. NTA Ans. (3)

Sol. $\log K = \frac{-Ea}{2.303RT} + \log A$

Acrroding to Arrhenius equation plot of 'log K' Vs. $\frac{1}{T}$ is linear with.

Slope = $\frac{-Ea}{2.303R}$

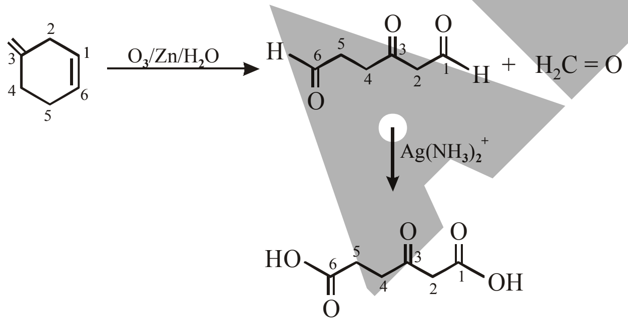
From plot we conclude :

$\underset{(\text{magnitude})}{|\text{slope}|}$: c > a > d > b

$\therefore\ E_c > E_a > E_d > E_b$

4. NTA Ans. (1)

Sol.



5. NTA Ans. (4)

Sol. If the given elements are arranged according to their position in periodic table

Atomic radius

C > O > F
^
Cl
^
Br

Br > Cl > C > O > F

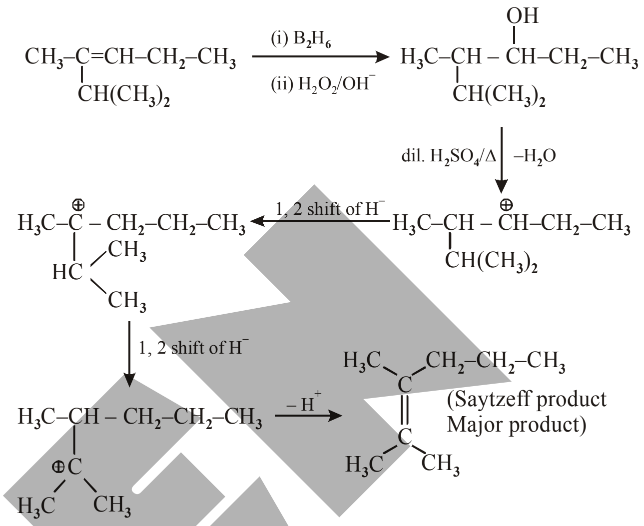
c < b < a < d < e

6. NTA Ans. (1)

Sol. Kjeldahl's method for estimation of nitrogen is not applicable for nitrobenzene $C_6H_5NO_2$. because reaction with H_2SO_4, nitrobenzene can not give ammonia.

7. NTA Ans. (1)

Sol.



8. NTA Ans. (1)

Sol. $\underset{(A)}{3Mg} + N_2 \xrightarrow{\Delta} \underset{(B)}{Mg_3N_2}$

$Mg_3N_2 \xrightarrow{6H_2O} 3Mg(OH)_2 + \underset{\text{colourless gas}}{2NH_3}$

$CuSO_4 + 4NH_3 \longrightarrow \underset{\text{deep blue solution}}{[Cu(NH_3)_4]SO_4}$

9. NTA Ans. (1)

Sol. Since AgBr has intermediate radius ratio

$\therefore$ it shows both schottky & Frenkel defects

ZnS $\rightarrow$ Frenkel defects

KBr, CsCl $\rightarrow$ Schottky defects

10. NTA Ans. (2)

Sol. $H_2O(\ell) \rightleftharpoons H^+_{(aq)} + OH^-_{(aq)}$

For ionization of H_2O : $\Delta H > O$

$\Rightarrow$ ENDOTHERMIC

On temperature increase reaction shifts forward

$\Rightarrow$ both $[H^+]$ and $[OH^-]$ increase

$\Rightarrow$ pH & pOH decreases.

11. NTA Ans. (3)

Sol. Bond length order in carbon halogen bonds are in the order of C – F < C – Cl < C – Br < C – I

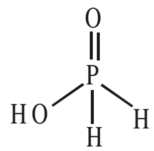
Hence, Bond energy order

C – F > C – Cl > C – Br > C – I

12. NTA Ans. (2)

Sol. $P_4 + 3NaOH + 3H_2O \longrightarrow \underset{(X)}{3NaH_2PO_2} + PH_3$

$NaH_2PO_2 + HCl \longrightarrow NaCl + \underset{(Y)}{H_3PO_2}$



(mono basic acid)

13. NTA Ans. (4)

Sol. $r_n = \dfrac{n^2 \times a_0}{z}$

For 2nd Bohr orbit of Li^{+2}

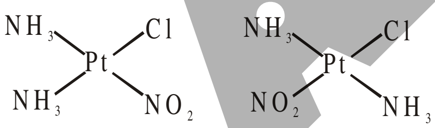
n = 2

z = 3

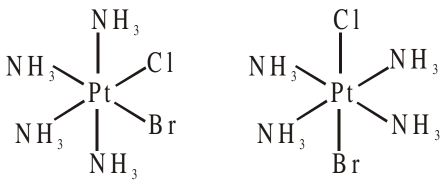
$\Rightarrow r_n = \dfrac{2^2 \times a_0}{3} = \dfrac{4a_0}{3}$

14. NTA Ans. (4)

Sol. $[Pt(NH_3)_3Cl]^+$ & $[Pt(NH_3)Cl_5]^-$ does not show geometrical isomerism

(c) 

cis trans

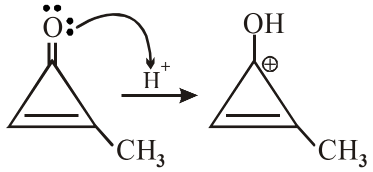
(d) 

cis trans

15. NTA Ans. (3)

Sol. Two monomers in maltose are α-D-glucose & α-D-glucose.

16. NTA Ans. (2)

Sol. 

(Aromatic stable product)

17. NTA Ans. (2)

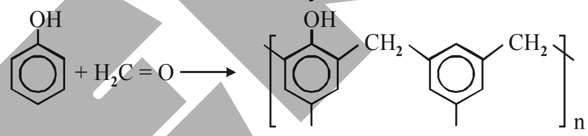
Sol. Hydrogen has three isotopes

Isotopes	Number of neutrons
Protium (^1_1H)	0
Deutrium (^2_1H)	1
Tritium (^3_1H)	2

Hence the sum of neutrons are 3

18. NTA Ans. (3)

Sol. Bakelite formation is example of electrophilic substitution and dehydration.



19. NTA Ans. (4)

20. NTA Ans. (1)

Sol.

$[Ni(CO)_4]$	μ_m = 0 B.M.
$[Ni(H_2O)_6]Cl_2$	μ_m = 2.8 B.M.
$Na_2[Ni(CN)_4]$	μ_m = 0 B.M.
$[PdCl_2(PPh_3)_2]$	μ_m = 0 B.M.

A ≈ C ≈ D < B

21. NTA Ans. (2.13 to 2.16)

Sol. Cell reaction is :

$Sn(s) + Pb^{+2}(aq) \longrightarrow Sn^{+2}(aq) + Pb(s)$

Apply Nernst equation :

$$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log\frac{[Sn^{+2}]}{[Pb^{+2}]} \quad ...(1)$$

$E^0_{cell} = -0.13 + 0.14 = 0.01$ V

At equilibrium : $E_{cell} = 0$

Substituting in (1)

$$0 = 0.01 - \frac{0.06}{2}\log\frac{[Sn^{+2}]}{[Pb^{+2}]}$$

$$\Rightarrow \frac{1}{3} = \log\frac{[Sn^{+2}]}{[Pb^{+2}]}$$

$$\Rightarrow \frac{[Sn^{+2}]}{[Pb^{+2}]} = 2.15$$

22. NTA Ans. (6.25)

Sol. For ideal gas :

$\Delta U = nC_V[T_2 - T_1]$

$\Rightarrow$ $5000 = 4 \times C_V[500 - 300]$

$\Rightarrow$ $C_V = \dfrac{5000}{800} = 6.25$ J mole^{-1} K^{-1}

23. NTA Ans. (2120 to 2140)

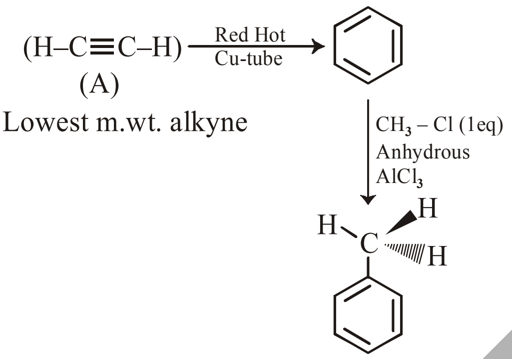
Sol. Mole of O_2 consumed $= \dfrac{1 \times 492}{0.082 \times 300} = 20$

Mole of $NaClO_3$ required = 20

Mass of $NaClO_3$ = 20 × 106.5 = 2130 gm

24. NTA Ans. (13)

Sol.



Lowest m.wt. alkyne

Total 13 atom are present in same plane (7 carbon & 6 hydrogen atoms.)

25. NTA Ans. (20)

MATHEMATICS

1. NTA Ans. (3)

Sol. $\vec{b} \times \vec{c} - \vec{b} \times \vec{a} = \vec{0}$

$\vec{b} \times (\vec{c} - \vec{a}) = \vec{0}$

$\vec{b} = \lambda(\vec{c} - \vec{a})$ ...(i)

$\vec{a} \cdot \vec{b} = \lambda(\vec{a} \cdot \vec{c} - \vec{a}^2)$

$4 = \lambda(0 - 6) \Rightarrow \lambda = \dfrac{-4}{6} = \dfrac{-2}{3}$

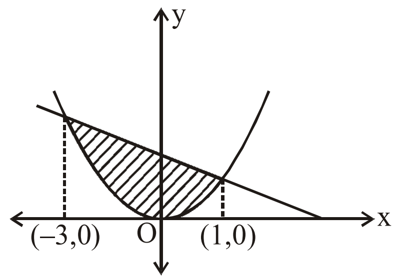
from (i) $\vec{b} = \dfrac{-2}{3}(\vec{c} - \vec{a})$

$\vec{c} = \dfrac{-3}{2}\vec{b} + \vec{a} = \dfrac{-1}{2}(\hat{i} + \hat{j} + \hat{k})$

$\boxed{\vec{b} \cdot \vec{c} = \dfrac{-1}{2}}$ (3) Option

2. NTA Ans. (4)

Sol.



Area $= \displaystyle\int_{-3}^{1} (3 - 2x - x^2)dx = \dfrac{32}{3}$

(4) option

3. NTA Ans. (2)

Sol. $x^2 + 2xy - 3y^2 = 0$

m_N = slope of normal drawn to curve at (2,2) is –1

L : x + y = 4.

perpendicular distance of L from (0,0)

$= \dfrac{|0 + 0 - 4|}{\sqrt{2}} = 2\sqrt{2}$

(2) Option

4. NTA Ans. (1)

Sol. $f(x) = \dfrac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$

$f'(x) = \dfrac{-6(x-1)(x-2)}{2(2x^3 - 9x^2 + 12x + 4)^{3/2}}$

$\therefore$ $f(x)$ is decreasing in (1,2)

$f(1) = \dfrac{1}{3}$; $f(2) = \dfrac{1}{\sqrt{8}}$

$\dfrac{1}{3} < I < \dfrac{1}{\sqrt{8}} \Rightarrow I^2 \in \left(\dfrac{1}{9}, \dfrac{1}{8}\right)$

(1) Option

5. NTA Ans. (2)

Sol. Slope of tangent to $x^2 + y^2 = 1$ at $P\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

$2x + 2yy' = 0 \Rightarrow m_T|_P = -1$

$y = mx + c$ is tangent to $(x - 3)^2 + y^2 = 1$

$y = x + c$ is tangent to $(x - 3)^2 + y^2 = 1$

$\left|\dfrac{c+3}{\sqrt{2}}\right| = 1 \Rightarrow c^2 + 6c + 7 = 0$

(2) Option

6. NTA Ans. (2)

ALLEN Ans. (Bonus)

Sol. option (1), (2), (3) are incorrect for $f(x)$ = constant and option (4) is incorrect

$\frac{f(1)-f(c)}{1-c} = f'(a)$ where c < a < 1 (use LMVT)

Also for $f(x) = x^2$ option (4) is incorrect.

7. NTA Ans. (1)

Sol. $\sim(p \vee \sim q) \rightarrow p \vee q$

$(\sim p \wedge q) \rightarrow p \vee q$

$\sim\{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$

$\sim(\sim p \wedge f)$

(1) Option

8. NTA Ans. (2)

Sol. $T_{10} = \frac{1}{20} = a + 9d$ ...(i)

$T_{20} = \frac{1}{10} = a + 19d$ ...(ii)

$a = \frac{1}{200} = d$

Hence, $S_{200} = \frac{200}{2}\left[\frac{2}{200} + \frac{199}{200}\right] = \frac{201}{2}$

(2) Option

9. NTA Ans. (4)

Sol. $f(x) = \begin{cases} \frac{x}{x^2+1} & ; \quad x \in (1,2) \\ \frac{2x}{x^2+1} & ; \quad x \in [2,3) \end{cases}$

$f(x)$ is decreasing function

$\therefore f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(4) Option

10. NTA Ans. (4)

Sol. $D = \begin{vmatrix} \lambda & 3 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = (\lambda+8)(2-\lambda)$

for $\lambda = 2$; $D_1 \neq 0$

Hence, no solution for $\lambda = 2$

(4) Option

11. NTA Ans. (3)

Sol. $2[{}^6C_0 x^6 + {}^6C_2 x^4(x^2-1) + {}^6C_4 x^2(x^2-1)^2 + {}^6C_6(x^2-1)^3]$

$\alpha = -96$ & $\beta = 36$

$\therefore \alpha - \beta = -132$

(3) Option

12. NTA Ans. (1)

Sol. Using L.H. Rule

$\lim_{x\to 0} \frac{x\sin(10x)}{1} = 0$ (1) Option

13. NTA Ans. (2)

Sol. $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$; $|A| = 8 - 18 = -10$

$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{pmatrix} 4 & -2 \\ -9 & 2 \end{pmatrix}}{-10}$

$10A^{-1} = \begin{pmatrix} -4 & 2 \\ 9 & -2 \end{pmatrix} = A - 6I$

(2) Option

14. NTA Ans. (1)

Sol. $\frac{\sum x_i}{20} = 10 \Rightarrow \Sigma x_i = 200$ ...(i)

$\frac{\sum x_i^2}{20} - 100 = 4 \Rightarrow \Sigma x_i^2 = 2080$ ...(ii)

Actual mean $= \frac{200-9+11}{20} = \frac{202}{20}$

Variance $= \frac{2080-81+121}{20} - \left(\frac{202}{20}\right)^2 = 3.99$

(1) Option

15. NTA Ans. (3)

Sol. $\frac{x^2}{36} - \frac{y^2}{b^2} = 1$ ...(i)

P(10,16) lies on (i) get $b^2 = 144$

$\frac{x^2}{36} - \frac{y^2}{144} = 1$

Equation of normal is

$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2e^2$

$2x + 5y = 100$

(3) Option

16. NTA Ans. (4)

Sol. $P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$

$P(A) + P(B) - P(A \cap B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{10}$

(4) Option

17. NTA Ans. (4)

Sol. Point on plane $R\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

Normal vector of plane is $\frac{10}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k}$

Equation of require plane is x + y + z = 1

Hence (1, –1, 1) lies on plane

(4) Option

18. NTA Ans. (4)

Sol. Let $3^x = t$; $t > 0$

$t(t-1) + 2 = |t-1| + |t-2|$

$t^2 - t + 2 = |t-1| + |t-2|$

Case-I : $t < 1$

$t^2 - t + 2 = 1 - t + 2 - t$

$t^2 + 2 = 3 - t$

$t^2 + t - 1 = 0$

$$t = \frac{-1 \pm \sqrt{5}}{2}$$

$$t = \frac{\sqrt{5}-1}{2} \quad \text{is only acceptable}$$

Case-II : $1 \le t < 2$

$t^2 - t + 2 = t - 1 + 2 - t$

$t^2 - t + 1 = 0$

$D < 0$ no real solution

Case-III : $t \ge 2$

$t^2 - t + 2 = t - 1 + t - 2$

$t^2 - 3t \quad 5 = 0 \Rightarrow D < 0$ no real solution

(4) Option

19. NTA Ans. (1)

Sol. $\alpha = \omega$

$a = (1+\omega)(1 + \omega^2 + \omega^4 + ... + \omega^{200})$

$$a = (1+\omega)\frac{\left(1-\left(\omega^2\right)^{101}\right)}{1-\omega^2} = 1$$

$b = 1 + \omega^3 + \omega^6 + ... + \omega^{300} = 101$

$x^2 - 102x + 101 = 0$

(1) Option

20. NTA Ans. (1)

Sol. $2x = 4by' \Rightarrow y' = \dfrac{2x}{4b}$

Required D.E. is $x^2 = \dfrac{2x}{y'}y + \left(\dfrac{x}{y'}\right)^2$

$x(y')^2 = 2yy' + x$

(1) Option

21. NTA Ans. (1)

Sol. $\dfrac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \dfrac{1}{7} \Rightarrow \tan\alpha = \dfrac{1}{7}$

$$\sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \tan\beta = \frac{1}{3} \Rightarrow \tan 2\beta = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta} = 1 \quad \text{Ans. } 1.00$$

22. NTA Ans. (3)

Sol. $f''(x) = \lambda(x-1)$

$$f'(x) = \frac{\lambda x^2}{2} - \lambda x + C \Rightarrow f'(-1) = 0 \Rightarrow c = \frac{-3\lambda}{2}$$

$$f(x) = \frac{\lambda x^3}{6} - \frac{\lambda x^2}{2} - \frac{3\lambda}{2}x + d$$

$f(1) = -6 \Rightarrow -11\lambda + 6d = -36$ ...(i)

$f(-1) = 10 \Rightarrow 5\lambda + 6d = 60$ ...(ii)

from (i) & (ii) $\lambda = 6$ & $d = 5$

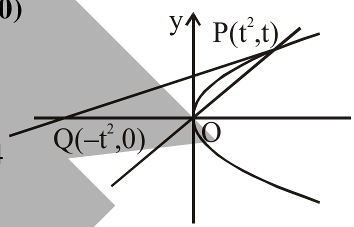
$f(x) = x^3 - 3x^2 - 9x + 5$

Which has minima at $x = 3$

Ans. 3.00

23. NTA Ans. (0.50)

Sol. $\Delta OPQ = 4$



$$\frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{matrix}\right\| = 4$$

$t = 2$ ($\because t > 0$)

$$\therefore\ m = \frac{1}{2}$$

Ans. 0.50

24. NTA Ans. (504)

Sol. $\dfrac{1}{4}\left(\displaystyle\sum_{n=1}^{7} 2n^3 + \sum_{n=1}^{7} 3n^2 + \sum_{n=1}^{7} n\right)$

$$= \frac{1}{4}\left(2\left(\frac{7\times 8}{2}\right)^2 + 3\left(\frac{7\times 8\times 15}{6}\right) + \frac{7\times 8}{2}\right)$$

$= 504$

Ans. 504.00

25. NTA Ans. (2454)

Sol. N → 2, A → 2, I → 2, E, X, M, T, O → 1

Category	Selection	Arrangement
2 alike of one kind and 2 alike of other kind	$^3C_2 = 3$	$3 \times \dfrac{4!}{2!\,2!} = 18$
2 alike and 2 different	$^3C_1 \times {}^7C_2$	$^3C_1 \times {}^7C_2 \times \dfrac{4!}{2!} = 756$
All 4 different	$^8C_4$	$^8C_4 \times 4! = 1680$

Total = 2454

Ans. 2454.00

SET # 05

PHYSICS

1. NTA Ans. (2)

Sol. $W = \int_{\vec{r}_i}^{\vec{r}_g} \vec{F}.d\vec{r}$

$$W = \int_1^0 -xdx + \int_0^1 ydy$$

$$W = \left.\frac{-x^2}{2}\right|_1^0 + \left.\frac{y^2}{2}\right|_0^1$$

$$= -\left(\frac{0^2}{2} - \frac{1^2}{2}\right) + \left(\frac{1^2}{2} - \frac{0^2}{2}\right)$$

W = 1J

2. NTA Ans. (3)

Sol. $[h] = M^1L^2T^{-1}$

$[C] = L^1T^{-1}$

$[G] = M^{-1}L^3T^{-2}$

$[f] = \sqrt{\frac{M^1L^2T^{-1} \times L^5T^{-5}}{M^{-1}L^3T^{-2}}} = M^1L^2T^{-2}$

3. NTA Ans. (1)

Sol. Initially, the body of mass m is moving in a circular orbit of radius R. So it must be moving with orbital speed.

$$v_0 = \sqrt{\frac{GM}{R}}$$

After collision, let the combined mass moves with speed v_1

$$mv_0 + \frac{m}{2}\frac{v_0}{2} = \left(\frac{3m}{2}\right)v_1$$

$$v_1 = \frac{5v_0}{6}$$

Since after collision, the speed is not equal to orbital speed at that point. So motion cannot be circular. Since velocity will remain tangential, so it cannot fall vertically towards the planet. Their speed after collision is less than escape speed $\sqrt{2}v_0$, so they cannot escape gravitational field.

So their motion will be elliptical around the planet.

4. NTA Ans. (3)

Sol. $\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx)$

Its corresponding magnetic field will be

$$\vec{B}_1 = \frac{E_0}{c}\hat{k}\cos(\omega t - kx)$$

$$\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$$

$$\vec{B}_2 = \frac{E_0}{c}\hat{i}\cos(\omega t - ky)$$

Net force on charge particle

$$= q\vec{E}_1 + q\vec{E}_2 + q\vec{v} \times B_1 + q\vec{v} \times \vec{B}_2$$

$$= qE_0\hat{j} + qE_0\hat{k} + q(0.8c\hat{j}) \times \left(\frac{E_0}{c}\hat{k}\right) + q(0.8c\hat{j}) \times \left(\frac{E_0}{c}\hat{i}\right)$$

$$= qE_0\hat{j} + qE_0\hat{k} + 0.8qE_0\hat{i} - 0.8qE_0\hat{k}$$

$$\vec{F} = qE_0[0.8\hat{i} + 1\hat{j} + 0.2\hat{k}]$$

5. NTA Ans. (4)

Sol. Fill the empty space with $+\rho$ and $-\rho$ charge density.

$$|E_A| = 0 + \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{R}{2}\right)^2} = k\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)$$

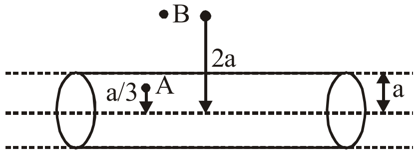
$$|E_B| = \frac{k\rho.\frac{4}{3}\pi R^3}{R^2} - \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$= k\rho\frac{4}{3}\pi R - k\rho\frac{4}{3}\pi\frac{R}{18} = k\rho.\frac{4}{3}\pi\left(\frac{17R}{18}\right)$$

$$\frac{E_A}{E_B} = \frac{9}{17} = \frac{18}{34}$$

6. **NTA Ans. (1)**

Sol.



Let current density be J.

$\therefore$ Applying Ampere's law.

$$\oint \vec{B}.d\vec{\ell} = \mu_0 i \Rightarrow B_A 2\pi\frac{a}{3} = \mu_0 J\pi\left(\frac{a}{3}\right)^2$$

$$\therefore \quad B_A = \frac{\mu_0 Ja}{6}$$

Similarly, $B_B = \frac{\mu_0 Ja}{4}$

$$\therefore \quad \frac{B_A}{B_B} = \frac{\mu_0 Ja \times 4}{\mu_0 J6a} = \frac{2}{3}$$

7. **NTA Ans. (2)**

Sol. Degree of freedom of a diatomic molecule if vibration is absent = 5

Degree of freedom of a diatomic molecule if vibration is present = 7

$$\therefore \quad C_v^A = \frac{f_A}{2}R = \frac{5}{2}R \;\&\; C_v^B = \frac{f_B}{2}R = \frac{7}{2}R$$

$$\therefore \quad \frac{C_v^A}{C_v^B} = \frac{5}{7}$$

8. **NTA Ans. (3)**

Sol. Given, de-Broglie wavelength = $\frac{h}{\sqrt{2mE}} = \lambda$

Also, $\frac{h}{\sqrt{2m(E+\Delta E)}} = \frac{\lambda}{2}$

$\therefore \quad \frac{E+\Delta E}{E} = 4 \Rightarrow \Delta E = 3E.$

9. **NTA Ans. (2)**

Sol. Given on six rotation, reading of main scale changes by 3mm.

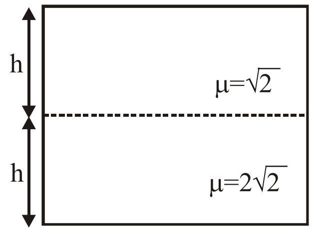
$\therefore$ 1 rotation corresponds to $\frac{1}{2}$mm

Also no. of division on circular scale = 50.

$\therefore$ Least count of the screw gauge will be $\frac{0.5}{50}$mm = 0.001 cm.

10. **NTA Ans. (2)**

Sol.



For near normal incidence,

$$h_{app} = \frac{h_{actual}}{\left(\frac{\mu_{in}}{\mu_{ref.}}\right)}$$

$$\therefore \quad h_{apparent} = \frac{\frac{h}{\left(\frac{2\sqrt{2}}{\sqrt{2}}\right)} + h}{\frac{\sqrt{2}}{1}} = \frac{3h}{2\sqrt{2}} = \frac{3}{4}h\sqrt{2}$$

11. **NTA Ans. (3)**

Sol. Let the work function be ϕ.

$$\therefore \quad KE_{max} = \frac{hc}{\lambda} - \phi$$

Again, $R_{max} = \frac{\sqrt{2mKE_{max}}}{qB} = \frac{\sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)}}{qB}$

$$\therefore \quad \frac{R_{max}^2 q^2 B^2}{2m} = \frac{hc}{\lambda} - \phi$$

$$\therefore \quad \phi = \frac{hc}{\lambda} - \frac{R_{max}^2 q^2 B^2}{2m} = 1.0899 \text{ eV} \approx 1.1\text{eV}$$

12. **NTA Ans. (3)**

Sol. Let distance is x then

$d\theta = \frac{1.22\lambda}{D}$ (D = diameter)

$\frac{x}{d} = \frac{1.22\lambda}{D}$ (d = distance between earth & moon)

$$x = \frac{1.22 \times (5500 \times 10^{-10}) \times (4 \times 10^8)}{5} = 53.68\,m$$

most appropriate is 60m.

13. NTA Ans. (2)

Sol. From momentum conservation

$$mu\hat{i} + mu\left(\frac{\hat{i}+\hat{j}}{2}\right) = (m + m)\bar{v}$$

$$\Rightarrow \bar{v} = \frac{3}{4}u\hat{i} + \frac{u}{4}\hat{j}$$

$$\Rightarrow |v| = \frac{u}{4}\sqrt{10}$$

Final kinetic energy = $\frac{1}{2}2m\left(\frac{u}{4}\sqrt{10}\right)^2 = \frac{5}{8}mu^2$

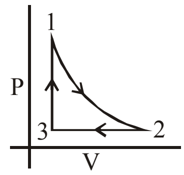
Initial kinetic energy

$$= \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{6}{8}mu^2$$

Loss in K.E. = $k_i - k_f = \frac{1}{8}mu^2$

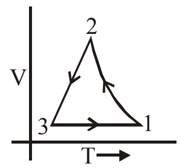
14. NTA Ans. (4)

Sol.



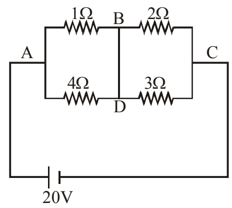
In process 2 to 3 pressure is constant & in process 3 to 1 volume is constant which is correct only in option 4.

Correct graph is



15. NTA Ans. (2)

Sol.



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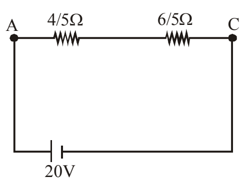

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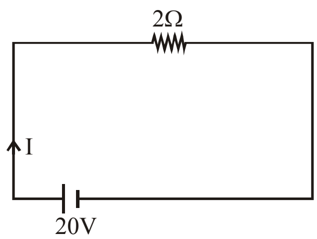
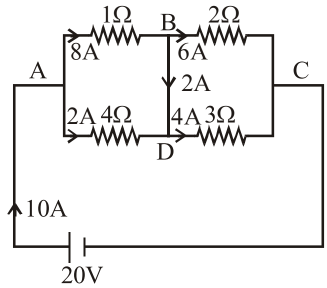


$$\Rightarrow I = \frac{20}{2} = 10A$$

Now,



16. NTA Ans. (2)

Sol. Option (A)

$W = k_f - k_i$

$$qE(2a - 0) = \frac{1}{2}m(2V)^2 - \frac{1}{2}mV^2$$

$$qE2a = \frac{3}{2}mV^2$$

$$E = \frac{3}{4}\frac{mv^2}{qa}$$

Option (B)

Rate of work done $P = \vec{F}.\vec{V} = FV\cos\theta = FV$

Power = qEV

$$\text{Power} = q\left(\frac{3}{4}\frac{mV^2}{qa}\right)V$$

$$\text{Power} = q\frac{3}{4}\frac{mV^3}{qa}$$

$$\text{Power} = \frac{3}{4}\frac{mV^3}{a}$$

Option (C)

Angle between electric force and velocity is 90º, hence rate of work done will be zero at Q.

Option (D)

Initial angular momentum L_i = mVa

Final angular momentum L_f = m(2V) (2a)

Change in angular momentum $L_f - L_i$ = 3mVa

(Note : angular momentum is calculated about O)

17. NTA Ans. (1)

Sol. Let amplitude of each wave is A.

Resultant wave equation

$$= A\sin\omega t + A\sin\left(\omega t - \frac{\pi}{4}\right) + A\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$= A\sin\omega t + \sqrt{2}A\sin\omega t$$

$$= \left(\sqrt{2}+1\right)A\sin\omega t$$

Resultant wave amplitude = $\left(\sqrt{2}+1\right)A$

as $I \propto A^2$

so $\dfrac{I}{I_0} = \left(\sqrt{2}+1\right)^2$

$I = 5.8\ I_0$

18. NTA Ans. (3)

Sol. Since $\vec{r}$ and $\vec{p}$ are perpendicular to each other therefore point lies on the equitorial plane. Therefore electric field at the point will be antiparallel to the dipole moment.

i.e. $\vec{E} \parallel -\vec{p}$

$\vec{E} \parallel (\hat{i} + 3\hat{j} - 2\hat{k})$

19. NTA Ans. (1)

Sol. From parallel axis theorem

$$I_0 = 3 \times \left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$$

$$I_A = I_0 + 3M\left(\frac{d}{\sqrt{3}}\right)^2$$

$$= \frac{13}{10}Md^2 + Md^2$$

$$= \frac{23}{10}Md^2$$

$$\frac{I_0}{I_A} = \frac{13}{23}$$

20. NTA Ans. (3)

Sol. Rate of flow of water = $A_AV_A = A_BV_B$

$(40)V_A = (20)V_B$

$V_B = 2V_A$ .... (1)

Using Bernoulli's theorem

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$700 = \frac{1}{2} \times 1000(4V_A^2 - V_A^2)$$

$V_A = 0.68$ m/s = 68 cm/s

Rate of flow = A_AV_A

= (40) (68) = 2720 cm^3/s

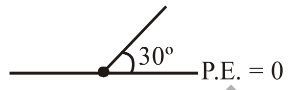
21. NTA Ans. (10.00)

Sol. $V = \left|L\dfrac{di}{dt}\right|$

$$\Rightarrow L = \frac{V}{\left|\frac{di}{dt}\right|} = \frac{100}{\frac{0.25}{0.025 \times 10^{-3}}} = 10\text{mH}$$

22. NTA Ans. (15.00)

Sol.



From mechanical energy conservation,

$U_i + K_i = U_f + K_f$

$$\Rightarrow mg\frac{\ell}{2}\sin 30° + 0 = 0 + \frac{1}{2}I\omega^2$$

$$\Rightarrow mg \times \frac{1}{2} \times \frac{1}{2} + 0 = 0 + \frac{1}{2} \times \frac{m(1)^2}{3}\omega^2$$

$$\Rightarrow \omega^2 = \frac{3g}{2} \Rightarrow \omega = \sqrt{15}$$

$\therefore$ n = 15

23. NTA Ans. (3.00)

Sol. $x = \sqrt{at^2 + 2bt + c}$

Differentiating w.r.t. time

$$\frac{dx}{dt} = v = \frac{1}{2\sqrt{at^2 + 2bt + c}} \times (2at + 2b)$$

$$\Rightarrow v = \frac{at + b}{x}$$

$\Rightarrow vx = at + b$

Differentiating w.r.t. x

$$\Rightarrow \frac{dv}{dx} \times x + v = a \times \frac{dt}{dx}$$

Multiply both side by v

$$\Rightarrow \left(v\frac{dv}{dx}\right)x + v^2 = a$$

$\Rightarrow a'x = a - v^2$ [Here a' is acceleration]

$$\Rightarrow a'x = a - \left(\frac{at + b}{x}\right)^2$$

$$\Rightarrow a'x = \frac{ax^2 - (at+b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{a(at^2+2bt+c)-(at+b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{ac-b^2}{x^2}$$

$$\Rightarrow a' = \frac{ac-b^2}{x^3}$$

$$\therefore a' \propto \frac{1}{x^3} \qquad \therefore n = 3$$

24. NTA Ans. (4.00)

Sol. $T = m\omega^2\ell$

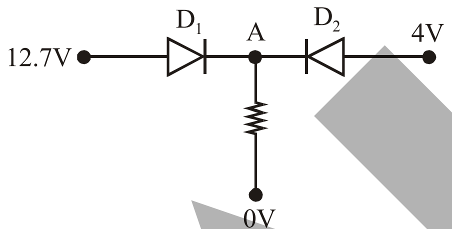
Breaking stress = $\frac{T}{A} = \frac{m\omega^2\ell}{A}$

$$\Rightarrow \omega^2 = \frac{4.8\times10^7\times(10^{-2}\times10^{-4})}{10\times0.3} = 16$$

$$\Rightarrow \omega = 4$$

25. NTA Ans. (12.00)

Sol.



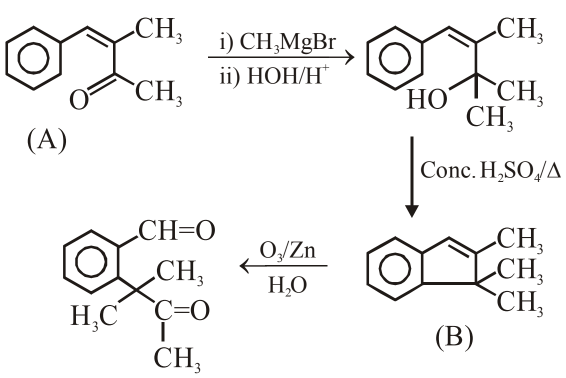
Diode D_1 is forward biased and D_2 is reverse biased.

$\therefore$ V_A = 12.7 – 0.7 = 12V.

CHEMISTRY

1. NTA Ans. (4)

Sol.



2. NTA Ans. (4)

Sol. $K_1 = Ae^{-\frac{Ea}{R\times700}}$

$K_2 = A\times e^{-\frac{(Ea-30)}{R\times500}}$

For same rate

$K_1 = K_2$

$$e^{-\frac{Ea}{700R}} = e^{-\frac{(Ea-30)}{R\times500}}$$

$$\frac{Ea}{700R} = \frac{Ea-30}{R\times500}$$

5Ea = 7Ea – 210

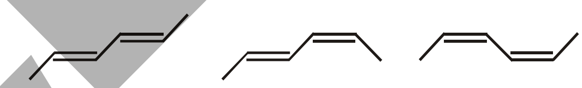
210 = 2Ea

E_a = 105 kJ/mole

E_a – 30 = 75

3. NTA Ans. (1)

Sol.



(a) (Trans, Trans) (b) (Trans, Cis) (c) (Cis, Cis)

$\therefore$ Generally trans is more stable then cis form.

Heat of combustion (HOC) $\propto \frac{1}{\text{Stability}}$

Stability : a > b > c

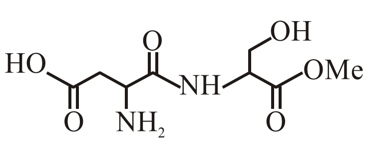
HOC : c > b > a

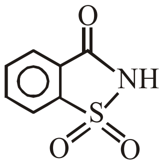
4. NTA Ans. (1)

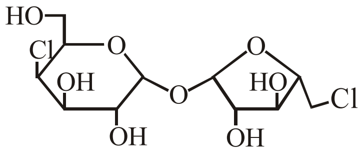
Sol. (i) Blue voilet color with Ninhydrine $\rightarrow$ amino acid derivative. So it cannot be saccharide or sucralose.

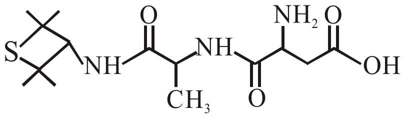
(ii) Lassaigne extract give +ve test with $AgNO_3$. So Cl is present, –ve test with $Fe_4[Fe(CN)_6]_3$ means N is absent. So it can't be Aspartame or Saccharine or Alitame, so C is sucralose.

(iii) Lassaigne solution of B and D given +ve sodium nitroprusside test, so it is having S, so it is Saccharine and Alitame.

(A) Aspartame 

(B) Saccharine 

(C) Sucralose 

(D) Alitame 

5. NTA Ans. (1)

Sol. CCl_4 is molecular solid so does not conduct electricity in liquid & solid state.

6. NTA Ans. (2)

Sol. A reduces BO_2 when temperature is above 1400°C because above 1400°C A has more –ve $\Delta G°$ for AO_2 formation than B to BO_2 formation.

7. NTA Ans. (2)

Sol. $\left[Pb^{2+}\right] = \dfrac{300 \times 0.134}{400}$

$= 1.005 \times 10^{-1}$ M

$\left[Cl^{-}\right] = \dfrac{100 \times 0.4}{400}$

$= 10^{-1}$ M

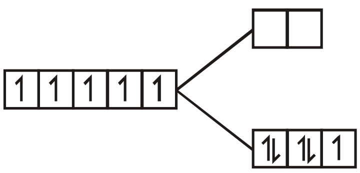
$PbCl_{2(s)} \rightleftharpoons Pb^{+2}_{(aq.)} + 2Cl^{-}_{(aq.)}$

$Q = [Pb^{2+}] \times [Cl^{-}]^2$

$= 1.005 \times 10^{-3} > k_{sp}$

8. NTA Ans. (2)

Sol. $[Pb(F)(Cl)(Br)(I)]^{2-}$ have three geometrical isomer so formula for $[Fe(CN)_6]^{n-6}$ is $[Fe(CN)_6]^{3-}$ and CFSE for this complex is $Fe^{3\oplus} \Rightarrow 3d^5 4s^\circ$



Magnetic Moment $= \sqrt{3}$

$= 1.73$ B.M

CFSE $= [(-0.4 \times 5) + (0.6 \times 0)]\ \Delta_0$

$= -2.0\ \Delta_0$

9. NTA Ans. (1)

Sol.

	number of unpaired electron	magnetic moment
$O_2^{\ominus}$	1	1.73 B.M
$O_2^{\oplus}$	1	1.73 B.M
O_2	2	2.83 BM

10. NTA Ans. (4)

Sol. Enthalpy of atomisation of $Br_2(l)$

$$Br_2(l) \xrightarrow{\Delta H_{vap}} Br_2(g) \xrightarrow{\Delta H_{BE}} 2Br(g)$$

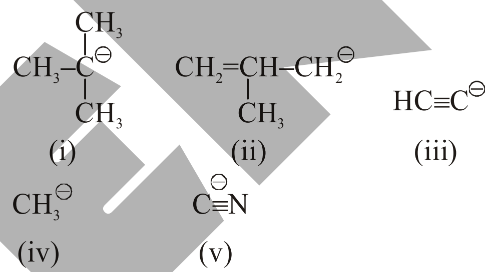
$$\Delta H_{atom.}$$

$\Delta H_{atom} = \Delta H_{vap} + \Delta H_{BE}$

$x = \Delta H_{vap} + y$

So, $x > y$

11. NTA Ans. (3)

Sol. 

Basic strength order : (i) > (iv) > (ii) > (iii) > (v)

12. NTA Ans. (1)

Sol. Be $\Rightarrow 1s^2\ 2s^2$

B $\Rightarrow 1s^2\ 2s^2\ 2p^1$

B has a smaller size than Be

it is easier to remove 2p electron than 2s electron due to less pentration effect of 2p than 2s.

2p electron of Boron is more shielded from the nucleus by the inner core of electron than the 2s electron of Be

B has a smaller size than Be

13. NTA Ans. (4)

Sol. 1. MgO Basic
Cl_2O Acidic
Al_2O_3 amphoteric

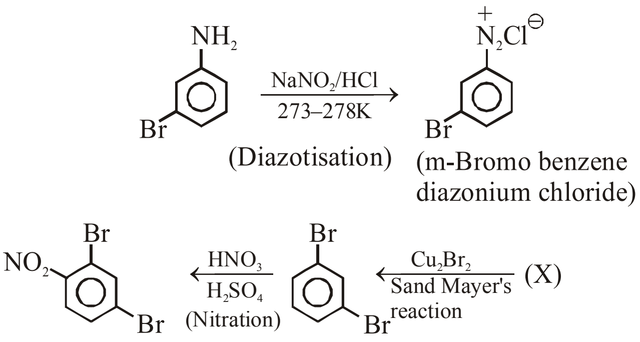
2. Cl_2O Acidic
CaO Basic
P_4O_{10} Acidic

3. Na_2O Basic
SO_3 Acidic
Al_2O_3 amphoteric

4. N_2O_3 Acidic
Li_2O Basic
Al_2O_3 amphoteric

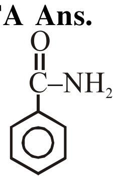
14. NTA Ans. (2)

Sol.

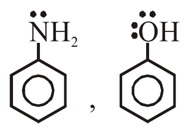


15. NTA Ans. (3)

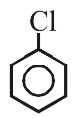
Sol. ∴



(Deactivated ring due to –R effect of amide)



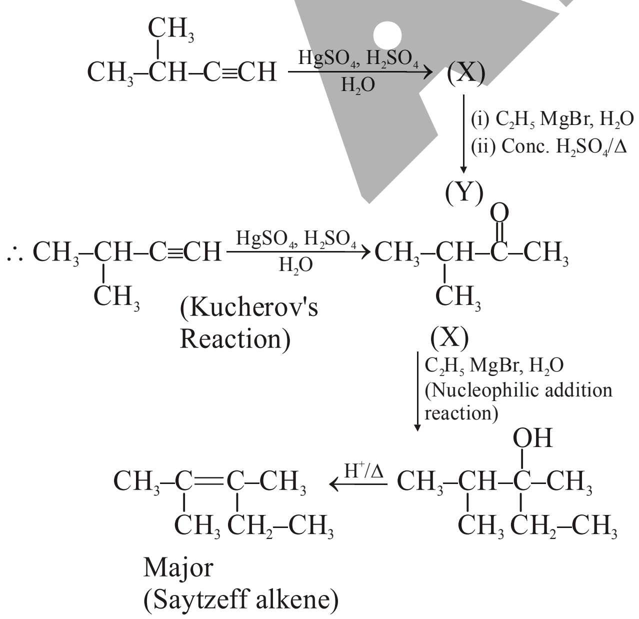
(l.p.e. of $-NH_2$ and $-\ddot{O}H$ group do acid-base reaction with lewis acid $AlCl_3$, so ring is deactivated)



(Highest yield produced)

16. NTA Ans. (3)

Sol.



17. NTA Ans. (1)

Sol. $Cr(H_2O)_6\ Cl_n$

if magnetic mement is 3.83 BM then it contain three unpaired electrons. It means chromium in +3 oxidation state so molecular formula is $Cr(H_2O)_6\ Cl_3$

∴ This formula have following isomers

(a) $[Cr(H_2O)_6]Cl_3$: react with $AgNO_3$ but does not show geometrical isomerism.

(b) $[Cr(H_2O)_5Cl]Cl_2.H_2O$ react with $AgNO_3$ but does not show geometrical isomerism.

(c) $[Cr(H_2O)_4Cl_2]Cl.2H_2O$ react with $AgNO_3$ & show geometrical isomerism.

(d) $[Cr(H_2O)_3Cl_3].3H_2O$ does not react with $AgNO_3$ & show geometrical isomerism.

$[Cr(H_2O)_4Cl_2]Cl.2H_2O$ react with $AgNO_3$ & show geometrical isomerism and it's IUPAC nomenclature is Tetraaquadichlorido chromium (III) Chloride dihydrate.

18. NTA Ans. (4)

Sol. (i) H_2O_2 act as oxidising agent as well as reducing agent depending on condition.

(ii) H_2SO_3 act as oxidising agent as well as reducing agent depending on condition.

(iii) HNO_2 act as oxidising agent as well as reducing agent depending on condition.

(iv) H_3PO_4 can not act both as oxidising and reducing agent.

H_3PO_4 can act as only oxidising agent.

$H_3PO_4 \rightleftharpoons 3H^+ + PO_4^{3-}$

19. NTA Ans. (1)

Sol. $2\pi r = n\lambda$

for $n = 1,\ r = a_0$

$n = 4,\ r = 16a_0$

So, $2\pi \times 16a_0 = 4 \times \lambda$

$\lambda = 8\pi a_0$

20. NTA Ans. (2)

Sol. $Eu_{63} \Rightarrow [Xe]\ 4f^7\ 5d^\circ\ 6s^2$

$Eu^{2\oplus} \Rightarrow [Xe]\ 4f^7$

$Ce_{58} \Rightarrow [Xe]\ 4f^1\ 5d^1\ 6s^2$

$Ce^{3\oplus} \Rightarrow [Xe]\ 4f^1$

21. NTA Ans. (100)

Sol. 1 Litre has 10^{-3} moles $MgSO_4$

So , 1000 litre has 1 mole $MgSO_4$

= 1 mole $CaCO_3$

= 100 ppm

22. NTA Ans. (14.00)

Sol. 100 gm soln $\rightarrow$ 63 gm HNO_3

$\frac{100}{1.4}$ mL $\rightarrow$ 1 mole HNO_3

$$\text{Molarity} = \frac{1}{\frac{100}{1.4}\times\frac{1}{1000}} = 14M$$

23. NTA Ans. (5.66 to 5.68)

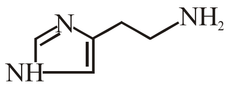
Sol. gm eq. of Ag $= \frac{108}{108} = 1$

gm eq. of $O_2(g) = 1$

Volume of $O_2(g) = 22.7\times\frac{1}{4}$ = 5.675 litre

24. NTA Ans. (37.80 to 38.20)

Sol.



M.F. of Histamine is $C_5H_9N_3$

Molecular mass of Histamine is 111

Now, mass % of nitrogen $= \left(\frac{42}{111}\right)\times 100$

$= 37.84\%$

25. NTA Ans. (1.74 to 1.76 or 0.03)

Sol. $\Delta T_f = i\times m\times K_f$

$0.2 = 2\times 2\times \frac{w/58.5}{600/1000}$

w = 1.755 gm

MATHEMATICS

1. NTA Ans. (3)

Sol. Let thickness of ice be 'h'.

Vol. of ice = $v = \frac{4\pi}{3}\left((10+h)^3 - 10^3\right)$

$$\frac{dv}{dt} = \frac{4\pi}{3}\left(3(10+h)^2\right).\frac{dh}{dt}$$

Given $\frac{dv}{dt} = 50cm^3/min$ and h = 5cm

$\Rightarrow \quad 50 = \frac{4\pi}{3}\left(3(10+5)^2\right)\frac{dh}{dt}$

$\Rightarrow \quad \frac{dh}{dt} = \frac{50}{4\pi\times 15^2} = \frac{1}{18\pi}\ cm/min$

2. NTA Ans. (1)

Sol. _ _ _ $\underline{2}$ _

No. of five digits numbers =

No. of ways of filling remaining 4 places

= 8 × 8 × 7 × 6

$\therefore \quad k = \frac{8\times 8\times 7\times 6}{336} = 8$

3. NTA Ans. (3)

Sol. $\left|\frac{z-i}{z+2i}\right| = 1$

$\Rightarrow$ |z – i| = |z + 2i|

$\Rightarrow$ z lies on perpendicular bisector of (0, 1) and (0, –2).

$\Rightarrow \quad Imz = -\frac{1}{2}$

Let $z = x - \frac{i}{2}$

$\because \quad |z| = \frac{5}{2} \quad \Rightarrow \quad x^2 = 6$

$\therefore \quad |z+3i| = \left|x+\frac{5i}{2}\right| = \sqrt{x^2+\frac{25}{4}}$

$= \sqrt{6+\frac{25}{4}} = \frac{7}{2}$

4. NTA Ans. (1)

Sol. A : Event when card A is drawn

B : Event when card B is drawn.

$P(A) = P(B) = \frac{1}{2}$

Required probability = P(AA or (AB)A or (BA)A or (ABB)A or (BAB)A or (BBA)A)

$= \frac{1}{2}\times\frac{1}{2} + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times 2 + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times 3$

$= \frac{1}{4}+\frac{1}{4}+\frac{3}{16} = \frac{11}{16}$

5. NTA Ans. (4)

Sol. $I = \int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ ....(1)

$= \left[\int_0^{\pi}\frac{x\sin^8 x}{\sin^8 x\cos^8 x}dx + \int_0^{\pi}\frac{(2\pi - x)\sin^8 x}{\sin^8 x+\cos^8 x}dx\right]$

$= 2\pi\int_0^{\pi}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$

$I = 2\pi\left[\int_0^{\pi/2}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx + \int_0^{\pi/2}\frac{\cos^8 x dx}{\sin^8 x+\cos^8 x}dx\right]$

$= 2\pi\int_0^{\pi/2} 1dx = 2\pi.\frac{\pi}{2} = \pi^2$

6. NTA Ans. (3)

Sol. $f'(x) = \tan^{-1}(\sec x + \tan x)$

$$f'(x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)\right)$$

$\because \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \Rightarrow 0 < \frac{\pi}{4} + \frac{x}{2} < \frac{\pi}{2}$

$\Rightarrow \quad f'(x) = \frac{\pi}{4} + \frac{x}{2}$

$\therefore \quad f(x) = \frac{\pi}{4}.x + \frac{x^2}{4} + c$

$\because \quad f(0) = 0 \Rightarrow c = 0$

$\Rightarrow \quad f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$

$\therefore \quad f(1) = \frac{\pi+1}{4}$

7. NTA Ans. (3)

Sol. $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

$\Rightarrow \quad |A| = 6$

$$\frac{|\text{adj}B|}{|c|} = \frac{|\text{adj}(\text{adj}A)|}{|9A|} = \frac{|A|^4}{3^3|A|} = \frac{|A|^3}{3^3}$$

$$= \frac{(6)^3}{(3)^3} = 8$$

8. NTA Ans. (4)

Sol. $e^{4x} + e^{3x} - 4e^x + e^x + 1 = 0$

Divide by e^{2x}

$\Rightarrow \quad e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$

$\Rightarrow \quad \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

$\Rightarrow \quad \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

Let $e^x + \frac{1}{e^x} = t \Rightarrow (e^x - 1)^2 = 0 \Rightarrow x = 0.$

$\therefore$ Number of real roots = 1

9. NTA Ans. (2)

Sol. $p = \sqrt{5}$ is an integer.

q : 5 is irrational

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$= \sqrt{5}$ is not an integer and 5 is not irrational.

10. NTA Ans. (4)

Sol. $\sum_{i=1}^{10}(x_i - 5) = 10$

$\Rightarrow$ Mean of observation $x_i - 5 = \frac{1}{10}\sum_{i=1}^{3}(x_i - 5) = 1$

$\Rightarrow \quad \mu$ = mean of observation $(x_i - 3)$

= (mean of observation $(x_i - 5)$) + 2

= 1 + 2 = 3

Variance of observation

$x_i - 5 = \frac{1}{10}\sum_{i=1}^{10}(x_i - 5)^2 -$ (Mean of $(x_i - 5))^2 = 3$

$\Rightarrow \quad \lambda$ = variance of observation $(x_i - 3)$

= variance of observation $(x_i - 5) = 3$

$\therefore \quad (\mu, \lambda) = (3, 3)$

11. NTA Ans. (1)

Sol. $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \ldots.\infty$

$= 2^{\frac{1}{4}} \cdot 2^{\frac{2}{16}} \cdot 2^{\frac{3}{48}} \cdot 2^{\frac{4}{128}} \cdot \ldots.\infty$

$= 2^{\frac{1}{4}} \cdot 2^{\frac{1}{8}} \cdot 2^{\frac{1}{16}} \cdot 2^{\frac{1}{32}} \cdot \ldots.\infty$

$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots..\infty} = (2)^{\left(\frac{1/4}{1-1/2}\right)} = 2^{1/2}$

12. NTA Ans. (3)

Sol. Equation of family of circle touching y-axis at (0, 4) is given by $(x - 0)^2 + (y - 4)^2 + \lambda x = 0.$

$\because$ It passes through (2, 0)

$\Rightarrow \quad \lambda = -10.$

$\Rightarrow$ Required circle is $(x - 0)^2 + (y - 4)^2 - 10x = 0$

$\Rightarrow \quad x^2 + y^2 - 10x - 8y + 16 = 0$

center of circle $\equiv$ (5, 4) and radius = 5

distance of $4x + 3y - 8 = 0$ from (5, 4)

$= \left|\frac{24}{5}\right| \neq$ radius

13. NTA Ans. (4)

Sol. $e_1 = \sqrt{1 - \frac{4}{18}} = \frac{\sqrt{7}}{3}$

$e_2 = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3}$

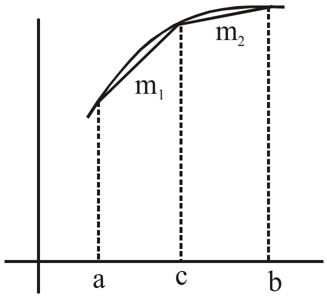
$\because$ (e_1, e_2) lies on $15x^2 + 3y^2 = k$

$\Rightarrow$ $15e_1^2 + 3e_2^2 = k$

$\Rightarrow$ $k = 16$

14. NTA Ans. (3)

Sol.



it is clear from graph that $m_1 > m_2$

$\Rightarrow$ $\frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c}$

$\Rightarrow$ $\frac{f(c) - f(a)}{f(b) - f(c)} > \frac{c - a}{b - c}$

15. NTA Ans. (1)

Sol. For planes to intersect on a line

$\Rightarrow$ there should be infinite solution of the given system of equations

for infinite solutions

$$\Delta = \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$$

$$\Delta_z = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 13 - \beta = 0 \Rightarrow \beta = 13$$

Also for $\alpha = -3$ and $b = 13$ $\Delta_x = \Delta_y = 0$

$\therefore$ $\alpha + \beta = -3 + 13 = 10$

16. NTA Ans. (1)

Sol. $I = \int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}} = \int \frac{dx}{\left(\frac{x+4}{x-3}\right)^{\frac{8}{7}}(x-3)^2}$

Let $\frac{x+4}{x-3} = t \Rightarrow \frac{dx}{(x-3)^2} = -\frac{1}{7}dt$

$\Rightarrow$ $I = -\frac{1}{7}\int \frac{dt}{t^{8/7}} = -\frac{1}{7}\int t^{-8/7}dt$

$= t^{-1/7} + C = +\left(\frac{x+4}{x-3}\right)^{-1/7} + C = \left(\frac{x-3}{x+4}\right)^{1/7} + C$

17. NTA Ans. (2)

Sol. Centroid of $\Delta = (2, 2)$

line passing through intersection of

$x + 3y - 1 = 0$ and

$3x - y + 1 = 0$, be given by

$(x + 3y - 1) + \lambda(3x - y + 1) = 0$

$\because$ It passes through (2, 2)

$\Rightarrow$ $7 + 5\lambda = 0 \Rightarrow \lambda = -\frac{7}{5}$

$\therefore$ Required line is $8x - 11y + 6 = 0$

$\because$ $(-9, -6)$ satisfies this equation.

18. NTA Ans. (4)

Sol. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0}\left(\frac{\sin(a+2)x}{x} + \frac{\sin x}{x}\right) = a + 3$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} \frac{(x + 3x^2)^{1/3} - x^{1/3}}{x^{4/3}}$

$= \lim_{x \to 0} \frac{(1 + 3x)^{1/3} - 1}{x} = 1$

$f(0) = b$

for continuity at $x = 0$

$\lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$

$\Rightarrow$ $a + 3 = b = 1$

$\therefore$ $a = -2$, $b = 1$

$\therefore$ $a + 2b = 0$

19. NTA Ans. (3)

Sol. $\cos^3\frac{\pi}{8}\cdot\sin\frac{\pi}{8}+\sin^3\frac{\pi}{8}\cdot\cos\frac{\pi}{8}$

$=\sin\frac{\pi}{8}.\cos\frac{\pi}{8}=\frac{1}{2}\sin\frac{\pi}{4}=\frac{1}{2\sqrt{2}}$

20. NTA Ans. (3)

Sol. $f(x) = a + bx + cx^2$

$\int_0^1 f(x)dx=\left[ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right]_0^1$

$=a+\frac{b}{2}+\frac{c}{3}=\frac{1}{6}[6a+3b+c]$

$=\frac{1}{6}\left[f(0)+f(1)+4f\left(\frac{1}{2}\right)\right]$

21. NTA Ans. (615.00)

Sol. $(1 + x + x^2)^{10}$

$= {}^{10}C_0 + {}^{10}C_1x(1 + x) + {}^{10}C_2x^2(1 + x)^2$

$+ {}^{10}C_3x^3(1 + x)^3 + {}^{10}C_4x^4(1 + x)^4 + ...$

Coeff. of $x^4 = {}^{10}C_2 + {}^{10}C_3 \times {}^3C_1 + {}^{10}C_4 = 615.$

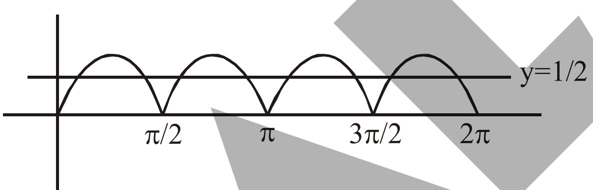
22. NTA Ans. (8.00)

Sol. $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|;\ x \in [0, 2\pi]$

$\Rightarrow \log_{1/2}|\sin x| + \log_{1/2}|\cos x| = 2$

$\Rightarrow \log_{1/2}(|\sin x\cos x|) = 2$

$\Rightarrow |\sin x\cos x| = \frac{1}{4} \Rightarrow |\sin 2x| = \frac{1}{2}$



$\Rightarrow$ 8 solutions

23. NTA Ans. (3.00)

Sol. $(x + 1)dy - ydx = ((x + 1)^2 - 3)dx$

$\Rightarrow \frac{(x+1)dy-ydx}{(x+1)^2}=\left(1-\frac{3}{(x+1)^2}\right)dx$

$\Rightarrow d\left(\frac{y}{(x+1)}\right)=\left(1-\frac{3}{(x+1)^2}\right)dx$

integrating both sides

$\frac{y}{x+1}=x+\frac{3}{(x+1)}+C$

Given $y(2) = 0 \quad \Rightarrow \quad c = -3$

$\therefore \quad y=(x+1)\left(x+\frac{3}{(x+1)}-3\right)$

$\therefore \quad y(3)=3.00$

24. NTA Ans. (1.00)

Sol. $\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$,

$\vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and

$\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$

$\because \quad \vec{p},\vec{q},\vec{r}$ are coplanar

$\Rightarrow \quad [\vec{p}\ \ \vec{q}\ \ \vec{r}]=0$

$\Rightarrow \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix}=0$

$\Rightarrow \quad 3a + 1 = 0 \qquad \Rightarrow \quad a=-\frac{1}{3}$

$\vec{p}.\vec{q}=-\frac{1}{3},\ \vec{r}.\vec{q}=-\frac{1}{3}$

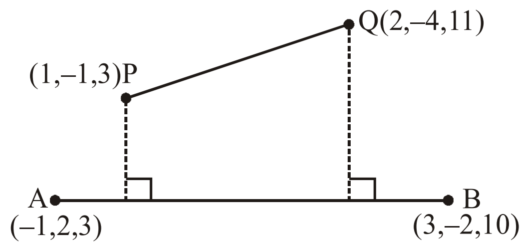
$|\vec{r}|^2=|\vec{q}|^2=\frac{2}{3}$

$\therefore \quad 3(\vec{p}.\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2=0$

$\Rightarrow \quad \lambda=\frac{3(\vec{p}.\vec{q})^2}{|\vec{r}\times\vec{q}|^2}=\frac{3(\vec{p}.\vec{q})^2}{|\vec{r}|^2|\vec{q}|^2-(\vec{r}.\vec{q})^2}=1.00$

25. NTA Ans. (8.00)

Sol.



Projection of $\overrightarrow{PQ}$ on $\overrightarrow{AB}=\left|\frac{\overrightarrow{PQ}\cdot\overrightarrow{AB}}{|\overrightarrow{AB}|}\right|$

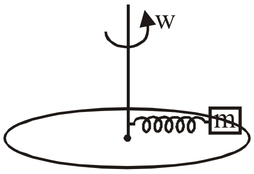
$=\left|\frac{(\hat{i}-3\hat{j}+8\hat{k})\cdot(4\hat{i}-4\hat{j}+7\hat{k})}{9}\right|=8$

SET # 06

PHYSICS

1. NTA Ans. (4)

Sol.



FBD of m in frame of disc/-

$k\Delta\ell \longleftarrow \boxed{m} \longrightarrow m\omega^2(\ell_0 + \Delta\ell)$

$k\Delta\ell = m\omega^2(\ell_0 + \Delta\ell)$

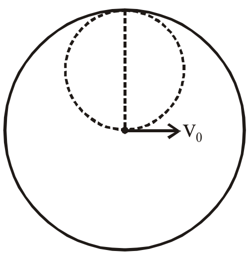
$$\Delta\ell = \frac{m\omega^2\ell_0}{k - m\omega^2} \approx \frac{m\omega\ell_0}{k}$$

$$\frac{\Delta\ell}{\ell_0} = \text{Relative change} = \frac{m\omega^2}{k}$$

$\therefore$ Correct answer (4)

2. NTA Ans. (2)

Sol. Top view of solenoid



Maximum possible radius of electron $= \dfrac{R}{2}$

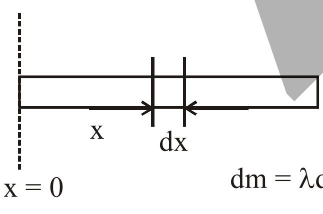
$$\therefore \frac{R}{2} = \frac{mv}{qB} = \frac{mv_{max}}{e(\mu_0 ni)}$$

$$v_{max} = \frac{R}{2}\frac{e\mu_0 ni}{m}$$

$\therefore$ Correct answer = 2

3. NTA Ans. (4)

Sol.



$dm = \lambda dx$

$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int(\lambda dx)x}{\int dm}$$

$$= \frac{\int_0^L \left(a + \frac{bx^2}{L^2}\right)x\,dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right)dx}$$

$$= \frac{\frac{aL^2}{2} + \frac{b}{L^2}\cdot\frac{L^4}{4}}{aL + \frac{b}{L^2}\cdot\frac{L^3}{3}}$$

$$= \frac{\left(\frac{4a+2b}{8}\right)L}{\frac{(3a+b)}{3}} = \frac{3}{4}\frac{(2a+b)L}{(3a+b)}$$

$\therefore$ correct answer 4

4. NTA Ans. (1)

Sol. Direction of polarisation $= \hat{E} = \hat{k}$

Direction of propagation $= \hat{E} \times \hat{B} = \dfrac{\hat{i} + \hat{j}}{\sqrt{2}}$

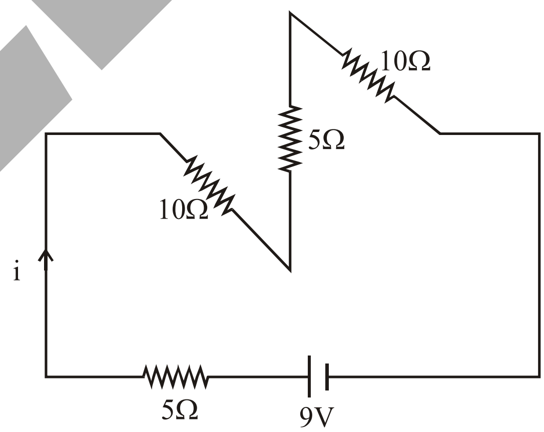
$$\therefore \hat{E} \times \hat{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

$$\hat{B} = \frac{\hat{i} - \hat{j}}{\sqrt{2}}$$

Correct answer (1)

5. NTA Ans. (3)

Sol.

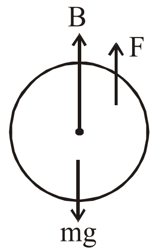


$$i = \frac{9}{(5 + 10 + 5 + 10)} = \frac{9}{30}\text{A}$$

$\therefore$ Correct answer (3)

6. NTA Ans. (2)

Sol. FBD of droplet



B + F = mg

$B = \left(\frac{2}{3}\pi R^3\right)\rho g$

F = T(2πR)

$m = d\left(\frac{4}{3}\pi R^3\right)$

$\left(\frac{2}{3}\pi R^3\right)\rho g + T(2\pi R) = d\left(\frac{4}{3}\pi R^3\right)g$

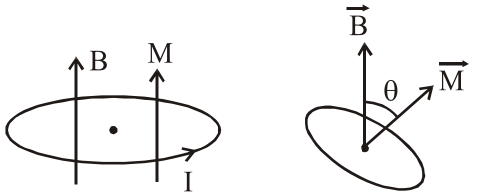
$T(2\pi R) = \left(\frac{2}{3}\pi R^3\right)g\left[2d - \rho\right]$

$R = \sqrt{\frac{3T}{(2d-\rho)g}}$

∴ Correct answer (2)

7. NTA Ans. (2)

Sol.



$\vec{T} = \vec{M} \times \vec{B} = -MB\sin\theta$

Iα = –MB sinθ

for small θ,

$\alpha = -\frac{MB}{I}\theta$

$\omega = \sqrt{\frac{MB}{I}} = \sqrt{\frac{(i)(\pi R^2)B}{\left(\frac{mR^2}{2}\right)}}$

$\omega = \sqrt{\frac{2i\pi B}{m}}$

$\therefore T = \frac{2\pi}{\omega} = \sqrt{\frac{2\pi m}{iB}}$

∴ Correct answer (2)

8. NTA Ans. (4)

Sol. $\frac{nv}{2\ell} = 420$

$\frac{(n+1)v}{2\ell} = 490$

$\frac{v}{2\ell} = 70$

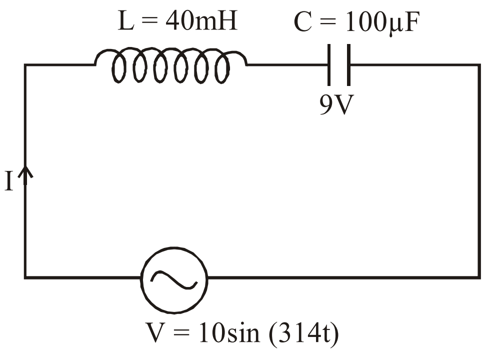
$\ell = \frac{v}{140} = \frac{1}{140}\sqrt{\frac{540}{6\times10^{-3}}} = \frac{1}{140}\sqrt{90\times10^3}$

$\ell = \frac{300}{140} = 2.142$

∴ Correct answer (4)

9. NTA Ans. (1)

Sol.

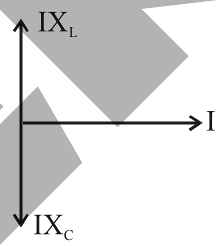


$X_L = \omega L = 314 \times 40 \times 10^{-3} = 12.56\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{314\times100\times10^{-6}}$

$= \frac{10^4}{314} = 31.84\Omega$

Phasor



$V_m = I_m(X_C - X_L)$

$10 = I_m(31.84 - 12.56)$

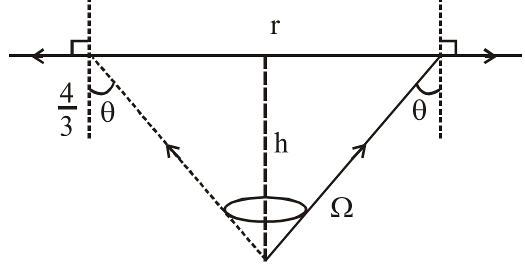
$I_m = \frac{10}{19.28} = 0.52A$

$I = 0.52 \sin\left(314t + \frac{\pi}{2}\right)$

∴ Correct answer (1)

10. NTA Ans. (1)

Sol.



$\frac{4}{3}\sin\theta = 1\sin 90°$

$\sin\theta = \frac{3}{4}$

Area of sphere in which light spread = $4\pi R^2$

Ω = 2π (1 – cos θ)

$$\Omega = 2\pi\left(1-\frac{\sqrt{7}}{4}\right)$$

P $\rightarrow$ 4π steradians

$$P' \rightarrow \frac{P}{4\pi}(1-\cos\theta)$$

$$\text{Ratio} = \frac{P'}{P} = \frac{2\pi(1-\cos\theta)}{4\pi} = \frac{(1-\cos\theta)}{2} = \frac{1-\frac{\sqrt{7}}{4}}{2}$$

$$= \frac{0.33}{2} = 0.17$$

$\therefore$ Correct answer (1)

11. NTA Ans. (1)
ALLEN Ans. (3)

Sol. $\lambda = \dfrac{1}{\sqrt{2}\pi n_v d^2}$

$$\tau = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\pi n_v d^2 v} = \frac{1}{\sqrt{2}\pi n_v d^2}\sqrt{\frac{M}{3RT}}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{M_1}{M_2}}\frac{d_2^2}{d_1^2}$$

$$= \sqrt{\frac{40}{140}}\frac{(0.1)^2}{(0.07)^2}$$

= 1.09

$\therefore$ Nearest possible answer (3)

12. NTA Ans. (3)

Sol. $x = u_x t + \frac{1}{2}a_x t^2$

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$32 = 0\times t + \frac{1}{2}(4)(t)^2$$

$t^2 = 16$

t = 4 sec

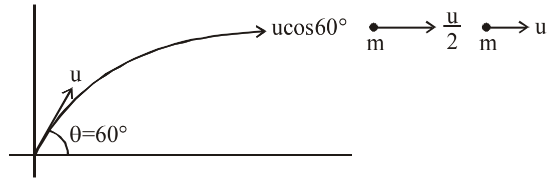
$$x = 3\times 4 + \frac{1}{2}\times 6\times 4^2$$

= 12 + 48 = 60 m

$\therefore$ Correct answer (3)

13. NTA Ans. (3)

Sol.



By momentum conservation,

$$\frac{mu}{2} + mu = 2mv'$$

$$v' = \frac{3v}{4}$$

$$\text{Range after collision} = \frac{3v}{4}\sqrt{\frac{2H}{g}}$$

$$= \frac{3v}{4}\sqrt{\frac{2\cdot u^2\sin^2 60^\circ}{g\,2g}}$$

$$= \frac{3}{4}\frac{\sqrt{3}}{2}\cdot\frac{u^2}{g} = \frac{3\sqrt{3}u^2}{8g}$$

$\therefore$ Correct answer (3)

14. NTA Ans. (4)

Sol. 1 Rydberg energy = 13.6 eV

So, ionisation energy = (13.6 Z^2)eV

= 9 × 13.6eV

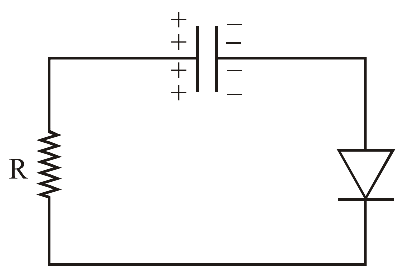
Z = 3

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2}-\frac{1}{3^2}\right) = 1.09\times 10^7\times 9\times\frac{8}{9}$$

λ = 11.4 nm

15. NTA Ans. (1)

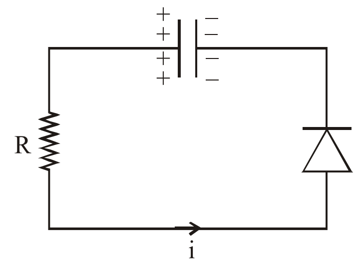
Sol. For (A)



No current flows

Hence Q_A = CV

For (B)



$$i = \frac{V}{R}e^{-\frac{t}{RC}} \quad ; \quad q = CVe^{-\frac{t}{RC}}$$

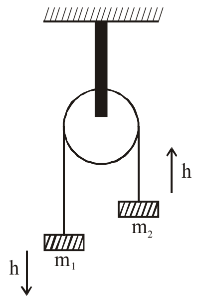
at t = CR

$$Q_B = CVe^{-1} = \frac{CV}{e}$$

$\therefore$ Correct answer (1)

16. NTA Ans. (2)

Sol.



by using work energy theorem

$W_g = \Delta KE$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)V^2 + \frac{1}{2}I\omega^2$$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)(\omega R)^2 + \frac{1}{2}I\omega^2$$

$$(m_1 - m_2)gh = \frac{\omega^2}{2}\left[(m_1 + m_2)R^2 + I\right]$$

$$\omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$$

$\therefore$ Correct answer (2)

17. NTA Ans. (1)

Sol. $V_e = \sqrt{\frac{2GM}{R}}$ (Escape velocity)

$$V_A = \sqrt{\frac{2GM}{R}}$$

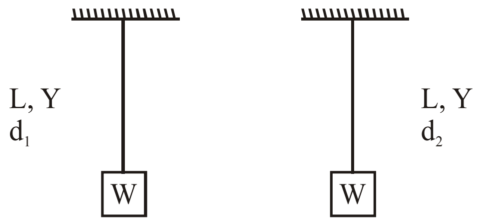
$$V_B = \sqrt{\frac{2G[M/2]}{R/2}} = \sqrt{\frac{2GM}{R}}$$

$$\frac{V_A}{V_B} = 1 = \frac{n}{4} \Rightarrow n = 4$$

$\therefore$ Correct answer (1)

18. NTA Ans. (4)

Sol.



$$\frac{\text{Energy stored}}{\text{Volume}} = \frac{1}{2}\frac{(\text{Stress})^2}{Y}$$

$$\frac{u_1}{u_2} = \frac{1}{4} \Rightarrow 4u_1 = u_2$$

$$4\frac{1}{2Y}\left[\frac{W \cdot 4}{\pi d_1^2}\right]^2 = \frac{1}{2Y}\left[\frac{W \cdot 4}{\pi d_2^2}\right]^2$$

$$4 = \left(\frac{d_1}{d_2}\right)^4$$

$$\Rightarrow \frac{d_1}{d_2} = \sqrt{2} : 1$$

$\therefore$ Correct answer (4)

19. NTA Ans. (Bonus)

Sol. $A_1 + B_1 + C_1 = 24.36 + 0.0724 + 256.2$

= 280.6324

= 280.6 (After rounding off)

$A_2 + B_2 + C_2 = 24.44 + 16.082 + 240.2$

= 280.722

= 280.7 (After rounding off)

$A_3 + B_3 + C_3 = 25.2 + 19.2812 + 236.183$

= 280.6642

= 280.7 (After rounding off)

$A_4 + B_4 + C_4 = 25 + 236.191 + 19.5$

= 280.691

= 281 (After rounding off)

$A_4 + B_4 + C_4 > A_3 + B_3 + C_3 = A_2 + B_2 + C_2 > A_1 + B_1 + C_1$

No option is matching Question should be (BONUS)

Best possible option is (2)

$\therefore$ Correct answer (2)

20. NTA Ans. (1)

Sol. $a=\dfrac{eE}{m}$

$v=u+at=\left(\dfrac{eE}{m}\right)t$

$\lambda=\dfrac{h}{mv}$

$\dfrac{d\lambda}{dt}=\dfrac{-(hm)\cdot\dfrac{dv}{dt}}{(mv)^2}=-\dfrac{ah}{mv^2}=-\dfrac{h}{|e|Et^2}$

$\therefore$ Correct answer (1)

21. NTA Ans. (1816.00 to 1820.00)

Sol. PV^{γ} = constant

$TV^{\gamma-1}=C$

$300\times V^{\frac{7}{5}-1}=T_2\left(\dfrac{V}{16}\right)^{\frac{7}{5}-1}$

$300\times 2^{4\times\frac{2}{5}}=T_2$

Isobaric process

$V=\dfrac{nRT}{P}$

$V_2=kT_2$ ... (1)

$2V_2=KT_f$ ... (2)

$\dfrac{1}{2}=\dfrac{T_2}{T_f}\Rightarrow T_f=2T_2$

$T_f=2\times300\times2^{\frac{8}{5}}$ = 1818.85

$\therefore$ Correct answer 1819

22. NTA Ans. (40.00)

Sol. In balancing

$\dfrac{R}{S}=\dfrac{25}{75}$

New resistance $R'=\dfrac{\rho\ell}{A}$

$=\dfrac{\rho\times\dfrac{\ell}{2}}{\dfrac{A}{4}}=\dfrac{\rho\ell}{2}\times4A$

$R'=2R$

$\dfrac{2R}{S}=\dfrac{\ell'}{100-\ell'}$

$2\times\dfrac{1}{3}=\dfrac{\ell'}{100-\ell'}=3\ell'=200-2\ell'$

$5\ell'=200$

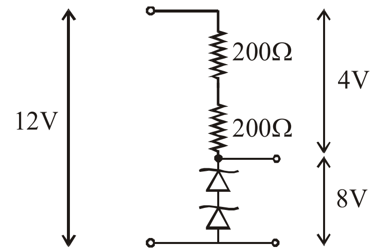
$\ell'=40$

$\therefore$ Correct answer 40

23. NTA Ans. (12.00)

ALLEN Ans. (40.00)

Sol.



Current in circuit = $\dfrac{4}{400}=\dfrac{1}{100}A$

So power dissipited in each diode = VI

$=4\times\dfrac{1}{100}W$

= 40×10^{-3} mW

$\therefore$ Correct answer 40

24. NTA Ans. (750.00)

Sol. The length of the screen used portion for 15 fringes, and also for ten fringes

$15\times500\times\dfrac{D}{\lambda}=10\times\dfrac{\lambda D}{\lambda}$

$15\times50=\lambda$

λ = 750 nm

$\therefore$ Correct answer 750

25. NTA Ans. (-48.00)

Sol. The flux passes through ABCD (x – y) plane is zero, because electric field parallel to surface. Flux of the electric field through surface BCGF (y - z)

At BCGF (electric field) $\Rightarrow\vec{E}=12\hat{i}-(y^2+1)\hat{j}$

(x = 3m)

Flux $\phi_{II}=12\times4=48\ Nm^2/C$

So $\phi_I-\phi_{II}=0-48=-48\ Nm^2/C$

$\therefore$ Correct answer –48

CHEMISTRY

1. NTA Ans. (2)

Sol. I $[Cr(H_2O)_6]^{2+}$

$Cr^{+2}\Rightarrow[Ar]\,3d^4$

$H_2O\rightarrow$ Weak field ligand

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\ t_{2g}$

Unpaired e^- = 4

Magnetic moment = $\sqrt{24}$ BM

= 4.89 BM

II $\left[Fe(CN)_6\right]^{4-}$

$Fe^{+2} \Rightarrow [Ar]\,3d^6$

CN^- →Strong field ligand

$\boxed{}\boxed{}\ e_g$

$\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\ t_{2g}$

Unpaired e^- = 0

Magnetic moment = 0 BM

= 0 BM

III $\left[Fe(C_2O_4)_3\right]^{3-}$

$Fe^{+3} \Rightarrow [Ar]\,3d^5$

As $\Delta_0 > P$

$\boxed{}\boxed{}\ e_g$

$\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\ t_{2g}$

Unpaired e^- = 1

Magnetic moment = $\sqrt{3}$ BM

= 1.73 BM

IV $(Et_4N)^+\left[CoCl_4\right]^{2-}$

$Co^{+2} \Rightarrow [Ar]\,3d^7$

$\left[CoCl_4\right]^{-2}$ = $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\ t_{2g}$

$\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\ e_g$

Unpaired electrons = 3

Magnetic moment = $\sqrt{15}$ BM

= 3.87 BM

Hence order of magnetic moment is

I > IV > III > II

2. NTA Ans. (1)

Sol. IE values indicate, that the metal belongs to I[st] group since second IE is very high

(∵ only one valence electron)

Metal hydroxide will be of type, MOH.

$MOH + HCl \rightarrow MCl + H_2O$

(1mol) (1mol)

$MOH + \frac{1}{2}H_2SO_4 \rightarrow \frac{1}{2}M_2SO_4 + H_2O$

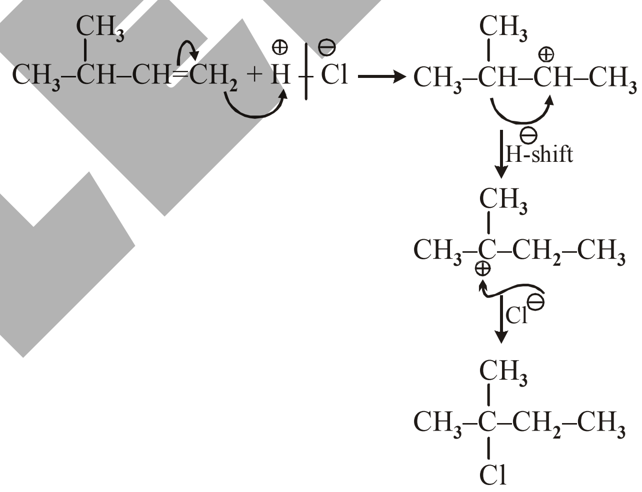
(1mol) ($\frac{1}{2}$ mol)

So one mole of HCl required to react with one mole MOH.

So $\frac{1}{2}$ mole of H_2SO_4 required to react with one mole MOH.

3. NTA Ans. (1)

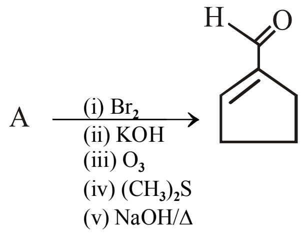
Sol.

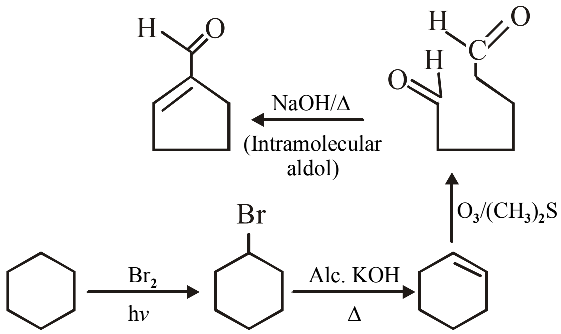


(No chiral centre, so no racemisation possible)

4. NTA Ans. (3)

Sol.





5. NTA Ans. (4)

Sol. Adsorption of Gases will decreases

6. NTA Ans. (4)

Sol. PHBV :

Poly β-hydroxy butyrate-co-β-hydroxy valerate

$$CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2-COOH$$

(3-hydroxy butanoic acid)

+

$$CH_3-CH_2-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2-\overset{\displaystyle O}{\overset{||}{C}}-OH$$

(3-hydroxy pentanoic acid)

7. NTA Ans. (3)

Sol. Biochemical oxygen demand (BOD) is amount of oxygen required by bacteria to break down organic waste in a certain volume of water sample.

8. NTA Ans. (3)

Sol. Lithium has highest hydration enthalpy among alkali metals due to its small size.

LiCl is soluble in pyridine because LiCl have more covalent character.

Li does not form ethynide with ethyne.

Both Li and Mg reacts slowly with H_2O

9. NTA Ans. (1)

Sol. Distilled water have lowest ionic conductance.

10. NTA Ans. (4)

Sol. $:\ddot{\text{Cl}}-CH=CH-N\begin{matrix}\nearrow O \\ \searrow O\end{matrix}$

Due to –M effect of $-NO_2$ and + M effect of Cl more D.B. character between C – Cl bond. So shortest bond length.

11. NTA Ans. (1)

ALLEN Ans. (1 or Bonus)

Sol. Bonus (no reaction is given)

$A \rightleftharpoons B$ (Assume reaction)

$$K = \frac{[B]}{[A]} = \frac{11}{6} \simeq 2$$

12. NTA Ans. (3)

13. NTA Ans. (1)

Sol. $Cr(OH)_3(s) \rightleftharpoons Cr^{3+}(aq.) + 3OH^-(aq.)$

(s) (3s)

$k_{sp} = 27(s)^4 = 6 \times 10^{-31}$

$\Rightarrow [3(s)]^4 = 18 \times 10^{-31}$

$[OH^-] = 3(s) = [18 \times 10^{-31}]^{1/4}$

14. NTA Ans. (4)

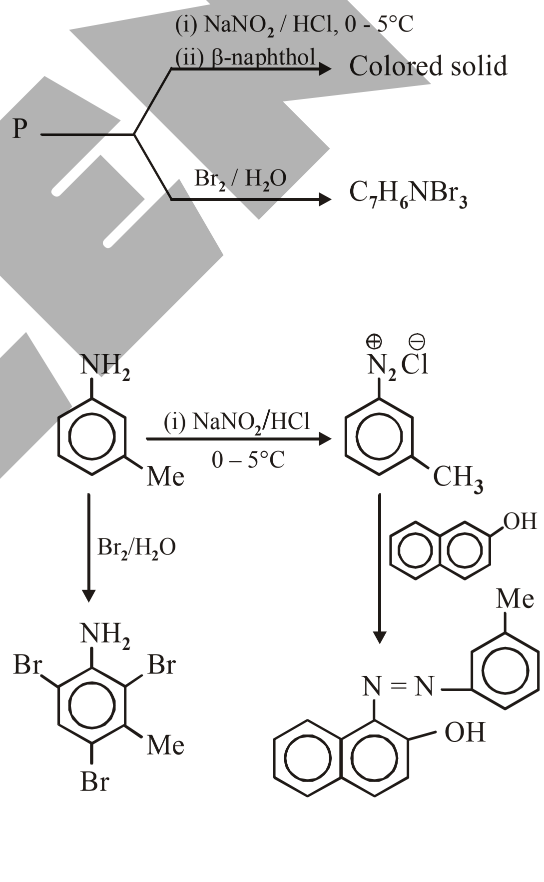
Sol. $Zn + 2HCl \longrightarrow ZnCl_2 + H_2$

$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2$

The ratio of the volume of H_2 is 1 : 1

15. NTA Ans. (2)

Sol.



(colored dye)

16. NTA Ans. (4)

Sol. Alanine does not show **Biuret test** because **Biuret test** is used for deduction of peptide linkage & alanine is amino acid.

Albumine is protein so have paptide linkage so it gives positive **Biuret test**.

Positive **Barfoed test** is shown by monosaccharide but not disaccharide. Positive **Molisch's test** is shown by glucose.

17. NTA Ans. (2)

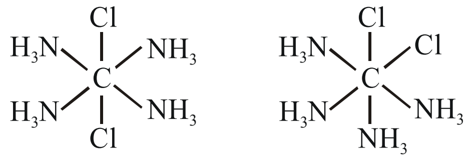
Sol.

$$\underset{(A)}{H_3N_3B_3Cl_3} + 3LiBH_4 \xrightarrow[\text{(T.H.F.)}]{\text{In tetrahydrofurane}} \underset{\substack{(B)\\ \text{Inorganic Benzene}\\ \text{(Borazene)}}}{H_3N_3B_3H_3} + 3LiCl + 3BH_3.THF$$

$$\underset{(A)}{H_3N_3B_3Cl_3} + \underset{(C)}{3CH_3MgBr} \xrightarrow{\rightarrow} H_3N_3B_3(CH_3)_3 + 3MgBrCl$$

18. NTA Ans. (4)

Sol. $[Co(NH_3)_4Cl_2]$ has 2 geometrical isomers

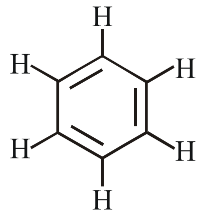


trans cis

cis isomer has Cl–Co–Cl angle of 90°

19. NTA Ans. (4)

Sol.



Each carbon atom is sp^2 hybridized

Therefore each carbon has 3 sp^2 hybrid orbitals.

Hence total sp^2 hybrid orbitals are 18.

20. NTA Ans. (1)

Sol. $ds = \int \frac{q_{rev.}}{T}$

21. NTA Ans. (2.17 to 2.23)

Sol. $0 - T_f' = 2 \times 0.5 = 1$

$T_f' = -1°C = 272$ K

for gas $P = \frac{0.1 \times 0.08 \times 272}{1}$

$P = 2.176$ atm

$P_1V_1 = P_2V_2$

$2.176 \times 1 = 1 \times V_2$

$V_2 = 2.176$ litre

22. NTA Ans. (10)

Sol. ppm $= \frac{10.3 \times 10^{-3}}{1030} \times 10^6 = 10$

23. NTA Ans. (3.98 to 4.00 or –3.98 to –4.00)

Sol. $\ln\left(\frac{t_1}{t_2}\right) = \frac{-Ea}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$

$\ln\left(\frac{60}{40}\right) = \frac{-Ea}{8.3}\left[\frac{1}{400} - \frac{1}{300}\right]$

$E = 0.4 \times 1200 \times 8.3$

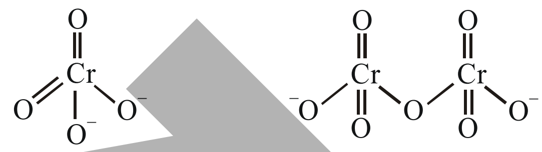
$= 3.984$ kJ/mole

24. NTA Ans. (12.00)

ALLEN Ans. (18.00)

Sol.

Chromate CrO_4^{2-} Dichromate $Cr_2O_7^{2-}$



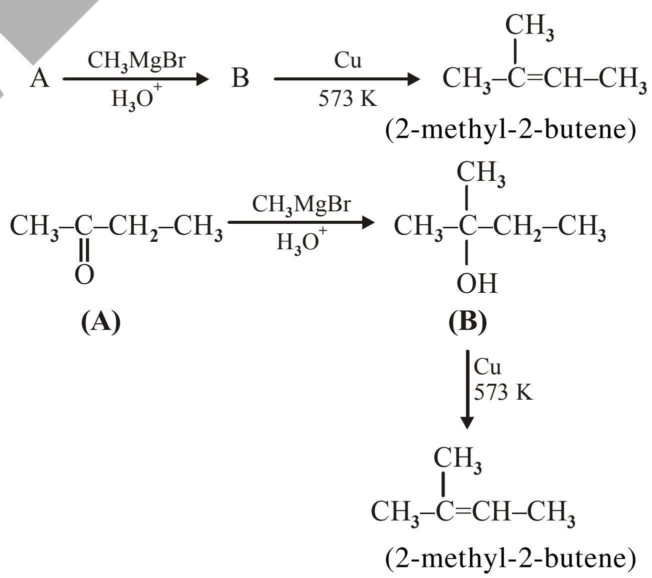
Total Cr-O bonds = 6 ($4\sigma + 2\pi$) Total Cr-O bonds = 12 ($8\sigma + 4\pi$)

Total number of bonds between chromium and oxygen in both structures are 18.

Note :- But answer of NTA is 12. They consider only linkages between Chromium and Oxygen but in question total no. of bonds are asked so σ and π bonds must be considered separately.

25. NTA Ans. (66.65 to 66.70)

Sol.



$C \Rightarrow 12 \times 4 = 48$

$H \Rightarrow 8 \times 1 = 8$

$O \Rightarrow 16 \times 1 = 16$

Total 72

% of C $= \frac{48}{72} \times 100 = 66.66\%$

MATHEMATICS

1. NTA Ans. (2)

Sol. $A = \lim_{x\to 0} x\left[\frac{4}{x}\right] = \lim_{x\to 0} x\left(\frac{4}{x}\right) - x\left\{\frac{4}{x}\right\} = 4$

$f(x) = [x^2]\sin(\pi x)$ will be discontinuous at nonintegers

$\therefore\ x = \sqrt{A+1}$ i.e. $\sqrt{5}$

2. NTA Ans. (1)

Sol. $7x + 6y - 2z = 0$ .... (1)

$3x + 4y + 2z = 0$ .... (2)

$x - 2y - 6z = 0$ .... (3)

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0 \Rightarrow \text{infinite solutions}$$

Now (1) + (2) $\Rightarrow y = -x$ put in (1), (2) & (3) all will lead to $x = 2z$

3. NTA Ans. (Bonus)

Sol. $x = 2\sin\theta - \sin 2\theta$

$\Rightarrow \frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta = 4\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)$

$y = 2\cos\theta - \cos 2\theta$

$\Rightarrow \frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta = 4\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$

$$\Rightarrow \frac{dy}{dx} = \cot\left(\frac{3\theta}{2}\right) \Rightarrow \frac{d^2y}{dx^2} = \frac{-\frac{3}{2}\operatorname{cosec}^2\left(\frac{3\theta}{2}\right)}{4\sin\left(\frac{\theta}{2}\right)\sin\frac{3\theta}{2}}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{\theta=\pi} = \frac{3}{8}$$

Alternate :-

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2\sin\theta + 2\sin 2\theta}{2\cos\theta - 2\cos 2\theta} = \frac{\sin\theta - \sin 2\theta}{-\cos\theta + \cos 2\theta}$$

$$\frac{d^2y}{dx^2}\cdot\frac{dx}{d\theta} = \frac{(-\cos\theta + \cos 2\theta)(\cos\theta - 2\cos 2\theta) - (\sin\theta - \sin 2\theta)(\sin\theta - 2\sin 2\theta)}{(-\cos\theta + \cos 2\theta)^2}$$

$$\frac{d^2y}{dx^2}.(-2-2) = \frac{(+1+1)(-1-2)-(0)}{(1+1)^2}$$

$$\frac{d^2y}{dx^2}(-4) = \frac{2\times -3}{4} = -\frac{3}{2}$$

$$\frac{d^2y}{dx^2} = \frac{3}{8}$$

Answer should be $\frac{3}{8}$. No options is correct.

4. NTA Ans. (2)

Sol. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $a > b$;

$2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}} \Rightarrow b^2 = \frac{4}{3}$

tangent $y = \frac{-x}{6} + \frac{4}{3}$ compare with

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$\Rightarrow m = \frac{-1}{6} \Rightarrow \sqrt{\frac{a^2}{36} + \frac{4}{3}} = \frac{4}{3} \Rightarrow a = 4;$

$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2}\sqrt{\frac{11}{3}}$

5. NTA Ans. (2)

Sol. $ax^2 - 2bx + 5 = 0 \begin{cases} \alpha \\ \alpha \end{cases}$

$\Rightarrow \alpha = \frac{b}{a};\ \alpha^2 = \frac{5}{a} \Rightarrow b^2 = 5a$

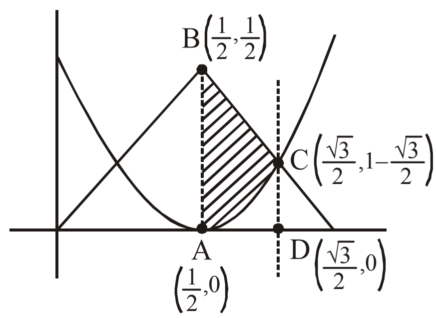
$x^2 - 2bx - 10 = 0 \begin{cases} \alpha \\ \beta \end{cases} \Rightarrow \alpha^2 - 2b\alpha - 10 = 0$

$\Rightarrow a = \frac{1}{4} \Rightarrow \alpha^2 = 20;\ \alpha\beta = -10 \Rightarrow \beta^2 = 5$

$\Rightarrow \alpha^2 + \beta^2 = 25$

6. NTA Ans. (2)

Sol.



Required area = Area of trepezium ABCD –

Area of parabola between $x = \frac{1}{2}$ & $x = \frac{\sqrt{3}}{2}$

$$A = \frac{1}{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right) - \int_{1/2}^{\sqrt{3}/2}\left(x-\frac{1}{2}\right)^2 dx = \frac{\sqrt{3}}{4}-\frac{1}{3}$$

7. NTA Ans. (2)

Sol. $\sum P(X) = 1 \Rightarrow K^2 + 2K + K + 2K + 5K^2 = 1$

$\Rightarrow 6K^2 + 5K - 1 = 0 \Rightarrow (6K - 1)(K + 1) = 0$

$\Rightarrow K = -1$ (rejected) $\Rightarrow K = \frac{1}{6}$

$P(X > 2) = K + 2K + 5K^2 = \frac{23}{36}$

8. NTA Ans. (3)

Sol. $x = \sum_{n=0}^{\infty}(-1)^n \tan^{2n}\theta = 1 - \tan^2\theta + \tan^4\theta + \ldots$

$\Rightarrow x = \cos^2\theta$

$y = \sum_{n=0}^{\infty}\cos^{2n}\theta \Rightarrow y = 1 + \cos^2\theta + \cos^4\theta + \ldots$

$\Rightarrow y = \frac{1}{\sin^2\theta} \Rightarrow y = \frac{1}{1-x}$

$\Rightarrow y(1 - x) = 1$

9. NTA Ans. (1)

Sol. $F'(x) = x^2 g(x) = x^2\int_1^x f(u)\,du \Rightarrow F'(1) = 0$

$F''(x) = x^2 f(x) - 2x\int_1^x f(u)\,du$

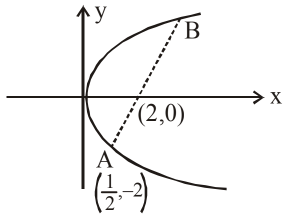
$F''(1) = 1.f(1) - 2 \times 0$

$F''(1) = 3$

$F'(1) = 0$ and $F''(1) = 3 > 0$ So, Minima

10. NTA Ans. (2)

Sol. $y^2 = 8x$



$4t_1 = -2 \Rightarrow t_1 = -\frac{1}{2}$,

$t_1.t_2 = -1$

$t_2 = -\frac{1}{t_1}$

$\Rightarrow t_2 = 2$

So coordinate of B is (8, 8)

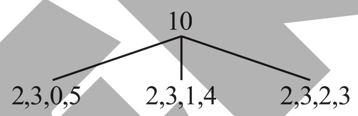
$\therefore$ Equation of tangent at B is

$8y = 4(x + 8) \Rightarrow 2y = x + 8$

11. NTA Ans. (3)

ALLEN Ans. (Bonus)

Sol. 10 different balls in 4 different boxes.



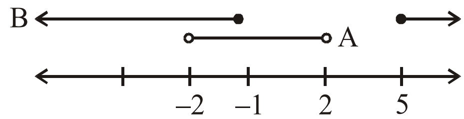
$$\frac{1}{4^{10}}\left(4!\times\frac{10!}{2!\times3!\times0!\times5!}+4!\times\frac{10!}{2!\times3!\times1!\times4!}+4!\times\frac{10!}{(2!)^2\times2!\times(3!)^2\times2!}\right)$$

$$= \frac{17\times945}{2^{15}}$$

12. NTA Ans. (3)

Sol. $A : x \in (-2, 2)$; $B : x \in (-\infty, -1] \cup [5, \infty)$

$\Rightarrow B - A = R - (-2, 5)$



13. NTA Ans. (4)

Sol. $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

Let $y = vx$

$\frac{dy}{dx} = v + x.\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{xvx}{x^2+v^2x^2} = \frac{v}{1+v^2}$

$x\frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{v-v-v^3}{1+v^2} = -\frac{v^3}{1+v^2}$

$$\int \frac{1+v^2}{v^3}.dv = \int -\frac{dx}{x}$$

$$\Rightarrow \int v^{-3}.dv + \int \frac{1}{v}\,dv = -\int \frac{dx}{x}$$

$$\Rightarrow \frac{v^{-2}}{-2} + \ell n\, v = -\ell n\, x + \lambda$$

$$\Rightarrow -\frac{1}{2v^2} + \ell n\left(\frac{y}{x}\right) = -\ell n\, x + \lambda$$

$$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + \ell n\, y - \ell n\, x = -\ell n\, x + \lambda$$

$$\Rightarrow -\frac{1}{2} + 0 = \lambda \Rightarrow \lambda = -\frac{1}{2}$$

$$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + \ell n\, y + \frac{1}{2} = 0 \text{ at } y = e$$

$$\Rightarrow -\frac{1}{2}\frac{x^2}{e^2} + 1 + \frac{1}{2} = 0 \Rightarrow \frac{x^2}{2e^2} = \frac{3}{2} \Rightarrow x^2 = 3e^2$$

$$\therefore x = \sqrt{3}e$$

14. NTA Ans. (1)

Sol. $$I = \int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)}$$

$$= \int \frac{\sec^2\theta\, d\theta}{\frac{2\tan\theta}{1-\tan^2\theta} + \frac{1+\tan^2\theta}{1-\tan^2\theta}} = \int \frac{(1-\tan^2\theta)\sec^2\theta\, d\theta}{(1+\tan\theta)^2}$$

$\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$

$$I = \int \frac{1-t^2}{(1+t)^2}dt = \int \frac{(1-t)(1+t)}{(1+t)^2}dt$$

$$= \int \frac{1}{1+t} - \frac{t}{1+t}\,dt$$

$$= \ell n|1+t| - \int\left(\frac{1+t}{1+t} - \frac{1}{1+t}\right)dt$$

$= \ell n|1 + t| - t + \ell n|1 + t|$

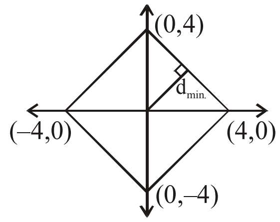
$= 2\ell n|1 + t| - t + C$

$= 2\ell n|1 + \tan\theta| - \tan\theta + C$

$\lambda = -1,\ f(\theta) = 1 + \tan\theta$

15. NTA Ans. (4)

Sol.



$z = x + iy$ $\qquad |x| + |y| = 4$

$|z| = \sqrt{x^2+y^2} \Rightarrow |z|_{min} = \sqrt{8}\ \&\ |z|_{max} = 4 = \sqrt{16}$

So $|z|$ cannot be $\sqrt{7}$

16. NTA Ans. (2)

Sol. $p \to (p \wedge \sim q)$ is F $\Rightarrow$ p is T & $p \wedge \sim q$ is F $\Rightarrow$ q is T

$\therefore$ p is T, q is T

17. NTA Ans. (3)

Sol. $R_1 \to R_1 + R_3 - 2R_2$

$$f(x) = \begin{vmatrix} a+c-2b & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$= (a + c - 2b)\ ((x + 3)^2 - (x + 2)(x + 4))$

$= x^2 + 6x + 9 - x^2 - 6x - 8 = 1$

$\Rightarrow f(x) = 1 \quad \Rightarrow f(50) = 1$

18. NTA Ans. (4)

Sol. $$T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^r$$

$$= {}^{16}C_r\,(x)^{16-2r} \times \frac{1}{(\cos\theta)^{16-r}(\sin\theta)^r}$$

For independent of x; $16 - 2r = 0 \Rightarrow r = 8$

$$\Rightarrow T_9 = {}^{16}C_8\frac{1}{\cos^8\theta\sin^8\theta}$$

$$= {}^{16}C_8\frac{2^8}{(\sin 2\theta)^8}$$

for $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ ℓ_1 is least for $\theta_1 = \frac{\pi}{4}$

for $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$ ℓ_2 is least for $\theta_2 = \frac{\pi}{8}$

$$\frac{\ell_2}{\ell_1} = \frac{(\sin 2\theta_1)^8}{(\sin 2\theta_2)^8} = (\sqrt{2})^8 = \frac{16}{1}$$

19. NTA Ans. (4)

Sol. $\sum_{n=1}^{100} a_{2n+1} = 200 \Rightarrow a_3 + a_5 + a_7 + ... + a_{201} = 200$

$$\Rightarrow ar^2 \frac{(r^{200}-1)}{(r^2-1)} = 200$$

$\sum_{n=1}^{100} a_{2n} = 100 \Rightarrow a_2 + a_4 + a_6 + ... + a_{200} = 100$

$$\Rightarrow \frac{ar(r^{200}-1)}{(r^2-1)} = 100$$

On dividing r = 2

on adding $a_2 + a_3 + a_4 + a_5 + ... + a_{200} + a_{201} = 300$

$\Rightarrow r(a_1 + a_2 + a_3 + ... + a_{200}) = 300$

$$\Rightarrow \sum_{n=1}^{200} a_n = 150$$

20. NTA Ans. (3)

Sol. $f(g(x)) = x$

$f'(g(x))\ g'(x) = 1$

put x = a

$\Rightarrow f'(b)\ g'(a) = 1$

$$f'(b) = \frac{1}{5}$$

21. NTA Ans. (14)

Sol. Common term are : 23, 51, 79, ... T_n

$T_n \le 407 \quad \Rightarrow 23 + (n-1)28 \le 407$

$\Rightarrow n \le 14.71$

n = 14

22. NTA Ans. (30)

Sol. $\vec{b}.\vec{c} = 10 \Rightarrow 5|\vec{c}|\cos\frac{\pi}{3} = 10 \Rightarrow |\vec{c}| = 4$

$$|\vec{a}\times(\vec{b}\times\vec{c})| = |\vec{a}|\,|\vec{b}\times\vec{c}|$$

$$= \sqrt{3}\,.\,5\,.\,4\,.\sin\frac{\pi}{4} = 30$$

23. NTA Ans. (3)

Sol. If $\lambda = -7$, then planes will be parallel & distance between them will be $\frac{3}{\sqrt{633}} \Rightarrow k = 3$

But if $\lambda \neq -7$, then planes will be intersecting & distance between them will be 0

24. NTA Ans. (51)

Sol. $S = 1.^{25}C_0 + 5.^{25}C_1 + 9.^{25}C_2 + ... + (101)^{25}C_{25}$

$S = 101^{25}C_{25} + 97^{25}C_1 + ... + 1^{25}C_{25}$

$2S = (102)(2^{25})$

$S = 51(2^{25})$

25. NTA Ans. (36)

Sol. Common tangent is $S_1 - S_2 = 0$

$\Rightarrow -6x + 8y - 8 + k = 0$

Use p = r for I[st] circle

$$\Rightarrow \frac{|-18-8+k|}{10} = 1$$

$\Rightarrow k = 36$ or $16 \quad \Rightarrow k_{max} = 36$

SET # 01

PHYSICS

1. NTA Ans. (1)

Sol. $f = \frac{-8}{2} = -4\text{cm}$

u = –10 cm
v = ?

as $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} + \left(\frac{1}{-10}\right) = \frac{1}{-4}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{4}$

$\frac{1}{v} = \frac{4-10}{40}$

$v = \frac{40}{-6}$

$v = \frac{-20}{3}$

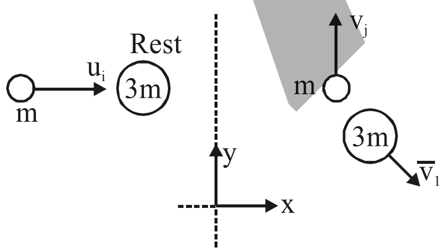
$m = \frac{-v}{u}$

$m = \frac{-\left(-\frac{20}{3}\right)}{-10} \Rightarrow m = \frac{-2}{3}$

or image will be real, inverted and unmagnified.

2. NTA Ans. (4)

Sol. Before collision After collision



From momentum conservation

$\vec{P}_i = \vec{P}_f$

m(ui) + 3m(0) = mvj + 3m$\overline{v}_1$

mui – mvj = 3m$\overline{v}_1$

$\overline{v}_1 = \frac{ui - vj}{3}$

or $|v_1| = \frac{\sqrt{u^2+v^2}}{3}$

or $v_1^2 = \frac{u^2+v^2}{9}$ ....(1)

As collision is perfectely elastic hence

$k_i = k_j$

$\frac{1}{2}mu^2 + \frac{1}{2}3m0^2 = \frac{1}{2}mv^2 + \frac{1}{2}3mv_1^2$

$\Rightarrow u^2 = v^2 + 3v_1^2$

$u^2 = v^2 + 3\frac{(u^2+v^2)}{9}$

$\Rightarrow 3u^2 = 3v^2 + u^2 + v^2$

$\Rightarrow 2u^2 = 4v^2$

$v = \frac{u}{\sqrt{2}}$

3. NTA Ans. (2)

Sol. Pitch = $\frac{2\pi m}{qB} v\cos\theta$

Pitch = $\frac{2(3.14)(1.67\times10^{-27})\times4\times10^5\times\cos 60}{(1.69\times10^{-19})(0.3)}$

Pitch = 0.04m = 4 cm

4. NTA Ans. (4)

Sol. $\rho_M > \rho_A > \rho_C$

5. NTA Ans. (4)

Sol. As for permanent magnet large retentivity and large coercivity required

6. NTA Ans. (4)

Sol. Least count = 1 mm or 0.01 cm

Zero error = 0 + 0.01 × 7 = 0.07 cm

Reading = 3.1 + (0.01 × 4) – 0.07

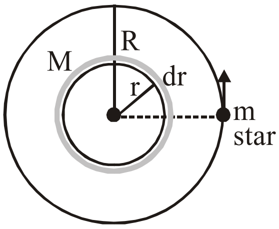
= 3.1 + 0.04 – 0.07

= 3.1 – 0.03

= 3.07 cm

7. NTA Ans. (3)

Sol.



$dm = \rho dv$

$$dm = \left(\frac{k}{r}\right)(4\pi r^2 dr)$$

$dm = 4\pi krdr$

$$M = \int_0^R dm = \int_0^R 4\pi krdr$$

$$M = 4\pi k \left.\frac{r^2}{2}\right|_0^R$$

$M = 2\pi k(R^2 - 0)$

$M = 2\pi kR^2$

for circular motion gravitational force will provide required centripital force or

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\frac{G(2\pi kR^2)m}{R^2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{2\pi GkR}$$

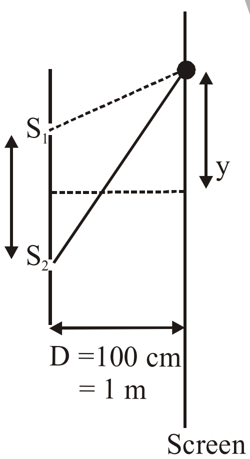
Time period $T = \frac{2\pi R}{v}$

$$T = \frac{2\pi R}{\sqrt{2\pi GkR}} \propto \sqrt{R}$$

or $T^2 \propto R$

8. NTA Ans. (1)

Sol.



$$y = \frac{nD\lambda}{d}$$

$$n = \frac{yd}{D\lambda} = \frac{1.27\times10^{-3}\times10^{-3}}{1\times632.8\times10^{-9}} = 2$$

Path difference $\Delta x = n\lambda$

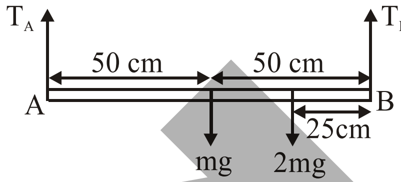
$= 2 \times 632.8$ nm

$= 1265.6$ nm

$= 1.27\ \mu$m

9. NTA Ans. (4)

Sol.



$\tau_B = 0$ (torque about point B is zero)

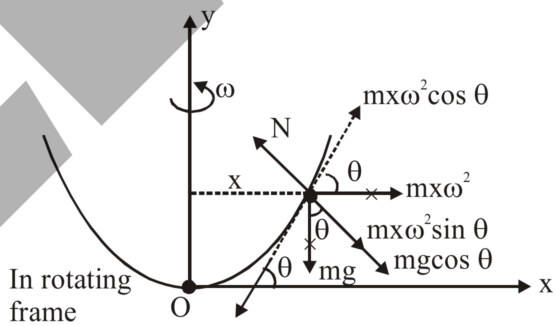
$(T_A) \times 100 - (mg) \times 50 - (2mg) \times 25 = 0$

$100\ T_A = 100$ mg

$T_A = 1$ mg

10. NTA Ans. (2)

Sol.



$mx\omega^2\cos\theta = mg\sin\theta$

$x\omega^2 = g\tan\theta$

$$x\omega^2 = g.\frac{dy}{dx}$$

$x\omega^2 = g.(8cx)$

$\omega^2 = 8\ gc$

$$\omega = 2\sqrt{2gc}$$

11. NTA Ans. (2)

Sol. Energy density $\frac{dU}{dV} = \frac{B_0^2}{2\mu_0}$

$$1.02\times10^{-8} = \frac{B_0^2}{2\times4\pi\times10^{-7}}$$

$$B_0^2 = (1.02\times10^{-8})\times(8\pi\times10^{-7})$$

$B_0 = 16 \times 10^{-8}$ T $= 160$ nT

12. NTA Ans. (2)

Sol. Number of uranium atoms in 2kg

$$=\frac{2\times 6.023\times 10^{26}}{235}$$

energy from one atom is 200 × 10^6 e.v. hence total energy from 2 kg uranium

$$=\frac{2\times 6.023\times 10^{26}}{235}\times 200\times 10^{6}\times 1.6\times 10^{-19}\,J$$

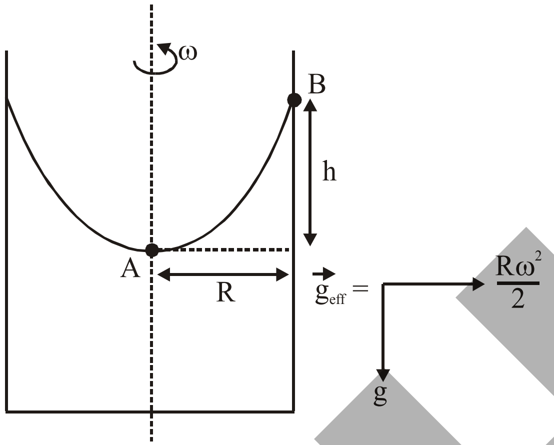
2 kg uranium is used in 30 days hence this energy is recieved in 30 days hence energy recived per second or power is

$$\text{Power}=\frac{2\times 6.023\times 10^{26}\times 200\times 10^{6}\times 1.6\times 10^{-19}}{235\times 30\times 24\times 3600}$$

Power = 63.2 × 10^6 watt or 63.2 Mega Watt

13. NTA Ans. (1)

Sol.



Applying pressure equation from A to B

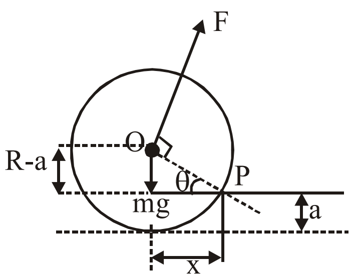
$$P_0+\rho.\frac{R\omega^2}{2}.R-\rho gh=P_0$$

$$\frac{\rho R^2\omega^2}{2}=\rho gh$$

$$h=\frac{R^2\omega^2}{2g}=(5)^2\frac{\omega^2}{2g}=\frac{25}{2}\frac{\omega^2}{g}$$

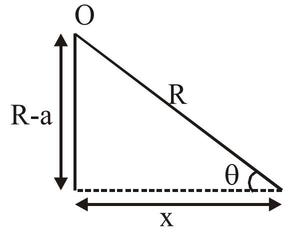
14. NTA Ans. (4)

Sol.



$(\tau)_P = 0$

F.R. – mgx = 0



$$x=\sqrt{R^2-(R-a)^2}$$

$$F=mg\frac{x}{R}$$

$$F=mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$$

= minimum value of force to pull

15. NTA Ans. (2)

Sol. $u=\frac{f_1n_1RT}{2}+\frac{f_2n_2RT}{2}$

$$u=\frac{5}{2}\times 3RT+\frac{3\times 5RT}{2}=15RT$$

16. NTA Ans. (1)

Sol. $Y = F^xA^yV^z$

$M^1L^{-1}T^{-2} = [MLT^{-2}]^x[L^2]^y[LT^{-1}]^z$

$M^1L^1T^{-2} = [M]^x[L]^{x+2y+z}[T]^{-2x-z}$

comparing power of ML and T

x = 1...(1)

x + 2y + z = –1 ....(2)

–2x – z = –2 ...(3)

after solving

x = 1

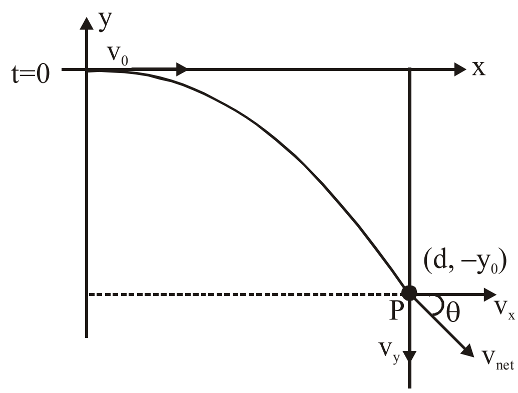
y = –1

z = 0

$Y = FA^{-1}V^0$

17. NTA Ans. (1)

Sol.



Let particle have charge q and mass 'm'
Solve for (q,m) mathematically
$F_x = 0$, $a_x = 0$, $(v)_x$ = constant

time taken to reach at 'P' = $\dfrac{d}{v_0} = t_0$ (let) ...(1)

(Along –y), $y_0 = 0 + \dfrac{1}{2}.\dfrac{qE}{m}.t_0^2$ ....(2)

$v_x = v_0$
$v = u + at$ (along -ve 'y')

speed $v_{y0} = \dfrac{qE}{m}.t_0$

$$\tan\theta = \frac{v_y}{v_x} = \frac{qEt_0}{m.v_0}, (t_0 = \frac{d}{v_0})$$

$$\tan\theta = \frac{qEd}{m.v_0^2}$$

$$\boxed{\text{slope} = \frac{-qEd}{mv_0^2}}$$

Now we have to find eq^n of straight line whose slope is $\dfrac{-qEd}{mv_0^2}$ and it pass through point $\rightarrow$ (d, $-y_0$)
Because after x > d
No electric field $\Rightarrow F_{net} = 0$, $\vec{v}$ = const.

$$y = mx + c, \left\{ \begin{matrix} m = \dfrac{qEd}{mv_0^2} \\ (d, -y_0) \end{matrix} \right\}$$

$$-y_0 = \frac{-qEd}{mv_0^2}.d + c \Rightarrow c = -y_0 + \frac{qEd^2}{mv_0^2}$$

Put the value

$$y = \frac{-qEd}{mv_0^2}x - y_0 + \frac{qEd^2}{mv_0^2}$$

$$y_0 = \frac{1}{2}.\frac{qE}{m}\left(\frac{d}{v_0}\right)^2 = \frac{1}{2}\frac{qEd^2}{mv_0^2}$$

$$y = \frac{-qEdx}{mv_0^2} - \frac{1}{2}\frac{qEd^2}{mv_0^2} + \frac{qEd^2}{mv_0^2}$$

$$y = \frac{-qEd}{mv_0^2}x + \frac{1}{2}\frac{qEd^2}{mv_0^2}$$

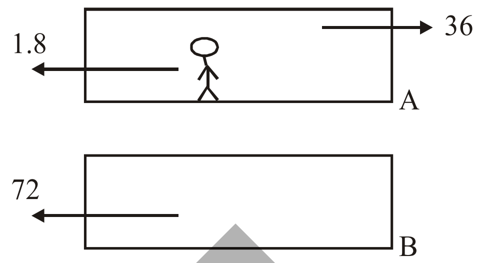
$$\boxed{y = \frac{qEd}{mv_0^2}\left(\frac{d}{2} - x\right)}$$

18. NTA Ans. (1,3)

Sol. $V_m = 5(1+0.6\cos 6280t)\sin(211 \times 10^4 t)$
$V_m = [5+3\cos 6820t]\sin(211 \times 10^4 t)$
$V_{max.} = 5 + 3 = 8$
$V_{min.} = 5 - 3 = 2$

19. NTA Ans. (2)

Sol.



Velocity of man with respect to ground

$\vec{V}_{m/g} = \vec{V}_{m/A} + \vec{V}_A = -1.8 + 36$

Velocity of man w.r.t. B

$\vec{V}_{m/B} = \vec{V}_m - \vec{V}_B$

= –1.8 + 36 – (–72) = 106.2 km/hr
= 29.5 m/s

20. NTA Ans. (3)

Sol. $f = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}$

For identical string l and μ will be same

$f \propto \sqrt{T}$

$$\frac{450}{300} = \sqrt{\frac{T_x}{T_y}}$$

$$\frac{T_x}{T_y} = \frac{9}{4} = 2.25$$

21. NTA Ans. (36.00)

Sol. $u_i = \dfrac{1}{2} \times 5 \times 10^{-6}(220)^2$

Final common potential

$$v = \frac{220 \times 5 + 0 \times 2.5}{5 + 2.5} = 220 \times \frac{2}{3}$$

$$u_f = \frac{1}{2}(5 + 2.5) \times 10^{-6}\left(220 \times \frac{2}{3}\right)^2$$

$\Delta u = u_f - u_i$
$\Delta u = -403.33 \times 10^{-4}$

$\Rightarrow -403.33 \times 10^{-4} = \dfrac{X}{100}$

X = –4.03
or magnitude or value of X is approximate 4

22. NTA Ans. (46.00)

Sol. Diatomic :

f = 5

$\gamma = 7/5$

$T_i = T = 273 + 20 = 293$ K

$V_i = V$

$V_f = V/10$

Adiabatic $\quad TV^{\gamma-1}$ = constant

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T.V^{7/5-1} = T_2\left(\frac{V}{10}\right)^{7/5-1}$$

$\Rightarrow T_2 = T.\ 10^{2/5}$

$$\Delta U = \frac{nfR(T_2 - T_1)}{2} = \frac{5\times5\times\frac{25}{3}\times(T.10^{2/5} - T)}{2}$$

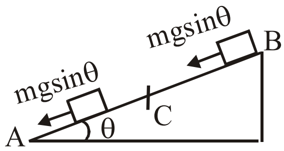
$$= \frac{25\times25\times}{6}T\,(10^{2/5} - 1)$$

$$= \frac{625\times293\times(10^{2/5}-1)}{6}$$

$$= 4.033\times10^3 \approx 46.1\text{kJ}$$

23. NTA Ans. (3)

Sol.



Apply work energy theorem

$mg\sin\theta\ (AC + 2AC) - \mu mg\cos\theta AC = 0$

$\mu = 3\tan\theta$

24. NTA Ans. (15)

Sol. r = 0.1 m $\quad \frac{T}{2} = 0.2$ sec

$B = 3\times10^{-5}$ m $\qquad$ T = 0.4 sec

At any time

flux $\phi = BA\cos\omega t$

$$|emf| = \left|\frac{d\phi}{dt}\right| = |BA\omega\ \sin\omega t|$$

$$(emf)_{max} = BA\omega = BA\ \frac{2\pi}{T}$$

$$= \frac{3\times10^{-5}\times\pi\times(0.1)^2\times2\pi}{0.4}$$

$$= \frac{6\pi^2}{4}\times10^{-6} \qquad \begin{pmatrix}\pi^2 \simeq 10\\ \text{take}\end{pmatrix}$$

$= 15\times10^{-6} = 15\ \mu V$

25. NTA Ans. (9)

Sol. $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV$ .....(i)

$\frac{hc}{3\lambda} = \frac{hc}{\lambda_0} + \frac{e\cdot V}{4}$ .....(ii)

(multiply by 4)

$\frac{4hc}{3\lambda} = \frac{4hc}{\lambda_0} + eV$ ....(iii)

From (i) & (iii)

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{4hc}{3\lambda} - \frac{4hc}{\lambda_0}$$

$$-\frac{hc}{3\lambda} = -\frac{3hc}{\lambda_0}$$

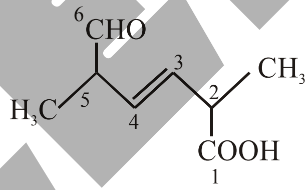
$$\boxed{9\lambda = \lambda_0}$$

n = 9

CHEMISTRY

1. NTA Ans. (4)

Sol.



IUPAC name

2, 5-dimethyl-6-oxo-hex-3-enoic acid

2. NTA Ans. (1)

Sol. Photoelectric effect (option 2), atomic spectrum (option 3) and Black body radiations (option 4) may be explained by quantum theory.

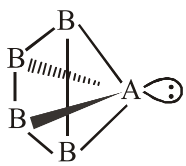
As on increasing temperature, all the values of internal energy becomes possible, it is not directly explained from quantum theory.

3. NTA Ans. (4)

Sol. Slow or fast process is kinetic parameter but extent less or more is thermodynamic parameter.

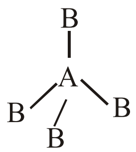
4. NTA Ans. (1)

Sol. (1) If AB_4 molecule is a square pyramidal then it has one lone pair and their structure should be



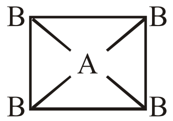
and it should be polar because dipole moment of lone pair of 'A' never be cancelled by others.

(2) If AB_4 molecule is a tetrahedral then it has no lone pair and their structure should be



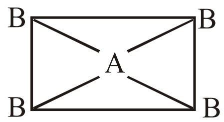
and it should be non polar due to perfect symmetry.

(3) If AB_4 molecule is a square planar then



it should be non polar because vector sum of dipole moment is zero.

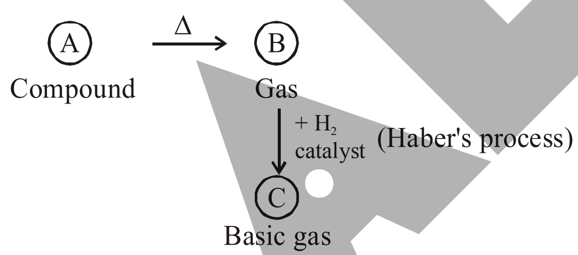
(4) If AB_4 molecule is a rectangular planar then



it should be non polar because vector sum of dipole moment is zero.

5. NTA Ans. (2)

Sol.



Basic gas (C) must be ammonia (NH_3).
It means (B) gas should be N_2 which is formed by heating of compound (A).

(1) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2\uparrow + Cr_2O_3 + 4H_2O\uparrow$

(2) $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2\uparrow + \frac{1}{2}O_2\uparrow$

(3) $2NaN_3 \xrightarrow{\Delta} 2Na + 3N_2\uparrow$

(4) $NH_4NO_2 \xrightarrow{\Delta} N_2\uparrow + 2H_2O\uparrow$

So, (A) should not be $Pb(NO_3)_2$

6. NTA Ans. (4)

Sol. In general across a period atomic radius decreases while ionisation enthalpy, electron gain enthalpy and electronegativity increases because effective nuclear charge (Z_{eff}) increases.

7. NTA Ans. (4)

Sol. In the stratosphere, CFCs release chlorine free radical ($\dot{Cl}$)

$CF_2Cl_2(g) \xrightarrow{UV} \dot{Cl}(g) + \dot{C}F_2Cl(g)$

which react with O_3 to give chlorine oxide ($Cl\dot{O}$) radical not chlorine dioxide ($\dot{Cl}O_2$) radical.

$\dot{Cl}(g) + O_3(g) \rightarrow Cl\dot{O}(g) + O_2(g)$

8. NTA Ans. (4)

Sol. Cs used in photoelectric cell as it has least ionisation energy.

9. NTA Ans. (3)

Sol. (I) Under weak field ligand, octahedral Mn(II) and tetrahedral Ni(II) both the complexes are high spin complex.

(II) Tetrahedral Ni(II) complex can very rarely be low spin because square planar (under strong ligand) complexes of Ni(II) are low spin complexes.

(III) With strong field ligands Mn (II) complexes can be low spin because they have less number of unpaired electron (unpaired electron = 1)

While with weak field ligands Mn(II) complexes can be high spin because they have more number of unpaired electron (unpaired electron = 5)

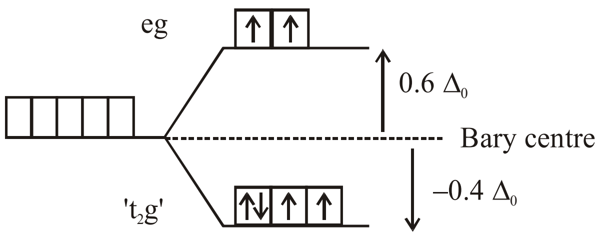
(IV) Aqueous solution of Mn(II) ions is pink in colour.

10. NTA Ans. (2)

Sol. If spin only magnetic moment of the complex is 4.90 BM, it means number of unpaired electrons should be 4.

(A) In octahedral complex : $[M(H_2O)_6]^{2+}$

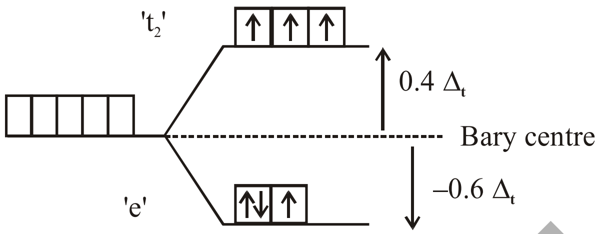
d^6



C.F.S.E. $= (-0.4\ \Delta_0) \times 4 + (+0.6\ \Delta_0) \times 2 + 0 \times P$
$= -0.4\ \Delta_0$

(B) In tetrahedral complex : $[M(H_2O)_4]^{2+}$

d^6



C.F.S.E. $= (-0.6\ \Delta_t) \times 3 + (+0.4\ \Delta_t) \times 3 + 0 \times P$
$= -0.6\ \Delta_t$

11. NTA Ans. (1)

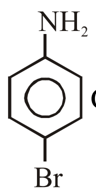
Sol. In Carius method

mass of organic compound = 0.172 gm

mass of Bromine = 0.08 gm

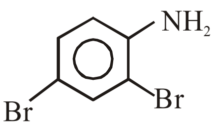
Hence % of Bromine $= \dfrac{0.08}{0.172} \times 100$

$= 46.51\%$

(1) 

C_6H_6NBr $\left[\%Br = \dfrac{80}{172} \times 100\right] = 46.51\%$

(2) CH_3CH_2Br C_2H_5Br $\%Br = \dfrac{80}{109} \times 100 = 73.33\%$

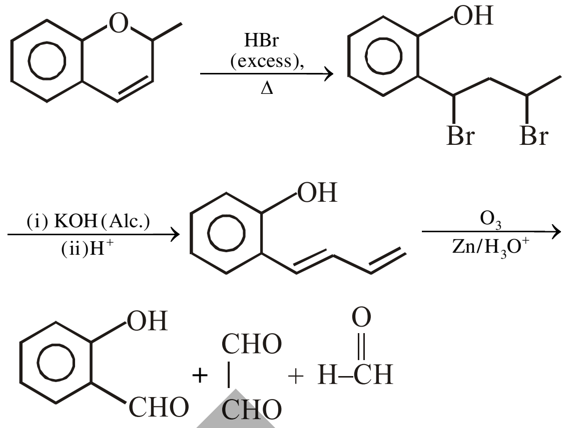
(3) 

$C_6H_5NBr_2$

(4) CH_3Br

12. NTA Ans. (2)

Sol.



13. NTA Ans. (1,3)

Sol. With addition of solute in solvent, surface area for vapourisation decreases causes lowering in vapour pressure

14. NTA Ans. (1)

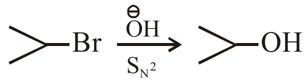
Sol. $PM = dRT \Rightarrow d \propto \dfrac{1}{T}$

15. NTA Ans. (1)

Sol. (1) $CH_3-\overset{*}{C}H(Et)-CH_2-Br \xrightarrow[S_N2]{\overset{\ominus}{O}H} CH_3-\overset{*}{C}H(Et)-CH_2OH$

* (NCERT)

(2) $CH_3-CH(Ph)-Br \xrightarrow[S_N2]{\overset{\ominus}{O}H} Ph-CH(CH_3)-OH$

(3) 

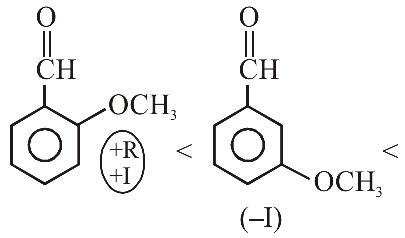
(4) $CH_3-CH(C_6H_{13})-Br \xrightarrow[S_N2]{\overset{\ominus}{O}H} CH_3-CH(C_6H_3)-OH$

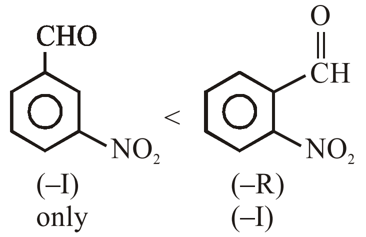
As language given, we have to go with option (1) as stereochemistry of chiral centre is not distorted.

16. NTA Ans. (3)

Sol. Increasing order of reactivity towards HCN addition

Greater the electrophilicity on $-\overset{O}{\overset{\|}{C}}-$ group greater the reactivity in nucleophilic addition.





(iii) < (i) < (iv) < (ii)

17. NTA Ans. (4)

Sol. Lab manual

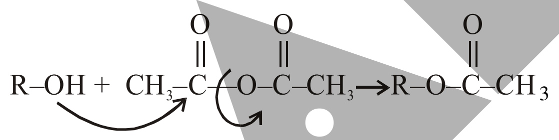
18. NTA Ans. (4)

Sol. (i) Glucose + dry HCl $\xrightarrow{\text{ROH}}$ Acetal $\xrightarrow[(CH_3CO)_2O]{\text{x Eq}}$ acetyl derivative

(ii) Glucose $\xrightarrow{Ni/H_2}$ A $\xrightarrow[(CH_3CO)_2O]{\text{y Eq}}$ Acetyl derivative

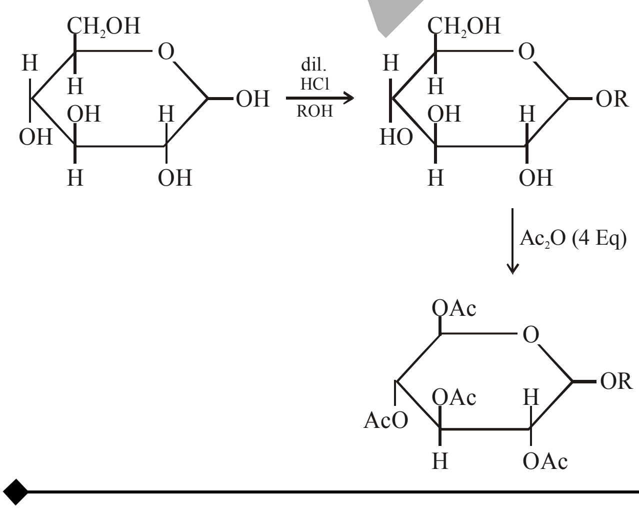
(iii) Glucose $\xrightarrow[(CH_3CO)_2O]{\text{z Eq}}$ Acetyl derivative

due to presence of –OH group in Glucose the reaction is

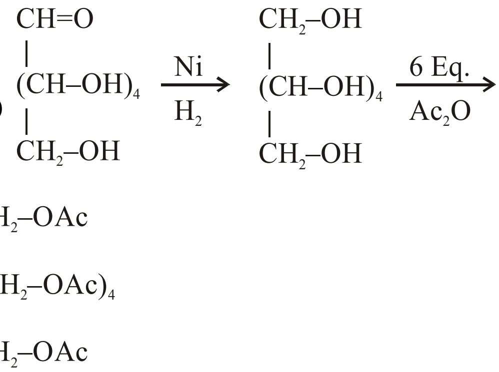


Acetyl derivative

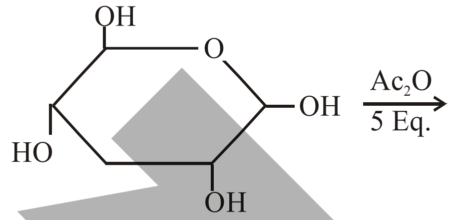
so for (i)



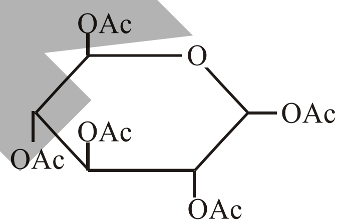
(ii)



(iii)

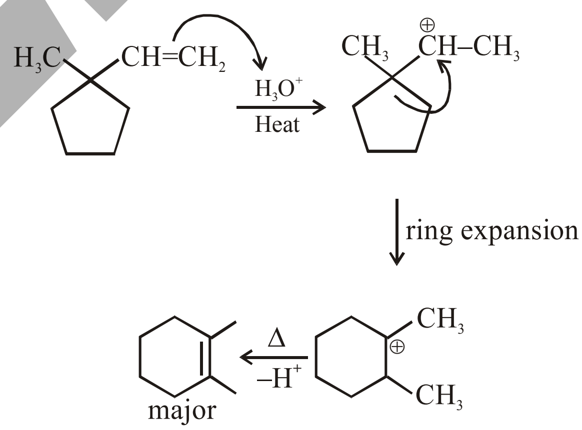


Glucose pentacetase



19. NTA Ans. (3)

Sol.



20. NTA Ans. (3)

Sol. Bredig's Arc method is used to from metal colloids.

21. NTA Ans. (96500.00)

Sol. $\Delta G = \Delta G^o + RT \ln \left[\frac{Sn^{+2}}{Cu^{+2}}\right]$

$= -2 \times 96500\ [(-0.16) - 0.34] + RT \ln \left(\frac{1}{1}\right)$

$= 96500$ J

22. NTA Ans. (48.00)

Sol. $\frac{x}{m} = kp^x$ ...(1)

$\Rightarrow \underbrace{\log\frac{x}{m}}_{y} = \underbrace{\log k}_{c} + \underbrace{x}_{m} \underbrace{\log p}_{x}$

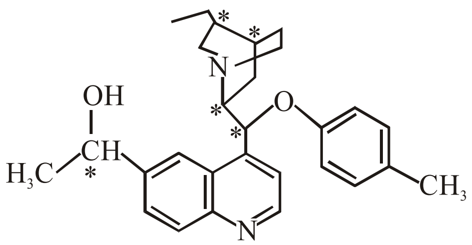
Given c = log k = 0.4771 or k = 3

slope x = 2

put in eq. (1) $\frac{x}{m} = 3 \times (4)^2 \Rightarrow 48$

23. NTA Ans. (5.00)

Sol. No. of chiral centres



24. NTA Ans. (6)

Sol. (A) $Na_4[\overset{x}{Fe}(CN)_5(NOS)]$

$(+1)4 + x + (-1)5 + (-1)\ 1 = 0$

$\boxed{x = +2}$

(B) $Na_4[\overset{y}{Fe}O_4]$

$(+1)4 + y + (-2)4 = 0$

$\boxed{y = +4}$

(C) $[\overset{z}{Fe}_2(CO)_9]$

$2z + 0 \times 9 = 0$

$\boxed{z = 0}$

so (x + y + z) = +2 + 4 + 0

= 6

25. NTA Ans. (189000.00 to 190000.00)

Sol. $H_2O(\ell) \rightleftharpoons H_2O(g)$ 90 gm of H_2O

$\Delta H = \Delta U + \Delta n_g RT$ $\Rightarrow$ 5 moles of H_2O

$5 \times 41000\ J = \Delta U + 1 \times 8.314 \times 373 \times 5$

$\Delta U = 189494.39$ Joule

MATHEMATICS

1. NTA Ans. (1)

Sol. $|x| < 1,\ |y| < 1,\ x \neq y$

$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3)$
$+ ....$

By multiplying and dividing x – y :

$$\frac{(x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4) + ...}{x - y}$$

$$= \frac{(x^2 + x^3 + x^4 + ...) - (y^2 + y^3 + y^4 + ...)}{x - y}$$

$$= \frac{\frac{x^2}{1-x} - \frac{y^2}{1-y}}{x - y}$$

$$= \frac{(x^2 - y^2) - xy(x - y)}{(1-x)(1-y)(x-y)}$$

$$= \boxed{\frac{x + y - xy}{(1-x)(1-y)}}$$

2. NTA Ans. (2)

Sol. Let t_{r+1} denotes

r + 1th term of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$

$$t_{r+1} = {}^{10}C_r \alpha^{10-r} (x)^{\frac{10-r}{9}} . \beta^r x^{-\frac{r}{6}}$$

$$= {}^{10}C_r\ \alpha^{10-r}\ \beta^r\ (x)^{\frac{10-r}{9} - \frac{r}{6}}$$

If t_{r+1} is independent of x

$\frac{10-r}{9} - \frac{r}{6} = 0 \quad \Rightarrow \quad r = 4$

maximum value of t_5 is 10 K (given)

$\Rightarrow\ {}^{10}C_4\ \alpha^6\ \beta^4$ is maximum

By AM $\geq$ GM (for positive numbers)

$$\frac{\frac{\alpha^3}{2} + \frac{\alpha^3}{2} + \frac{\beta^2}{2} + \frac{\beta^2}{2}}{4} \geq \left(\frac{\alpha^6\beta^4}{16}\right)^{\frac{1}{4}}$$

$\Rightarrow \boxed{\alpha^6\beta^4 \leq 16}$

So, 10 K = ${}^{10}C_4 16$

$\Rightarrow$ K = 336

3. NTA Ans. (4)

Sol. $$f(x)=\begin{cases} ae^{x}+be^{-x}, & -1\le x<1 \\ cx^2, & 1\le x\le 3 \\ ax^2+2cx, & 3<x\le 4 \end{cases}$$

For continuity at x = 1

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)$$

$\Rightarrow$ $\boxed{ae+be^{-1}=c} \Rightarrow \boxed{b=ce-ae^2}$ ...(1)

For continuity at x = 3

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)$$

$\Rightarrow$ 9c = 9a + 6c

$\Rightarrow$ c = 3a ...(2)

$f'(0) + f'(2) = e$

$(ae^x - be^x)_{x=0} + (2cx)_{x=2} = e$

$\Rightarrow$ $\boxed{a-b+4c=e}$ ...(3)

From (1), (2) & (3)

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow$ $a(e^2 + 13 - 3e) = e$

$$\Rightarrow a=\frac{e}{e^2-3e+13}$$

4. NTA Ans. (1)

Sol. Let B_1 be the event where Box–I is selected.

& $B_2 \to$ where box-II selected

$$P(B_1) = P(B_2) = \frac{1}{2}$$

Let E be the event where selected card is non prime.

For B_1 : Prime numbers :

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

For B_2 : Prime numbers :

{31, 37, 41, 43, 47}

$$P(E) = P(B_1)\times P\left(\frac{E}{B_1}\right) + P(B_2)P\left(\frac{E}{B_2}\right)$$

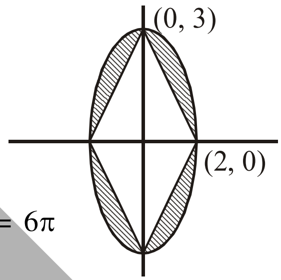
$$=\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}$$

Required probability :

$$P\left(\frac{B_1}{E}\right)=\frac{\frac{1}{2}\times\frac{20}{30}}{\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}}=\frac{\frac{2}{3}}{\frac{2}{3}+\frac{3}{4}}=\frac{8}{17}$$

5. NTA Ans. (2)

Sol. $$\frac{|x|}{2}+\frac{|y|}{3}=1$$



$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

Area of Ellipse = $\pi ab = 6\pi$

Required area,

= $\pi \times 2 \times 3$ – (Area of quadrilateral)

$$= 6\pi-\frac{1}{2}6\times 4$$

= $6\pi - 12$

= $6(\pi - 2)$

6. NTA Ans. (3)

Sol. $2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

For no solution :

$$D=\begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix}=0$$

$\Rightarrow$ $2(-2-\lambda^2) + 1(1-\lambda) + 2(\lambda + 2) = 0$

$\Rightarrow$ $-2\lambda^2 + \lambda + 1 = 0$

$$\Rightarrow \lambda = 1,\ -\frac{1}{2}$$

$$D_x=\begin{vmatrix} 2 & -1 & 2 \\ -4 & 2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix}=2\begin{vmatrix} 1 & -1 & 2 \\ -2 & -2 & \lambda \\ \lambda & \lambda & 1 \end{vmatrix}$$

= $2(1 + \lambda)$

whichis not equal to zero for

$$\lambda = 1,\ -\frac{1}{2}$$

7. NTA Ans. (4)

Sol. $|A| \neq 0$

For (P) : $A \neq I_2$

$$\text{So, } A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$|A|$ can be -1 or 1

So (P) is false.

For (Q); $|A| = 1$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$\Rightarrow$ $tr(A) = 2$

$\Rightarrow$ Q is true

8. NTA Ans. (3)

Sol. Let p denotes statement

p : I reach the station in time.

q : I will catch the train.

Contrapositive of $p \to q$

is $\sim q \to \sim p$

$\sim q \to \sim p$: I will not catch the train, then I do not reach the station in time.

9. NTA Ans. (4)

Sol. $\dfrac{2+\sin x}{y+1}\dfrac{dy}{dx} = -\cos x,\ y > 0$

$$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$$

By integrating both sides :

$$\ell n|y+1| = -\ell n|2+\sin x| + \ell nK$$

$$\Rightarrow y + 1 = \frac{K}{2+\sin x} \qquad (y + 1 > 0)$$

$$\Rightarrow y(x) = \frac{K}{2+\sin x} - 1$$

Given $y(0) = 1 \Rightarrow K = 4$

$$\text{So, } y(x) = \frac{4}{2+\sin x} - 1$$

$a = y(\pi) = 1$

$$b = \left.\frac{dy}{dx}\right]_{x=\pi} = \left.\frac{-\cos x}{2+\sin x}\left(y(x)+1\right)\right]_{x=\pi} = 1$$

So, $(a, b) = (1, 1)$

10. NTA Ans. (1)

Sol. σ^2 = variance

μ = mean

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$$

$\mu = 17$

$$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b)}{17} = 17$$

$\Rightarrow$ $9a + b = 17$ ....(1)

$\sigma^2 = 216$

$$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b-17)^2}{17} = 216$$

$$\Rightarrow \frac{\sum_{x=1}^{17}a^2(x-9)^2}{17} = 216$$

$\Rightarrow$ $a^2 81 - 18 \times 9a^2 + a^2\ 3 \times (35) = 216$

$\Rightarrow$ $a^2 = \dfrac{216}{24} = 9$ $\Rightarrow$ $a = 3\ (a > 0)$

$\Rightarrow$ From (1), $b = -10$

So, $a + b = -7$

11. NTA Ans. (3)

Sol. Slope of tangent to the curve $y = x + \sin y$

at (a, b) is $\dfrac{2-\frac{3}{2}}{\frac{1}{2}-0} = 1$

$\Rightarrow \left.\dfrac{dy}{dx}\right]_{x=a} = 1$

$\dfrac{dy}{dx} = 1 + \cos y.\dfrac{dy}{dx}$ (from equation of curve)

$\left.\dfrac{dy}{dx}\right]_{x=a} = 1 + \cos b.\left.\dfrac{dy}{dx}\right]_{x=a}$

$\Rightarrow$ cos b = 0

$\Rightarrow$ sin b = ±1

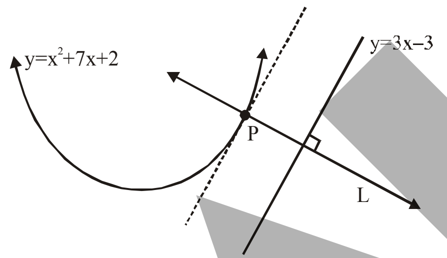
Now, from curve y = x + sin y

b = a + sin b

$\Rightarrow$ |b – a| = |sin b| = 1

12. NTA Ans. (4)

Sol.



Let L be the common normal to parabola $y = x^2 + 7x + 2$ and line $y = 3x - 3$

$\Rightarrow$ slope of tangent of $y = x^2 + 7x + 2$ at P = 3

$\Rightarrow \left.\dfrac{dy}{dx}\right]_{\text{For P}} = 3$

$\Rightarrow 2x + 7 = 3 \Rightarrow x = -2 \Rightarrow y = -8$

So P(–2, –8)

Normal at P : x + 3y + C = 0

$\Rightarrow$ C = 26 (P satisfies the line)

$\boxed{\text{Normal}: x + 3y + 26 = 0}$

13. NTA Ans. (2)

Sol. Two points on the line (L say) $\dfrac{x}{3} = \dfrac{y}{2}, z = 1$ are (0, 0, 1) & (3, 2, 1)

So dr's of the line is < 3, 2, 0 >

Line passing through (1, 2, 1), parallel to L and coplanar with given plane is

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + t(3\hat{i} + 2\hat{j}), t \in R$ (–2, 0, 1) satisfies the line (for t = –1)

$\Rightarrow$ (–2, 0, 1) lies on given plane.

Answer of the question is (2)

We can check other options by finding eqution of plane

Equation plane : $\begin{vmatrix} x-1 & y-2 & z-1 \\ 1+2 & 2-0 & 1-1 \\ 2+2 & 1-0 & 2-1 \end{vmatrix} = 0$

$\Rightarrow 2(x - 1) - 3(y - 2) - 5(z - 1) = 0$

$\Rightarrow 2x - 3y - 5z + 9 = 0$

14. NTA Ans. (1)

Sol. α and β are roots of $5x^2 + 6x - 2 = 0$

$\Rightarrow 5\alpha^2 + 6\alpha - 2 = 0$

$\Rightarrow 5\alpha^{n+2} + 6\alpha^{n+1} - 2\alpha^n = 0$ …(1)

(By multiplying α^n)

Similarly $5\beta^{n+2} + 6\beta^{n+1} - 2\beta^n = 0$ …(2)

By adding (1) & (2)

$5S_{n+2} + 6S_{n+1} - 2S_n = 0$

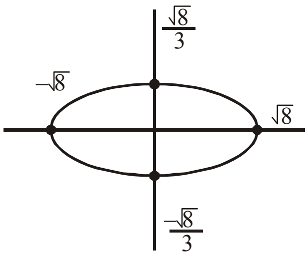
For n = 4

$\boxed{5S_6 + 6S_5 = 2S_4}$

15. NTA Ans. (2)

Sol. $R = \{(x, y) : x, y \in z, x^2 + 3y^2 \le 8\}$

For domain of R^{-1}



Collection of all integral of y's

For $x = 0$, $3y^2 \le 8$

$\Rightarrow y \in \{-1, 0, 1\}$

16. NTA Ans. (4)

Sol. Let three terms of G.P. are $\frac{a}{r}$, a, ar

product = 27

$\Rightarrow a^3 = 27 \Rightarrow a = 3$

$$S = \frac{3}{r} + 3r + 3$$

For $r > 0$

$$\frac{\frac{3}{r} + 3r}{2} \ge \sqrt{3^2} \quad \text{(By AM} \ge \text{GM)}$$

$$\Rightarrow \frac{3}{r} + 3r \ge 6 \quad \ldots(1)$$

For $r < 0$ $\quad \frac{3}{r} + 3r \le -6 \quad \ldots(2)$

From (1) & (2)

$S \in (-\infty - 3] \cup [9, \infty]$

17. NTA Ans. (2)

Sol. Slope of tangent is 2, Tangent of hyperbola

$\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1) is

$$\frac{xx_1}{4} - \frac{yy_1}{2} = 1 \quad (T = 0)$$

$$\text{Slope : } \frac{1}{2}\frac{x_1}{y_1} = 2 \Rightarrow \boxed{x_1 = 4y_1} \quad \ldots(1)$$

(x_1, y_1) lies on hyperbola

$$\Rightarrow \boxed{\frac{x_1^2}{4} - \frac{y_1^2}{2} = 1} \quad \ldots(2)$$

From (1) & (2)

$$\frac{(4y_1)^2}{4} - \frac{y_1^2}{2} = 1 \Rightarrow 4y_1^2 - \frac{y_1^2}{2} = 1$$

$$\Rightarrow 7y_1^2 = 2 \Rightarrow \boxed{y_1^2 = 2/7}$$

Now $x_1^2 + 5y_1^2 = (4y_1)^2 + 5y_1^2$

$$= (21)y_1^2 = 21 \times \frac{2}{7} = 6$$

18. NTA Ans. (1)

Sol. $f(x) = \sin\left(\frac{|x|+5}{x^2+1}\right)$

For domain :

$$-1 \le \frac{|x|+5}{x^2+1} \le 1$$

Since $|x| + 5$ & $x^2 + 1$ is always positive

So $\frac{|x|+5}{x^2+1} \geq 0 \; \forall x \in R$

So for domain :

$$\frac{|x|+5}{x^2+1} \leq 1$$

$\Rightarrow |x| + 5 \leq x^2 + 1$

$\Rightarrow 0 \leq x^2 - |x| - 4$

$$\Rightarrow 0 \leq \left(|x| - \frac{1+\sqrt{17}}{2}\right)\left(|x| - \frac{1-\sqrt{17}}{2}\right)$$

$$\Rightarrow |x| \geq \frac{1+\sqrt{17}}{2} \text{ or } |x| \leq \frac{1-\sqrt{17}}{2} \quad \text{(Rejected)}$$

$$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$$

So, $a = \frac{1+\sqrt{17}}{2}$

19. NTA Ans. (2)

Sol. The value of $\left(\frac{1+\sin 2\pi/9 + i\cos 2\pi/9}{1+\sin\frac{2\pi}{9} - i\cos\frac{2\pi}{9}}\right)$

$$= \left(\frac{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)+i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)-i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}\right)^3$$

$$= \left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$$

$$= \left(\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3$$

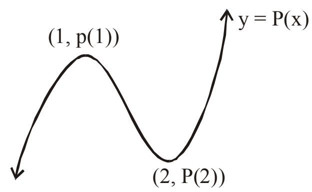
$$= \left(\frac{e^{i5\pi/36}}{e^{-i5\pi/36}}\right)^3 = \left(e^{i5\pi/18}\right)^3$$

$$= \cos\frac{5\pi}{6} + i\sin 5\pi/6$$

$$= -\frac{\sqrt{3}}{2} + i/2$$

20. NTA Ans. (4)

Sol.



Since p(x) has realtive extreme at x = 1 & 2

so p'(x) = 0 at x = 1 & 2

$\Rightarrow p'(x) = A(x-1)(x-2)$

$\Rightarrow p(x) = \int A(x^2-3x+2)dx$

$$p(x) = A\left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)+C \quad \ldots(1)$$

P(1) = 8

From (1)

$$8 = A\left(\frac{1}{3}-\frac{3}{2}+2\right)+C$$

$$\Rightarrow 8 = \frac{5A}{6}+C \Rightarrow \boxed{48 = 5A+6C} \quad \ldots(3)$$

P(2) = 4

$$\Rightarrow 4 = A\left(\frac{8}{3}-6+4\right)+C$$

$$\Rightarrow 4 = \frac{2A}{3}+C \Rightarrow \boxed{12 = 2A+3C} \quad \ldots(4)$$

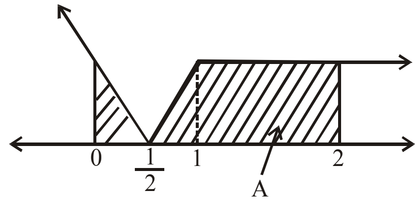
From 3 & 4, C = –12

So P(0) = C = $\boxed{-12}$

21. NTA Ans. (1.50)

Sol. $\int_0^2 ||x-1|-|x||dx$

Let f(x) ||x – 1|–x| = $\begin{cases} 1, & x \geq 1 \\ |1-2x|, & x \leq 1 \end{cases}$



$$A=\frac{1}{2}+1=\frac{3}{2}$$

or

$$\int_0^{1/2}(1-2x)dx+\int_{1/2}^{1}(2x-1)+\int_0^2 1dx$$

$$=\left[x-x^2\right]_0^{\frac{1}{2}}+\left[x^2-x\right]_{1/2}^{1}+\left[x\right]_1^2$$

$$=\boxed{3/2}$$

22. NTA Ans. (2.00)

Sol. $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

$$\left|\vec{a}-\vec{b}\right|^2+\left|\vec{a}-\vec{b}\right|^2=8$$

$$\Rightarrow\ |\vec{a}|^2+|\vec{b}|^2-2\vec{a}.\vec{b}+|\vec{a}|^2+|\vec{c}|^2-2\vec{a}.\vec{c}=8$$

$$\Rightarrow\ 4-2(\vec{a}.\vec{b}+\vec{a}.\vec{c})=8$$

$$\Rightarrow\ \boxed{\vec{a}.\vec{b}+\vec{a}.\vec{c}=-2}$$

$$|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$$

$$=|a^2|+4|\vec{b}|^2+4\vec{a}.\vec{b}+|\vec{a}|^2+4|\vec{c}|^2+4\vec{a}.\vec{c}$$

$$=10+4(\vec{a}.\vec{b}+\vec{a}.\vec{c})$$

$$=10-8$$

$$=\boxed{2}$$

23. NTA Ans. (40.00)

Sol. $\lim\limits_{x\to 1}\dfrac{x+x^2+.......+x^2-n}{x-1}=820$

$$\Rightarrow\ \lim_{x\to 1}\left(\frac{x-1}{x-1}+\frac{x^2-1}{x-1}+......\frac{x^n-1}{x-1}\right)=820$$

$\Rightarrow$ $1+2+.....+n=820$

$\Rightarrow$ $n(n+1)=2\times 820$

$\Rightarrow$ $n(n+1)=40\times 41$

Since $n\in N$, so $\boxed{n=40}$

24. NTA Ans. (309.00)

Sol. MOTHER

$1\to E$

$2\to H$

$3\to M$

$4\to O$

$5\to R$

$6\to T$

So position of word MOTHER in dictionary

$2\times 5!+2\times 4!+3\times 3!+2!+1$

$=240+48+18+2+1$

$=\boxed{309}$

25. NTA Ans. (9.00)

Sol. Circle $x^2+y^2-2x-4y+4=0$

$\Rightarrow$ $(x-1)^2+(y-2)^2=1$

Centre : $(1, 2)$ radius $=1$

line $3x+4y-k=0$ intersects the circle at two distinct points.

$\Rightarrow$ distance of centre from the line < radius

$$\Rightarrow\ \left|\frac{3\times 1+4\times 2-k}{\sqrt{3^2+4^2}}\right|<1$$

$\Rightarrow$ $|11-k|<5$

$\Rightarrow$ $6<k<16$

$\Rightarrow$ $k\in\{7, 8, 9, ...15\}$ since $k\in I$

Number of K is $\boxed{9}$

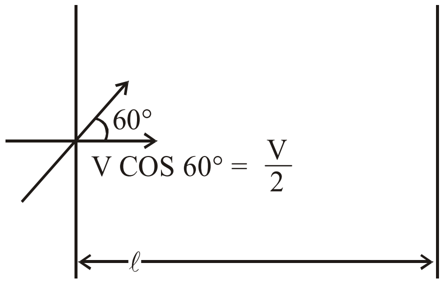
SET # 02

PHYSICS

1. NTA Ans. (3)

Sol. $T = \frac{2\pi m}{qB}$

total time t = 10 T



Kinematics

$$\ell = \frac{V}{2}t$$

$$\ell = \frac{V}{2}10 \times \frac{2\pi m}{qB}$$

$$= 4 \times 10^5 \times 10 \times \frac{3.14 \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 0.3}$$

= 0.439

2. NTA Ans. (4)

Sol. (A) F = ma $a = -\omega^2 x$

at $\frac{3T}{4}$ displacement zero (x = 0), so a = 0

F = 0

(B) at t = T displacement (x) = A

x maximum, So acceleration is maximum.

(C) $V = \omega\sqrt{A^2 - x^2}$

V_{max} at x = 0

$V_{max} = A\omega$

at t = $\frac{T}{4}$, x = 0, So $V_{max.}$

(D) KE = PE

$\therefore$ at $x = \frac{A}{\sqrt{2}}$.

at $t = \frac{T}{2}$ $x = -A$ (So not possible)

3. NTA Ans. (1)

Sol. M = NIA

N = 1

For ABCD

$$\vec{M}_1 = abI\ \hat{K}.$$

For DEFA

$$\vec{M}_2 = abI\ \hat{j}$$

$\vec{M} = \vec{M}_1 + \vec{M}_2$

$= ab\ I\ \left(\hat{k} + \hat{j}\right)$

$= ab\ I\ \sqrt{2}\ \left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

4. NTA Ans. (1)

Sol. Balancing length is measured from P.

So 100 – 49 = 51 cm

$E_2 = \phi \times 51$

Where ϕ = Potential gradient

$1.02 = \phi \times 51$

ϕ = 0.02 V/cm

5. NTA Ans. (3)

Sol. $\eta = \frac{\text{Work done}}{\text{Heat supplied}}$

$$\frac{1}{2} = \eta = \frac{1915 - 40 + 125 - Q}{1915 + 125}$$

$$\frac{1}{2} = \frac{2000 - Q}{2040}$$

2040 = 4000 – 2Q

2Q = 1960

Q = 980 J

6. NTA Ans. (1)

Sol. Let the length of segment is "ℓ"

Let N is the no. of fringes in "ℓ"

and w is fringe width.

$\rightarrow$ We can write

$N w = \ell$

$$N\left(\frac{\lambda D}{d}\right) = \ell$$

$$\frac{N_1\lambda_1 D}{d} = \ell$$

$$\frac{N_2\lambda_2 D}{d} = \ell$$

$N_1\lambda_1 = N_2\lambda_2$

$16 \times 700 = N_2 \times 400$

$N_2 = 28$

7. NTA Ans. (2)

Sol. In hydrogen atom,

$$E_n = \frac{-E_0}{n^2}$$

Where E_0 is Ionisation Energy of H.

$\rightarrow$ For transition from (n + 1) to n, the energy of emitted radiation is equal to the difference in energies of levels.

$$\Delta E = E_{n+1} - E_n$$

$$\Delta E = E_0\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$\Delta E = h\nu = E_0\left(\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right)$$

$$h\nu = E_0\left[\frac{2n+1}{n^4\left(1+\frac{1}{n}\right)^2}\right]$$

$$h\nu = E_0\left[\frac{n\left(2+\frac{1}{n}\right)}{n^4\left(1+\frac{1}{n}\right)^2}\right]$$

Since n >>> 1

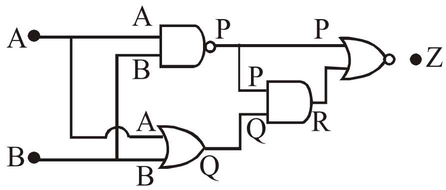
Hence, $\frac{1}{n} \simeq 0$

$$h\nu = E_0\left[\frac{2}{n^3}\right]$$

$$\nu \,\alpha\, \frac{1}{n^3}$$

8. NTA Ans. (3)

Sol.



$$Z = \overline{(P+R)}$$

$$Z = \overline{(P+PQ)}$$

$$Z = \overline{(P(1+Q))}$$

$$Z = (\overline{P}) \text{ [Using Identity } (1+A)=1]$$

$$Z = \overline{(\overline{AB})}$$

Z = AB

Truth table for Z = AB

A	B	Z
1	0	0
0	0	0
1	1	1
0	1	0

9. NTA Ans. (2)

Sol. Let $[E] = [P]^x [A]^y [T]^z$

$ML^2T^{-2} = [MLT^{-1}]^x [L^2]^y [T]^z$

$ML^2T^{-2} = M^x L^{x+2y} T^{-x+z}$

$\rightarrow x = 1$

$\rightarrow x + 2y = 2$

$1 + 2y = 2$

$y = \frac{1}{2}$

$\rightarrow -x + z = -2$

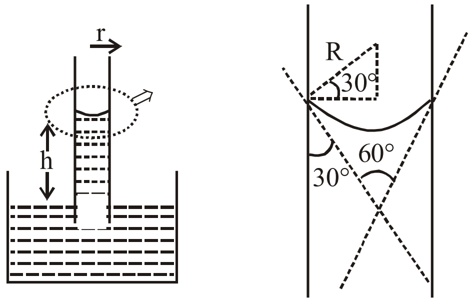
$-1 + z = -2$

$z = -1$

$[E] = [PA^{1/2} T^{-1}]$

10. NTA Ans. (3)

Sol.



r $\rightarrow$ radius of capillary

R $\rightarrow$ Radius of meniscus.

From figure, $\frac{r}{R} = \cos 30°$

$$R = \frac{2r}{\sqrt{3}} = \frac{2 \times 0.15 \times 10^{-3}}{\sqrt{3}}$$

$$= \frac{0.3}{\sqrt{3}} \times 10^{-3} \text{m}$$

Height of capillary

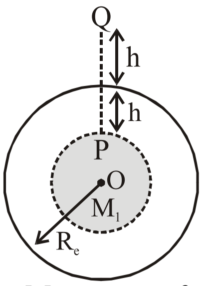
$$h = \frac{2T}{\rho g R} = 2\sqrt{3}\ T$$

$$h = \frac{2 \times 0.05}{667 \times 10 \times \left(\frac{0.3 \times 10^{-3}}{\sqrt{3}}\right)}$$

h = 0.087 m

11. NTA Ans. (1)

Sol.



♦ M = mass of earth

M_1 = mass of shaded portion

R = Radius of earth

♦ $M_1 = \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi (R-h)^3$

$$= \frac{M(R-h)^3}{R}$$

♦ Weight of body is same at P and Q

i.e. $mg_P = mg_Q$

$g_P = g_Q$

$$\frac{GM_1}{(R-h)^2} = \frac{GM}{(R+h)^2}$$

$$\frac{GM(R-h)^3}{(R-h)^2 R^3} = \frac{GM}{(R+h)^2}$$

$(R - h)(R + h)^2 = R^3$

$R^3 - hR^2 - h^2R - h^3 + 2R^2 h - 2Rh^2 = R^3$

$R^2 - Rh^2 - h^3 = 0$

$R^2 - Rh - h^2 = 0$

$$h^2 + Rh - R^2 = 0 \Rightarrow h = \frac{-R \pm \sqrt{R^2 + 4R^2}}{2}$$

$$\text{i.e. } h = \frac{-R + \sqrt{5}R}{2} = \left(\frac{\sqrt{5} - 1}{2}\right) R$$

12. NTA Ans. (4)

Sol. The mean free path of molecules of an ideal gas is given as :

$$\lambda = \frac{V}{\sqrt{2}\pi d^2 N}$$

where : V = Volume of container
N = No of molecules

Hence with increasing temp since volume of container does not change (closed container), so mean free path is unchanged.

Average collision time

$$= \frac{\text{mean free path}}{V_{av}} = \frac{\lambda}{(\text{avg speed of molecules})}$$

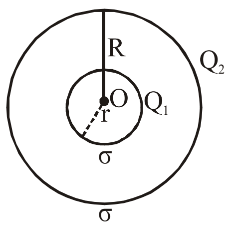
$\because$ avg speed $\alpha \sqrt{T}$

$\therefore$ Avg coll. time $\alpha \frac{1}{\sqrt{T}}$

Hence with increase in temperature the average collision time decreases.

13. NTA Ans. (3)

Sol. Let the charges on inner and outer spheres are Q_1 and Q_2.



Since charge density 'σ' is same for both spheres, so

$$\sigma = \frac{Q_1}{4\pi r^2} = \frac{Q_2}{4\pi R^2} \Rightarrow \frac{Q_1}{Q_2} = \frac{r^2}{R^2}$$

$$Q_1 + Q_2 = Q \Rightarrow \frac{Q_2 r^2}{R^2} + Q_2 = Q$$

$$\Rightarrow Q_2 = \frac{QR^2}{(r^2 + R^2)}$$

$$Q_1 = \frac{r^2}{R^2} \cdot \frac{QR^2}{(R^2 + r^2)} = \frac{Qr^2}{(R^2 + r^2)}$$

Potential at centre 'O' = $\dfrac{kQ_1}{r} + \dfrac{kQ_2}{R}$

$$= k\left[\frac{Qr^2}{r(R^2 + r^2)} + \frac{QR^2}{R(R^2 + r^2)}\right]$$

$$= \frac{kQ(r + R)}{(R^2 + r^2)} = \frac{1}{4\pi \in_0} \frac{(R + r)}{(R^2 + r^2)} Q$$

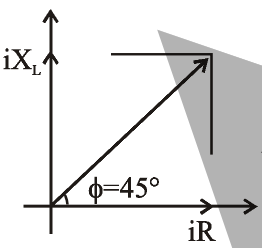
14. NTA Ans. (1)

Sol. 
R,L

- Reactance of inductance coil

$= \sqrt{R^2 + x_L^2} = 100$ .....(i)

- f = 1000 Hz of applied AC signal
- Voltage leads current by 45°

ie 

$\tan 45° = \dfrac{iX_L}{iR} = \dfrac{\omega L}{R}$

ie $R = X_L = \omega L$

Putting in eqn (i) : $\sqrt{X_L^2 + X_L^2} = 100$

$$\sqrt{2} X_L = 100 \Rightarrow X_L = 50\sqrt{2}$$

ie $\omega L = 50\sqrt{2}$

$$L = \frac{50\sqrt{2}}{\omega} = \frac{50\sqrt{2}}{2\pi f} = \frac{25\sqrt{2}}{\pi \times 1000} \text{ H}$$

$= 1.125 \times 10^{-2}$ H

15. NTA Ans. (4)

Sol.
- Both discs are rotating in same sense
- Angular momentum conserved for the system

i.e. $L_1 + L_2 = L_{final}$

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

$0.1 \times 10 + 0.2 \times 5 = (0.1+0.2) \times \omega_f$

$$\omega_f = \frac{20}{3}$$

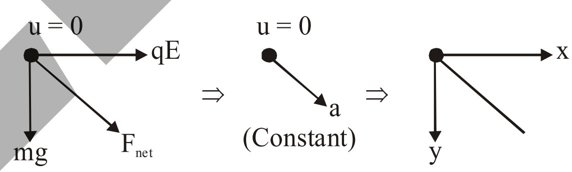
- Kinetic energy of combined disc system

$$\Rightarrow \frac{1}{2}(I_1 + I_2)\omega_f^2$$

$$= \frac{1}{2}(0.1 + 0.2).\left(\frac{20}{3}\right)^2$$

$$= \frac{0.3}{2} \times \frac{400}{9} = \frac{120}{18} = \frac{20}{3} \text{J}$$

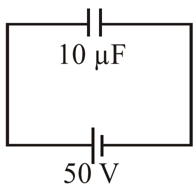
16. NTA Ans. (4)

Sol. 

Since initial velocity is zero and acceleration of particle will be constant, so particle will travel on a straight line path.

17. NTA Ans. (2)

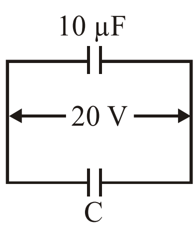
Sol. Initially



- Charge on capacitor 10 μF

Q = CV = (10 μF) (50V)

Q = 500 μC•



- Final Charge on 10 μF capacitor

$Q = CV = (10\ \mu F)\ (20V)$

$Q = 200\ \mu C$

- From charge conservation,

Charge on unknown capacitor

$C = 500\ \mu C - 200\ \mu C = 300\ \mu C$

$\Rightarrow \text{Capacitance } (C) = \dfrac{Q}{V} = \dfrac{300\ \mu C}{20\ V} = 15\ \mu F$

18. NTA Ans. (2)

Sol. Given $\dfrac{\Delta L}{L} = 0.02\%$

$\therefore \Delta L = L\alpha\Delta T \Rightarrow \dfrac{\Delta L}{L} = \alpha\Delta T = 0.02\%$

$\therefore\ \beta = 2\alpha$ (Areal coefficient of expansion)

$\Rightarrow \beta\Delta T = 2\alpha\Delta T = 0.04\%$

Volume = Area × Length

$\text{Density}(\rho) = \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{\text{Mass}}{\text{Area} \times \text{Length}} = \dfrac{M}{AL}$

$\Rightarrow \dfrac{\Delta\rho}{\rho} = \overbrace{\cancel{\dfrac{\Delta M}{M}}}^{0} - \dfrac{\Delta A}{A} - \dfrac{\Delta L}{L}$ (Mass remains constant)

$\Rightarrow \left(\dfrac{\Delta\rho}{\rho}\right) = \dfrac{\Delta A}{A} + \dfrac{\Delta L}{L} = \beta\Delta T + \alpha\Delta T$

$= 0.04\% + 0.02\%$

$= 0.06\%$

19. NTA Ans. (1)

Sol. $\hat{E} = \hat{k}$

$\vec{B} = 2\hat{i} - 2\hat{j} \Rightarrow \hat{B} = \dfrac{\vec{B}}{|B|} = \dfrac{2\hat{i} - 2\hat{j}}{2\sqrt{2}}$

$\Rightarrow \hat{B} = \dfrac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

Direction of wave propagation $= \hat{C} = \hat{E} \times \hat{B}$

$$\hat{C} = \hat{k} \times \left[\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})\right]$$

$$\hat{C} = \frac{1}{\sqrt{2}}(\hat{k} \times \hat{i} - \hat{k} \times \hat{j})$$

$$\hat{C} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$

20. NTA Ans. (4)

Sol. Let mass of particle = m

Let speed of $e^- = V$

$\Rightarrow$ speed of particle = 5V

Debroglie wavelength $\lambda_d = \dfrac{h}{P} = \dfrac{h}{mv}$

$\Rightarrow (\lambda_d)_P = \dfrac{h}{m(5V)}$ .....(1)

$\Rightarrow (\lambda_d)_e = \dfrac{h}{m_e.V}$ .....(2)

According to question

$\dfrac{(1)}{(2)} = \dfrac{m_e}{5m} = 1.878 \times 10^{-4}$

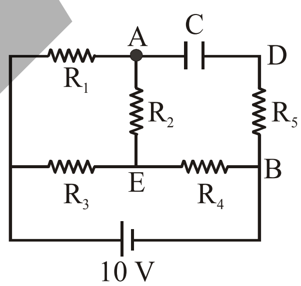
$\Rightarrow m = \dfrac{m_e}{5 \times 1.878 \times 10^{-4}}$

$\Rightarrow m = \dfrac{9.1 \times 10^{-31}}{5 \times 1.878 \times 10^{-4}}$

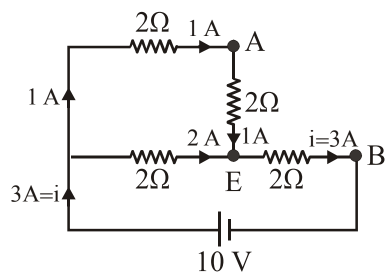
$\Rightarrow m = 9.7 \times 10^{-28}$ kg

21. NTA Ans. (8.00)

Sol.



- R_1 to $R_5 \rightarrow$ each 2Ω
- Cap. is fully charged
- So no current is there in branch ADB
- Effective circuit of current flow :



$R_{eq} = \left(\dfrac{4 \times 2}{4 + 2}\right) + 2$

$$R_{eq} = \frac{4}{3} + 2 = \frac{10}{3}\Omega$$

$$i = \frac{10}{10/3} = 3A$$

So potential different across AEB

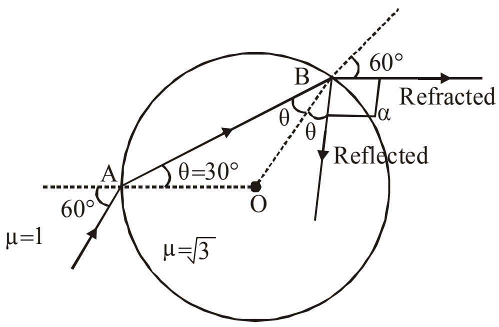
$\Rightarrow 2 \times 1 + 2 \times 3 = 8V$

Hence potential difference across

Capacitor = $\Delta V = V_{AEB} = 8V$

22. NTA Ans. (90.00)

Sol.



By Snell's law at A :

$1 \times \sin 60° = \sqrt{3} \times \sin\theta$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin\theta$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

So at B :

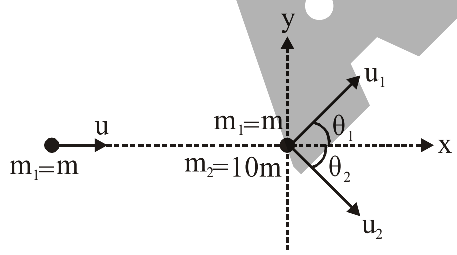
$\theta + 60° + \alpha = 180°$

$30° + 60° + \alpha = 180°$

$\alpha = 90°$

23. NTA Ans. (10.00)

Sol.



By momentum conservation along y :

$m_1u_1\sin\theta_1 = m_2u_2\sin\theta_2$

i.e. $mu_1\sin\theta_1 = 10mu_2\sin\theta_2$

$$\Rightarrow \boxed{u_1\sin\theta_1 = 10u_2\sin\theta_2} \quad .....(i)$$

$$kf_{m_1} = \frac{1}{2}ki_{m_1} \quad \text{i.e. } \frac{1}{2}mu_1^2 = \frac{1}{2}\times\frac{1}{2}mu^2$$

i.e. $\boxed{u_1 = \frac{u}{\sqrt{2}}}$ .....(ii)

Also collision is elastic : $k_i = k_f$

$$\frac{1}{2}mu^2 = \frac{1}{2}mu_1^2 + \frac{1}{2}.10m.u_2^2$$

$$\frac{1}{2}mu^2 = \frac{1}{2}\times\frac{1}{2}mu^2 + \frac{1}{2}\times 10m.u_2^2$$

$$\frac{1}{4}mu^2 = \frac{1}{2}\times 10\times mu_2^2$$

$$\boxed{u_2 = \frac{u}{\sqrt{20}}} \quad ....(iii)$$

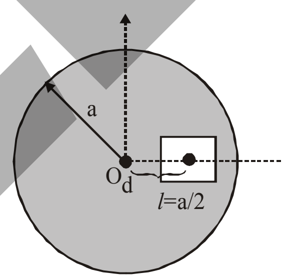
Putting (ii) & (iii) in (i)

$$\frac{u}{\sqrt{2}}\sin\theta_1 = 10.\frac{u}{\sqrt{20}}\sin\theta_2$$

$\boxed{\sin\theta_1 = \sqrt{10}\sin\theta_2}$ $\rightarrow$ Hence n = 10

24. NTA Ans. (23.00)

Sol.



$$X_{com} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

where :

- m_1 = mass of complete disc
- m_2 = removed mass
- Let σ = surface mass density of disc material

$$\text{wrt 'O' : } X_{com} = \frac{\sigma\pi a^2(O) - \sigma.\frac{a^2}{4}.d}{\sigma\pi a^2 - \sigma\frac{a^2}{4}} = \frac{-\frac{a^2}{4}d}{\pi a^2 - \frac{a^2}{4}}$$

$$= \frac{-d}{4\pi - 1} = -\frac{a}{2(4\pi - 1)}$$

So, X = 2(4π–1) = (8π–2) = 23.12

So, nearest integer value of X = 23

25. NTA Ans. (35.00)

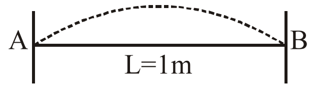
Sol. $\rho_{wire} = 9\times10^{-3}\dfrac{kg}{cm^3} = \dfrac{9\times10^{-3}}{10^{-6}}\,kg/m^3$

$= 9000\ kg/m^2$

(A = CSA of wire)

(Y = 9×10^{10} Nm^2)

(Strain = 4.9×10^{-4})



$$\Rightarrow L = 1m = \frac{\lambda}{2} \Rightarrow \lambda = 2m$$

$$\Rightarrow v = f\lambda \Rightarrow \sqrt{\frac{T}{\mu}} = f\,\lambda$$

Where $Y = \dfrac{T/A}{strain} \Rightarrow T = Y.A.\,strain$

$$\Rightarrow \sqrt{\frac{Y.A.\,strain}{m/L}} = f\times2 \Rightarrow \sqrt{\frac{Y.A.L.\,strain}{M}} = f\times2$$

$$\Rightarrow \sqrt{\frac{Y\times V\times strain}{M}} = f\times2 \Rightarrow \sqrt{\frac{Y\times strain}{\rho}} = f\times2$$

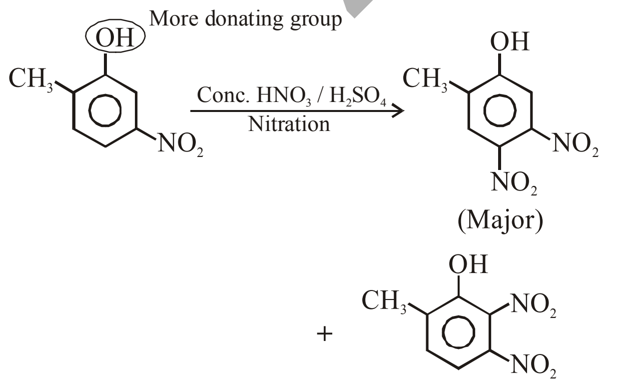
$$f = \frac{1}{2}.\sqrt{\frac{Y\times strain}{\rho}} = \frac{1}{2}\sqrt{\frac{9\times10^{10}\times4.9\times10^{-4}}{9000}}$$

$$f = \frac{1}{2}.\sqrt{\frac{9\times10^{3}}{9}\times4.9} = \frac{1}{2}\sqrt{4900} = \frac{70}{2} = 35\,Hz$$

CHEMISTRY

1. NTA Ans. (3)

Sol.



[Minor due to crowding]

2. NTA Ans. (3)

Sol. Toilet cleaning liquid has about 10.5% w/v HCl; to neutralise its affect aqueous $NaHCO_3$ is used while NaOH is avoid for this purpose because its highly corosive in nature and can burn body.

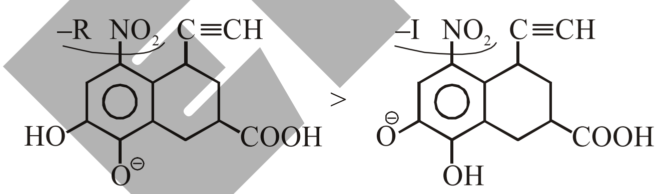
3. NTA Ans. (1)

Sol. Acidic strength order :

$$R-\overset{\overset{O}{||}}{C}-OH > R-OH > R-C\equiv CH$$

Reason : $R-\overset{\overset{O}{||}}{C}-O^{\ominus}$ stable by equivalent resonance.

Stable :



So answer is b > c > d > a.

4. NTA Ans. (2)

Sol. Cast iron is used for manufacturing of wrought iron and steel.

5. NTA Ans. (3)

Sol.

$$\underset{(A)}{CH_3-CH_2-\overset{\overset{O}{||}}{C}-H} \xrightarrow{CH_3MgBr} \underset{(C)}{CH_3-CH_2-\overset{\overset{OH}{|}}{CH}-CH_3}$$

2° Alcohol

CAN test for alcohol : ✓

Iodoform test : ✓

$$\underset{(B)}{CH_3-\overset{\overset{O}{||}}{C}-CH_3} \xrightarrow{CH_3MgBr} CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{OH}{|}}{C}}-CH_3$$

3° Alcohol

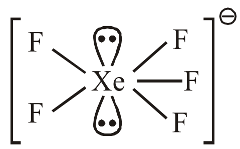
CAN test for alcohol : ✓

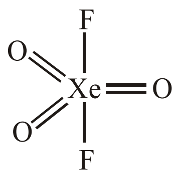
Lucas test : Immediately

Iodoform test : ×

6. NTA Ans. (1)

Sol.



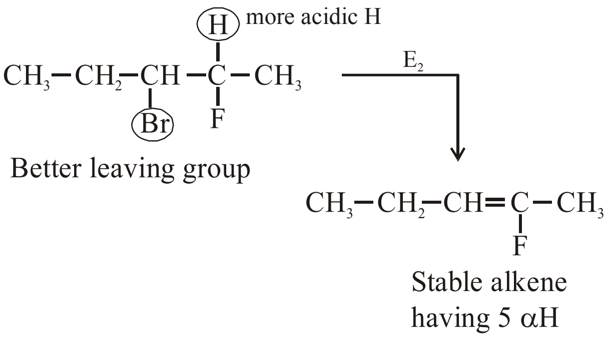


XeF_5^- — sp^3d^3 — Pentagonal planar

XeO_3F_2 — sp^3d — Trigonal bipyramidal

7. NTA Ans. (4)

Sol.



8. NTA Ans. (3)

Sol. When we are moving from left to right in a periodic table acidic character of oxides increases (as well as atomic number of atom increases)

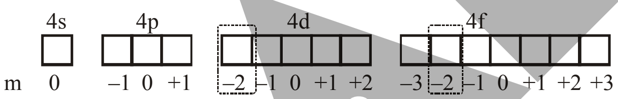
$\therefore$ X < Y < Z (acidic character)

X < Y < Z (atomic number)

9. NTA Ans. (4)

Sol. For n = 4

ℓ = 0, 1, 2, 3



$\therefore$ 4d & 4f subshell associated with n = 4, m = –2

10. NTA Ans. (2)

Sol. Strength of ligand $F^- < NCS^- < NH_3$

$[MF_6]^{(-6+n)}$ (ii) $[M(NCS)]^{(-6+n)}$ (i) $[M(NH_3)_6]^{n+}$ (iii)

$\longrightarrow \Delta_{Oh} \uparrow$

$\longrightarrow \lambda_{max} \downarrow$

As given in graph : A < B < C (λ_{max})

$\therefore$ Correct matching is A–(iii), B-(i), C-(ii)

11. NTA Ans. (1)

Sol. Reaction 1 : SN_1

Reaction 2 : E_2

SN_1 is independent of concentration of nucleophile/base

12. NTA Ans. (1)

Sol. From rate law

$$r = -\frac{1}{2}\frac{d[A]}{dt} = \frac{-d[B]}{dt}$$

$= K[A]^x [B]^y$

$6 \times 10^{-3} = K(0.1)^x (0.1)^y$ ......(1)

$2.4 \times 10^{-2} = K(0.1)^x (0.2)^y$ ......(2)

$1.2 \times 10^{-2} = K(0.2)^x (0.1)^y$ ......(3)

(3) ÷ (1) $\Rightarrow$ x = 1

(2) ÷ (3) $\Rightarrow$ x = 2

So, order with respect to A = 1

Order with respect to B = 2

(4) ÷ (3)

$$\left(\frac{x}{0.2}\right)\times\left(\frac{0.2}{0.1}\right)^2 = \frac{7.2\times10^{-2}}{1.2\times10^{-2}}$$

$$x = \frac{6\times0.2}{4}$$

x = 0.3 M

(5) ÷ (4)

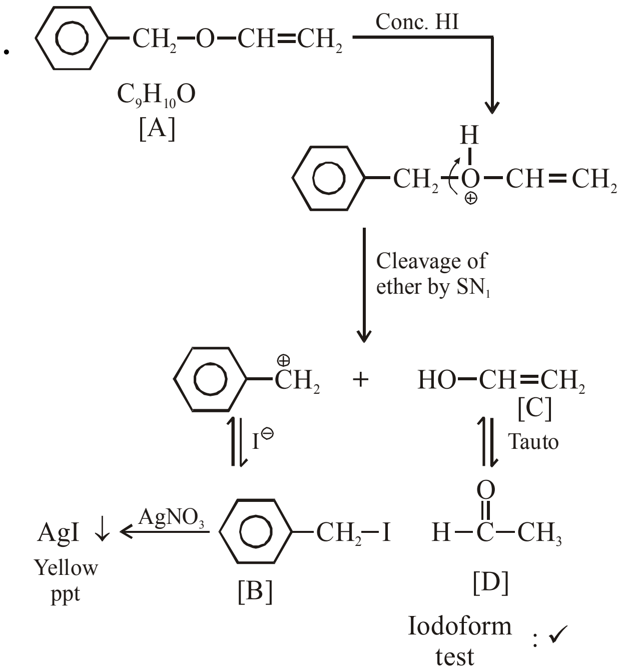
$$\left(\frac{y}{0.2}\right)^2 = \frac{2.88\times10^{-1}}{7.2\times10^{-2}}$$

$y^2 = 4 \times 0.2^2$

y = 0.4 M

13. NTA Ans. (2)

Sol



14. NTA Ans. (3)

Sol. Raw mango shrink in salt solution due to net transfer of water molecules from mango to salt solution due to phenomenon of osmosis.

15. NTA Ans. (2)

Sol. Both Li and Mg form nitride when reacts directly with nitrogen.

The hydrogen carbonate of both Li and Mg does not exist in solid state.

All alkali metal hydrogen carbonate exist in solid state except $LiHCO_3$.

16. NTA Ans. (2)

Sol. $[Ni(NH_3)_2Cl_2]$ is tetrahedral complex, therefore does not show geometrical and optical isomerism.

$[Ni(NH_3)_2Cl_2]$ does not show structural isomerism

$[Ni(NH_3)_4(H_2O)_2]^{2+}$ & $[Pt(NH_3)_2Cl_2]$ show geometrical isomerism

$[Ni(en)_3]^{2+}$ show optical isomerism

17. NTA Ans. (2)

Sol.(a) Since adsorption is exothermic process, as adsorption proceeds number of active sites present over adsorbent decreases, so less heat is evolved.

(b) Since NH_3 has higher force of attraction on adsorbent due to its polar nature (high value of 'a').

(c) As the adsorption increases, residual forces over surface decreases.

(d) Since process is exothermic, on increasing temperature it shift to backward direction, so concentration of adsorbate particle decreases.

18. NTA Ans. (3)

Sol.

Type of interaction	Interaction Energy(E)
ion - ion	$E \propto \frac{1}{r}$
dipole - dipole	$E \propto \frac{1}{r^3}$
London dispersion	$E \propto \frac{1}{r^6}$

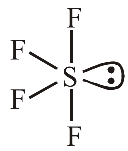
19. NTA Ans. (3)

Sol. Seliwanoff's test is used to distinguish between aldose and ketose sugars; when added to a solution containing ketose, red colour is formed rapidly.

$$\text{Sucrose} \xrightarrow{\text{Hydrolysis}} \text{Glucose} + \text{Fructose} \xrightarrow[\text{reagent}]{\text{Seliwanoff's}} \text{Red colour}$$

20. NTA Ans. (1)

Sol.



4σ bonds +1 lone pair

$\therefore$ Shape (including lone pair of electrons) is Trigonal bipyramidal

21. NTA Ans. (–326400.00)

Sol. $C_2H_5OH_{(\ell)} + 3O_{2(g)} \longrightarrow 2CO_{2(g)} + 3H_2O_{(\ell)}$

$\Delta n_g = 2 - 3 = -1$

$\Delta_c H = \Delta_c U + (\Delta n_g)\,RT$

$\Delta_c H = \Delta_c U - RT$

$\Delta_c U = \Delta_c H + RT$

$= -327 \times 10^3 + 2 \times 300$

$= -326400$ cal.

$\therefore$ Heat evolved

$= 326400$ cal.

22. NTA Ans. (144.00)

Sol.

$$\begin{array}{l} Cu^+ \longrightarrow Cu^{2+} + e^- \\ Cu^+ + e^- \longrightarrow Cu(s) \\ \hline 2Cu^+ \longrightarrow Cu^{2+} + Cu \end{array}$$

$E^o_{cell} = E^o_{Cu^+/Cu} - E^o_{Cu^{2+}/Cu^+}$

$= 0.52 - 0.16$

$= 0.36$ V

At equilibrium $\rightarrow E_{cell} = 0$

$$E^o_{cell} = \frac{RT}{nF} \ln K$$

$$\ln K = \frac{E^o_{cell} \times nF}{RT}$$

$$\ln K = \frac{0.36 \times 1}{0.025} = 14.4 = 144 \times 10^{-1}$$

23. NTA Ans. (19.00)

Sol. $K_2Cr_2O_7$

$2(+1) + 2x + 7(-2) = 0$

$x = +6$

In $K_2Cr_2O_7$, Transition metal (Cr) present in +6 oxidation state.

$KMnO_4$

$(+1) + y + 4(-2) = 0$

$x = +7$

In $KMnO_4$, transition metal (Mn) present in +7 oxidation state

K_2FeO_4

$2(+1) + z + 4(-2) = 0$

$x = +6$

In K_2FeO_4, transition metal (Fe) present in +6 oxidation state

So, $x = +6$
$y = +7$
$z = +6$

$x + y + z = 19$

24. NTA Ans. (5.00)

Sol. C : H = 4 : 1

C : O = 3 : 4

Mass ratio

C : H : O = 12 : 3 : 16

Mole ratio

C : H : O = 1 : 3 : 1

Empirical formula = CH_3O

Molecular formula = $C_2H_6O_2$

(saturated acyclic organic compound)

$$C_2H_6O_2 + \frac{5}{2}O_2 \longrightarrow 2CO_2 + 3H_2O$$

2 mole   5 mol

Moles of O_2 required = 5 moles

25. NTA Ans. (222.00)

Sol. $E = W + K{\cdot}E_{max}$

$K{\cdot}E_{max} = E - W$

$$= \frac{hc}{\lambda} - 4.41 \times 10^{-19}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{300\times10^{-9}} - 4.41 \times 10^{-19}$$

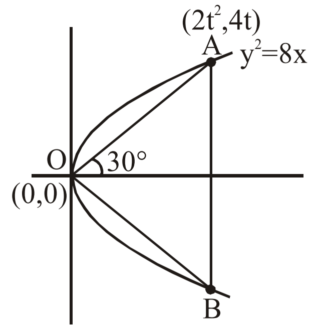
$= 2.22 \times 10^{-19}$ J

$= 222 \times 10^{-21}$ J

MATHEMATICS

1. NTA Ans. (3)

Sol.



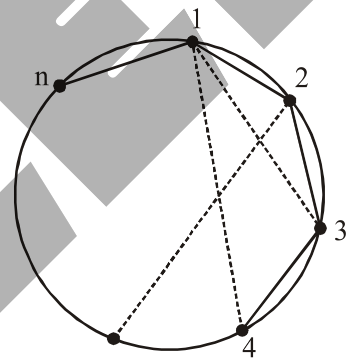
$$\tan 30° = \frac{4t}{2t^2} = \frac{2}{t} \Rightarrow t = 2\sqrt{3}$$

$AB = 8t = 16\sqrt{3}$

$$\text{Area} = 256.3\cdot\frac{\sqrt{3}}{4} = 192\sqrt{3}$$

2. NTA Ans. (3)

Sol.



Number of blue lines = Number of sides = n

Number of red lines = number of diagonals $= {}^nC_2 - n$

$${}^nC_2 - n = 99\,n \Rightarrow \frac{n(n-1)}{2} - n = 99\,n$$

$$\frac{n-1}{2} - 1 = 99 \Rightarrow n = 201$$

3. NTA Ans. (4)

Sol. $\lambda = -(\sin^4\theta + \cos^4\theta)$

$\lambda = -[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$$\lambda = \frac{\sin^2 2\theta}{2} - 1$$

$$\frac{\sin^2 2\theta}{2} \in \left[0, \frac{1}{2}\right]$$

$$\lambda \in \left[-1, -\frac{1}{2}\right]$$

4. NTA Ans. (3)

Sol. $f(x) = a(x - 3)(x - \alpha)$

$f(2) = a(\alpha - 2)$

$f(-1) = 4a(1 + \alpha)$

$f(-1) + f(2) = 0 \Rightarrow a(\alpha - 2 + 4 + 4\alpha) = 0$

$a \neq 0 \Rightarrow 5\alpha = -2$

$\alpha = -\frac{2}{5} = -0.4$

$\alpha \in (-1, 0)$

5. NTA Ans. (1)

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow f(n) = nf(1)$

$f(n) = 2n$

$g(n) = \sum_{k=1}^{n-1} 2n = 2\left(\frac{(n-1)n}{2}\right) = n(n-1)$

$g(n) = 20 \Rightarrow n(n-1) = 20$

$n = 5$

6. NTA Ans. (4)

Sol. $A^TA = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$

and $ab + bc + ca = 0$

Now, $(a + b + c)^2 = 1$

$\Rightarrow a + b + c = \pm 1$

So, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \pm 1(1 - 0) = \pm 1$

$\Rightarrow 3abc = 2 \pm 1 = 3, 1$

$\Rightarrow abc = 1, \frac{1}{3}$

7. NTA Ans. (1)

Sol. $f'(x) = \dfrac{\frac{x}{1+x} - \ell n(1+x)}{x^2}$

$= \dfrac{x - (1+x)\,\ell n(1+x)}{x^2(1+x)}$

Suppose $h(x) = x - (1 + x)\,\ell n(1+x)$

$\Rightarrow h'(x) = 1 - \ell n(1+x) - 1 = -\ell n(1+x)$

$h'(x) > 0, \forall x \in (-1, 0)$

$h'(x) < 0, \forall x \in (0, \infty)$

$h(0) = 0 \Rightarrow h'(x) < 0 \ \forall x \in (-1, \infty)$

$\Rightarrow f'(x) < 0 \ \forall x \in (-1, \infty)$

$\Rightarrow f(x)$ is a decreasing function for all $x \in (-1, \infty)$

8. NTA Ans. (2)

Sol. $a_1 + a_2 + a_3 + \ldots\ldots + a_{11} = 0$

$\Rightarrow (a_1 + a_{11}) \times \frac{11}{2} = 0$

$\Rightarrow a_1 + a_{11} = 0$

$\Rightarrow a_1 + a_1 + 10d = 0$

where d is common difference

$\Rightarrow \boxed{a_1 = -5d}$

$a_1 + a_3 + a_5 + \ldots\ldots + a_{23}$

$= (a_1 + a_{23}) \times \frac{12}{2} = (a_1 + a_1 + 22d) \times 6$

$= \left(2a_1 + 22\left(\frac{-a_1}{5}\right)\right) \times 6$

$= -\frac{72}{5}a_1 \Rightarrow K = \frac{-72}{5}$

9. NTA Ans. (1)

Sol. $\left(3 + 2\sqrt{-54}\right) = 3 + 2 \times 3 \times \sqrt{6}\,i$

$= \left(3 + \sqrt{6}\,i\right)^2$

$\left(3 - 2\sqrt{54}\right) = \left(3 - \sqrt{6}\,i\right)^2$

$\left(3 + 2\sqrt{-54}\right)^{1/2} + \left(3 - 2\sqrt{-54}\right)^{1/2}$

$= \pm\left(3 + \sqrt{6}\,i\right) \pm \left(3 - \sqrt{6}\,i\right)$

$= 6, -6, 2\sqrt{6}i, -2\sqrt{6}i$, (four possible answer)

10. NTA Ans. (4)

Sol. $\lim_{x \to 0}\left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{1/x}$

$= e^{\lim_{x\to 0}\frac{1}{x}\left\{\tan\left(\frac{\pi}{4}+x\right)-1\right\}}$

$= e^{\lim_{x\to 0}\left(\frac{1+\tan x - 1 + \tan x}{x(1-\tan x)}\right)}$

$= e^{\lim_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}$

$= e^2$

11. NTA Ans. (2)

Sol. Given equation of curve

$y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$

at $\boxed{x=0}$

$y = (1 + 0)^{2y} + \cos^2(\sin^{-1}0)$

$y = 1 + 1$

$\boxed{y=2}$

So we have to find the normal at (0, 2)

Now $y = e^{2y\ln(1+x)} + \cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$$y = e^{2y\ln(1+x)} + \left(\sqrt{1-x^2}\right)^2$$

$y = e^{2y\ln(1+x)} + (1-x^2)$ ...(1)

Now differentiate w.r.t. x

$$y' = e^{2y\ln(1+x)}\left[2y\cdot\left(\frac{1}{1+x}\right) + \ln(1+x).2y'\right] - 2x$$

Put x = 0 & y = 2

$$y' = e^{2\times 2\ln 1}\left[2\times 2\left(\frac{1}{1+0}\right) + \ln(1+0).2y'\right] - 2\times 0$$

$y' = e^0[4 + 0] - 0$

$y' = 4$ = slope of tangent to the curve

so slope of normal to the curve $= -\frac{1}{4}$ $\{m_1m_2=-1\}$

Hence equation of normal at (0, 2) is

$$y - 2 = -\frac{1}{4}(x-0)$$

$\Rightarrow 4y - 8 = -x$

$\Rightarrow \boxed{x+4y=8}$

12. NTA Ans. (2)

Sol. Given $\theta \in \left(0, \frac{\pi}{2}\right)$

equation of hyperbola $\Rightarrow x^2 - y^2\sec^2\theta = 10$

$$\Rightarrow \frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1$$

Hence eccentricity of hyperbola

$$(e_H) = \sqrt{1+\frac{10\cos^2\theta}{10}} \quad \ldots(1) \quad \left\{e = \sqrt{1+\frac{b^2}{a^2}}\right\}$$

Now equation of ellipse $\Rightarrow x^2\sec^2\theta + y^2 = 5$

$$\Rightarrow \frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1 \qquad \left\{e = \sqrt{1-\frac{a^2}{b^2}}\right\}$$

Hence eccenticity of ellipse

$$(e_E) = \sqrt{1-\frac{5\cos^2\theta}{5}}$$

$$(e_E) = \sqrt{1-\cos^2\theta} = |\sin\theta| = \sin\theta \quad \ldots(2)$$

$$\left\{\because \theta \in \left(0, \frac{\pi}{2}\right)\right\}$$

given $\Rightarrow e_H = \sqrt{5}\, e_e$

Hence $1 + \cos^2\theta = 5\sin^2\theta$

$1 + \cos^2\theta = 5(1 - \cos^2\theta)$

$1 + \cos^2\theta = 5 - 5\cos^2\theta$

$6\cos^2\theta = 4$

$\cos^2\theta = \frac{2}{3}$ ...(3)

Now length of latus rectum of ellipse

$$= \frac{2a^2}{b} = \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$$

13. NTA Ans. (1)

Sol. Option (1) is

$\sim p\wedge(p\vee q) \to q$

$\equiv (\sim p\wedge p)\vee(\sim p\wedge q) \to q$

$\equiv C\vee(\sim p\wedge q) \to q$

$\equiv (\sim p\wedge q)\to q$

$\equiv \sim(\sim p\wedge q)\vee q$

$\equiv (p\vee\sim q)\vee q$

$\equiv (p\vee q)\vee(\sim q\vee q)$

$\equiv (p\vee q) \vee t$

so $\sim p\wedge(p\vee q) \to q$ is a tautology

14. NTA Ans. (2)

Sol. Hence normal is $\perp^r$ to both the lines so normal vector to the plane is

$$\vec{n} = (\hat{i} - 2\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$$

Now equation of plane passing through (3,1,1) is

$\Rightarrow -4(x-3) + 5(y - 1) + 7(z - 1) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0$ ...(1)

Plane is also passing through $(\alpha, -3, 5)$ so this point satisfies the equation of plane so put in equation (1)

$-4\alpha + 5 \times (-3) + 7 \times (5) = 0$

$\Rightarrow -4\alpha - 15 + 35 = 0$

$\Rightarrow \boxed{\alpha = 5}$

15. NTA Ans. (1)

Sol. Given E_1, E_2, E_3 are pairwise indepedent events

so $P(E_1 \cap E_2) = P(E_1).P(E_2)$

and $P(E_2 \cap E_3) = P(E_2).P(E_3)$

and $P(E_3 \cap E_1) = P(E_3).P(E_1)$

& $P(E_1 \cap E_2 \cap E_3) = 0$

Now $P\left(\dfrac{\bar{E}_2 \cap \bar{E}_3}{E_1}\right) = \dfrac{P\left[E_1 \cap (\bar{E}_2 \cap \bar{E}_3)\right]}{P(E_1)}$

$$= \frac{P(E_1) - \left[P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)\right]}{P(E_1)}$$

$$= \frac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) - 0}{P(E_1)}$$

$= 1 - P(E_2) - P(E_3)$

$= [1 - P(E_3)] - P(E_2)$

$= P(E_3^C) - P(E_2)$

16. NTA Ans. (2)

Sol. Given $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}$, Here $|P| = 0$ & also

given PX = 0

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{aligned} x + 2y + z &= 0 \\ -2x + 3y - 4z &= 0 \\ x + 9y - z &= 0 \end{aligned}\right\} \because D = 0, \text{ so system have}$$

infinite many solutions,

By solving these equation

we get $x = \dfrac{-11\lambda}{2}; y = \lambda; z = \dfrac{7\lambda}{2}$

Also given, $x^2 + y^2 + z^2 = 1$

$$\Rightarrow \left(\frac{-11\lambda}{2}\right)^2 + (\lambda)^2 + \left(\frac{7\lambda}{2}\right)^2 = 1$$

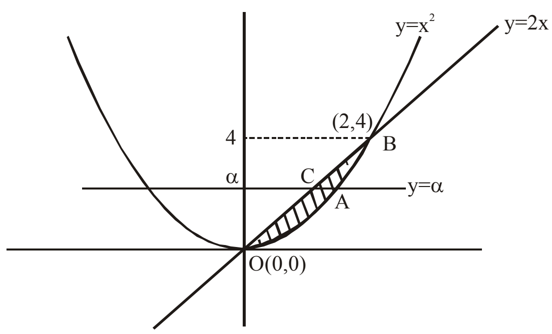
$$\Rightarrow \lambda = \pm\frac{1}{\sqrt{\frac{121}{4} + 1 + \frac{49}{4}}}$$

so, there are 2 values of λ.

$\therefore$ so, there are 2 solution set of (x,y,z).

17. NTA Ans. (4)

Sol.



* $y \geq x^2 \Rightarrow$ upper region of $y = x^2$

$y \leq 2x \Rightarrow$ lower region of $y = 2x$

According to ques, area of OABC = 2 area of OAC

$$\Rightarrow \int_0^4\left(\sqrt{y}-\frac{y}{2}\right)dy = 2\int_0^\alpha\left(\sqrt{y}-\frac{y}{2}\right).dy$$

$$\Rightarrow \frac{4}{3} = 2\left[\frac{2}{3}\alpha^{3/2}-\frac{1}{4}.\alpha^2\right]$$

$$\Rightarrow \boxed{3\alpha^2-8\alpha^{3/2}+8=0}$$

18. NTA Ans. (2)

Sol. $2x^2dy = (2xy + y^2)\,dx$

$$\Rightarrow \frac{dy}{dx}=\frac{2xy+y^2}{2x^2} \quad \{\text{Homogeneous D.E.}\}$$

$$\left\{\begin{array}{l}\text{let } y = xt\\ \Rightarrow \dfrac{dy}{dx} = t + x\dfrac{dt}{dx}\end{array}\right\}$$

$$\Rightarrow t + x\frac{dt}{dx} = \frac{2x^2t+x^2t^2}{2x^2}$$

$$\Rightarrow t + x\frac{dt}{dx} = t + \frac{t^2}{2}$$

$$\Rightarrow x\frac{dt}{dx} = \frac{t^2}{2}$$

$$\Rightarrow 2\int\frac{dt}{t^2} = \int\frac{dx}{x}$$

$$\Rightarrow 2\left(-\frac{1}{t}\right) = \ell n(x) + C \quad \left\{\text{Put } t = \frac{y}{x}\right\}$$

$$\Rightarrow -\frac{2x}{y} = \ell n x + C \quad \left\{\begin{array}{l}\text{Put } x = 1 \,\&\, y = 2\\ \text{then we get } C = -1\end{array}\right\}$$

$$\Rightarrow \frac{-2x}{y} = \ell n(x) - 1$$

$$\Rightarrow y = \frac{2x}{1-\ell n x}$$

$$\Rightarrow f(x) = \frac{2x}{1-\log_e x}$$

so, $\boxed{f\left(\frac{1}{2}\right) = \frac{1}{1+\log_e 2}}$

19. NTA Ans. (3)

Sol. $S = [x + ka + 0] + [x^2 + ka + 2a] + [x^3 + ka + 4a] + [x^4 + ka + 6a] + \ldots\ldots 9$ terms

$\Rightarrow S = (x + x^2 + x^3 + x^4 + \ldots.. 9 \text{ terms}) + (ka + ka + ka + ka + \ldots\ldots. 9 \text{ terms}) + (0 + 2a + 4a + 6a + \ldots\ldots. 9 \text{ terms})$

$$\Rightarrow S = x\left[\frac{x^9-1}{x-1}\right] + 9ka + 72a$$

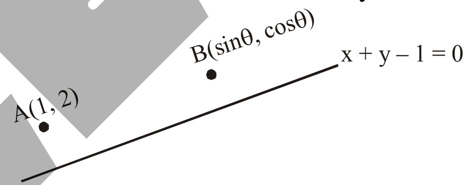
$$\Rightarrow S = \frac{(x^{10}-x)+(9k+72)a(x-1)}{(x-1)}$$

Compare with given sum, then we get, $(9k + 72) = 45$

$$\Rightarrow \boxed{k=-3}$$

20. NTA Ans. (4)

Sol. Given that both points (1, 2) & (sinθ, cosθ) lie on same side of the line $x + y - 1 = 0$



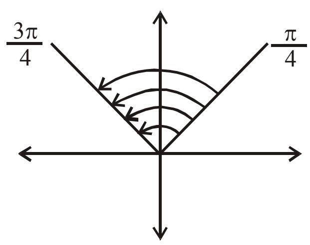
$$\text{So, } \left(\begin{array}{l}\text{Put } (1,2) \text{ in}\\ \text{given line}\end{array}\right)\left(\begin{array}{l}\text{Put } (\sin\theta, \cos\theta \text{ in}\\ \text{given line}\end{array}\right) > 0$$

$$\Rightarrow (1 + 2 - 1)(\sin\theta + \cos\theta - 1) > 0$$

$$\Rightarrow \sin\theta + \cos\theta > 1 \quad \{\div \text{by}\sqrt{2}\}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$$



$$\Rightarrow \frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4}$$

$$\Rightarrow \boxed{0<\theta<\frac{\pi}{2}}$$

21. NTA Ans. (3.00)

Sol. Let a be the first term and d be the common difference of the given A.P. Where d > 0

$$\bar{X} = a + \frac{0 + d + 2d + ... + 10d}{11}$$

$= a + 5d$

$\Rightarrow$ varience $= \frac{\Sigma(\bar{X} - x_i)^2}{11}$

$\Rightarrow 90 \times 11 = (25d^2 + 16d^2 + 9d^2 + 4d^2) \times 2$

$\Rightarrow d = \pm 3 \Rightarrow d = 3$

22. NTA Ans. (91)

Sol. Put $\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5} \quad 0 < \alpha < \frac{\pi}{2}$

Now $\frac{3}{5}\cos kx - \frac{4}{5}\sin kx$

$= \cos\alpha \cdot \cos kx - \sin\alpha \cdot \sin kx$

$= \cos(\alpha + kx)$

As we have to find derivate at x = 0

We have $\cos^{-1}(\cos(\alpha + kx))$

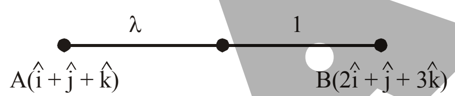
$= (\alpha + kx)$

$$\Rightarrow y = \sum_{k=1}^{6} (\alpha + kx)$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{\text{at } x=0} = \sum_{k=x}^{6} k = \frac{6 \times 7 \times 13}{6} = 91$$

23. NTA Ans. (0.8)

Sol.



Using section formula we get

$$\overline{OP} = \frac{2\lambda+1}{\lambda+1}\hat{i} + \frac{\lambda+1}{\lambda+1}\hat{j} + \frac{3\lambda+1}{\lambda+1}\hat{k}$$

Now $\overline{OB} \cdot \overline{OP} = \frac{4\lambda + 2 + \lambda + 1 + 9\lambda + 3}{\lambda + 1}$

$$= \frac{14\lambda + 6}{\lambda + 1}$$

$$\overline{OA} \times \overline{OP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \frac{2\lambda+1}{\lambda+1} & 1 & \frac{3\lambda+1}{\lambda+1} \end{vmatrix}$$

$$= \frac{2\lambda+1}{\lambda+1}\hat{i} + \frac{-\lambda}{\lambda+1}\hat{j} + \frac{-\lambda}{\lambda+1}\hat{k}$$

$$|\overline{OA} \times \overline{OP}|^2 = \frac{(2\lambda+1)^2 + \lambda^2 + \lambda^2}{(\lambda+1)^2}$$

$$= \frac{6\lambda^2 + 1}{(\lambda+1)^2}$$

$$\Rightarrow \frac{14\lambda + 6}{\lambda + 1} - 3 \times \frac{(6\lambda^2 + 1)}{(\lambda+1)^2} = 6$$

$\Rightarrow 10\lambda^2 - 8\lambda = 0$

$\Rightarrow \lambda = 0, \frac{8}{10} = 0.8$

$\Rightarrow \lambda = 0.8$

24. NTA Ans. (118)

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12$

Now $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5}$

$\Rightarrow 7r = 2n + 2 \qquad ...(1)$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$$

$\Rightarrow 17r = 5n - 12 \qquad ...(2)$

On solving (1) & (2)

$\Rightarrow n = 118$

25. NTA Ans. (1.0)

Sol. $3 < 3x < 6$

Take cases when $3 < 3x < 4$, $4 < 3x < 5$, $5 < 3x < 6$;

Now $\int_1^2 |2x - [3x]|\, dx$

$$= \int_1^{4/3} (3 - 2x)dx + \int_{4/3}^{5/3} (4 - 2x)\, dx + \int_{5/3}^{2} (5 - 2x)dx$$

$$= \frac{2}{9} + \frac{3}{9} + \frac{4}{9} = 1$$

SET # 03

PHYSICS

1. NTA Ans. (4)

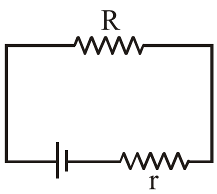
Sol. $LC = \frac{\text{pitch}}{\text{CSD}} = \frac{0.1\text{ cm}}{50} = 0.002\text{ cm}$

So any measurement will be integral Multiple of LC.

So ans. will be 2.124 cm

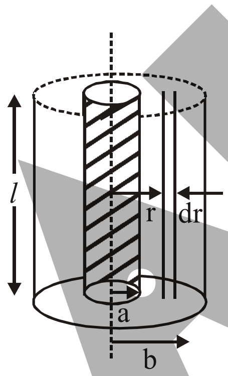
2. NTA Ans. (4)

Sol. Maximum power in external resistance is generated when it is equal to internal resistance of battery.



$P_R = \left(\frac{\in}{r+R}\right)^2 R$

P_R is max. when r = R



$\int dr = \int_a^b \frac{\rho dr}{2\pi r l} \Rightarrow r = \frac{\rho}{2\pi l} ln \frac{b}{a}$

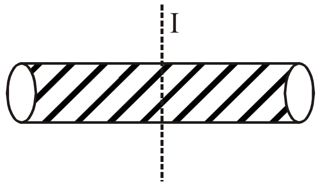
3. NTA Ans. (1)

Sol. $\frac{3}{1} = \frac{\frac{hc}{200\text{ nm}} - \phi}{\frac{hc}{500\text{ nm}} - \phi}$, hc = 1240 eV-nm

On solving ϕ = 0.61 eV

4. NTA Ans. (3)

Sol.



$$I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right) \quad .....(1)$$

as mass is constant $\Rightarrow$ m = ρV = constant

V = constant

$\pi^2 Rl$ = constant $\Rightarrow$ R^2L = constant

$$2RL + R^2\frac{dL}{dR} = 0 \quad .....(2)$$

From equation (1)

$$\frac{dI}{dR} = M\left(\frac{2R}{4} + \frac{2L}{12} \times \frac{dL}{dr}\right) = 0$$

$$\frac{R}{2} + \frac{L}{6}\frac{dL}{dR} = 0$$

Substituting value of $\frac{dL}{dR}$ from eqution (2)

$$\frac{R}{2} + \frac{L}{6}\left(\frac{-2L}{R}\right) = 0$$

$$\frac{R}{2} = \frac{L^2}{3R} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

5. NTA Ans. (2)

Sol. $\vec{B} = 3 \times 10^{-8} \sin[200\pi(y+ct)]\hat{i}\ T$

$E_0 = CB_0 \Rightarrow E_0 = 3 \times 10^8 \times 3 \times 10^{-8}$ = 9 V/m

and direction of wave propagation is given as

$\left(\vec{E} \times \vec{B}\right) \parallel \vec{C}$

$\hat{B} = \hat{i}$ & $\hat{C} = -\hat{j}$

so $\hat{E} = -\hat{k}$

$\therefore \vec{E} = E_0 \sin[200\pi(y+ct)](-\hat{k})\ V/m$

6. NTA Ans. (3)

Sol. $\vec{F} = 9\left(\vec{V} \times \vec{B}\right)$ (Force on charge particle moving in magnetic field)

$\vec{V} \times \vec{B} = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times (5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3}$

$$= \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 3 & -6 \end{pmatrix} \times 10^{-3}$$

$$= \left[\hat{i}[-18-12] - \hat{j}[-12-20] + \hat{k}[6-15]\right] \times 10^{-3}$$

$$= \left[\hat{i}[-30] + \hat{j}[32] + \hat{k}[-9]\right] \times 10^{-3}$$

$$\text{Force} = 10^{-6}[-30\hat{i} + 32\hat{j} - 9\hat{k}] \times 10^{-3}$$

$$= 10^{-9}[-30\hat{i} + 32\hat{j} - 9\hat{k}]$$

7. NTA Ans. (3)

Sol. f = 750 Hz, V_{rms} = 20V,
R = 100 Ω, L = 0.1803 H,
C = 10 μF, S = 2 J/°C

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$= \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$

Putting values
|Z| = 834Ω
In AC power P = $V_{rms}\, i_{rms} \cos\phi$

$$\text{Cos}\phi = \frac{R}{|Z|} \qquad i_{rms} = \frac{V_{rms}}{|Z|}$$

$$= \frac{V_{rms}^2 R}{(|Z|)^2}$$

$$= \left(\frac{20}{834}\right)^2 \times 100 = 0.0575\ \text{J/s}$$

H = Pt = SΔθ

$$t = \frac{2(10)}{0.0575} = 348\ \text{sec}$$

8. NTA Ans. (1)

Sol. First order decay
$N(t) = N_0 e^{-\lambda t}$
Given $N(t) / N_0 = 9/16 = e^{-\lambda t}$
Now, $N(t/2) = N_0\, e^{-\lambda t/2}$

$$\frac{N(t/2)}{N_0} = \sqrt{e^{-\lambda t}} = \sqrt{9/16}$$

$N(t/2) = 3/4\ N_0$

9. NTA Ans. (2)

Sol. Bursting of helium balloon is irreversible & adiabatic.

10. NTA Ans. (1)

Sol. $\Delta P_1 = 0.01 = 4T/R_1$ .....(1)
$\Delta P_2 = 0.02 = 4T/R_2$ .....(2)
Equation (1) ÷ (2)

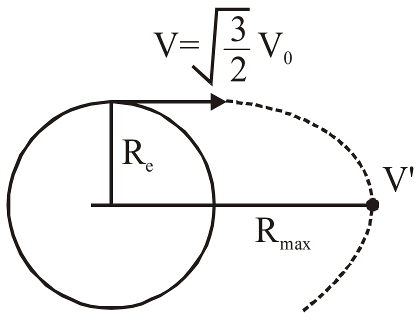
$$\frac{1}{2} = \frac{R_2}{R_1}$$

$R_1 = 2R_2$

$$\frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = 8$$

11. NTA Ans. (2)

Sol.



$$V_0 = \sqrt{\frac{GM}{R_e}}$$

$$\frac{-GMm}{R_e} + \frac{1}{2}mv^2 = \frac{-GMm}{R_{max}} + \frac{1}{2}mv'^2 \quad ....(i)$$

$VR_e = V'R_{max}$ ....(ii)
Solving (i) & (ii)

$$\boxed{R_{max} = 3R_e}$$

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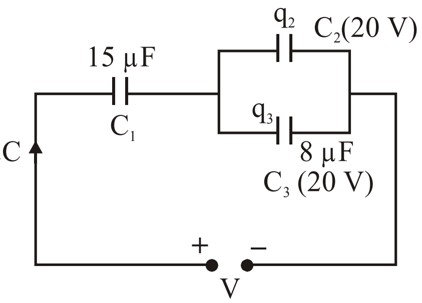
12. NTA Ans. (1)

Sol.


$q_3 = 20 \times 8 = 160\ \mu C$

$\therefore\ q_2 = 750 - 160 = 590\ \mu C$

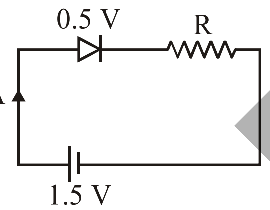
13. NTA Ans. (1)

Sol. $\in = NAB\omega\cos\omega t$ $\boxed{N=1}$

$$P_{avg} = < \frac{\in^2}{R} > = < \frac{(AB\omega\cos\omega t)^2}{R} >$$

$$= \frac{A^2B^2\omega^2}{R}\frac{1}{2} = \frac{\pi^2a^2b^2B^2\omega^2}{2R}$$

14. NTA Ans. (1)

Sol.


$1.5 - 0.5 - R \times 10 \times 10^{-3} = 0$

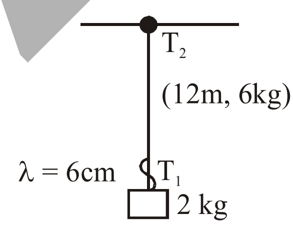
$\therefore\ R = 100\ \Omega$

15. NTA Ans. (2)

Sol. $V \propto \lambda$ $T_2 = 8g$

$T_1 = 2g$

$$\frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$



$$\lambda_2 = \frac{V_2}{V_1}\lambda_1 = \sqrt{\frac{T_2}{T_1}} \times \lambda_1$$

$$= \sqrt{\frac{8g}{2g}}\lambda_1 = 2 \times 6 = 12\text{ cm}$$

16. NTA Ans. (2)

Sol. Angular momentum conservation

$$mvl = \frac{Ml^2}{3}\omega + ml^2\omega$$

$$\Rightarrow \omega = \frac{1 \times 6 \times 1}{\frac{2}{3}+1} = \frac{18}{5}$$

Now using energy consevation

$$\frac{1}{2}\left(M\frac{l^2}{3}\right)\omega^2 + \frac{1}{2}(ml^2)\omega^2$$

$$= (m+M)r_{cm}(1-\cos\theta)$$

$$= (m+M)\left(\frac{ml + \frac{Ml}{2}}{m+M}\right)g(1-\cos\theta)$$

$$\frac{5}{6}\times\left(\frac{18}{5}\right)^2 = 20(1-\cos\theta)$$

$$\Rightarrow 1-\cos\theta = \frac{18}{5}\times\frac{3}{20}$$

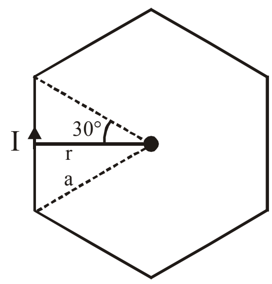
$$\cos\theta = 1 - \frac{27}{50}$$

$$\cos\theta = \frac{23}{50} \Rightarrow \theta \simeq 63°$$

17. NTA Ans. (4)

Sol. $\Delta\theta_0 = \left(\frac{\lambda}{d}\times\frac{180}{\pi}\right)^0 = 0.57°$

18. NTA Ans. (3)

Sol.


r = acos30°

$$B = \frac{6\mu_0 I}{4\pi a\cos 30°}\times 2\sin 30° \times 50$$

$$= \frac{\mu_0 I}{\pi}\frac{150}{\sqrt{3}a} = \frac{50\sqrt{3}}{0.1}\frac{\mu_0 I}{\pi}$$

$$= 500\sqrt{3}\frac{\mu_0 I}{\pi}$$

19. NTA Ans. (4)

Sol. DOF = 3 + 3 = 6

$$U = \frac{f}{2}nRT = 3RT$$

20. NTA Ans. (1)

Sol. Now

$Q_1 + Q_2 = Q'_1 + Q'_2 = 12\ \mu C - 3\ \mu C = 9\ \mu C$

$$\&\ V_1 = V_2 \Rightarrow \frac{KQ'_1}{\frac{2R}{3}} = \frac{KQ'_2}{\frac{R}{3}}$$

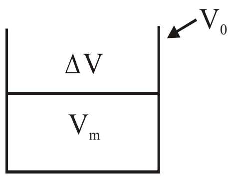
$$Q'_1 = 2Q'_2 \Rightarrow 2Q'_2 + Q'_2 = 9\mu C$$

$\Rightarrow Q'_2 = 3\ \mu C$

& $Q'_1 = 6\ \mu C$

21. NTA Ans. (20)

Sol.



$$\boxed{\Delta V = (V_0 - V_m)}$$

After increasing temperature

$\Delta V' = (V'_0 - V'_m)$

$\Delta V' = \Delta V$

$V_0 - V_m = V_0(1+\gamma_b\Delta T) - V_m\ (1+\gamma_M\Delta T)$

$V_0\gamma_b = V_m\gamma_m$

$$V_m = \frac{V_0\gamma_b}{\gamma_m} = \frac{(500)(6\times10^{-6})}{(1.5\times10^{-4})}$$

= 20 CC

22. NTA Ans. (150)

Sol. $W_F = \frac{1}{2}mv^2 = mgh$

F(S) = mgh

F(0.2) = (0.15) (10) (20)

$$\boxed{F = 150N}$$

23. NTA Ans. (101)

Sol. Capillary rise

$$h = \frac{2S\cos\theta}{\rho gr}$$

$$S = \frac{\rho grh}{2\cos\theta}$$

$$= \frac{(900)(10)(15\times10^{-5})(15\times10^{-2})}{2}$$

$S = 1012.5 \times 10^{-4}$

$S = 101.25 \times 10^{-3} = 101.25$ mN/m

In question closest integer is asked

so closest integer = 101.00 Ans.

24. NTA Ans. (9)

Sol. $L_i = L_f$

$$\left(80R^2 + \frac{200R^2}{2}\right)\omega = \left(0 + \frac{200R^2}{2}\right)\omega_1$$

$180\omega_0 = 100\omega_1$

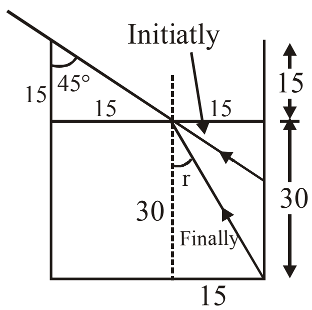
$\omega_1 = 1.8\omega_0 = 1.8 \times 5$

= 9 rpm

25. NTA Ans. (158)

Sol. $\tan r = \frac{15}{30} = \frac{1}{2}$

$$\sin r = \frac{1}{\sqrt{5}}$$



$1\sin45° = \mu\sin r$

$$\frac{1}{\sqrt{2}} = \mu\left(\frac{1}{\sqrt{5}}\right)$$

$$\boxed{\mu = \sqrt{\frac{5}{2}} = 1.581}$$

$$\frac{N}{100} = \mu$$

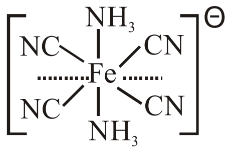
$N = 100\ \mu$

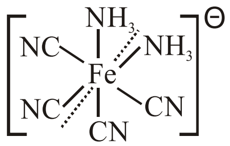
N = 158.11

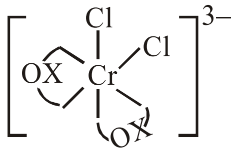
So integer value of N = 158

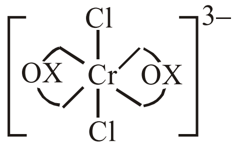
CHEMISTRY

1. NTA Ans. (3)

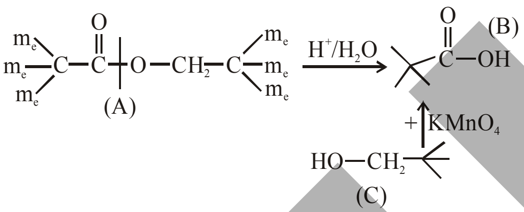
Sol. (1)  optically inactive

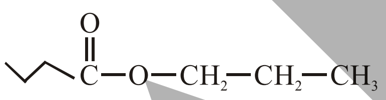
(2)  optically inactive

(3)  optically active

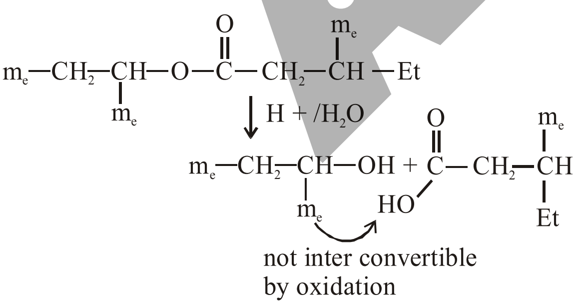
(4)  optically inactive

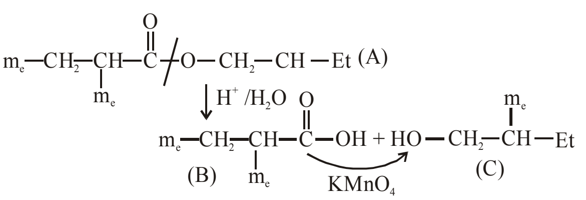
2. NTA Ans. (3)

Sol. (1) 

(2) 

Total 8 'C' $\rightarrow$ so molecular formula not matched.

(3) 

(4) 

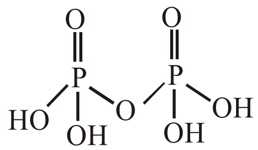
3. NTA Ans. (2)

Sol. Boiling point of H_2S < Boiling point of H_2O

(213 K) (373 K)

4. NTA Ans. (4)

Sol. Pyrophosphoric acid.



P – OH linkages = 4

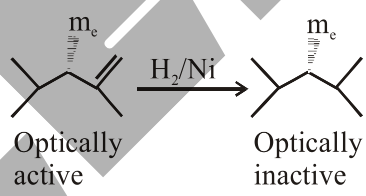
P = O linkages = 2

P–O–P linkages = 1

5. NTA Ans. (4)

Sol. Zero order reaction is multiple step reaction.

6. NTA Ans. (2)

Sol. 

7. NTA Ans. (2)

Sol. (1) $P_\gamma = K_H X_Y$

$$P_\gamma = 2\times10^{-15}\times\frac{55.5}{55.5+\frac{1000}{18}} = 2\times10^{-5}\text{K bar}$$

$= 2 \times 10^{-2}$ bar

(2) $P_\delta = K_H X_\delta$

$$P_\delta = 0.5\times\frac{55.5}{55.5+\frac{1000}{18}} = .249\text{ K bar} = 249\text{ bar}$$

(3) On increasing temperature solubility of gases decreases

(4) $K_H \downarrow$ solubility $\uparrow$ and lowest K_H is for γ.

8. NTA Ans. (3)

Sol. The diameter of disperseed particles is similar to wavelength of light used.

9. NTA Ans. (3)

Sol. Thermal power plants lead to acid rain.

10. NTA Ans. (4)

Sol. CFSE = $0.4\ \Delta_0$

$$= 0.4 \times \frac{20300}{83.7}$$

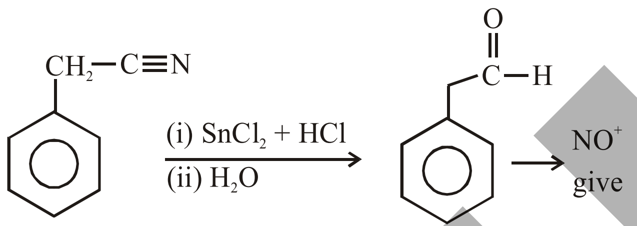
$$= 97 \text{ kJ/mol}$$

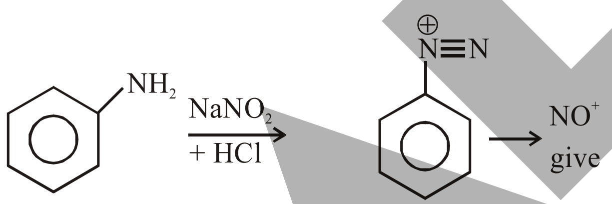
11. NTA Ans. (3)

Sol. $Au + HNO_3 + 4HCl \rightarrow HAuCl_4 + NO + 2H_2O$

12. NTA Ans. (2)

Sol. Kjeldahl method is used for N estimation But not given by 'Diazo' compounds



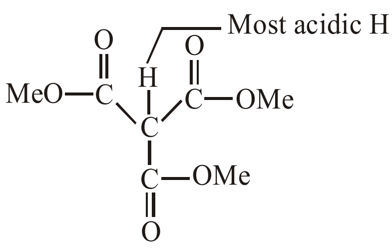


13. NTA Ans. (2)

Sol. $S_N{}^1$ favours
(a) The reaction is favoured by weak nucleophiles
(b) $R^{\oplus}$ would be easily formed if the substituents are bulky
(c) The reaction is accompained by recemization

14. NTA Ans. (4)

Sol.



Due to presence of 3 (–R) groups

15. NTA Ans. (3)

Sol. Glycerol is separated by reduced pressure distillation in soap industries.

16. NTA Ans. (4)

Sol. Bond order of NO^{2+} = 2.5
Bond order of NO^+ = 3
Bond order of NO = 2.5
Bond order of NO^- = 2
Bond order α bond strength.

17. NTA Ans. (4)

Sol. 1 0 9
un nil enn
Hence correct name → unnilennium

18. NTA Ans. (3)

Sol.

$$\underset{\substack{10\text{ milli mol}\\ —}}{HCl} + \underset{\substack{20\text{ milli mol}\\ 10\text{ milli mol}}}{CH_3COONa} \rightarrow \underset{\substack{—\\ 10\text{ milli mol}}}{CH_3COOH} + \underset{\substack{—\\ 10\text{ milli mol}}}{NaCl}$$

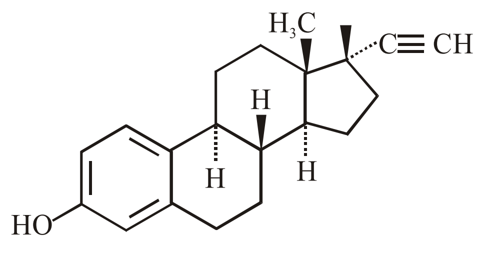
So finally we get mixture of $CH_3COOH + CH_3COONa$ that will work like acidic buffer solution.

19. NTA Ans. (Bonus)

Sol. Dissolution of $BaSO_4$ is an endothermic reaction 50 on increasing temperature number of ions of $BaSO_4$ decrease so it's conduction also decrease.

20. NTA Ans. (1)

Sol.



Ethynylestradiol (novestrol)

gives (1) $Br_2 + H_2O$ test
(2) Lucas test with $ZnCl_2$ + HCl
(3) $FeCl_3$ test of phenolic group.

21. NTA Ans. (100)

Sol. Volume strength of H_2O_2 at 1 atm
273 kelvin = M × 11.2 = 8.9 × 11.2 = 99.68
Ans : 100

22. **NTA Ans. (47)**

Sol. $X_{C_6H_{12}O_6} = 0.1$

Let total mole is 1 mol then mole of glucose will be 0.1 and mole of water will be 0.9

so mass % of water $= \dfrac{0.9\times18}{0.1\times180+0.9\times18}\times100$

$= 47.36$

Ans : 47

23. **NTA Ans. (58.00)**

Sol. $\frac{1}{2}H_2 \rightarrow H^+ + e^{\ominus}$

$e^{\ominus} + AgCl_{(s)} \rightarrow Ag_{(s)} + Cl^{\ominus}$

$\frac{1}{2}H_2 + AgCl_{(s)} \rightarrow H^+_{(aq)} + Ag_{(s)} + Cl^{\ominus}_{(aq)}$

$$E = \varepsilon^0 - \frac{.06}{1}\log\frac{[H^+][Cl^{\ominus}]}{P_{H_2}^{\frac{1}{2}}}$$

$$E = 0.22 - .06\log\frac{(10^{-1})(10^{-1})}{1^{\frac{1}{2}}}$$

E = 0.22 + .12 = .34 volt

$\Rightarrow$ total energy of photon will be (for Na)

= 2.3 + 0.34 = 2.64 eV

$\Rightarrow$ stopping potential required for K

= 2.64 – 2.25 = 0.39 volt

$$E = \varepsilon^0 - \frac{.06}{1}\log\frac{[H^+][Cl^-]}{P_{H_2}^{\frac{1}{2}}}$$

as $[H^+] = [Cl^{\ominus}]$ so

$$0.39 = 0.22 - .06\log\frac{[H^+]^2}{1^{\frac{1}{2}}}$$

0.17 = + .12 pH

pH = 1.4166 $\Rightarrow$ 1.42

24. **NTA Ans. (143)**

Sol. $$d = \frac{z\left(\frac{M}{N_A}\right)}{a^3}$$

$$2.7\times10^3 = z\frac{\left(\frac{2.7\times10^{-2}}{6\times10^{23}}\right)}{\left(405\times10^{-12}\right)^3}$$

$$2.7\times10^3 = z\frac{\left(2.7\times10^{-2}\right)}{6\times10^{23}\left(4.05\times10^{-10}\right)^3}$$

$$2.7\times10^3 = z\frac{\left(2.7\times10^{-2}\right)}{6\times10^{23}\times66.43\times10^{-30}}$$

3.98 = z

$z \approx 4$ structure is fcc

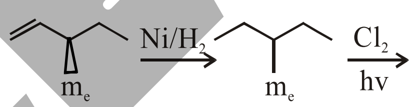
$$\frac{a}{\sqrt{2}} = 2r$$

$$r = \frac{a}{2\sqrt{2}} = \frac{\sqrt{2}a}{4} = \frac{1.414\times405\times10^{-12}}{4}$$

r = 143.16 × 10^{-12}

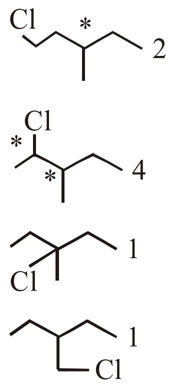
25. **NTA Ans. (8)**

Sol.



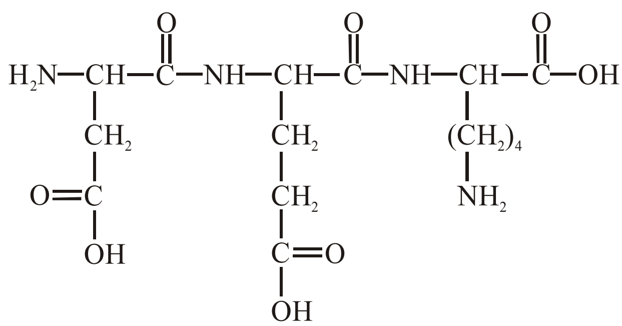
Simplest

O.A. Alkene



Alter

Str. of Tri peptide



MATHEMATICS

1. NTA Ans. (2)

Sol. A : Sum obtained is a multiple of 4.

A = {(1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)}

B : Score of 4 has appeared at least once.

B = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}

Required probability = $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}$

$$=\frac{1/36}{9/36}=\frac{1}{9}$$

2. NTA Ans. (3)

Sol. $\vec{r}=\hat{i}(1+2\ell)+\hat{j}(-1)+\hat{k}(\ell)$

$\vec{r}=\hat{i}(2+m)+\hat{j}(m-1)+\hat{k}(-m)$

For intersection

$1 + 2\ell = 2 + m$ ...... (i)

$-1 = m - 1$ ...... (ii)

$\ell = -m$ ...... (iii)

from (ii) m = 0

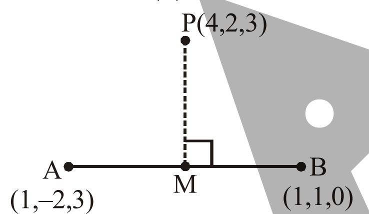
from (iii) $\ell = 0$

These values of m and ℓ do not satisfy equation (1).

Hence the two lines do not intersect for any values of ℓ and m.

3. NTA Ans. (4)

Sol.



Equation of AB = $\vec{r}=(\hat{i}+\hat{j})+\lambda(3\hat{j}-3\hat{k})$

Let coordinates of M = (1, (1 + 3λ), –3λ).

$\overline{PM}=-3\hat{i}+(3\lambda-1)\hat{j}-3(\lambda+1)\hat{k}$

$\overline{AB}=3\hat{j}-3\hat{k}$

$\because$ $\overline{PM}\perp\overline{AB} \Rightarrow \overline{PM}\,.\,\overline{AB}=0$

$\Rightarrow$ $3(3\lambda - 1) + 9(\lambda + 1) = 0$

$\Rightarrow \lambda=-\frac{1}{3}$

$\therefore$ M = (1, 0, 1)

Clearly M lies on 2x + y – z = 1.

4. NTA Ans. (2)

Sol. Ellipse : $\frac{x^2}{4}+\frac{y^2}{3}=1$

eccentricity = $\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

$\therefore$ foci = (±1, 0)

for hyperbola, given 2a = $\sqrt{2}$ $\Rightarrow$ a = $\frac{1}{\sqrt{2}}$

$\therefore$ hyperbola will be

$$\frac{x^2}{1/2}-\frac{y^2}{b^2}=1$$

eccentricity = $\sqrt{1+2b^2}$

$\therefore$ foci = $\left(\pm\sqrt{\frac{1+2b^2}{2}},0\right)$

$\because$ Ellipse and hyperbola have same foci

$\Rightarrow$ $\sqrt{\frac{1+2b^2}{2}}=1$

$\Rightarrow$ $b^2=\frac{1}{2}$

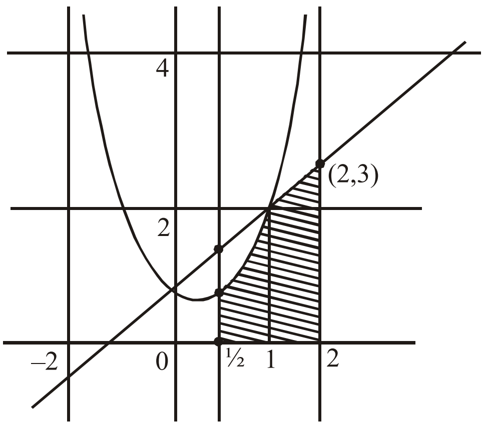
$\therefore$ Equation of hyperbola : $\frac{x^2}{1/2}-\frac{y^2}{1/2}=1$

$\Rightarrow$ $x^2 - y^2 = \frac{1}{2}$

Clearly $\left(\sqrt{\frac{3}{2}},\frac{1}{\sqrt{2}}\right)$ does not lie on it.

5. NTA Ans. (3)

Sol. $0 \le y \le x^2 + 1,\ 0 \le y \le x + 1,\ \frac{1}{2} \le x \le 2$



Required area = $\int_{1/2}^{1}(x^2+1)dx+\frac{1}{2}(2+3)\times 1$

$$=\frac{19}{24}+\frac{5}{2}=\frac{79}{24}$$

6. **NTA Ans. (4)**

Sol. Sum of 1st 25 terms = sum of its next 15 terms

$\Rightarrow$ $(T_1 + ... + T_{25}) = (T_{26} + ... + T_{40})$

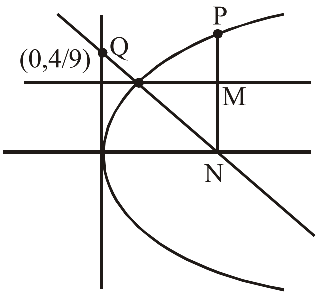
$\Rightarrow$ $(T_1 + ... + T_{40}) = 2(T_1 + ... + T_{25})$

$$\Rightarrow \frac{40}{2}[2 \times 3 + (39d)] = 2 \times \frac{25}{2}[2 \times 2 + 24d]$$

$$\Rightarrow d = \frac{1}{6}$$

7. **NTA Ans. (3)**

Sol.



Let $P = (3t^2, 6t)$; $N = (3t^2, 0)$

$M = (3t^2, 3t)$

Equation of MQ : $y = 3t$

$$\therefore \quad Q = \left(\frac{3}{4}t^2, 3t\right)$$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3}{4}t^2 - 3t^2\right)}(x - 3t^2)$$

y-intercept of NQ = $4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$

$$\therefore \quad MQ = \frac{9}{4}t^2 = \frac{1}{4}$$

$PN = 6t = 2$

8. **NTA Ans. (4)**

Sol. $\because \sigma^2 \le \frac{1}{4}(M - m)^2$

Where M and m are upper and lower bounds of values of any random variable.

$$\therefore \quad \sigma^2 < \frac{1}{4}(10 - 0)^2$$

$\Rightarrow$ $0 < \sigma < 5$

$\therefore$ $\sigma \ne 6$.

9. **NTA Ans. (1)**

Sol. $$\int_{-\pi}^{\pi} \big|\pi - |x|\big| dx = 2\int_0^{\pi} |\pi - x| dx$$

$$= 2\int_0^{\pi} (\pi - x) dx$$

$$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \pi^2$$

10. **NTA Ans. (1)**

Sol. A : $D \ge 0$

$\Rightarrow$ $(m + 1)^2 - 4(m + 4) \ge 0$

$\Rightarrow$ $m^2 + 2m + 1 - 4m - 16 \ge 0$

$\Rightarrow$ $m^2 - 2m - 15 \ge 0$

$\Rightarrow$ $(m - 5)(m + 3) \ge 0$

$\Rightarrow$ $m \in (-\infty, -3] \cup [5, \infty)$

$\therefore$ $A = (-\infty, -3] \cup [5, \infty)$

$B = [-3, 5)$

$A - B = (-\infty, -3) \cup [5, \infty)$

$A \cap B = \{-3\}$

$B - A = (-3, 5)$

$A \cup B = R$

11. **NTA Ans. (1)**

Sol. $y^2 + \ln(\cos^2 x) = y \qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

for $x = 0$ $\qquad$ $y = 0$ or 1

Differentiating wrt x

$\Rightarrow$ $2yy' - 2\tan x = y'$

At $(0, 0)$ $y' = 0$

At $(0, 1)$ $y' = 0$

Differentiating wrt x

$2yy'' + 2(y')^2 - 2\sec^2 x = y''$

At $(0, 0)$ $y'' = -2$

At $(0, 1)$ $y'' = 2$

$\therefore$ $|y''(0)| = 2$

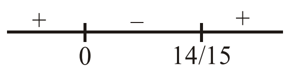
12. NTA Ans. (2)

Sol. $f(x) = (3x - 7)x^{2/3}$

$\Rightarrow f(x) = 3x^{5/3} - 7x^{2/3}$

$\Rightarrow f'(x) = 5x^{2/3} - \dfrac{14}{3x^{1/3}}$

$$= \frac{15x - 14}{3x^{1/3}} > 0$$



$\therefore f'(x) > 0 \ \forall \ x \in (-\infty, 0) \cup \left(\dfrac{14}{15}, \infty\right)$

13. NTA Ans. (3)

Sol. $S = (2.{}^1p_0 - 3.{}^2p_1 + 4.{}^3p_2$ .... upto 51 terms)

+ (1! + 2! + 3! .... upto 51 terms)

$[\because {}^np_{n-1} = n!]$

$\therefore S = (2 \times 1! - 3 \times 2! + 4 \times 3! \ .... + 52.51!)$

$+ (1! - 2! + 3! \ .... (51)!)$

$= (2! - 3! + 4! \ .... + 52!)$

$+ (1! - 2! + 3! - 4! + \ ...... + (51)!)$

$= 1! + 52!.$

14. NTA Ans. (3)

Sol. $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}$

$= Ax^3 + Bx^2 + Cx + D.$

$R_2 \to R_2 - R_1 \qquad R_3 \to R_3 - R_2$

$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-1 & x-1 & x-1 \\ x-2 & 2(x-2) & 6(x-2) \end{vmatrix}$

$= (x-1)(x-2) \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix}$

$= -3(x-1)^2(x-2) = -3x^3 + 12x^2 - 15x + 6$

$\therefore B + C = 12 - 15 = -3$

15. NTA Ans. (1)

Sol. $(1 + e^{-x})(1 + y^2)\dfrac{dy}{dx} = y^2$

$\Rightarrow (1 + y^{-2})\,dy = \left(\dfrac{e^x}{1+e^x}\right)dx$

$\Rightarrow \left(y - \dfrac{1}{y}\right) = \ell n(1+e^x) + c$

$\therefore$ It passes through $(0, 1) \Rightarrow c = -\ell n 2$

$\Rightarrow y^2 = 1 + y\,\ell n\left(\dfrac{1+e^x}{2}\right)$

16. NTA Ans. (2)

Sol. $T_{r+1} = {}^nC_r (3)^{\frac{n-r}{2}} (5)^{\frac{r}{8}} \qquad (n \geq r)$

Clearly r should be a multiple of 8.

$\because$ there are exactly 33 integral terms

Possible values of r can be

0, 8, 16, ... , 32×8

$\therefore$ least value of n = 256.

17. NTA Ans. (3)

Sol. α, β are roots of $x^2 + px + 2 = 0$

$\Rightarrow \alpha^2 + p\alpha + 2 = 0 \ \& \ \beta^2 + p\beta + 2 = 0$

$\Rightarrow \dfrac{1}{\alpha}, \dfrac{1}{\beta}$ are roots of $2x^2 + px + 1 = 0$

But $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ are roots of $2x^2 + 2qx + 1 = 0$

$\Rightarrow p = 2q$

Also $\alpha + \beta = -p \qquad \alpha\beta = 2$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \left(\frac{\alpha^2 - 1}{\alpha}\right)\left(\frac{\beta^2 - 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)$$

$$= \frac{(-p\alpha - 3)(-p\beta - 3)(\alpha\beta + 1)^2}{(\alpha\beta)^2}$$

$$= \frac{9}{4}(p\alpha\beta + 3p(\alpha + \beta) + 9)$$

$$= \frac{9}{4}(9 - p^2) = \frac{9}{4}(9 - 4q^2)$$

18. NTA Ans. (2)

Sol. LHL : $\lim\limits_{x\to 0^-}\left|\dfrac{1-x-x}{\lambda-x-1}\right| = \left|\dfrac{1}{\lambda-1}\right|$

RHL : $\lim\limits_{x\to 0^+}\left|\dfrac{1-x+x}{\lambda-x+1}\right| = \left|\dfrac{1}{\lambda}\right|$

For existence of limit

LHL = RHL

$\Rightarrow \dfrac{1}{|\lambda-1|} = \dfrac{1}{|\lambda|} \Rightarrow \lambda = \dfrac{1}{2}$

$\therefore\ L = \dfrac{1}{|\lambda|} = 2$

19. NTA Ans. (3)

Sol. $2\pi - \left(\sin^{-1}\left(\dfrac{4}{5}\right) + \sin^{-1}\left(\dfrac{5}{13}\right) + \sin^{-1}\left(\dfrac{16}{65}\right)\right)$

$= 2\pi - \left(\tan^{-1}\left(\dfrac{4}{3}\right) + \tan^{-1}\left(\dfrac{5}{12}\right) + \tan^{-1}\left(\dfrac{16}{63}\right)\right)$

$= 2\pi - \left(\tan^{-1}\left(\dfrac{63}{16}\right) + \tan^{-1}\left(\dfrac{16}{63}\right)\right)$

$= 2\pi - \dfrac{\pi}{2} = \dfrac{3\pi}{2}$

20. NTA Ans. (1)

Sol. $p \to \sim(p \wedge \sim q)$

$= \sim p \vee \sim(p \wedge \sim q)$

$= \sim p \vee \sim p \vee q$

$= \sim(p \wedge q) \vee q$

$= \sim p \vee q$

21. NTA Ans. (10)

Sol. $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$= \begin{bmatrix} (x^2+1)^2 + x^2 & x(x^2+1)+x \\ x(x^2+1)+x & x^2+1 \end{bmatrix}$

$a_{11} = (x^2+1)^2 + x^2 = 109$

$\Rightarrow x = \pm 3$

$a_{22} = x^2 + 1 = 10$

22. NTA Ans. (8)

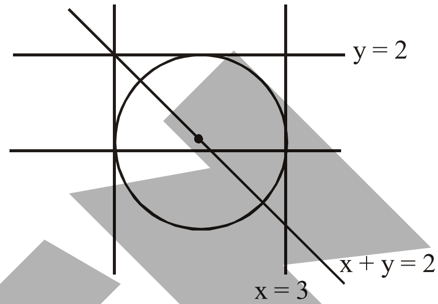
Sol. $\lim\limits_{x\to 0}\left\{\dfrac{1}{x^8}\left(1-\cos\dfrac{x^2}{2} - \cos\dfrac{x^2}{4} + \cos\dfrac{x^2}{2}\cos\dfrac{x^2}{4}\right)\right\} = 2^{-k}$

$\Rightarrow \lim\limits_{x\to 0}\dfrac{\left(1-\cos\dfrac{x^2}{2}\right)}{4\left(\dfrac{x^2}{2}\right)^2}\dfrac{\left(1-\cos\dfrac{x^2}{4}\right)}{16\left(\dfrac{x^2}{4}\right)^2} = \dfrac{1}{8}\times\dfrac{1}{32} = 2^{-k}$

$\Rightarrow 2^{-8} = 2^{-k} \Rightarrow k = 8.$

23. NTA Ans. (3)

Sol.



$\because$ center lies on x + y = 2 and in 1st quadrant

center = $(\alpha, 2-\alpha)$

where $\alpha > 0$ and $2 - \alpha > 0 \Rightarrow 0 < \alpha < 2$

$\because$ circle touches x = 3 and y = 2

$\Rightarrow |3-\alpha| = |2-(2-\alpha)| =$ radius

$\Rightarrow |3-\alpha| = |\alpha| \Rightarrow \alpha = \dfrac{3}{2}$

$\therefore$ radius = α

$\Rightarrow$ Diameter = 2α = 3.

24. NTA Ans. (4)

Sol. $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\ldots\ldots\ldots\text{to }\infty\right)}$

$= \left(\dfrac{4}{25}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)} = \left(\dfrac{1}{2}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{4}{25}\right)} = \left(\dfrac{1}{2}\right)^{-2} = 4$

25. NTA Ans. (4)

Sol. $\left(\dfrac{1+i}{1-i}\right)^{m/2} = \left(\dfrac{1+i}{i-1}\right)^{n/3} = 1$

$\Rightarrow \left(\dfrac{(1+i)^2}{2}\right)^{m/2} = \left(\dfrac{(1+i)^2}{-2}\right)^{n/3} = 1$

$\Rightarrow (i)^{m/2} = (-i)^{n/3} = 1$

$\Rightarrow \dfrac{m}{2} = 4k_1$ and $\dfrac{n}{3} = 4k_2$

$\Rightarrow m = 8k_1$ and $n = 12k_2$

Least value of m = 8 and n = 12.

$\therefore$ GCD = 4

SET # 04

PHYSICS

1. NTA Ans. (1)

Sol. A perfect diamagnetic substance will completely expel the magnetic field. Therefore, there will be no magnetic field inside the cavity of sphere. Hence the paramagnetic substance kept inside the cavity will experience no force.

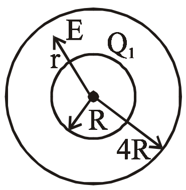
2. NTA Ans. (3)

Sol. $\rho_{nucleus} = \frac{mass}{volume} = \frac{A}{(4/3)\pi r_0^3 A}$

$= \frac{3}{4\pi r_0^3} = 2.3 \times 10^{17}\, kg/m^3$

3. NTA Ans. (1)

Sol.



$E = \frac{KQ_1}{r^2}$

$\Delta V = \int_R^{4R} E\, dr = \frac{3KQ_1}{4R}$

4. NTA Ans. (4)

Sol. $q\Delta V = \frac{1}{2}mV^2 \Rightarrow v = \sqrt{\frac{2q\Delta V}{m}}$

$\therefore \frac{V_1}{V_2} = \sqrt{\frac{e}{m}\frac{4m}{e}} = 2$

5. NTA Ans. (2)

Sol. $E\, 4\pi r^2 = \int \rho_0 4\pi r^2 dr$

$\Rightarrow Er^2 = 4\pi G \int_0^r \rho_0 \left(1 - \frac{r^2}{R^2}\right) r^2 dr$

$\Rightarrow E = 4\pi G \rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$

$\frac{dE}{dr} = 0 \;\therefore\; r = \sqrt{\frac{5}{9}}\, R$

6. NTA Ans. (3)

Sol. $\frac{dK}{dE} = P = cost \Rightarrow K = Pt = \frac{1}{2}mV^2$

$\therefore\; V = \sqrt{\frac{2Pt}{m}} = \frac{ds}{dt} \therefore S = \sqrt{\frac{2P}{m}}\frac{2}{3}t^{\frac{3}{2}}$

7. NTA Ans. (3)

Sol. $\Delta E = \frac{\lambda c}{\lambda e} = 3.1 eV$

8. NTA Ans. (3)

Sol. $nC_P(50) = 160$

$nC_v\ (100) = 240$

$\Rightarrow \frac{C_p}{2C_v} = \frac{160}{240} = \frac{\gamma}{2}$

$\therefore\; \gamma = \frac{4}{3}$ and $f = \frac{2}{\gamma - 1} = 6$

9. NTA Ans. (4)

Sol. At equilibrium position

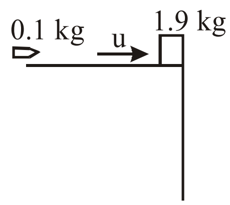
$V_0 = \omega_0 A = \sqrt{\frac{K}{m}}A$ .....(i)

$V = \omega A^1 = \sqrt{\frac{K}{\frac{m}{2}}}A^1$ .....(ii)

$\therefore\; A^1 = \frac{A}{\sqrt{2}}$

10. NTA Ans. (1)

Sol.



$p_i = p_f \Rightarrow 0.1 \times 20 = 2v$

$\therefore\; v = 1$ m/s

$KE_f = mgh + \frac{1}{2}mv^2 = 21J$

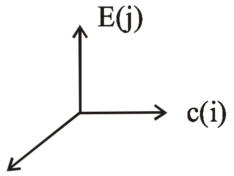
11. NTA Ans. (1)

Sol. $\Delta p = n_1 L_1 - n_2 L_2$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta p$$

12. NTA Ans. (3)

Sol.



$\therefore \vec{B}(\hat{k})$

$\Rightarrow \vec{B} = B_0 \cos(wt - kx)\hat{k}$

Now put t = 0.

13. NTA Ans. (1)

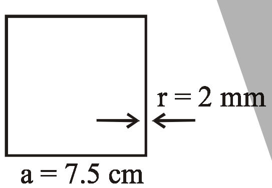
Sol. $\frac{50-40}{300} = \beta\left(\frac{50+40}{2} - 20\right)$

$$\frac{40-T}{300} = \beta\left(\frac{40+T}{2} - 20\right)$$

$$\therefore \ T = \frac{100}{3}$$

14. NTA Ans. (1)

Sol.



$$q_i = \frac{d(Ba^2)}{dt} = a^2\frac{dB}{dt}$$

$$i = \frac{q}{R} = \frac{a^2 dB/dt}{\frac{\rho(40)}{\pi r^2}}$$

15. NTA Ans. (4)

Sol. $S = \frac{P}{A} = \frac{ML^2T^{-3}}{L^2} = MT^{-3}$

16. NTA Ans. (1)

Sol. (1) Multimeter shows deflection when it connects with capacitor

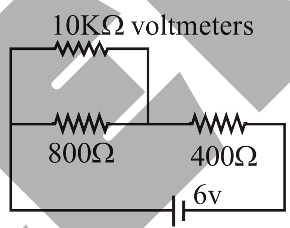
(2) If we assume that LED has negligiable resistance then multimeter shows no deflection for the forward blas but when it connects in reverse direction, it break down occurs so splash of light out.

(3) The resistance of metal wire may be taken zero, so no deflection in multimeter

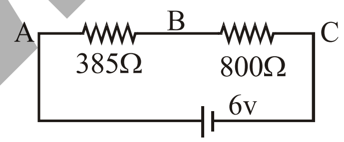
(4) No matter, how we connect the resistance across multimeter It shows same deflection.

17. NTA Ans. (2)

Sol.



$\Rightarrow$



So the potential difference in voltmeter across the points A and B is $\frac{6}{1185}\times 385 = 1.949$ V

18. NTA Ans. (1)

Sol. $P = \frac{nhc}{\lambda t}$

$$\therefore \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1}{500}$$

19. NTA Ans. (2)

Sol.

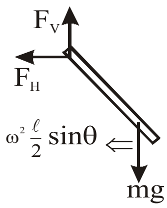
Cal	H_2O	Sterm
20gm	180gm	m
25°C	25°C	100°C

200 × 1 × (31 – 25)

= m × 540 + m × 1 × (100 – 31)

20. NTA Ans. (2)

Sol.



$F_V = mg$

$F_H = m\omega^2 \frac{\ell}{2} \sin\theta$

$$mg\frac{\ell}{2}\sin\theta - m\omega^2\frac{\ell}{2}\sin\theta\frac{\ell}{2}\cos\theta = \frac{m\ell^2}{12}\omega^2\sin\theta\cos\theta$$

$$\cos\theta = \frac{3}{2}\frac{g}{\omega^2\ell} \qquad .....(ii)$$

21. NTA Ans. (1)

Sol. $\left|\left(\frac{dv}{dt}\right)\right| = \left|\frac{v^2}{4^2}\right|\left|\frac{du}{dt}\right| = \left(\frac{10}{30}\right)2\times 9 = 1m/s$

22. NTA Ans. (20)

Sol. $\vec{\tau} = \vec{m}\times\vec{B}$

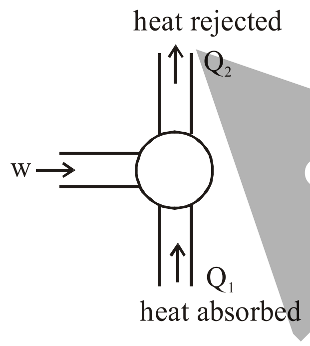
$\tau = NI \times A \times B$

$105 = 500 \times 3\times10^{-4} \times \frac{1}{2}\times B$

$B = 20$

23. NTA Ans. (8791)

Sol.



$w + Q_1 = Q_2$

$w = Q_2 - Q_1$

$$C.O.P. = \frac{Q_1}{w} = \frac{Q_1}{Q_2 - Q_1} = \frac{273}{300-273} = \frac{Q_1}{W}$$

$$w = \frac{27}{273}\times 80\times 100\times 4.2$$

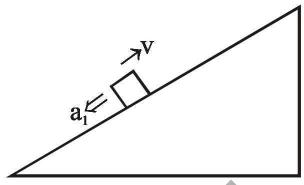
$Q_2 = w + \theta_1$

$$Q_2 = \frac{27}{273}\times 80\times 100\times 4.2 + 80\times 100\times 4.2$$

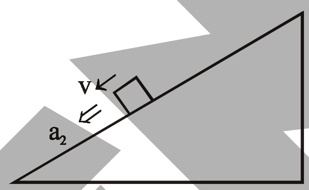
$$Q_2 = \frac{300}{273}\times 80\times 100 = 8791.2 \text{ cal}$$

24. NTA Ans. (346)

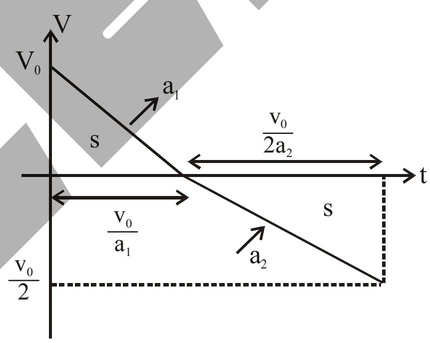
Sol.



$a_1 = g(\sin\theta + \mu\cos\theta)$



$a_2 = g(\sin\theta + \mu\cos\theta)$



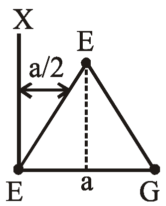
$$\therefore \frac{1}{2}v_0\frac{v_0}{a_1} = \frac{1}{2}\left(\frac{v_0}{2}\right)\left(\frac{v_0}{2a_1}\right)$$

$\Rightarrow 3\sin\theta = 5\mu\cos\theta$

$\therefore \mu = \sqrt{3}/5$

25. NTA Ans. (25)

Sol.



$$I = 0 + m\left(\frac{a}{2}\right)^2 + ma^2$$

$$= \frac{5}{4}ma^2$$

CHEMISTRY

1. NTA Ans. (3)

Sol. I, A_N : Be < Mg

II IE : Be > Al

III Charge/radius ratio of Be is less than that of Al

IV Be, Al mainly form covalent compounds

2. NTA Ans. (2)

Sol. Volume strength = 11.2 × molarity

$\Rightarrow$ molarity = $\frac{5.6}{11.2} = 0.5$

Assuming 1 litre solution;

mass of solution = 1000 ml × 1 g/ml = 1000 g

mass of solute = moles × molar mass

= 0.5 mol × 34 g/mol

= 17 gm.

$\Rightarrow$ mass% = $\frac{17}{1000} \times 100 = 1.7\%$

3. NTA Ans. (1)

Sol. l = 0 to (n + 1)

n = 1 $\quad$ n = 2

l = 0, 1, 2 $\quad$ l = 0, 1, 2, 3

$(n + l) \Rightarrow \frac{1s}{1}\frac{1p}{2}\frac{1d}{3} \qquad \frac{2s}{2}\frac{2p}{3}\frac{2d}{4}\frac{2f}{5}$

n = 3

l = 0, 1, 2, 3, 4

$\frac{3s}{3}\frac{3p}{4}\frac{3d}{5}\frac{3f}{6}\frac{3g}{7}$

Now, in order to write electronic configuration, we need to apply (n + l) rule

Energy order : 1s < 1p < 2s < 1d < 2p < 3s < 2d…

Option 1) 13 : $1s^2 1p^6 2s^2 1d^3$ is not half filled

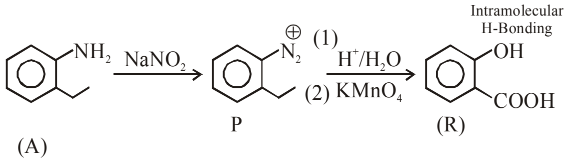
Option 2) 9 : $1s^2 1p^6 2s^1$ is the first alkali metal because after losing one electron, it will achieve first noble gas configuration

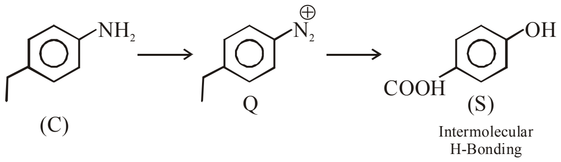
Option 3) 8 : $1s^2 1p^6$ is the first noble gas because after $1p^6$ e^- will enter 2s hence new period

Option 4) 6 : $1s^2$ $1p^4$ has 1p valence subshell.

4. NTA Ans. (2)

Sol.





$$Ph-CH_2-NH-Me \xrightarrow{Ph-SO_2Cl} Ph-S-N\begin{matrix} Me \\ CH_2-Ph \end{matrix}$$

Solid sulphonamide (not soluble in Aq. NaOH)

5. NTA Ans. (3)

Sol. Steep rise in pH around the equivalence point for titration of strong acid with strong base.

6. NTA Ans. (2)

Sol. (1) Acid rain corrodes water pipes resulting in the leaching of heavy of heavy metals such as iron, lead and copper into the drinking water.

(2) Acid rain damages buildings and other structures made of stone or metal.

(3) It causes respiratory aliments in human beings and animals.

(4) It is harmful for agriculture, trees and plants as it wasshes down the nutrients needed for its growth.

7. NTA Ans. (2)

Sol. Let suppose element X $\Rightarrow$

$$X_{(g)} \xrightarrow[800]{IE_1} \overset{+}{X}(g) \xrightarrow[2427]{IE_2} \overset{+2}{X}(g) \xrightarrow[3658]{IE_3}$$

$$\overset{+3}{X}(g) \xrightarrow[25024]{IE_4} \overset{+4}{X}(g) \xrightarrow[32824]{IE_5} \overset{+5}{X}(g)$$

X^{+3} has stable inert gas configuration as there is high jump after IE_3

So valence electrons are 3

8. NTA Ans. (3)

Sol. According to Dalton's law of partial pressure

$p_i = x_i \times P_T$

p_i = partial pressure of the i^{th} component

x_i = mole fraction of the i^{th} component

p_T = total pressure of mixture

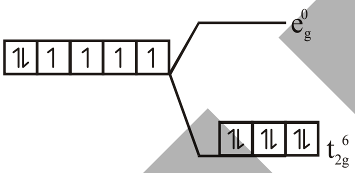
$$\Rightarrow \; 2\text{ atm} = \left(\frac{n_{H_2}}{n_{H_2} + n_{H_e} + n_{O_2}}\right) \times p_T$$

$$\Rightarrow \; p_T = 2\text{ atm} \times \frac{3}{1} = 6\text{ atm}$$

9. NTA Ans. (3)

Sol. $[Ru(en)_3]Cl_2$ — Ru ⇒ 4d series; en ⇒ chelating ligand hence large splitting of d-subshell

CN = 6, octahedral splitting

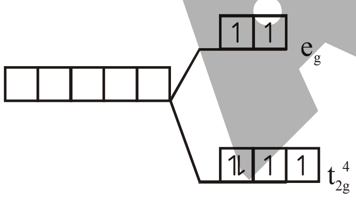
$Ru^{+2} \Rightarrow [Kr]4d^6 5s^0$ 

$[Fe(H_2O)_6]Cl_2 \Rightarrow H_2O \Rightarrow$ Weak filled ligand

$Fe^{+2} \Rightarrow [Ar]\ 3d^6 4s^0$

less splitting

CN = 6 octahedral splitting

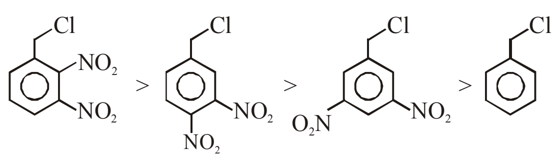


10. NTA Ans. (2)

Sol. Correct Ans. is (2)

11. NTA Ans. (2)

Sol.

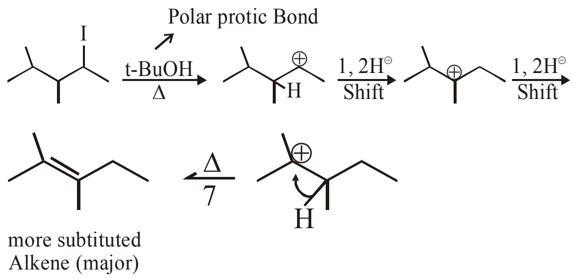


–I & –R　　less –I & –R　　Only 2 – I

rate of SN^2 reaction

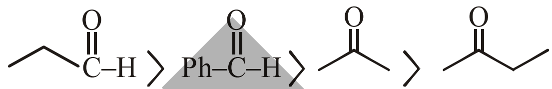
12. NTA Ans. (4)

Sol.



13. NTA Ans. (1)

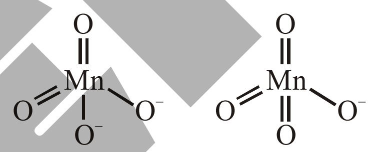
Sol. Reactivity order of various carbonyl compounds → Aldehydes > Ketones



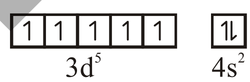
14. NTA Ans. (3)

Sol. Option 1) Manganate ⇒ MnO_4^{2-},

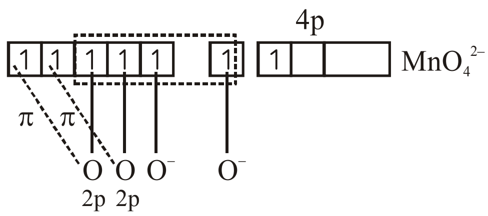
Permanganate ⇒ MnO_4^-



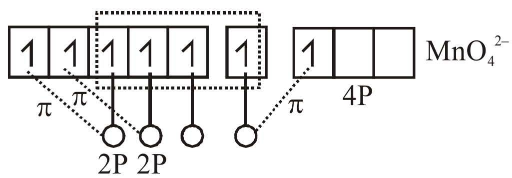
hybridisation of Mn ⇒ d^3s　　hybridisation of Mn ⇒ d^3s

Mn ⇒ 

After excitation

Mn ⇒ 

$2 \times 2p_\pi - 3d_{\pi\sigma}$



$2 \times 2P_\pi - 3d_\pi$

$1 \times 2P_\pi - 4P_\pi$

(2) $MnO_4^{2-} \Rightarrow$ green

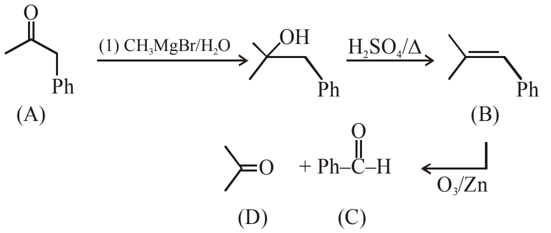
$MnO_4^- \Rightarrow$ purple/violet

(3) Manganate contains 1 unpaired electron hence it is paramagnetic where as permanganetic contains no unpaired electrons hence it is diamagnetic.

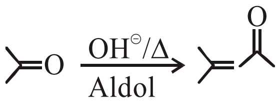
(4) Both have d^3s hybridisation hence both have tetrahedral geometry.

15. NTA Ans. (3)

Sol.



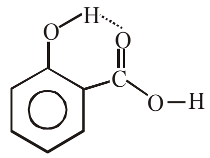
$$Ph{-}\overset{O}{\overset{\|}{C}}{-}H \xrightarrow[\text{NaOH (cannizaro)}]{\text{Conc.}} Ph{-}COOK + Ph{-}CH_2OH$$



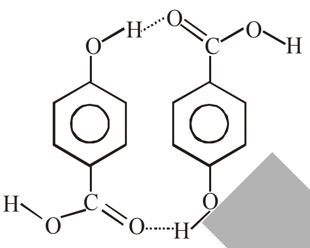
16. NTA Ans. (3)

Official Ans. by ALLEN (2, 3 & 4)

Sol.



O-salicylic acid intra molecular H-bonding



p-salicylic acid inter molecular H-bonding

(a) B will be more crystalline due to more inter molecular interactions hence more efficient packing.

(b) B will have higher boiling point due to higher intermolecular interactions.

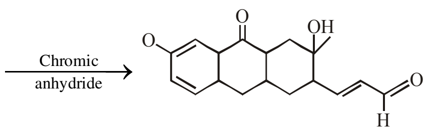
(c) B will be more soluble in water than A as B will have more extent of H-bonding in water

So all three statements are correct

{Solubility date $\Rightarrow$ O-salicylic acid = 2g/L

P-salicylic acid = 5g/L}

17. NTA Ans. (2)

Sol. Compound 

due to pressure of b

18. NTA Ans. (3)

Sol. Ranitidine $\rightarrow$ Antacid

Nardil $\rightarrow$ Antidepressant

Chloramphenicol $\rightarrow$ Antibiotic

Dimetane $\rightarrow$ Antihistamine

19. NTA Ans. (4)

Sol. For aA + bB $\rightarrow$ cC;

$$\frac{-1}{a}\frac{d[A]}{dt} = \frac{-1}{b}\frac{d[B]}{dt} = \frac{1}{c}\frac{d[C]}{dt}$$

$$\therefore \quad \frac{-1}{2}\frac{d[A]}{dt} = \frac{-1}{3}\frac{d[B]}{dt} = \frac{-2}{3}\frac{d[C]}{dt} = \frac{1}{3}\frac{d[P]}{dt}$$

20. NTA Ans. (3)

Sol. % mass of water

$$= \frac{x \times 18}{(12 + 6 \times 16 + 35 \times 3 + 52)} \times 100 = 13.5$$

$$\Rightarrow \quad x = \frac{265 \times 13.5}{18 \times 100} \simeq 2$$

21. NTA Ans. (60)

Sol. Moles of $e^{\ominus} = \left(\frac{8 \times 60 \times 2}{96000}\right)$

Using stoichiometry; theoritically

$$\frac{n_{e^{\ominus}}\text{ used}}{6} = \frac{n_{cr^{+3}}\text{ produced}}{2}$$

$$\Rightarrow n_{cr^{+3}}\text{ produced} = \frac{2}{6} \times \frac{8 \times 60 \times 2}{96000}$$

$$= \frac{0.02}{6}$$

$$\Rightarrow \text{wt}_{cr^{+3}}\text{ theoritically produced}$$

$$= \left(\frac{0.02}{6} \times 52\right)g$$

$$\Rightarrow \% \text{ efficiency} = \frac{0.104g}{\left(\frac{0.02 \times 52}{6}\right)g} \times 100$$

$$= 60\%$$

22. NTA Ans. (25)

Sol. moles $= \dfrac{\text{number of molecules}}{6\times10^{23}} = \dfrac{\text{given mass}}{\text{molar mass}}$

$\Rightarrow$ molar mass $= \dfrac{10\times6.023\times10^{23}}{6.023\times10^{22}} = 100\,g/mol$

$\Rightarrow$ molarity $= \dfrac{\text{moles of solute}}{\text{volume of sol}^n(\ell)} = \dfrac{(5/100)}{2}$

$= 0.025$

23. NTA Ans. (10)

Sol. $H_3PO_2 + NaOH \rightarrow NaH_2PO_2 + H_2O$

$$\frac{n_{H_3PO_2}\text{reacted}}{1} = \frac{n_{NaOH}\text{reacted}}{1}$$

$$\Rightarrow \frac{0.1\times10}{1} = 0.1\times V_{NaOH}$$

$\Rightarrow V_{NaOH} = 10$ ml.

24. NTA Ans. (177)

Sol. Let molar mass of protein A = x g/mol
Let molar mass of protein B = y g/mol

$$\pi_A = \text{osmotic pressure of protein A} = \frac{\left(\frac{0.73}{x}\right)}{0.25}RT$$

$$\pi_B = \text{osmotic pressure of protein B} = \frac{\left(\frac{1.65}{y}\right)}{1}RT$$

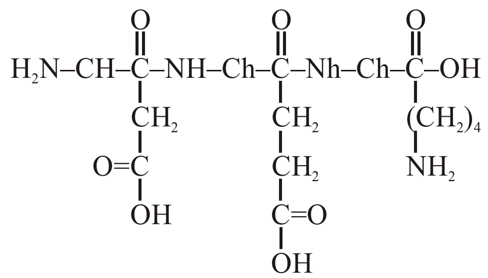
$$\pi_A = \pi_B$$

$$\Rightarrow \left(\frac{0.73}{x\times0.25}\right)RT = \left(\frac{1.65}{y}\right)RT$$

$$\Rightarrow \left(\frac{x}{y}\right) = \frac{0.73}{0.25\times1.65} = 1.769 \cong 1.77$$

25. NTA Ans. (5)

Sol. Structure of Tri peptide Asp – Glu – Lys



MATHEMATICS

1. NTA Ans. (1)

Sol. $\dfrac{d}{dt}(6a^2) = 3.6 \Rightarrow 12a\dfrac{da}{dt} = 3.6$

$$a\frac{da}{dt} = 0.3$$

$$\frac{dv}{dt} = \frac{d}{dt}(a^3) = 3a\left(a\frac{da}{dt}\right)$$

$= 3 \times 10 \times 0.3 = 9$

2. NTA Ans. (1)

Sol. $\displaystyle\int_0^{1/2}\frac{((x^2-1)+1)}{(1-x^2)^{3/2}}dx$

$$\int_0^{1/2}\frac{dx}{(1-x^2)^{3/2}} - \int_0^{1/2}\frac{dx}{\sqrt{1-x^2}}$$

$$\int_0^{1/2}\frac{x^{-3}}{(x^{-2}-1)^{3/2}}dx - (\sin^{-1}x)_0^{1/2}$$

Let $x^{-2} - 1 = t^2 \Rightarrow x^{-3}\,dx = -\,t\,dt$

$$\int_\infty^{\sqrt3}\frac{-t\,dt}{t^3} - \frac{\pi}{6} = \int_{\sqrt3}^{\infty}\frac{dt}{t^2} - \frac{\pi}{6} = \frac{1}{\sqrt3} - \frac{\pi}{6} = \frac{k}{6}$$

$$k = 2\sqrt3 - \pi$$

3. NTA Ans. (4)

Sol. Let $a^2 + b^2 \in Q$ & $b^2 + c^2 \in Q$

eg. $a = 2 + \sqrt3$ & $b = 2 - \sqrt3$

$a^2 + b^2 = 14 \in Q$

Let $c = \left(1 + 2\sqrt3\right)$

$b^2 + c^2 = 20 \in Q$

But $a^2 + c^2 = \left(2+\sqrt3\right)^2 + \left(1+2\sqrt3\right)^2 \notin Q$

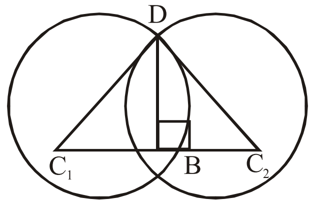
for R_2 Let $a^2 = 1$, $b^2 = \sqrt3$ & $c^2 = 2$

$a^2 + b^2 \notin Q$ & $b^2 + c^2 \notin Q$

But $a^2 + c^2 \in Q$

4. NTA Ans. (1)

Sol. Length of latus rectum = 4



DB = 2

$C_1B = \sqrt{(C_1D)^2 - (DB)^2} = 4$

$C_1C_2 = 8$

5. NTA Ans. (3)

Sol. Put $x = \tan^2\theta \Rightarrow dx = 2\tan\theta\sec^2\theta\, d\theta$

$$\int \underset{\text{I}}{\underset{\downarrow}{\theta}}.\underset{\text{II}}{\underset{\downarrow}{(2\tan\theta\cdot\sec^2\theta)}}d\theta \qquad \text{(By parts)}$$

$$= \theta.\tan^2\theta - \int \tan^2\theta\, d\theta$$

$$= \theta.\tan^2\theta - \int (\sec^2\theta - 1)\, d\theta$$

$$= \theta(1 + \tan^2\theta) - \tan\theta + C$$

$$= \tan^{-1}\left(\sqrt{x}\right)(1 + x) - \sqrt{x} + C$$

6. NTA Ans. (3)

Sol. First Case: Choose two non-zero digits ${}^9C_2$

Now, number of 5-digit numbers containing both digits = $2^5 - 2$

Second Case: Choose one non-zero & one zero as digit ${}^9C_1$.

Number of 5-digit numbers containg one non zero and one zero both = $(2^4 - 1)$

Required prob.

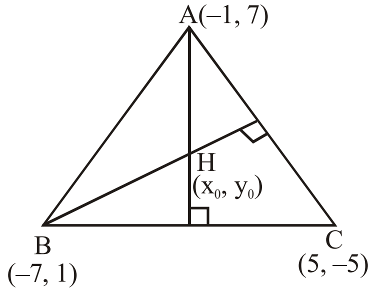
$$= \frac{\left({}^9C_2 \times \left(2^5 - 2\right) + {}^9C_1 \times \left(2^4 - 1\right)\right)}{9\times10^4}$$

$$= \frac{36\times(32-2)+9\times(16-1)}{9\times10^4}$$

$$= \frac{4\times30+15}{10^4} = \frac{135}{10^4}$$

7. NTA Ans. (3)

Sol. Let orthocentre is $H(x_0, y_0)$



$m_{AH}.m_{BC} = -1$

$$\Rightarrow \left(\frac{y_0 - 7}{x_0 + 1}\right)\left(\frac{1+5}{-7-5}\right) = -1$$

$\Rightarrow \quad 2x_0 - y_0 + 9 = 0$ .... (1)

and $\quad m_{BH}.m_{AC} = -1$

$$\Rightarrow \left(\frac{y_0 - 1}{x_0 + 7}\right)\left(\frac{7-(-5)}{-1-5}\right) = -1$$

$\Rightarrow \quad x_0 - 2y_0 + 9 = 0$ .... (2)

Solving equation (1) and (2) we get

$(x_0, y_0) \equiv (-3, 3)$

8. NTA Ans. (4)

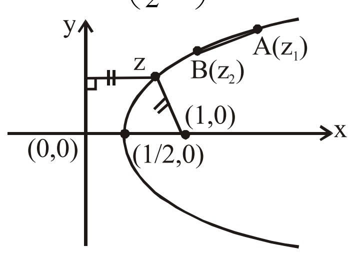
Sol. $\text{Re}(z) = |z - 1|$

$\Rightarrow \quad x = \sqrt{(x-1)^2 + (y-0)^2} \qquad (x > 0)$

$\Rightarrow \quad y^2 = 2x - 1 = 4.\frac{1}{2}\left(x - \frac{1}{2}\right)$

$\Rightarrow$ a parabola with focus (1, 0) & directrix as imaginary axis.

$\therefore \quad \text{Vertex} = \left(\frac{1}{2}, 0\right)$



$A(z_1)$ & $B(z_2)$ are two points on it such that

slope of AB = $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$(\arg(z_1 - z_2) = \frac{\pi}{6})$

for $\quad y^2 = 4ax$

Let $A(at_1^2, 2at_1)$ & $B(at_2^2, 2at_2)$

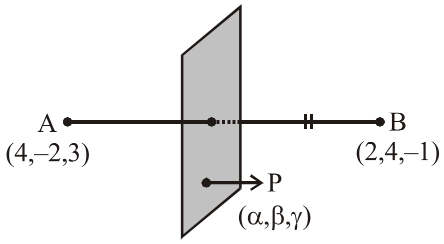
$$m_{AB} = \frac{2}{t_1 + t_2} = \frac{4a}{y_1 + y_2} = \frac{1}{\sqrt{3}}$$

$$\left(\text{Here } a = \frac{1}{2}\right)$$

$\Rightarrow$ $y_1 + y_2 = 4a\sqrt{3} = 2\sqrt{3}$

9. NTA Ans. (1)

Sol.



PA = PB

$\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow$ $(\alpha - 4)^2 + (\beta + 2)^2 + (\gamma - 3)^2$

$= (\alpha - 2)^2 + (\beta - 4)^2 + (\gamma + 1)^2$

$\Rightarrow$ $-4\alpha + 12\beta - 8\gamma = -8$

$\Rightarrow$ $2x - 6y + 4z = 4$

10. NTA Ans. (1)

Sol. Required limit

$$L = \lim_{h\to 0} \frac{(a + 2(a+h))^{1/3} - (3(a+h))^{1/3}}{(3a + a + h)^{1/3} - (4(a+h))^{1/3}}$$

$$= \lim_{h\to 0} \frac{(3a)^{1/3}\left(1 + \frac{2h}{3a}\right)^{1/3} - (3a)^{1/3}\left(1 + \frac{h}{a}\right)^{1/3}}{(4a)^{1/3}\left(1 + \frac{h}{4a}\right)^{1/3} - (4a)^{1/3}\left(1 + \frac{h}{a}\right)^{1/3}}$$

$$= \lim_{h\to 0}\left(\frac{3^{1/3}}{4^{1/3}}\right)\left[\frac{\left(1 + \frac{2h}{9a}\right) - \left(1 + \frac{h}{3a}\right)}{\left(1 + \frac{h}{12a}\right) - \left(1 + \frac{h}{3a}\right)}\right]$$

$$= \left(\frac{3}{4}\right)^{1/3}\frac{\left(\frac{2}{9} - \frac{1}{3}\right)}{\left(\frac{1}{12} - \frac{1}{3}\right)} = \left(\frac{3}{4}\right)^{1/3}\left(\frac{8-12}{3-12}\right)$$

$$= \left(\frac{3}{4}\right)^{1/3}\left(\frac{-4}{-9}\right) = \frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}} = \frac{4^{2/3}}{3^{5/3}}$$

$$= \frac{(8\times 2)^{1/3}}{(27\times 9)^{1/3}} = \frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

11. NTA Ans. (3)

Sol. $C = \text{adj } A = \begin{vmatrix} +2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{vmatrix}$

$|C| = |\text{adj } A| = +2(0 + 4) + 1.(1 - 2) + 1.(2, 4)$

$= +8 - 1 + 2$

$|\text{adj } A| = |A|^2 = 9 = 9$

$\lambda = |A| = \pm 3$

$|\lambda| = 3$

$B = \text{adj } C$

$|B| = |\text{adj } C| = |C|^2 = 81$

$|(B^{-1})^T| = |B|^{-1} = \frac{1}{81}$

$(|\lambda|, \mu) = \left(3, \frac{1}{81}\right)$

12. NTA Ans. (3)

Sol. $f'(x) = x(x + 1)(x - 1) = x^3 - x$

$\int df(x) = \int x^3 - x\, dx$

$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + C$

$f(x) = f(0)$

$\frac{x^4}{4} - \frac{x^2}{2} = 0$

$x^2(x^2 - 2) = 0$

$x = 0, 0, \sqrt{2}, -\sqrt{2}$

$x_1^2 + x_2^2 + x_3^2 = 0 + 2 + 2 = 4$

13. NTA Ans. (1)

Sol. $\cos\phi = \frac{\bar{p}.\bar{q}}{|\bar{p}||\bar{q}|} = \frac{ab + bc + ca}{a^2 + b^2 + c^2} = \frac{\Sigma ab}{1}$

$$= abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

$$= \frac{abc}{\lambda}\left(\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)\right)$$

$$= \frac{abc}{\lambda}\left(\cos + 2\cos(\theta + \pi)\cos\frac{\pi}{3}\right)$$

$$= \frac{abc}{\lambda}(\cos\theta - \cos\theta) = 0$$

$$\phi = \frac{\pi}{2}$$

14. NTA Ans. (3)

Sol. $S=\frac{100}{5}+\frac{98}{5}+\frac{96}{5}+\frac{94}{5}+.....n$

$S_n=\frac{n}{2}\left(2\times\frac{100}{5}+(n-1)\left(-\frac{2}{5}\right)\right)=188$

n(100 – n + 1) = 488 × 5

$n^2 - 101n + 488 \times 5 = 0$

n = 61, 40

$T_n = a + (n-1)d = \frac{100}{5}-\frac{2}{5}\times 60$

= 20 – 24 = –4

15. NTA Ans. (3)

Sol. $\text{Variance}=\frac{\Sigma(x_i-p)^2}{n}-\left(\frac{\Sigma(x_i-p)}{n}\right)^2$

$=\frac{9}{10}-\left(\frac{3}{10}\right)^2=\frac{81}{100}$

S.D. = $\frac{9}{10}$

16. NTA Ans. (2)

Sol. $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$

$\Rightarrow$ $dy(x^3 - x^2) = dx\,(2y - xy)$

$\Rightarrow$ $-\int\frac{1}{y}dy=\int\frac{x-2}{x^2(x-1)}dx$

$\Rightarrow$ $-\ell ny=\int\left(\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x-1)}\right)dx$

Where A = 1, B = +2, C = –1

$\Rightarrow$ $-\ell ny=\ell n\,x-\frac{2}{x}-\ell n(x-1)+\lambda$

$\Rightarrow$ y(2) = e

$\Rightarrow$ $-1 = \ell n\ 2 - 1 - 0 + \lambda$

$\therefore$ $\lambda = -\ \ell n 2$

$\Rightarrow$ $\ell n\,y=-\ \ell nx+\frac{2}{x}+\ell n(x-1)+\ell n2$

Now put x = 4 in equation

$\Rightarrow$ $\ell n\,y=-\ell n4+\frac{1}{2}+\ell n3+\ell n2$

$\Rightarrow$ $\ell n\,y=\ell n\left(\frac{3}{2}\right)+\frac{1}{2}\ell n\,e$

$\Rightarrow$ $y=\frac{3}{2}\sqrt{e}$

17. NTA Ans. (1)

Sol. For ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ (b < 5)

Let e_1 is eccentricity of ellipse

$\therefore$ $b^2 = 25\,(1 - e_1^2)$ .... (1)

Again for hyperbola

$\frac{x^2}{16}-\frac{y^2}{b^2}=1$

Let e_2 is eccentricity of hyperbola.

$\therefore$ $b^2 = 16(e_2^2 - 1)$ .... (2)

by (1) & (2)

$25(1 - e_1^2) = 16(e_2^2 - 1)$

Now $e_1.e_2 = 1$ (given)

$\therefore$ $25(1 - e_1^2) = 16\left(\frac{1-e_1^2}{e_1^2}\right)$

or $e_1=\frac{4}{5}$ $\therefore$ $e_2=\frac{5}{4}$

Now distance between foci is 2ae

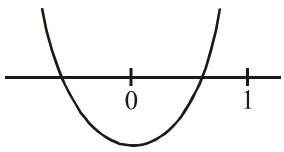
$\therefore$ distance for ellipse $= 2 \times 5 \times \frac{4}{5} = 8 = \alpha$

distance for hyperbola $= 2 \times 4 \times \frac{5}{4} = 10 = \beta$

$\therefore$ $(\alpha, \beta) \equiv (8, 10)$

18. NTA Ans. (2)

Sol. If exactly one root in (0, 1) then



$\Rightarrow \quad f(0).f(1) < 0$

$\Rightarrow \quad 2(\lambda^2 - 4\lambda + 3) < 0$

$\Rightarrow \quad 1 < \lambda < 3$

Now for $\lambda = 1$, $2x^2 - 4x + 2 = 0$

$(x - 1)^2 = 0$, $x = 1, 1$

So both roots doesn't lie between (0, 1)

$\therefore \lambda \neq 1$

Again for $\lambda = 3$

$10x^2 - 12x + 2 = 0$

$\Rightarrow \quad x = 1, \frac{1}{5}$

so if one root is 1 then second root lie between (0, 1)

so $\lambda = 3$ is correct.

$\therefore \quad \lambda \in (1, 3]$.

19. NTA Ans. (4)

Sol. $T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r$

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$$

For independent of x

$18 - 3r = 0$, $r = 6$

$\therefore \quad T_7 = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6 = \frac{21}{54} = k$

$\therefore \; 18k = \frac{21}{54} \times 18 = 7$

20. NTA Ans. (3)

Sol. $(p \wedge q) \rightarrow (\sim q \vee r) = \text{false}$

when $(p \wedge q) = T$

and $(\sim q \vee r) = F$

So $(p \wedge q) = T$ is possible when $p = q = \text{true}$

$\therefore \quad \sim q = \text{False}$ (q = true)

So $(\sim q \vee r) = \text{False}$ is possible if r is false

$\therefore \; p = T, q = T, r = F$

21. NTA Ans. (39)

Sol. $3, A_1, A_2 \, ... \, A_m, 243$

$$d = \frac{243 - 3}{m + 1} = \frac{240}{m + 1}$$

Now $3, G_1, G_2, G_3, 243$

$$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = 3$$

$\therefore \quad A_4 = G_2$

$\Rightarrow \quad a + 4d = ar^2$

$$3 + 4\left(\frac{240}{m+1}\right) = 3(3)^2$$

$m = 39$

22. NTA Ans. (4)

Sol. $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

$$m = \left(\frac{dy}{dx}\right)_{(c, e^c)} = e^c$$

$\Rightarrow \quad$ Tangent at (c, e^c)

$y - e^c = e^c (x - c)$

it intersect x-axis

Put $y = 0 \Rightarrow x = c - 1$ ......(1)

Now $y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,\,2)} = 1$

$\Rightarrow$ Slope of normal = –1

Equation of normal $y - 2 = -1(x - 1)$

$x + y = 3$ it intersect x-axis

Put $y = 0 \Rightarrow x = 3$ ......(2)

Points are same

$\Rightarrow$ $x = c - 1 = 3$

$\Rightarrow$ $c = 4$

23. NTA Ans. (5)

Sol. Dr's normal to plane

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

Equation of plane

$-1(x - 1) + 1(y - 0) + 1(z - 0) = 0$

$x - y - z - 1 = 0$ ......(1)

Now $\frac{\alpha - 1}{1} = \frac{\beta - 0}{-1} = \frac{\gamma - 1}{-1} = -\frac{(1 - 0 - 1 - 1)}{3}$

$$\frac{\alpha - 1}{1} = \frac{\beta}{-1} = \frac{\gamma - 1}{-1} = \frac{1}{3}$$

$$\alpha = \frac{4}{3}, \beta = -\frac{1}{3}, \gamma = \frac{2}{3}$$

$$3(\alpha + \beta + \gamma) = 3\left(\frac{4}{3} - \frac{1}{3} + \frac{2}{3}\right) = 5$$

24. NTA Ans. (8)

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let $x = k$

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0,\ y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k,\ y = \frac{k}{2},\ z = 0$ put in condition

$$15 \le k^2 + \left(\frac{k}{2}\right)^2 + 0 \le 150$$

$12 \le k^2 \le 120$

$\Rightarrow$ $k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

25. NTA Ans. (54)

Sol. Let three digit number is xyz

$x + y + z = 10$; $x \ge 1, y \ge 0\ z \ge 0$ ..... (1)

Let $T = x - 1 \Rightarrow x = T + 1$ where $T \ge 0$

Put in (1)

$T + y + z = 9$; $0 \le T \le 8,\ 0 \le y, z \le 9$

No. of non negative integral solution

$= {}^{9+3-1}C_{3-1} - 1$ (when T = 9)

$= 55 - 1 = 54$

SET # 05

PHYSICS

1. NTA Ans. (3)

Sol. Intensity, $I = 3.3\ Wm^{-2}$

Area, $A = 3 \times 10–4\ m^2$

Angular speed, $\omega = 31.4$ rad/s

$\because <\cos^2\theta> = \frac{1}{2}$, in one time period

$\therefore$ Average energy $= I_0 A \times \frac{1}{2} \times \frac{2\pi}{\omega}$

$= \frac{(3.3)(3\times10^{-4})}{2}$

$\simeq 1\times10^{-4}$ J

2. NTA Ans. (1)

Sol. $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

where 'f' is degree of freedom

(A) Monoatomic $f = 3$, $\gamma = 1 + \frac{2}{3} = \frac{5}{3}$

(B) Diatomic rigid molecules,

$f = 5$, $\gamma = 1 + \frac{2}{5} = \frac{7}{5}$

(C) Diatomic non-rigid molecules

$f = 7$, $\gamma = 1 + \frac{2}{7} = \frac{9}{7}$

(D) Triatomic rigid molecules

$f = 6$, $\gamma = 1 + \frac{2}{6} = \frac{4}{3}$

3. NTA Ans. (3)

Sol. Information based

$\lambda_{radiowaves} > \lambda_{microwaves} > \lambda_{visible} > \lambda_{x\text{-rays}}$

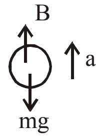
4. NTA Ans. (3)

Sol. Volume $V = \frac{4\pi}{3} r^3 = \frac{4\pi}{3} \times (1)^3 = 4.19 cm^3$

$a = 9.8\ cm/s^2$

$B - mg = ma$

$m = \frac{B}{g+a}$



$m = \frac{(V\rho_\omega g)}{g+a} = \frac{V\rho_\omega}{1+\frac{a}{g}}$

$= \frac{(4.19)\times1}{1+\frac{9.8}{980}} = \frac{4.19}{1.01} = 4.15 gm$

5. NTA Ans. (4)

Sol. $\because \frac{d\theta}{dt} = kA\frac{dT}{dx}$

$k = \frac{\left(\frac{d\theta}{dt}\right)}{A\left(\frac{dT}{dx}\right)}$

$[k] = \frac{[ML^2T^{-3}]}{[L^2][KL^{-1}]} = [MLT^{-3}K^{-1}]$

6. NTA Ans. (1)

Sol. Given $E_G = \frac{Ax}{(x^2+a^2)^{3/2}}$, $V_\infty = 0$

$\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G.\vec{d}_x$

$V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2+a^2)^{3/2}} dx$

put $x^2 + a^2 = z$

$2x\ dx = dz$

$V_x - 0 = -\int_\infty^x \frac{A\,dz}{2(z)^{3/2}} = \left[\frac{A}{z^{1/2}}\right]_\infty^x = \left[\frac{A}{(x^2+a^2)^{1/2}}\right]_\infty^x$

$V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$

7. NTA Ans. (3)

Sol. Given $\vec{u} = 5\hat{j}\ m/s$, $\vec{a} = 10\hat{i} + 4\hat{j}$, final coordinate $(20, y_0)$ in time t

$S_x = 4_x t + \frac{1}{2} a_x t^2$

$20 - 0 = 0 + \frac{1}{2} \times 10 \times t^2$

$t = 2sec$

$S_y = u_y \times t + \frac{1}{2} a_y t^2$

$y_0 = 5 \times 2 + \frac{1}{2} 4 \times 2^2 = 18m$

2 sec and 18 m

8. NTA Ans. (1)

Sol. Graph of V_s and f given (B 5.5, 0)

$h\nu = \phi + eV_s$

at B $V_s = 0$, $\nu = 5.5$

$\Rightarrow h \times 5.5 \times 10^{14} = \phi$

$$\phi = \frac{6.62\times10^{-34}\times5.5\times10^{14}}{1.6\times10^{-19}}\text{eV} = 2.27\text{eV}$$

9. NTA Ans. (3)

Sol. Given $i_A = 2$, $r_A = 2$ cm, $\theta_A = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

$i_B = 3$, $r_B = 4$ cm, $\theta_B = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$B = \frac{\mu_0 I\theta}{4\pi R}$$

$$\frac{B_A}{B_B} = \frac{I_A}{I_B}\times\frac{\theta_A R_B}{\theta_B R_A} = \frac{6}{5}$$

10. NTA Ans. (4)

Sol. Given T to C 1.5 m

C to C 5m

T to C = $(2n_1 + 1)\frac{\lambda}{2}$

C to C = $n_2\lambda$

$$\frac{1.5}{5} = \frac{(2n_1+1)}{2n_2} \Rightarrow 3n_2 = 10n_1 + 5$$

$n_1 = 1$, $n_2 = 5 \rightarrow \lambda = 1$

$n_1 = 4$, $n_2 = 15 \rightarrow \lambda = 1/3$

$n_1 = 7$, $n_2 = 25 \rightarrow \lambda = 1/5$

11. NTA Ans. (4)

Sol. Torque on a bar magnet : $I = MB\sin\theta$

Here, $\theta = 30°$, $I = 0.018$ N-m, $B = 0.06$ T

$\Rightarrow 0.018 = M \times 0.06 \times \sin 30°$

$\Rightarrow 0.018 = M \times 0.06 \times \frac{1}{2}$

$\Rightarrow M = 0.6$ A-m^2

Now $v = -MB\cos\theta$

Position of stable equilibrium ($\theta = 0°$) : $u_i = -MB$

Position of unstable equilibrium ($\theta = 180°$) : $u_f = MB$

$\Rightarrow$ work done : ΔU

$\Rightarrow W = 2MB$

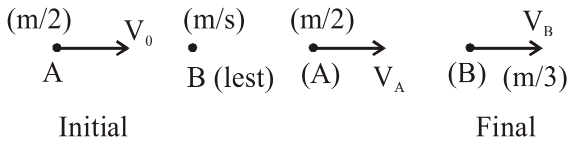
$\Rightarrow W = 2 \times 0.6 \times 0.06$

$\Rightarrow W = 7.2 \times 10^{-2}$ J

option (4) is correct

12. NTA Ans. (1)

Sol.



Applying momentum conservation

$$\frac{m}{2}\times V_0 + \frac{m}{3}\times(0) = \frac{m}{2}V_A + \frac{m}{3}V_B$$

$$= \frac{V_0}{2} = \frac{V_A}{2} + \frac{V_B}{3} \quad .... (1)$$

Since, collision is elastic (e = 1)

$$e = 1 = \frac{V_B - V_A}{V_0} \Rightarrow V_0 = V_B - V_A \quad ... (2)$$

On solving (1) & (2) : $V_A = \frac{V_0}{5}$

Now, De-Broglie wavelength of A before collision :

$$\lambda_0 = \frac{h}{m_A V_0} = \frac{h}{\left(\frac{m}{2}\right)V_0}$$

$$\Rightarrow \lambda_0 = \frac{2h}{mV_0}$$

Final De-Broglie wavelength :

$$\lambda_f = \frac{h}{m_A V_0} = \frac{h}{\frac{m}{2}\times\frac{V_0}{5}} \Rightarrow \lambda_f = \frac{10h}{mV_0}$$

Now $\Delta\lambda = \lambda_f - \lambda_0$

$$\Delta\lambda = \frac{10h}{mV_0} - \frac{2h}{mV_0}$$

$$\Rightarrow \Delta\lambda = \frac{8h}{mv_0} \Rightarrow \Delta\lambda = 4 \times \frac{2h}{mv_0}$$

$\Rightarrow \Delta\lambda = 4\lambda_0$

option (1) is correct.

13. NTA Ans. (3)

Sol. When bar magnet is entering with constant speed, flux will change and an e.m.f. is induced, so galvanometer will deflect in positive direction.

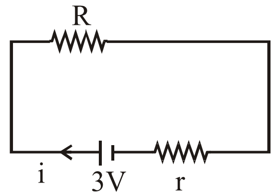
When magnet is completely inside, flux will not change, so reading of galvanometer will be zero.

When bar magnet is making on exit, again flux will change and on e.m.f. is induced in opposite direction to not of (a), so galvanometer will deflect in negative direction.

Looking at options, option (3) is correct.

14. NTA Ans. (4)

Sol.



$P_R = 0.5W$

$\Rightarrow i^2R = 0.5W$

Also, $V = E - ir$

$2.5 = 3 - ir$

$\Rightarrow ir = 0.5$

Power dissipated across 'r' : $P_r = i^2r$

Now $iR = 2.5$

$ir = 0.5$

On dividing : $\frac{R}{r} = 5$

Now $\frac{P_R}{P_r} = \frac{i^2R}{i^2r} \Rightarrow \frac{P_R}{P_r} = \frac{R}{r} \Rightarrow \frac{P_R}{P_r} = 5$

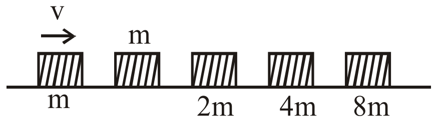
$\Rightarrow P_r = \frac{P_R}{5}$

$\Rightarrow P_r = \frac{0.50}{5} \Rightarrow P_r = 0.10$ W

option (4) is correct.

15. NTA Ans. (4)

Sol.



All collisions are perfectly inelastic, so after the final collision, all blocks are moving together. So let the final velocity be v', so on applying momentum conservation:

$mv = 16m\ v' \Rightarrow v' = v/16$

Now initial energy $E_i = \frac{1}{2}mv^2$

Final energy : $E_f = \frac{1}{2}\times 16m \times \left(\frac{v}{16}\right)^2$

$\Rightarrow E_f = \frac{1}{2}m\frac{v^2}{16}$

Energy loss : $E_i - E_f$

$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{16}$

$\Rightarrow \frac{1}{2}mv^2\left[1-\frac{1}{16}\right] \Rightarrow \frac{1}{2}mv^2\left[\frac{15}{16}\right]$

$$\%p = \frac{\text{Energy loss}}{\text{Original energy}}\times 100$$

$$= \frac{\frac{1}{2}mv^2\left[\frac{15}{16}\right]}{\frac{1}{2}mv^2}\times 100 = 93.75\%$$

$\Rightarrow$ Value of P is close to 94.

16. NTA Ans. (1)

Sol. Here the water will provide heat for ice to melt therefore

$m_w\ s_w\ \Delta\theta = m_{ice}\ L_{ice}$

$m_{ice} = \frac{0.2\times 4200\times 25}{3.4\times 10^5}$

= 0.0617 kg

= 61.7 gm

Remaining ice will remain un-melted

so correct answer is 1

17. NTA Ans. (3)

Sol. Velocity at ground (means zero height) is non-zero therefore one is incorrect and velocity versus height is non-linear therefore two is also incorrect.

$v^2 = 2gh$

$$v\frac{dv}{dh} = 2g = \text{const.}$$

$$\frac{dv}{dh} = \frac{\text{constant}}{v}$$

Here we can see slope is very high when velocity is low therefore at Maximum height the slope should be very large which is in option 3 and as velocity increases slope must decrease there for option 3 is correct.

18. NTA Ans. (3)

Sol. Potential of –q is same as initial and final point of the path therefore potential due to 4q will only change and as potential is decreasing the energy will decrease

Decrease in potential energy = $q\,(V_i - V_f)$

Decrease in potential energy

$$= q\left[\frac{k4q}{d/2} - \frac{k4q}{3d/2}\right] = \frac{4q^2}{3\pi\varepsilon_0 d}$$

Therefore correct answer is 3.

19. NTA Ans. (1)

Sol. Thin infinite uniformly charged planes produces uniform electric field therefore option 2 and option 3 are obviously wrong.

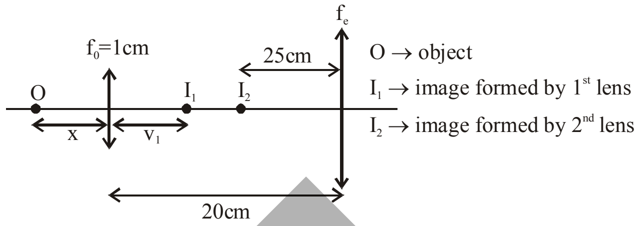
And as positive charge density is bigger in magnitude so its field along Y direction will be bigger than field of negative charge in X direction and this is evident in option 1 so it is correct.

20. NTA Ans. (2)

Sol. As there are two zener diodes in reverse polarity so if one is in forward bias the other will be in reverse bias and above 6V the reverse bias will too be in conduction mode. Therefore when voltage is more than 6V the output will be constant. And when it is less than 6V it will follow the input voltage so correct answer is two.

21. Ans. by NTA (6.25)

Sol.



for first lens = $\dfrac{1}{v_1} - \dfrac{1}{-x} = \dfrac{1}{1} \Rightarrow v_1 = \dfrac{x}{x-1}$

also magnification $|m_1| = \left|\dfrac{v_1}{u_1}\right| = \dfrac{1}{x-1}$

for 2nd lens this is acting as object

so $u_2 = -(20 - v_1) = -\left(20 - \dfrac{x}{x-1}\right)$

and $v_2 = -25$cm

angular magnification $|m_A| = \left|\dfrac{D}{u_2}\right| = \dfrac{25}{|u_2|}$

Total magnification $m = m_1 m_A = 100$

$$\left(\frac{1}{x-1}\right)\left(\frac{25}{20 - \frac{x}{x-1}}\right) = 100$$

$$\frac{25}{20(x-1)-x} = 100 \Rightarrow 1 = 80(x-1) - 4x$$

$$\Rightarrow 76x = 81 \Rightarrow x = \frac{81}{76}$$

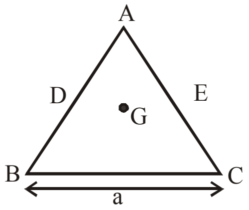
$$\Rightarrow u_2 = -\left(20 - \frac{81/76}{81/76 - 1}\right) = \frac{-19}{5}$$

now by lens formula

$$\frac{1}{-25} - \frac{1}{-19/5} = \frac{1}{f_e} \Rightarrow f_e = \frac{25\times 19}{106} \approx 4.48\text{cm}$$

22. NTA Ans. (11)

Sol. Let side of triangle is a and mass is m



MOI of plate ABC about centroid

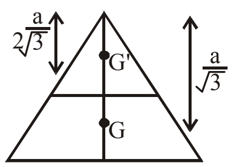
$$I_0 = \frac{m}{3}\left(\left(\frac{a}{2\sqrt{3}}\right)^2 \times 3\right) = \frac{ma^2}{12}$$

triangle ADE is also an equilateral triangle of side a/2.

Let moment of inertia of triangular plate ADE about it's centroid (G') is I_1 and mass is m_1

$$m_1 = \frac{m}{\frac{\sqrt{3}a^2}{4}} \times \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{m}{4}$$

$$I_1 = \frac{m_1}{12}\left(\frac{a}{2}\right)^2 = \frac{m}{4\times 12}\frac{a^2}{4} = \frac{ma^2}{192}$$



distance GG' $= \frac{a}{\sqrt{3}} - \frac{a}{2\sqrt{3}} = \frac{a}{2\sqrt{3}}$

so MOI of part ADE about centroid G is

$$I_2 = I_1 + m_1\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{ma^2}{192} + \frac{m}{4}.\frac{a^2}{12} = \frac{5ma^2}{192}$$

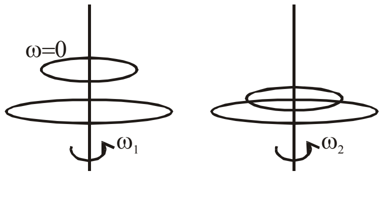
now MOI of remaining part

$$= \frac{ma^2}{12} - \frac{5ma^2}{192} = \frac{11ma^2}{12\times 16} = \frac{11I_0}{16}$$

$\Rightarrow$ N = 11

23. NTA Ans. (20)

Sol.



Let moment of inertia of bigger disc is $I = \frac{MR^2}{2}$

$\Rightarrow$ MOI of small disc $I_2 = \frac{M\left(\frac{R}{2}\right)^2}{2} = \frac{I}{4}$

by angular momentum conservation

$$I\omega_1 + \frac{I}{4}(0) = I\omega_2 + \frac{I}{4}\omega_2 \Rightarrow \omega_2 = \frac{4\omega_1}{5}$$

initial kinetic energy $K_1 = \frac{1}{2}I\omega_1^2$

final kinetic energy K_2

$$= \frac{1}{2}\left(I + \frac{I}{4}\right)\left(\frac{4\omega_1}{5}\right)^2 = \frac{1}{2}I\omega_1^2\left(\frac{4}{5}\right)$$

$$P\% = \frac{K_1 - K_2}{K_1}\times 100\% = \frac{1 - 4/5}{1}\times 100 = 20\%$$

24. NTA Ans. (266.00 to 267.00)

Sol. As work done on gas and heat supplied to the gas are zero,

total internal energy of gases remain same

$u_1 + u_2 = u_1' + u_2'$

$(0.1)\,C_v\,(200) + (0.05)\,C_v\,(400) = (0.15)C_v\,T$

$$T = \frac{800}{3}k = 266.67\,k$$

25. NTA Ans. (10553 to 10554)

Sol. $$\lambda = \frac{c}{\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

for lyman series

$$\lambda_1 = \frac{c}{\frac{1}{1^2} - \frac{1}{\infty^2}} = c \quad (n = \infty \text{ to } n = 1)$$

$$\lambda_2 = \frac{c}{\frac{1}{1^2} - \frac{1}{2^2}} = \frac{4c}{3} \quad (n = 2 \text{ to } n = 1)$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{c}{3} = 304\text{Å} \Rightarrow c = 912\text{Å}$$

for paschen series

$$\lambda_1 = \frac{c}{\frac{1}{3^2} - \frac{1}{\infty^2}} = 9c \quad (n = \infty \text{ to } n = 3)$$

$$\lambda_2 = \frac{c}{\frac{1}{3^2} - \frac{1}{4^2}} = \frac{144c}{7} \quad (n = 4 \text{ to } n = 3)$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{144c}{7} - 9c = \frac{81c}{7} = \frac{81\times 912}{7}$$

= 10553.14 Å

CHEMISTRY

1. NTA Ans. (4)

Sol. $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + \underset{\substack{\text{Brown}\\\text{gas}\\\text{(A)}}}{2NO_2} + \frac{1}{2}O_2(g)$

$NO_2(g) \xrightarrow{\text{Cooling}} \underset{\text{(B)}}{N_2O_4}$

$N_2O_4 + NO \xrightarrow{\Delta} \underset{\substack{\text{Blue Solid}\\\text{(C)}}}{N_2O_3}$

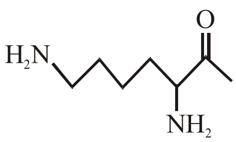
O.S. of nitrogen in N_2O_3 is + 3

N_2O_3 $2x + 3(-2) = 0$

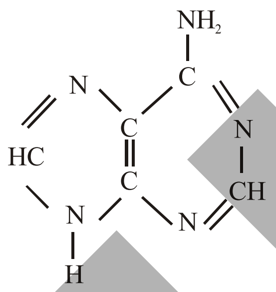
$x = +3$

2. NTA Ans. (1)

Sol. Adenine and lysine Both have primary amine react with $CHCl_3$ + alc. KOH



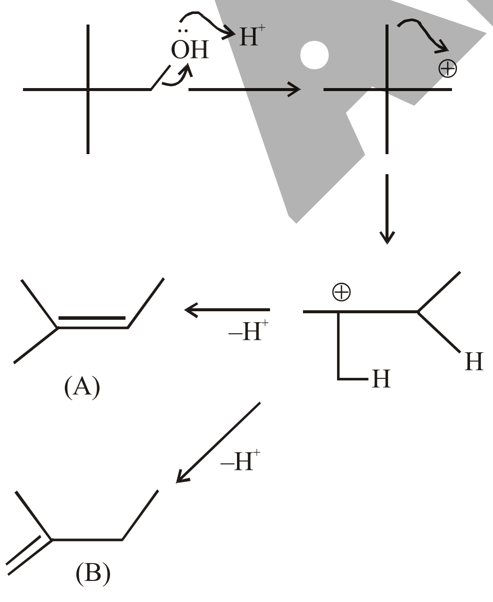
Lysine



Adenine

3. NTA Ans. (4)

Sol.



4. NTA Ans. (4)

Sol.

(a) $CaCO_3 \xrightarrow{\Delta} CaO + CO_2$ {In Blast furnace}
lime stone

(b) Ag form cyanide complex $[Ag(CN)_2]^-$ during cyaride process

$Ag / Ag_2S + CN^{\ominus} \rightarrow [Ag(CN)_2]^-$

(c) Ni is purified by mond's process

(d) Zr and Ti are purified by van arkel method All (a), (b), (c), (d) are correct statements

Thus correct option is (4)

5. NTA Ans. (1)

Sol. $E^o_{cell} = 0.34 - (-0.76)$

$= 1.10$ volt

If $E_{ext} > 1.10$ volt

Cu → Anode

Zn → Cathode

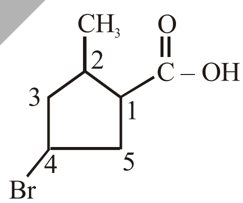
If $E_{ext} = 1.10$ volt

Zn → Anode

Cu → Cathode

6. NTA Ans. (1)

Sol.



4-bromo-2-methyl cyclopentane carboxylic Acid

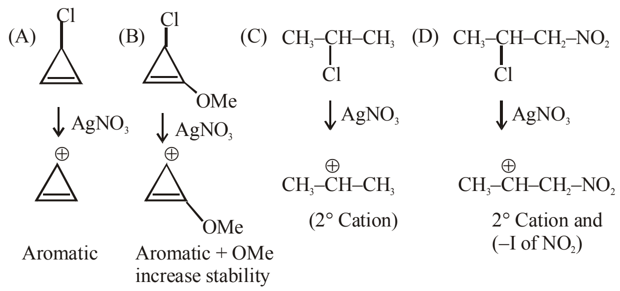
7. NTA Ans. (3)

Sol. at equilibrium

$r_a = r_b$

8. NTA Ans. (4)

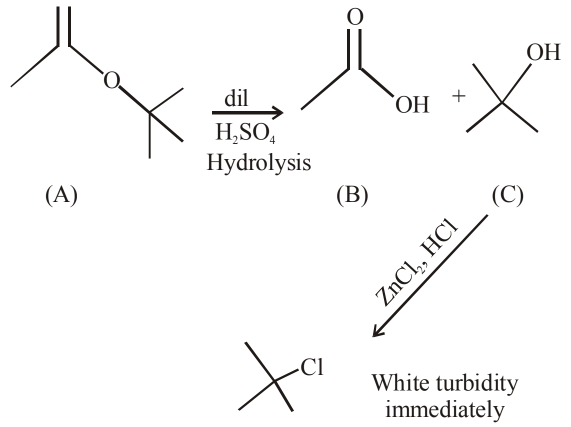
Sol.



∴ Stability Cation B > A > C > D

9. NTA Ans. (1)

Sol.



10. NTA Ans. (3)

Sol. Foam - Froth

Gel → Jellies

Aerosol → Smoke

Sol → Cell fluids

Solid sol → rubber

11. NTA Ans. (2)

Sol. Element with atomic no. 101 is an Actinoid element.

12. NTA Ans. (3)

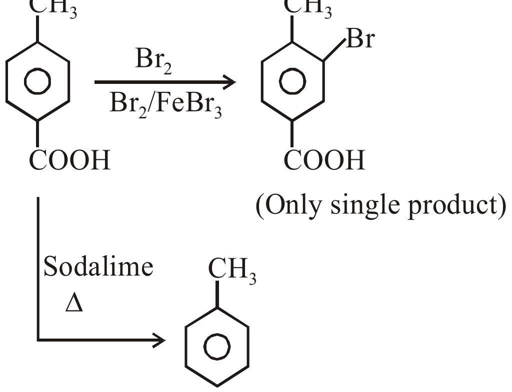
Sol. $Li + O_2 \rightarrow Li_2O$ (Major Oxides)
excess

$Na + '' \rightarrow Na_2O_2$ ('')

$K + '' \rightarrow KO_2$ ('')

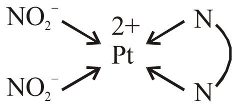
13. NTA Ans. (4)

Sol.



14. NTA Ans. (1)

Sol. $[Pt\ (en)\ (NO_2)_2] \Rightarrow$ does not show G.I. as well as optical isomerism.



This complex will have three linkage isomers as follows :-

$[Pt\ (en)\ (NO_2)_2]$ I

$[Pt\ (en)\ (NO_2)(ONO)]$ II

$[Pt\ (en)\ (ONO)_2]$ III

15. NTA Ans. (4)

Sol.

	O^{-2}	F^-	Na^+	Mg^{2+}
z	8	9	11	12
e^-	10	10	10	10
$\frac{z}{e}$	0.8	0.9	1.1	1.2

as $\frac{z}{e}$ ratio increases size decreases.

Thus correct ionic radii order is

$O^{-2} > F^- > Na^+ > Mg^{2+}$

Therefore correct option is (4)

16. NTA Ans. (1)

Sol. Balmer series give visible lines For H-atom

17. NTA Ans. (3)

Sol. From the given graph, potential energy of A-B molecule is minimum.

Thus A-B bond is most stable and have strongest bond amongst these.

B → Most electronegative

D → Least electronegative

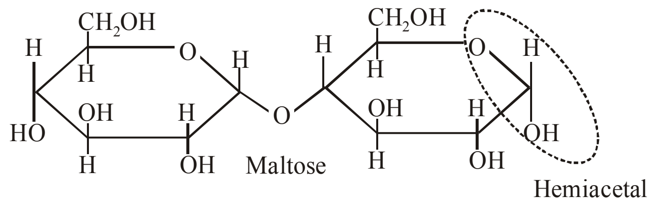
A-B → Shortest bond length

A-B → Largest bond enthalpy

Therefore correct option is (3).

18. NTA Ans. (2)

Sol.



19. NTA Ans. (1)

Sol. For ideal Gas

\# U = f(T), H = f(T)

\# Z = 1

\# $C_P - C_V = R$

\# $dU = C_V\, dT$

20. NTA Ans. (3)

Sol.

Complex	e^- configuration	no. of unpaired e^-
$[Mn(H_2O)_6]^{2+}$ WFL	↿ ↿ eg ↿ ↿ ↿ t2g	5
$[Cr(H_2O)_6]^{2+}$ WFL	↿ eg ↿ ↿ ↿	4
$[COCl_4]^{2-}$ Tetrahedral	↿ ↿ ↿ t_2 ⥮ ⥮ e	3
$[Fe(H_2O)_6]^{2+}$ WFL	↿ ↿ eg ⥮ ↿ ↿ t_2g	4
$[Co(OH)_4]^{2-}$ WFL	↿ ↿ ↿ t_2 ⥮ ⥮ e	3
Tetrahedral $[Fe(NH_3)_6]^{2+}$	↿ ↿ ⥮ ↿ ↿	4

Thus complex $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ have same no. of unpaired e^- and hence same magnetic moment (spin only).

21. NTA Ans. (3400)

Sol.

N_2	+ $3H_2$ →	$2NH_3$
$\frac{2.8}{28}$ K mol	$\frac{1}{2}$ K mol	
= 0.1 K mol	0.5 K mol	—
0	0.2 K mol	0.2 K mol

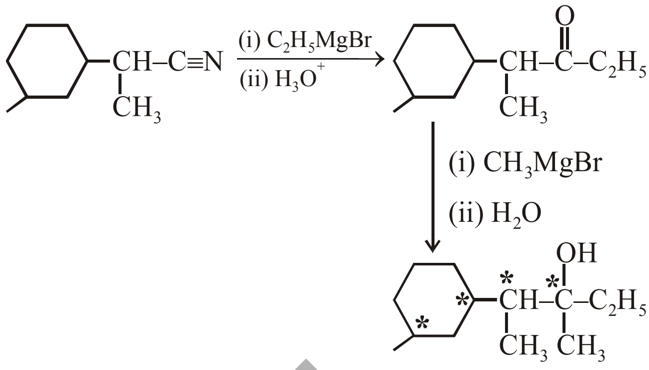
mass (NH_3) = 0.2 × 17 Kg

= 3.4 Kg

= 3400 gm

22. NTA Ans. (4)

Sol.



23. NTA Ans. (85)

Sol. Eq of H_2O_2 = Eq of $KMnO_4$

$$x \times 2 = \frac{0.316}{158} \times 5$$

$$x = 5 \times 10^{-3} \text{ mol}$$

$$m_{H_2O_2} = 5 \times 10^{-3} \times 34 = 0.17\text{gm}$$

$$\%H_2O_2 = \frac{0.17}{0.2} \times 100 = 85$$

24. NTA Ans. (60)

Sol. $t_{0.75} = 2 \times \frac{\ln 2}{k} = 90$

$$k = \frac{\ln 2}{45}\text{min}^{-1}$$

$$kt = \ln\frac{1}{1-0.6} = \ln 2.5$$

$$\frac{\ln 2}{45} \times t = \ln 2.5$$

$$t = 45 \times \frac{\log 2.5}{\log 2} = 45 \times \frac{0.4}{0.3} = 60\,\text{min}$$

25. NTA Ans. (600)

Sol. $550 = P_A^o \times \frac{1}{4} + P_B^o \times \frac{3}{4}$

$2200 = P_A^o + 3P_B^o$ .....(i)

$2800 = P_A^o + 4P_B^o$ ....(ii)

$560 = P_A^o \times \frac{1}{5} + P_B^o \times \frac{4}{5}$

$P_B^o = 600, P_A^o = 400$

MATHEMATICS

1. NTA Ans. (3)

Sol. $A^2 = \begin{pmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{pmatrix}$

Similarly, $A^5 = \begin{pmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(1) $a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos 75°$

(2) $a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 0$

(3) $a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

(4) $a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

2. NTA Ans. (4)

Sol. $[x]^2 + 2[x + 2] - 7 = 0$

$\Rightarrow [x]^2 + 2[x] + 4 - 7 = 0$

$\Rightarrow [x] = 1, -3$

$\Rightarrow x \in [1, 2) \cup [-3, -2)$

3. NTA Ans. (3)

Sol. $x^2 - 3x + p = 0 \genfrac{}{}{0pt}{}{\alpha}{\beta}$

$\alpha, \beta, \gamma, \delta$ in G.P.

$\alpha + \alpha r = 3$ ....(1)

$x^2 - 6x + q = 0 \genfrac{}{}{0pt}{}{\gamma}{\delta}$

$\alpha r^2 + \alpha r^3 = 6$ ...(2)

(2) ÷ (1)

$r^2 = 2$

So, $\dfrac{2q + p}{2q - p} = \dfrac{2r^5 + r}{2r^5 - r} = \dfrac{2r^4 + 1}{2r^4 - 1} = \dfrac{9}{7}$

4. NTA Ans. (1)

Sol. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a > b)$; $\dfrac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$ ...(i)

Now, $\phi(t) = \dfrac{5}{12} + t - t^2 = \dfrac{8}{12} - \left(t - \dfrac{1}{2}\right)^2$

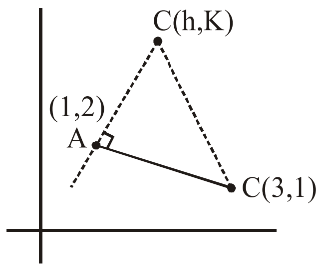
$\phi(t)_{max} = \dfrac{8}{12} = \dfrac{2}{3} = e \Rightarrow e^2 = 1 - \dfrac{b^2}{a^2} = \dfrac{4}{9}$ ... (ii)

$\Rightarrow a^2 = 81$ (from (i) & (ii))

So, $a^2 + b^2 = 81 + 45 = 126$

5. NTA Ans. (3)

Sol.



$\left(\dfrac{K-2}{h-1}\right)\left(\dfrac{1-2}{3-1}\right) = -1 \Rightarrow K = 2h$ ...(1)

$\sqrt{5}\,|h - 1| = 10$

$\because [\Delta ABC] = 5\sqrt{5}$

$\Rightarrow \dfrac{1}{2}\left(\sqrt{5}\right)\sqrt{(h-1)^2 + (K-2)^2} = 5\sqrt{5}$ ....(2)

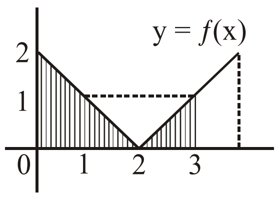
$\Rightarrow h = 2\sqrt{5} + 1$ $(h > 0)$

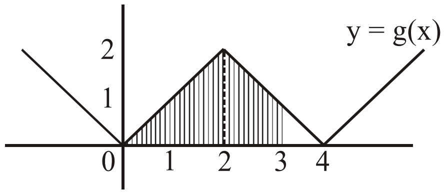
6. NTA Ans. (4)

Sol. $\int_0^3 g(x) - f(x) = \int_0^3 \big||x-2| - 2\big|dx - \int_0^3 |x-2|dx$

$= \left(\dfrac{1}{2} \times 2 \times 2 + 1 + \dfrac{1}{2} \times 1 \times 1\right) - \left(\dfrac{1}{2} \times 2 \times 2 + \dfrac{1}{2} \times 1 \times 1\right)$

$= \left(2 + 1 + \dfrac{1}{2}\right) - \left(2 + \dfrac{1}{2}\right) = 1$





7. NTA Ans. (3)

Sol. Let TV(r) denotes truth value of a statement r.

Now, if TV(p) = TV(q) = T

$\Rightarrow TV(S_1) = F$

Also, if TV(p) = T & TV(q) = F

$\Rightarrow TV(S_2) = T$

8. NTA Ans. (1)

Sol. Since, (3, 3) lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{9}{a^2}-\frac{9}{b^2}=1$ ....(1)

Now, normal at (3, 3) is $y-3=-\frac{a^2}{b^2}(x-3)$,

which passes through (9, 0) $\Rightarrow$ $b^2 = 2a^2$ ....(2)

So, $e^2 = 1 + \frac{b^2}{a^2} = 3$

Also, $a^2 = \frac{9}{2}$ (from (i) & (ii))

Thus, $(a^2, e^2) = \left(\frac{9}{2}, 3\right)$

9. NTA Ans. (4)

Sol. $f(x) = \int_1^3 \frac{\sqrt{x}\,dx}{(1+x)^2} = \int_1^{\sqrt{3}} \frac{t.2t\,dt}{(1+t^2)^2}$ $\left(\text{put } \sqrt{x}=t\right)$

$= \left(-\frac{t}{1+t^2}\right)_1^{\sqrt{3}} + (\tan^{-1}t)_1^{\sqrt{3}}$ [Appling by parts]

$= -\left(\frac{\sqrt{3}}{4}-\frac{1}{2}\right)+\frac{\pi}{3}-\frac{\pi}{4}$

$= \frac{1}{2}-\frac{\sqrt{3}}{4}+\frac{\pi}{12}$

10. NTA Ans. (4)

Sol. $n(B) \le n(A \cup B) \le n(U)$

$\Rightarrow 76 \le 76 + 63 - x \le 100$

$\Rightarrow -63 \le -x \le -39$

$\Rightarrow 63 \ge x \ge 39$

11. NTA Ans. (3)

Sol. $u = \frac{2z+i}{z-ki}$

$= \frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2} + i\frac{\left(x(2y+1)-2x(y-k)\right)}{x^2+(y-k)^2}$

Since Re(u) + Im(u) = 1

$\Rightarrow 2x^2 + (2y+1)(y-k)+x(2y+1)-2x(y-k)$

$= x^2 + (y-k)^2$

$\left.\begin{matrix}P(0,y_1)\\Q(0,y_2)\end{matrix}\right\rangle \Rightarrow y^2+y-k-k^2=0 \left\langle\begin{matrix}y_1+y_2=-1\\y_1y_2=-k-k^2\end{matrix}\right.$

$\because$ PQ = 5

$\Rightarrow |y_1-y_2|=5 \Rightarrow k^2+k-6=0$

$\Rightarrow k = -3, 2$

So, k = 2 (k > 0)

12. NTA Ans. (4)

Sol. $f(x)=\vec{a}.\left(\vec{b}\times\vec{c}\right)=\begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} = x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$

and $f''(-3) < 0$

$\Rightarrow$ local maxima at $x = x_0 = -3$

Thus, $\vec{a}=-3\hat{i}-2\hat{j}+3\hat{k}$,

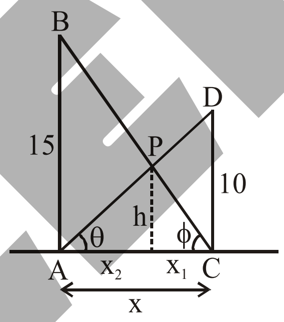
$\vec{b}=-2\hat{i}-3\hat{j}-\hat{k}$,

and $\vec{c}=7\hat{i}-2\hat{j}-3\hat{k}$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=9-5-26=-22$

13. NTA Ans. (4)

Sol.



$\tan\theta=\frac{10}{x}=\frac{h}{x_2}\Rightarrow x_2=\frac{hx}{10}$

$\tan\phi=\frac{15}{x}=\frac{h}{x_1}\Rightarrow x_1=\frac{hx}{15}$

Now, $x_1 + x_2 = x = \frac{hx}{15}+\frac{hx}{10}$

$\Rightarrow 1 = \frac{h}{10}+\frac{h}{15} \Rightarrow h = 6$

14. NTA Ans. (1)

Sol. $\bar{x}=10$

$\Rightarrow \bar{x}=\frac{63+a+b}{8}=10 \Rightarrow a + b = 17$ ...(1)

Since, variance is independent of origin.

So, we subtract 10 from each observation.

So, $\sigma^2 = 13.5 = \frac{79+(a-10)^2+(b-10)^2}{8}-(10-10)^2$

$\Rightarrow a^2 + b^2 - 20(a + b) = -171$

$\Rightarrow a^2 + b^2 = 169$ ...(2)

From (i) & (ii) ; a = 12 & b = 5

15. NTA Ans. (4)

Sol. $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx = \int\left(\frac{x}{\cos x}\right).\frac{x\cos x\,dx}{(x\sin x+\cos x)^2}$

$= \frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)$

$+\int\left(\frac{\cos x + x\sin x}{\cos^2 x}\right)\left(\frac{1}{x\sin x+\cos x}\right)dx$

$= -\frac{x\sec x}{x\sin x+\cos x}+\int \sec^2 x\,dx$

$= -\frac{x\sec x}{x\sin x+\cos x}+\tan x + C$

16. NTA Ans. (2)

Sol. $1 + (1 - 2^2.1) + (1 - 4^2.3) + ... + (1 - 20^2.19)$
$= \alpha - 220\,\beta$
$= 11 - (2^2.1 + 4^2.3 + ... + 20^2.19)$

$= 11 - 2^2.\sum_{r=1}^{10} r^2(2r-1) = 11 - 4\left(\frac{110^2}{2} - 35\times 11\right)$

$= 11 - 220(103)$
$\Rightarrow \alpha = 11,\ \beta = 103$

17. NTA Ans. (1)

Sol. $x\frac{dy}{dx} - y = x^2(x\cos x+\sin x),\ x>0$

$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x) \Rightarrow \frac{dy}{dx} + Py = Q$

so, I.F. $= e^{\int -\frac{1}{x}dx} = \frac{1}{|x|} = \frac{1}{x}$ $(x > 0)$

Thus, $\frac{y}{x} = \int \frac{1}{x}\big(x(x\cos x + \sin x)\big)dx$

$\Rightarrow \frac{y}{x} = x\sin x + C$

$\because\ y(\pi) = \pi \Rightarrow C = 1$

so, $y = x^2\sin x + x \Rightarrow (y)_{\pi/2} = \frac{\pi^2}{4} + \frac{\pi}{2}$

Also, $\frac{dy}{dx} = x^2\cos x + 2x\sin x + 1$

$\Rightarrow \frac{d^2y}{dx^2} = -x^2\sin x + 4x\cos x + 2\sin x$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\frac{\pi}{2}} = -\frac{\pi^2}{4} + 2$

Thus, $y_{\left(\frac{\pi}{2}\right)} + \frac{d^2y}{dx^2}_{\left(\frac{\pi}{2}\right)} = \frac{\pi}{2} + 2$

18. NTA Ans. (2)

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + {}^{30}C_6$

$= {}^{50}C_6 + {}^{49}C_6 + ... + {}^{31}C_6 + \left({}^{30}C_6 + {}^{30}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{49}C_6 + ... + \left({}^{31}C_6 + {}^{31}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{50}C_7 - {}^{30}C_7$

$= {}^{51}C_7 - {}^{30}C_7$

$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$

19. NTA Ans. (3)

Sol. $f(2) = 8,\ f'(2) = 5,\ f'(x)\geq 1,\ f''(x) \geq 4,\ \forall x\in(1,6)$

$f''(x) = \frac{f'(5)-f'(2)}{5-2} \geq 4 \Rightarrow f'(5) \geq 17$ ...(1)

$f'(x) = \frac{f(5)-f(2)}{5-2} \geq 1 \Rightarrow f(5) \geq 11$ ...(2)

$\overline{f'(5) + f(5) \geq 28}$

20. NTA Ans. (2)

Sol. $\left(a+\sqrt{2}b\cos x\right)\left(a-\sqrt{2}b\cos y\right) = a^2 - b^2$

$\Rightarrow a^2 - \sqrt{2}\,ab\cos y + \sqrt{2}\,ab\cos x$

$-2b^2\cos x\cos y = a^2 - b^2$

Differentiating both sides :

$0 - \sqrt{2}\,ab\left(-\sin y\frac{dy}{dx}\right) + \sqrt{2}\,ab(-\sin x)$

$-2b^2\left[\cos x\left(-\sin y\frac{dy}{dx}\right) + \cos y(-\sin x)\right] = 0$

At $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$:

$ab\frac{dy}{dx} - ab - 2b^2\left(-\frac{1}{2}\frac{dy}{dx} - \frac{1}{2}\right) = 0$

$\Rightarrow \frac{dx}{dy} = \frac{ab+b^2}{ab-b^2} = \frac{a+b}{a-b}$; a, b > 0

21. NTA Ans. (5)

Sol. $D=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & 1\\ 1 & -7 & a\end{vmatrix}=0\Rightarrow a=8$

also, $D_1=\begin{vmatrix}9 & -2 & 3\\ b & 1 & 1\\ 24 & -7 & 8\end{vmatrix}=0\Rightarrow b=3$

hence, a – b = 8 – 3 = 5

22. NTA Ans. (3)

Sol. We have, 1 – (probability of all shots result in failure) > $\frac{1}{4}$

$\Rightarrow 1-\left(\frac{9}{10}\right)^n>\frac{1}{4}$

$\Rightarrow \frac{3}{4}>\left(\frac{9}{10}\right)^n \Rightarrow n \geq 3$

23. NTA Ans. (10)

Sol. Since, $\lim\limits_{x\to 0}\frac{f(x)}{x}$ exist $\Rightarrow f(0) = 0$

Now, $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\frac{f(h)+xh^2+x^2h}{h}$ (take y = h)

$=\lim\limits_{h\to 0}\frac{f(h)}{h}+\lim\limits_{h\to 0}(xh)+x^2$

$\Rightarrow f'(x) = 1 + 0 + x^2$

$\Rightarrow f'(3) = 10$

24. NTA Ans. (8)

Sol. Given $(2x^2 + 3x + 4)^{10} = \sum\limits_{r=0}^{20} a_r\, x^r$ .... (1)

replace x by $\frac{2}{x}$ in above identity :-

$\frac{2^{10}\left(2x^2+3x+4\right)^{10}}{x^{20}}=\sum\limits_{r=0}^{20}\frac{a_r\, 2^r}{x^r}$

$\Rightarrow 2^{10}\sum\limits_{r=0}^{20} a_r\, x^r=\sum\limits_{r=0}^{20} a_r\, 2^r\, x^{(20-r)}$ (from (i))

now, comparing coefficient of x^7 from both sides

(take r = 7 in L.H.S. & r = 13 in R.H.S.)

$2^{10}\, a_7 = a_{13}\, 2^{13} \Rightarrow \frac{a_7}{a_{13}}=2^3=8$

25. NTA Ans. (3)

Sol. $D_1=\begin{vmatrix}-7 & 4 & -1\\ 8 & 1 & 5\\ 15 & b & 6\end{vmatrix}=0\Rightarrow b=-3$

$D=\begin{vmatrix}1 & 4 & -1\\ 3 & 1 & 5\\ a & b & 6\end{vmatrix}=0\Rightarrow 21a-8b-66=0$ .... (1)

P : 2x – 3y + 6z = 15

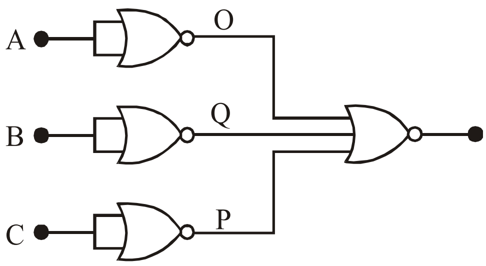
so required distance $=\frac{21}{7}=3$

SET # 06

PHYSICS

1. NTA Ans. (1)

Sol.



A	B	C	
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
1	0	1	0
0	1	1	0
1	1	1	1

2. NTA Ans. (2)

Sol. $I_1 = \dfrac{MR^2}{2} = \dfrac{\rho(\pi R^2)t.R^2}{2}$

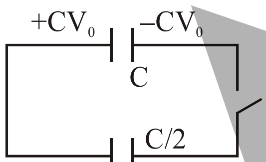
$I \propto R^4$

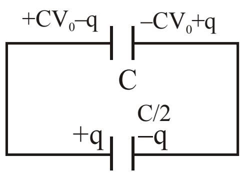
$\dfrac{I_1}{I_2} = \dfrac{R_1^4}{R_2^4} = \dfrac{1}{16}$

$\therefore \dfrac{R_1}{R_2} = \dfrac{1}{2}$

3. NTA Ans. (1)

Sol.





$\dfrac{CV_0 - q}{C} = \dfrac{q}{C/2} = \dfrac{2q}{C}$

$V_0 = \dfrac{3q}{C} \quad \Rightarrow q = \dfrac{CV_0}{3}$

$U_i = \dfrac{1}{2}CV_0^2$

$$U_f = \frac{\left(\frac{2CV_0}{3}\right)^2}{2C} + \frac{\left(\frac{CV_0}{3}\right)^2}{2\left(\frac{C}{2}\right)}$$

$$= \frac{1}{2}CV_0^2\left[\frac{4}{9}+\frac{2}{9}\right] = \frac{1}{2}CV_0^2\left(\frac{2}{3}\right)$$

$$\text{Heat loss} = \frac{1}{2}CV_0^2 - \left(\frac{2}{3}\right)\left(\frac{1}{2}CV_0^2\right)$$

$$= \frac{1}{6}CV_0^2$$

4. NTA Ans. (2)

Sol. $F = 200\ N \qquad \text{for } 0 \le x \le 15$

$= 200 - \dfrac{100}{15}(x-15) \quad \text{for} \quad 15 \le x < 30$

$W = \int F\,dx$

$$= \int_0^{15} 200\,dx + \int_{15}^{30}\left(300 - \frac{100}{15}x\right)dx$$

$$= 200 \times 15 + 300 \times 15 - \frac{100}{15} \times \frac{(30^2 - 15^2)}{2}$$

$= 3000 + 4500 - 2250$

$= 5250$ J

5. NTA Ans. (2)

Sol. $\vec{E} = E_0(\hat{x} + \hat{y})\sin(kz - \omega t)$

direction of propagation $= +\hat{k}$

$\hat{E} = \dfrac{\hat{i} + \hat{j}}{\sqrt{2}}$

$\hat{k} = \hat{E} \times \hat{B}$

$\hat{k} = \left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right) \times \hat{B} \qquad \Rightarrow \hat{B} = \dfrac{-\hat{i}+\hat{j}}{\sqrt{2}}$

$\therefore \vec{B} = \dfrac{E_0}{C}(-\hat{x} + \hat{y})\sin(kz - \omega t)$

6. NTA Ans. (1)

Sol. B.E. = $[\Delta m].c^2$

$M_{expected} = ZM_p + (A - Z)M_n$

$= 50\,[1.00783] + 70\,[1.00867]$

$M_{actual} = 119.902199$

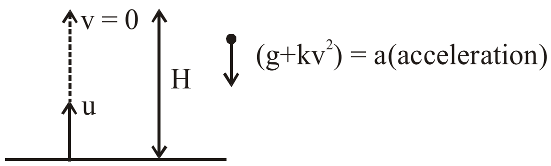
$$\text{B.E.} = \left[50[1.00783]+70[1.00867]-119.902199\right] \times 931$$

$= 1020.56$

$$\frac{BE}{\text{nucleon}} = \frac{1020.56}{120} = 8.5 \text{ MeV}$$

7. NTA Ans. (2)

Sol.



$$\vec{F} = mkv^2 - mg$$

$$\vec{a} = \frac{\vec{F}}{m} = -\left[kv^2 + g\right]$$

$$\Rightarrow \quad v.\frac{dv}{dh} = -[kv^2 + g]$$

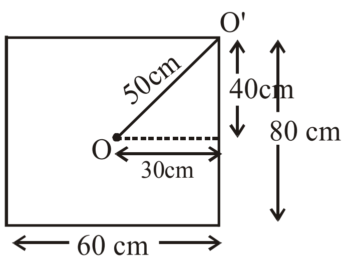
$$\Rightarrow \int_u^0 \frac{v.dv}{kv^2+g} = -\int_0^H dh$$

$$\frac{1}{2K}\ln\left[kv^2+g\right]_u^0 = -H$$

$$\Rightarrow \frac{1}{2K}\ln\left[\frac{ku^2+g}{g}\right] = H$$

8. NTA Ans. (4)

Sol.



Rectangular sheet

$$I_O = \frac{M}{12}[L^2+B^2] = \frac{M}{12}[80^2+60^2]$$

$$I_{O'} = I_0 + Md^2 \{\text{parallel axis theorem}\}$$

$$= \frac{M}{12}\left[80^2+60^2\right] + M\,[50]^2$$

$$\frac{I_O}{I_{O'}} = \frac{M/12[80^2+60^2]}{\frac{M}{12}[80^2+60^2]+M[50]^2} = \frac{1}{4}$$

9. NTA Ans. (1)

Sol. (I) Adiabatic process $\Rightarrow \Delta Q = 0$

No exchange of heat takes place with surroundings

(II) Isothermal proess $\Rightarrow$ Temperature remains constant $(\Delta T = 0)$

$$\Delta u = \frac{F}{2}nR\Delta T \Rightarrow \Delta u = 0$$

No change in internal energy $[\Delta u = 0]$

(III) Isochoric process Volume remains constant

$\Delta V = 0$

$$W = \int P.dV = 0$$

Hence work done is zero.

(IV) Isobaric process $\Rightarrow$ Pressure remains constant

$$W = P.\ \Delta V \neq 0$$

$$\Delta u = \frac{F}{2}nR\Delta T = \frac{F}{2}[P\Delta V] \neq 0$$

$$\Delta Q = nC_P\Delta T \neq 0$$

10. NTA Ans. (2)

Sol. For paramagnetic material

According to curies law

$$\chi \propto \frac{1}{T} \quad \Rightarrow \quad \chi_1 T_1 = \chi_2 T_2$$

$$\Rightarrow \frac{6}{0.4}\times 4 = \frac{I}{0.3}\times 24$$

$$I = \frac{0.3}{0.4} = 0.75 \text{ A/m}$$

11. NTA Ans. (3)

Sol. $U_{max} = \frac{1}{2}LI_{max}^2$

$i = I_{max}\left(1-e^{-Rt/L}\right)$

For U to be $\frac{U_{max}}{n}$; i has to be $\frac{I_{max}}{\sqrt{n}}$

$\frac{I_{max}}{\sqrt{n}} = I_{max}(1-e^{-Rt/L})$

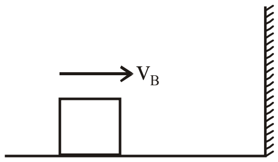
$e^{-Rt/L} = 1-\frac{1}{\sqrt{n}} = \frac{\sqrt{n}-1}{\sqrt{n}}$

$-\frac{Rt}{L} = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$

$t = \frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$

12. NTA Ans. (1)

Sol.



$f_1 = \left(\frac{330}{330-v_B}\right)420$

$f_2 = \left(\frac{330+v_0}{330}\right)\left(\frac{330}{330-v_B}\right)420$

$490 = \left(\frac{330+v_B}{330-v_B}\right)420$

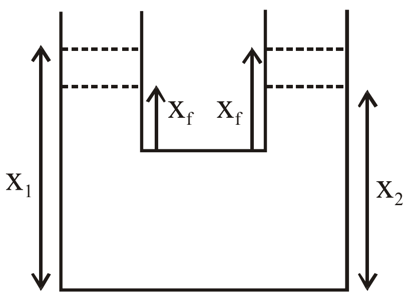
$\frac{7}{6} = \frac{330+v_B}{330-v_B}$

$v_B = \frac{330}{13}$ m / s

$= \frac{330}{13}\times\frac{18}{5} \approx 91$ km / hr

13. NTA Ans. (3)

Sol.



$U_i = (\rho Sx_1)g\,.\,\frac{x_1}{2} + (\rho Sx_2)g.\frac{x_2}{2}$

$U_f = (\rho Sx_f)g.\frac{x_f}{2}\times 2$

By volume conservation

$Sx_1 + Sx_2 = S(2x_f)$

$x_f = \frac{x_1+x_2}{2}$

$\Delta U = \rho Sg\left[\left(\frac{x_1^2}{2}+\frac{x_2^2}{2}\right)-x_f^2\right]$

$= \rho Sg\left[\frac{x_1^2}{2}+\frac{x_2^2}{2}-\left(\frac{x_1+x_2}{2}\right)^2\right]$

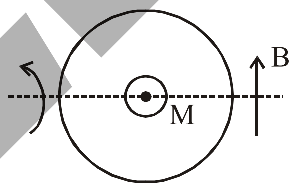
$= \frac{\rho Sg}{2}\left[\frac{x_1^2}{2}+\frac{x_2^2}{2}-x_1x_2\right]$

$= \frac{\rho Sg}{4}(x_1-x_2)^2$

14. NTA Ans. (1)

Sol. $I_{dia} = 0.8$ kg/m^2

$M = 20$ Am^2



$U_i + K_i = U_f + K_f$

$0 + 0 = -MB\cos 30° + \frac{1}{2}I\omega^2$

$20 \times 4 \times \frac{\sqrt{3}}{2} = \frac{1}{2}(0.8)\,\omega^2$

$\omega = \sqrt{100\sqrt{3}} = 10(3)^{1/4}$

15. NTA Ans. (4)

Sol. $E = E_0(1 - ax^2)$

$W = \int qE\,dx = qE_0\int_0^{x_0}(1-ax^2)\,dx$

$= qE_0\left[x_0 - \frac{ax_0^3}{3}\right]$

For $\Delta KE = 0$, $W = 0$

Hence $x_0 = \sqrt{\frac{3}{a}}$

16. NTA Ans. (2)

Sol. $B = -\dfrac{\Delta P}{\frac{\Delta V}{V}}$

$\left|\dfrac{\Delta V}{V}\right| = \dfrac{\Delta P}{B}$

$= \dfrac{4\times10^9}{8\times10^{10}} = \dfrac{1}{20}$

$\dfrac{\Delta \ell}{\ell} = \dfrac{1}{3} \times \dfrac{\Delta V}{V} = \dfrac{1}{60}$

Percentage change $= \dfrac{\Delta\ell}{\ell}\times100\%$

$= \dfrac{100}{60}\% = 1.67\%$

17. NTA Ans. (4)

Sol. Voltage across AC = 8V

$R_{AC} = 4 + 4 = 8\Omega$

$i_1 = \dfrac{V}{R_{AC}} = \dfrac{8}{8} = 1\ A$

18. NTA Ans. (3)

Sol. $V_{orbit} = \sqrt{\dfrac{GM}{R}}$

$V_{escape} = \sqrt{\dfrac{2GM}{R}}$

$\dfrac{V_{orbit}}{V_{escape}} = \dfrac{1}{\sqrt{2}}$

19. NTA Ans. (3)

Sol. $x = \dfrac{IFV^2}{WL^4}$

$[x] = \dfrac{[ML^2][MLT^{-2}][LT^{-1}]^2}{[ML^2T^{-2}][L]^4}$

$[x] = [ML^{-1}T^{-2}]$

$[\text{Energy density}] = \left[\dfrac{E}{V}\right]$

$= \left[\dfrac{ML^2T^{-2}}{L^3}\right]$

$= [ML^{-1}T^{-2}]$

Same as x

20. NTA Ans. (3)

Sol. $eV = \dfrac{hc}{\lambda} - \phi$

$V = \left(\dfrac{hc}{e}\right)\left(\dfrac{1}{\lambda}\right) - \phi$

Slope of the line in above equation and all other terms are independent of intensity.

The graph does not change.

21. NTA Ans. (200.00)

Sol. Condition for minimum,

$d\sin\theta = n\lambda$

$\therefore\ \sin\theta = \dfrac{n\lambda}{d} < 1$

$n < \dfrac{d}{\lambda} = \dfrac{6\times10^{-5}}{6\times10^{-7}} = 100$

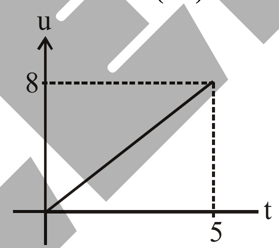
$\therefore$ Total number of minima on one side = 99

Total number of minima = 198

Correct Answer is 198

22. NTA Ans. (20)

Sol.

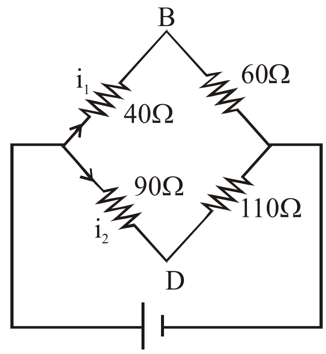


Distance $= \int v\,dt$

Area under graph $= \dfrac{1}{2}\times5\times8 = 20$

23. NTA Ans. (2)

Sol.



$i_1 = \dfrac{40}{40+60} = 0.4$

$i_2 = \dfrac{40}{90+110} = \dfrac{1}{5}$

$v_B + i_1(40) - i_2(90) = v_D$

$v_B - v_D = \dfrac{1}{5}(90) - \dfrac{4}{10}\times40$

$v_B - v_D = 18 - 16 = 2$

24. NTA Ans. (5.00)

Sol. Using displacement method

$$f = \frac{D^2 - d^2}{4D}$$

Here, D = 100 cm

d = 40 cm

$$f = \frac{100^2 - 40^2}{4(100)} = 21 \text{ cm}$$

$$P = \frac{1}{f} = \frac{100}{21} D$$

$$\frac{N}{100} = \frac{100}{21}$$

N = 476

25. NTA Ans. (150)

Sol. PV = nRT

$P\Delta V + V\Delta P = 0$ (for constant temp.)

$P\Delta V = nR\Delta T$ (for constant pressure)

$$\Delta T = \frac{P\Delta V}{nR}$$

$$\Delta P = -\frac{P\Delta V}{V} \quad (\Delta V \text{ is same in both cases})$$

$$\frac{\Delta T}{\Delta P} = \frac{P\Delta V}{nR}\frac{V}{-P\Delta V} = \frac{-V}{nR} = -\frac{T}{P}$$

(PV = nRT)

$$\left(\frac{V}{nR} = \frac{T}{P}\right)$$

$$\left|\frac{\Delta T}{\Delta P}\right| = \left|\frac{-300}{2}\right| = 150 \text{ a}$$

CHEMISTRY

1. NTA Ans. (2)

Sol. $A \rightleftharpoons B + C \quad K_{eq}^{(1)} = \frac{[B][C]}{[A]}$ .....(1)

$B + C \rightleftharpoons P \quad K_{eq}^{(2)} = \frac{[P]}{[B][C]}$ .....(2)

For

$A \rightleftharpoons P \quad K_{eq} = \frac{[P]}{[A]}$

Multiplying equation (1) & (2)

$$K_{eq}^{(1)} \times K_{eq}^{(2)} = \frac{[P]}{[A]} = K_{eq}$$

2. NTA Ans. (4)

Sol. As the expansion is done in vaccum that is in absence of p_{ext} so

W = zero

3. NTA Ans. (2)

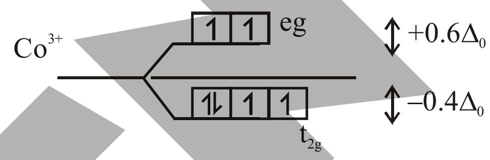
Sol. $H_{(g)} + e^- \rightarrow H^-$ is exothermic

rest of all endothermic process.

4. NTA Ans. (4)

Sol. $[CoF_3(H_2O)_3]$ $\quad \Delta_0 < P$

Means all ligands behaves as weak field ligands



$= [-0.4 \times 4 + 0.6 \times 2]\Delta_0$

$= [-1.6 + 1.2]\Delta_0$

$= [-0.4\Delta_0]$

5. NTA Ans. (3)

Sol. Seldane is an antihistamine drugs it inhibits the action of histamine receptor.

6. NTA Ans. (2)

Sol. [Be]

$BeSO_4$ is water soluble

$Be(OH)_2$ is water insoluble

BeO is stable to heat

7. NTA Ans. (2)

Sol. [Be]

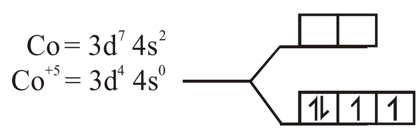
$BeSO_4$ is water soluble

$Be(OH)_2$ is water insoluble

BeO is stable to heat

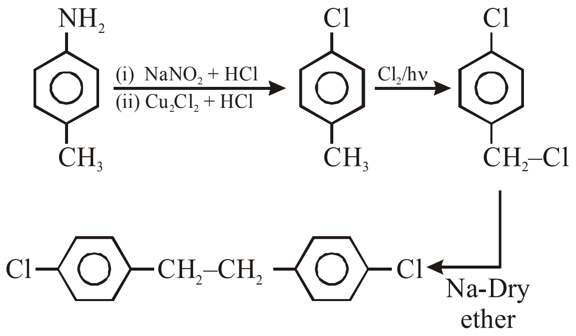
8. NTA Ans. (3)

Sol. $[Co(OX)_2(OH)_2]^-$ $\quad \Delta_0 > P$ [S.F.L]



It has highest number of unpaired e^-s. so it is most paramagnetic.

9. NTA Ans. (3)

Sol. 

10. NTA Ans. (3)

11. NTA Ans. (1)

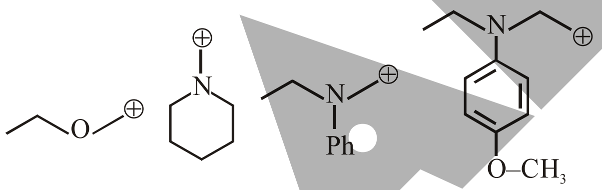
Sol. Due to industrial process SO_2 gas is released which is responsible for acid rain & global warming.

12. NTA Ans. (2)

Sol. $R-x+aq.AgNO_3 \xrightarrow{R.D.S} R^{\oplus} + \underset{(PPT)}{Agx} \quad (1)$

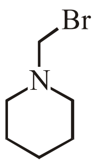
So rate of P.P.T formation of Agx depend's on stability of carbocation (R^+)

In given question formed carbocation will be



(a) (b) (c) (d)

Most stable carbocation is (b) so

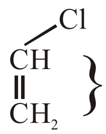
 give fastest P.P.T of AgBr with aq $AgNO_3$

13. NTA Ans. (1)

Sol. $[Ni(CN)_4]^{2-}$

dsp^2 hybridisation.

14. NTA Ans. (3)

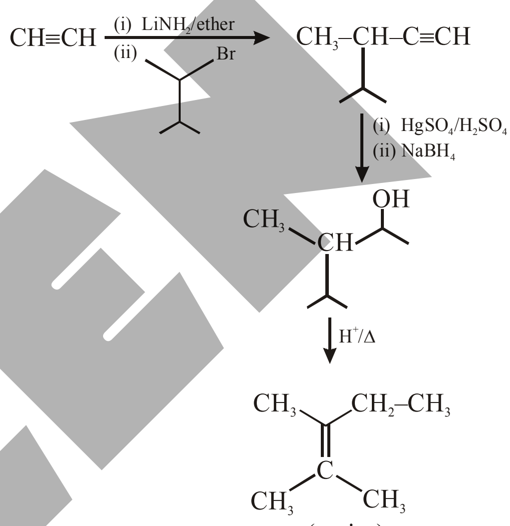
Sol. 

In option (3) C—Cl bond is shortest due to resonance of lone pair of –Cl.

Due to resonance C—Cl bond acquire partial double bond character.

Hence C—Cl bond length is least.

15. NTA Ans. (2)

Sol. 

(major)

Now :- (i) $HgSO_4$/dil.H_2SO_4

(ii) $NaBH_4$

is convert triple bond into ketone and formed ketone is reduced by $NaBH_4$ and convert into Alcohol.

16. NTA Ans. (4)

Sol. $KMnO_4$ will not give satisfactory result when it is titrated by HCl.

17. NTA Ans. (4)

Sol. As voltage is '2V' so both Ag^+ & Au^+ will reduce and their equal gm equivalent will reduce so

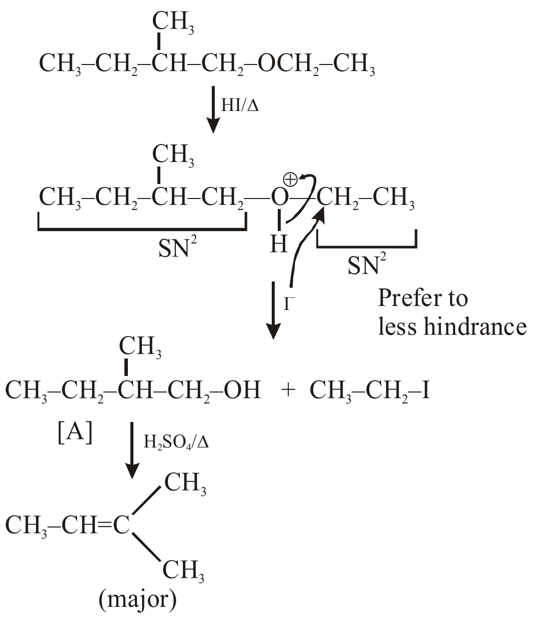
gmeq Ag = gmeq of Au

$$\frac{Wt_{Ag}}{E_{qwt_{Ag}}} = \frac{Wt_{Au}}{E_{qwt_{Au}}}$$

$$\text{So} \quad \frac{wt_{Ag}}{wt_{Au}} = \frac{E_{qwt_{Ag}}}{E_{qwt_{Au}}} = \frac{At\,wt_{Ag}}{Atwt_{Au}}$$

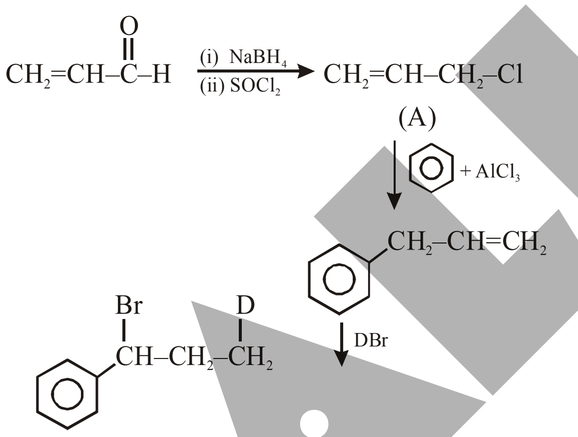
18. NTA Ans. (4)

Sol.



19. NTA Ans. (3)

Sol.



20. NTA Ans. (3)

Sol. As we know $\Delta E = \dfrac{hc}{\lambda}$

So $\lambda = \dfrac{hc}{\Delta E}$ for λ minimum i.e.

shortest; ΔE = maximum

for Lyman series n = 1 & for ΔE_{max}

Transition must be form n = ∞ to n = 1

So $\dfrac{1}{\lambda} = R_H Z^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$$\frac{1}{\lambda} = R_H Z^2 (1-0)$$

$$\frac{1}{\lambda} = R \times (1)^2 \Rightarrow \lambda_1 = \frac{1}{R}$$

For longest wavelength ΔE = minimum for Balmer series n = 3 to n = 2 will have ΔE minimum

for He^+ Z = 2

So $\dfrac{1}{\lambda_2} = R_H \times Z^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$$\frac{1}{\lambda_2} = R_H \times 4\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda_2} = R_H \times \frac{5}{9}$$

$$\lambda_2 = \lambda_1 \times \frac{9}{5}$$

21. NTA Ans. (10)

Sol. Molar mass of $Na_2CO_3 \cdot xH_2O$

⇒ 23 × 2 + 12 + 48 + 18x

⇒ 46 + 12 + 48 + 18x

⇒ (106 + 18x)

$\text{Eqwt} = \dfrac{M}{2} = (53 + 9x)$

As n_{factor} in dissolution will be determined from net cationic or anionic charge; which is 2 so

$\text{Eqwt} = \dfrac{M}{2} = 53 + 9x$

$\text{Gmeq} = \dfrac{\text{wt}}{\text{Eqwt}} = \dfrac{1.43}{53+9x}$

$\text{Normality} = \dfrac{\text{Gmeq}}{V_{litre}}$

$\text{Normality} = 0.1 = \dfrac{\frac{1.43}{53+9x}}{0.1}$

As volume = 100 ml
= 0.1 Litre

So $10^{-2} = \dfrac{1.43}{53+9x}$

53 + 9x = 143

9x = 90

x = 10.00

22. NTA Ans. (167)

Sol. Osmotic pressure = $\pi = i \times C \times RT$

For NaCl i = 2 so

$\pi_{NaCl} = i \times C_{NaCl} \times RT$ C_{NaCl} = conc. of NaCl

$0.1 = 2 \times C_{NaCl} \times RT$

$C_{NaCl} = \dfrac{0.05}{RT}$ $C_{glucose}$= conc. of glucose

For glucose i = 1 so

$\pi_{Glucose} = i \times C_{glucose} \times RT$

$0.2 = 1 \times C_{glucose} \times RT$

$C_{Glucose} = \dfrac{0.2}{RT}$ η_{NaCl} = No. of moles NaCl

η_{NaCl} in 1 L = $C_{NaCl} \times V_{Litre}$

$= \dfrac{0.05}{RT}$ $\eta_{glucose}$= No. of moles glucose

$\eta_{glucose}$ in 2 L = $C_{glucose} \times V_{Litre}$

$= \dfrac{0.4}{RT}$

$V_{Total} = 1 + 2 = 3L$

so Final conc. NaCl = $\dfrac{0.05}{3RT}$

Final conc. glucose = $\dfrac{0.4}{3RT}$

$\pi_{Total} = \pi_{NaCl} + \pi_{glucose}$

$= \left[i \times C_{NaCl} + C_{glucose}\right] \times RT$

$= \left(\dfrac{2 \times 0.05}{3RT} + \dfrac{0.4}{3RT}\right) \times RT$

$= \dfrac{0.5}{3}$ atm

= 0.1666 atm

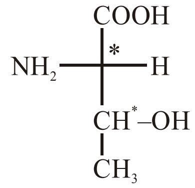
= 166.6×10^{-3} atm

$\Rightarrow 167.00 \times 10^{-3}$ atm

so x = 167.00

23. NTA Ans. (2)

Sol. Structure of Threonine is :



S. 2-chiral center is present

24. NTA Ans. (19)

Sol. $\left[Fe^{2+} \rightarrow Fe^{3+} + e^{-}\right] \times 2$

$$\frac{H_2O_2 + 2e^- \rightarrow 2HO^{\ominus}}{2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2HO^{\ominus}_{(aq)}}$$

x = 2 y = 2

$\left[8H^+ + MnO_4^- + 5e^- \rightarrow Mn^{2+} + 4H_2O\right] \times 2$

$\left[H_2O_2 \rightarrow O_{2(g)} + 2H^+ + 2e^-\right] \times 5$

$\Rightarrow 16H^+ + 2MnO_4^- + 5H_2O_2$

$\rightarrow 2Mn^{2+} + 8H_2O + 5O_{2(g)} + 10H^+$

$\Rightarrow 6H^+ + 2MnO_4^- + 5H_2O_2$

$\rightarrow 2Mn^{2+} + 8H_2O + 5O_{2(g)}$

So x' = 2 y' = 8 z' = 5

so x + y + x' + y' + z'

$\Rightarrow$ 2 + 2 + 2 + 8 + 5

$\Rightarrow$ 19

25. NTA Ans. (84290.00 to 84300.00)

Sol. T_1 = 300K T_2 = 315K

As per question $K_{T_2} = 5K_{T_1}$ as molecules activated are increased five times so k will increases 5 times

Now

$$\ln\left(\frac{K_{T_2}}{K_{T_1}}\right) = \frac{Ea}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\ln 5 = \frac{Ea}{R}\left(\frac{15}{300 \times 315}\right)$$

So $Ea = \dfrac{1.6094 \times 8.314 \times 300 \times 315}{15}$

Ea = 84297.47 Joules/mole

MATHEMATICS

1. NTA Ans. (1)

Sol. $f(x)=\begin{cases} \frac{\pi}{4}+\tan^{-1}x & , \quad x\in(-\infty,-1]\cup[1,\infty) \\ -\frac{(x+1)}{2} & , \quad x\in(-1,0] \\ \frac{x-1}{2} & , \quad x\in(0,1) \end{cases}$

for continuity at x = –1

L.H.L. = $\frac{\pi}{4}-\frac{\pi}{4}=0$

R.H.L. = 0

so, continuous at x = –1

for continuity at x = 1

L.H.L. = 0

R.H.L. = $\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

so, not continuous at x = 1

For differentiability at x = –1

L.H.D. = $\frac{1}{1+1}=\frac{1}{2}$

R.H.D. = $-\frac{1}{2}$

so, non differentiable at x = –1

2. NTA Ans. (4)

Sol. $n(X_i) = 10.\ \bigcup_{i=1}^{50} X_i = T, \Rightarrow\ n(T) = 500$

each element of T belongs to exactly 20

elements of $X_i \Rightarrow \frac{500}{20}$ = 25 distinct elements

so $\frac{5n}{6} = 25 \Rightarrow n = 30$

3. NTA Ans. (4)

Sol. $\alpha + \beta = 1,\ \alpha\beta = 2\lambda$

$\alpha + \beta = \frac{10}{3}, \qquad \alpha\gamma = \frac{27\lambda}{3} = 9\lambda$

$\gamma - \beta = \frac{7}{3}$,

$\frac{\gamma}{\beta}=\frac{9}{2}\Rightarrow\gamma = \frac{9}{2}\beta=\frac{9}{2}\times\frac{2}{3} \Rightarrow \gamma = 3$

$\frac{9}{2}\beta-\beta=\frac{7}{3}$

$\frac{9}{2}\beta=\frac{7}{3}\Rightarrow\beta=\frac{2}{3}$

$\alpha = 1 - \frac{2}{3}=\frac{1}{3}$

$2\lambda = \frac{2}{9}\Rightarrow\lambda=\frac{1}{9}$

$\frac{\beta\gamma}{\lambda}=\frac{\frac{2}{3}\times3}{\frac{1}{9}} = 18$

4. NTA Ans. (3)

Sol. $\ell n(y + 3x) = z$ (let)

$\frac{1}{y+3x}\cdot\left(\frac{dy}{dx}+3\right)=\frac{dz}{dx}$ ..(1)

$\frac{dy}{dx}+3=\frac{y+3x}{\ell n(y+3x)}$ (given)

$\frac{dz}{dx}=\frac{1}{z}$

$\Rightarrow z\,dz = dx \Rightarrow \frac{z^2}{2}=x+C$

$\Rightarrow \frac{1}{2}\ell n^2(y+3x) = x+C$

$\Rightarrow x - \frac{1}{2}(\ell n(y + 3x))^2 = C$

5. NTA Ans. (3)

Sol. $a_n = a_1 + (n - 1)d$

$\Rightarrow 300 = 1 + (n - 1)d$

$\Rightarrow (n - 1)d = 299 = 13 \times 23$

since, $n \in [15, 50]$

$\therefore$ n = 24 and d = 13

$a_{n-4} = a_{20} = 1 + 19 \times 13 = 248$

$\Rightarrow a_{n-4} = 248$

$S_{n-4} = \frac{20}{2}\{1+248\} = 2490$

6. NTA Ans. (2)

Sol. equation of line parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ passes through (1, –2, 3) is

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=r$$

$x = 2r + 1$

$y = 3r - 2,$

$z = -6r + 3$

So $2r + 1 - 3r + 2 - 6r + 3 = 5$

$\Rightarrow -7r + 1 = 0$

$$r = \frac{1}{7}$$

$$x = \frac{9}{7},\ y = \frac{-11}{7},,\ z = \frac{15}{7}$$

$$\text{Distance is} = \sqrt{\left(\frac{9}{7}-1\right)^2+\left(2-\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$$

$$= \sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{6}{7}\right)^2}$$

$$= \frac{1}{7}\sqrt{4+9+36}$$

$$= \frac{1}{7}\sqrt{49} = 1$$

7. NTA Ans. (4)

Sol. $L = \lim\limits_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}$

using L.H. rule

$$L = \lim\limits_{t\to x}\frac{2tf^2(x)-x^2\cdot 2f'(t).f(t)}{1}$$

$\Rightarrow L = 2xf(x)\ (f(x) - x\ f'(x)) = 0$ (given)

$$\Rightarrow f(x) = xf'(x) \Rightarrow \int\frac{f'(x)dx}{f(x)}=\int\frac{dx}{x}$$

$\Rightarrow \ell n\ |f(x)| = \ell n\ |x| + C$

$\because f(1) = e,\ x > 0,\ f(x) > 0$

$\Rightarrow f(x) = ex,\quad$ if $f(x) = 1 \Rightarrow x = \frac{1}{e}$

8. NTA Ans. (3)

Sol. For infinite solutions

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

$$\text{Now } \Delta = 0 \Rightarrow \begin{vmatrix}1 & 1 & 1\\ 2 & 4 & -1\\ 3 & 2 & \lambda\end{vmatrix}=0$$

$$\Rightarrow \lambda = \frac{9}{2}$$

$$\Delta_{x=0} \Rightarrow \begin{vmatrix}2 & 1 & 1\\ 6 & 4 & -1\\ \mu & 2 & -\frac{9}{2}\end{vmatrix} = 0$$

$\Rightarrow \mu = 5$

For $\lambda = \frac{9}{2}$ & $\mu = 5$, $\Delta_y = \Delta_z = 0$

Now check option $2\lambda + \mu = 14$

9. NTA Ans. (1)

Sol. Usnign AM $\geq$ GM

$$\Rightarrow \frac{2^{\sin x}+2^{\cos x}}{2}\geq\sqrt{2^{\sin x}\cdot 2^{\cos x}}$$

$$\Rightarrow 2^{\sin x}+2^{\cos x}\geq 2^{1+\left(\frac{\sin x+\cos x}{2}\right)}$$

$$\Rightarrow \min\left(2^{\sin x}+2^{\cos x}\right)=2^{1-\frac{1}{\sqrt{2}}}$$

10. NTA Ans. (3)

Sol. $I = \int_{\pi/6}^{\pi/3}((2\tan^3 x\cdot\sec^2 x\cdot\sin^4 3x)+(3\tan^4 x\cdot\sin^3 3x\cdot\cos 3x))dx$

$$\Rightarrow I = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}d((\sin 3x)^4(\tan x)^4)$$

$$\Rightarrow I = ((\sin 3x)^4(\tan x)^4)_{\pi/6}^{\pi/3}$$

$$\Rightarrow I = -\frac{1}{18}$$

11. NTA Ans. (4)

Sol. Let S be the circle pasing through point of intersection of S_1 & S_2

$\therefore\ S = S_1 + \lambda S_2 = 0$

$\Rightarrow S : (x^2 + y^2 - 6x) + \lambda\ (x^2 + y^2 - 4y) = 0$

$\Rightarrow S : x^2 + y^2 - \left(\frac{6}{1+\lambda}\right)x - \left(\frac{4\lambda}{1+\lambda}\right)\ y = 0$ ...(1)

Centre $\left(\frac{3}{1+\lambda}, \frac{2\lambda}{1+\lambda}\right)$ lies on

$2x - 3y + 12 = 0 \Rightarrow \lambda = -3$

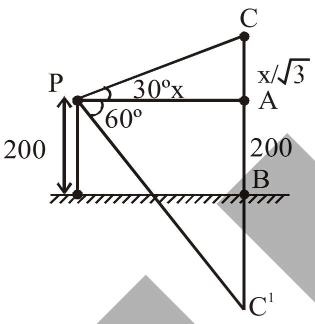
put in (1) $\Rightarrow S : x^2 + y^2 + 3x - 6y = 0$

Now check options point (–3, 6) lies on S.

12. NTA Ans. (1)

Sol. Let PA = x

For ΔAPC



$AC = \frac{PA}{\sqrt{3}} = \frac{x}{\sqrt{3}}$

$AC^1 = AB + BC^1$

$AC^1 = AB + BC$

$AC^1 = 400 + \frac{x}{\sqrt{3}}$

From ΔC^1PA : $AC^1 = \sqrt{3}\,PA$

$\Rightarrow \left(400 + \frac{x}{\sqrt{3}}\right) = \sqrt{3}x \Rightarrow x = (200)(\sqrt{3})$

from Δ APC : $PC = \frac{2x}{\sqrt{3}} \Rightarrow PC = 400$

13. NTA Ans. (4)

Sol. $\alpha = \omega \qquad (\omega^3 = 1)$

$\Rightarrow (2 + \omega)^4 = a + b\omega$

$\Rightarrow 2^4 + 4.2^3\ \omega + 6.2^2\omega^3 + 4.2\ .\ \omega^3 + \omega^4 = a + b\omega$

$\Rightarrow 16 + 32\ \omega + 24\ \omega^2 + 8 + \omega = a + b\omega$

$\Rightarrow 24 + 24\ \omega^2 + 33\omega = a + b\omega$

$\Rightarrow -24\omega + 33\omega = a + b\omega$

$\Rightarrow a = 0,\ b = 9$

14. NTA Ans. (4)

Sol. $P(6) = \frac{5}{36}$, $P(7) = \frac{1}{6}$

P(A) = W + FFW + FFFFW + ...

$= \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right) \times \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right)^2 \times \frac{5}{36} + \ldots$

$= \frac{\frac{5}{36}}{1 - \frac{155}{216}} = \frac{5}{36} \times \frac{216}{61} = \frac{30}{61}$

15. NTA Ans. (2)

Sol. Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

directrix : $x = \frac{a}{e} = 4$ & $e = \frac{1}{2}$

$\Rightarrow a = 2$ & $b^2 = a^2\ (1 - e^2) = 3$

$\Rightarrow$ Ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

P is $\left(1, \frac{3}{2}\right)$

Normal is : $\frac{4x}{1} - \frac{3y}{3/2} = 4 - 3$

$\Rightarrow 4x - 2y = 1$

16. NTA Ans. (3)

Sol. p = function is differantiable at a

q = function is continuous at a

contrapositive of statement $p \rightarrow q$ is

$\sim q \rightarrow \sim p$

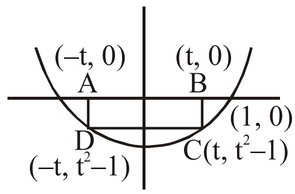
17. NTA Ans. (1)

Sol. Area (A) = $2t \cdot (1 - t^2)$

$(0 < t < 1)$

$A = 2t - 2t^3$

$\frac{dA}{dt} = 2 - 6t^2$



$t = \frac{1}{\sqrt{3}}$

$\Rightarrow A_{max} = \frac{2}{\sqrt{3}}\left(1 - \frac{1}{3}\right) = \frac{4}{3\sqrt{3}}$

18. NTA Ans. (3)

Sol. Let n + 5 = N

$^{N}C_{r-1} : {}^{N}C_{r} : {}^{N}C_{r+1} = 5 : 10 : 14$

$$\Rightarrow \frac{^{N}C_{r}}{^{N}C_{r-1}} = \frac{N+1-r}{r} = 2$$

$$\frac{^{N}C_{r+1}}{^{N}C_{r}} = \frac{N-r}{r+1} = \frac{7}{5}$$

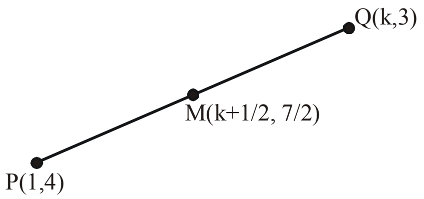
$\Rightarrow$ r = 4, N = 11

$\Rightarrow$ $(1 + x)^{11}$

Largest coefficient = $^{11}C_6 = 462$

19. NTA Ans. (4)

Sol.



Slope = $m = \frac{1}{1-k}$

Equation of $\perp^r$ bisector is

y + 4 = (k –1) (x– 0)

$\Rightarrow$ y + 4 = x(k –1)

$$\Rightarrow \frac{7}{2} + 4 = \frac{k+1}{2}(k-1)$$

$$\Rightarrow \frac{15}{2} = \frac{k^2-1}{2} \Rightarrow k^2 = 16 \Rightarrow k = 4, -4$$

20. NTA Ans. (4)

Sol. $Ax_1 = b_1$

$Ax_2 = b_2$

$Ax_3 = b_3$

$$\Rightarrow |A|\begin{vmatrix}1 & 0 & 0\\1 & 2 & 0\\1 & 1 & 1\end{vmatrix} = \begin{vmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{vmatrix}$$

$$\Rightarrow |A| = \frac{4}{2} = 2$$

21. NTA Ans. (135)

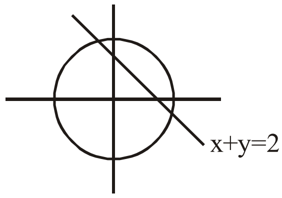
Sol. Ways = $^{6}C_4 \cdot 1^4 \cdot 3^2$

= 15 × 9

= 135

22. NTA Ans. (7)

Sol.



Let P (3cosθ, 3 sinθ)

Q (–3 cosθ, –3 sinθ)

$$\Rightarrow \alpha\beta = \frac{|(3\cos\theta + 3\sin\theta)^2 - 4|}{2}$$

$$\Rightarrow \alpha\beta = \frac{5 + 9\sin 2\theta}{2} \le 7$$

23. NTA Ans. (21)

Sol. $\int_0^n \{x\}dx = n\int_0^1 \{x\}dx = n\int_0^1 x\,dx = \frac{n}{2}$

$$\int_0^n [x]dx = \int_0^n (x - \{x\})dx = \frac{n^2}{2} - \frac{n}{2}$$

$$\Rightarrow \left(\frac{n^2-n}{2}\right)^2 = \frac{n}{2}\cdot 10\cdot n(n-1) \quad (\text{where } n > 1)$$

$$\Rightarrow \frac{n-1}{4} = 5 \Rightarrow n = 21$$

24. NTA Ans. (18)

Sol. $\Sigma|\vec{a} - (\vec{a}\cdot i)i|^2$

$\Rightarrow$ $\Sigma\left(|a|^2 + (\vec{a}\cdot i)^2 - 2(\vec{a}\cdot i)^2\right)$

$\Rightarrow$ $3|\vec{a}|^2 - \Sigma(\vec{a}\cdot i)^2$

$\Rightarrow$ $2|\vec{a}|^2$

$\Rightarrow$ 18

25. NTA Ans. (4)

Sol. ∵ Variance is independent of shifting of origin

$\Rightarrow$ x_i : 15 25 35 or –10 0 10

f_i : 2 x 2 2 x 2

$$\Rightarrow \text{Variance } (\sigma^2) = \frac{\Sigma x_i^2 f_i}{\Sigma f_i} - (\bar{x})^2$$

$$\Rightarrow 50 = \frac{200 + 0 + 200}{x+4} - 0 \qquad \{\bar{x} = 0\}$$

$\Rightarrow$ 200 + 50x = 200 + 200

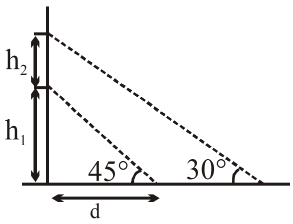
$\Rightarrow$ x = 4

SET # 07

PHYSICS

1. NTA Ans. (1)

Sol.



$\frac{h_1}{d} = \tan 45° \Rightarrow h_1 = d$ ... (1)

$\frac{h_1 + h_2}{d + 2.464d} = \tan 30°$

$\Rightarrow (h_1 + h_2) \times \sqrt{3} = 3.46$ d

$(h_1 + h_2) = \frac{3.46d}{\sqrt{3}}$

$\Rightarrow d + h_2 = \frac{3.46d}{\sqrt{3}}$

$h_2 = d$

2. **NTA Ans. (3)**

Sol. $\Rightarrow \lambda = 2\,(l_2 - l_1) \Rightarrow 2 \times (24.5 - 17)$

$\Rightarrow 2 \times 7.5 = 15$ cm

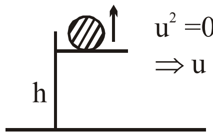
& $v = f\lambda \Rightarrow 330 = \lambda \times 15 \times 10^{-2}$

$\lambda = \frac{330}{15} \times 100 \Rightarrow \frac{1100 \times 100}{5}$

$\Rightarrow 2200$ Hz

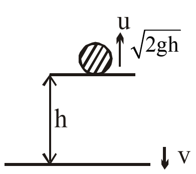
3. **NTA Ans. (3)**

Sol.



$u^2 = 0 + 2gh$

$\Rightarrow u = \sqrt{2gh}$



$v^2 = u^2 + 2as$

$v^2 = 2gh + 2gh$

$v = \sqrt{4gh}$

$\Rightarrow \sqrt{4gh} = \sqrt{2gh} + gt$

$\Rightarrow t = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}} \Rightarrow 3.4\sqrt{\frac{h}{g}}$

4. **NTA Ans. (3)**

Sol. $R = R_0 e^{-\lambda t}$

$ln R = \ln R_0 - \lambda t$

$\lambda_A = \frac{6}{10} \Rightarrow T_A = \frac{10}{6}\, ln\, 2$

$\lambda_B = \frac{6}{5} \Rightarrow T_B = \frac{5\, ln 2}{6}$

$\lambda_C = \frac{2}{5} \Rightarrow T_C = \frac{5\, ln 2}{2}$

$\frac{10}{6} : \frac{5}{6} : \frac{15}{6} :: 2 : 1 : 3$

5. **NTA Ans. (2)**

Sol. $\frac{4}{3}\pi\left(R^3 - r^3\right) \rho_m g = \frac{4}{3}\pi R^3 \rho_w g$

$1 - \left(\frac{r}{R}\right)^3 = \frac{8}{27}$

$\Rightarrow \frac{r}{R} = \left(\frac{19}{27}\right)^{1/3} = \frac{19^{1/3}}{3}$

$= 0.88 \simeq \frac{8}{9}$

6. **NTA Ans. (1)**

Sol. $\Delta U = nC_v\, \Delta T$ = same

AB $\rightarrow$ volume is increasing $\Rightarrow W > 0$

AD $\rightarrow$ volume is decreasing $\Rightarrow W < 0$

AC $\rightarrow$ volume is constant $\Rightarrow W = 0$

7. **NTA Ans. (1)**

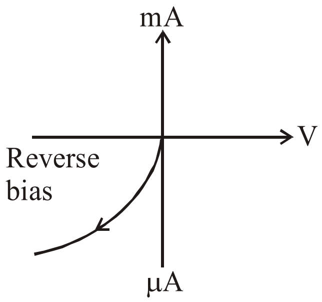
Sol. $\frac{kQq}{R} + mgy$

$= \frac{kQq}{R + y} + \frac{1}{2}mv^2$

$v^2 = 2gy + \frac{2kQqy}{mR(R + y)}$

8. **NTA Ans. (1)**

Sol. I-V characteristic of a photodiode is as follows:



On increasing the potential difference the current first increases and then attains a saturation.

9. **NTA Ans. (4)**

Sol. $v = \dfrac{uf}{u+f}$

Case-I

If $v = u$

$\Rightarrow f + u = f$

$\Rightarrow u = 0$

Case-II

If $u = \infty$

then $v = f$

Only option (4) satisfies this condition.

10. **NTA Ans. (2)**

Sol. $vi = 10^3$

$i = \dfrac{1000}{220}$

loss $= i^2R = \left(\dfrac{50}{11}\right)^2 \times 2$

efficiency $= \dfrac{1000}{1000+i^2R} \times 100 = 96\%$

11. **NTA Ans. (2)**

Sol. $\Delta p = BkS_0$

$= \rho v^2 \times \dfrac{\omega}{v} \times S_0$

$\Rightarrow S_0 = \dfrac{\Delta p}{\rho v \omega}$

$\approx \dfrac{10}{1 \times 300 \times 1000} m$

$= \dfrac{1}{30} mm \approx \dfrac{3}{100} mm$

12. **NTA Ans. (2)**

Sol. $\dfrac{1}{2} mv^2 \times \dfrac{1}{2} = ms\Delta T$

$\Delta T = \dfrac{v^2}{4 \times 5} = \dfrac{210^2}{4 \times 30 \times 4.200}$

$= 87.5°C$

13. **NTA Ans. (3)**

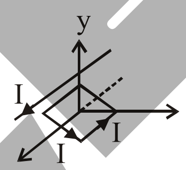
Sol. $n = \dfrac{PV}{RT}, \dfrac{3}{2} kT = 4 \times 10^{-14}$

$N = \dfrac{PV}{RT} \times Na$

$= \dfrac{2 \times 13.6 \times 980 \times 4}{\dfrac{8}{3} \times 10^{-14}} = 3.99 \times 10^{18}$

14. **NTA Ans. (1)**

Sol.



$\vec{\tau} = \vec{M} \times \vec{B}$

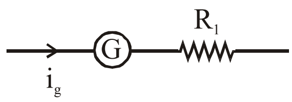
$= 4a^2 I \times \dfrac{\mu_0 I}{2\pi b}$

15. **NTA Ans. (2)**

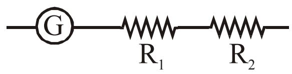
Sol. $\dfrac{\Delta Z}{Z} = \dfrac{2\Delta a}{a} + \dfrac{2}{3}\dfrac{\Delta b}{b} + \dfrac{1}{2}\dfrac{\Delta c}{c} + \dfrac{3\Delta d}{d} = 14.5\%$

16. **NTA Ans. (2)**

Sol.



$\Rightarrow 1 = i_g(G + R_1)$ ... (1)



$\Rightarrow 2 = i_g(R_1 + R_2 + G)$ ...(2)

(1) % (2)

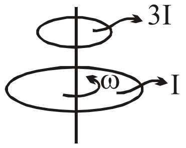
$\Rightarrow \dfrac{1}{2} = \dfrac{G + R_1}{G + R_1 + R_2}$

$G + R_1 + R_2 = 2G + 2h_1$

$(R_2 = G + R_1)$

17. NTA Ans. (3)

Sol.



By anglar momentum conservation

$\omega I + 3I \times 0 = 4I\omega' \Rightarrow \omega' = \dfrac{\omega}{4}$

$(KE)_i = \dfrac{1}{2} I\omega^2$

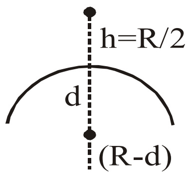
$(KE)_f = \dfrac{1}{2} \times (4I) \times \left(\dfrac{\omega}{4}\right)^2 = \dfrac{I\omega^2}{8}$

$\Delta KE = \dfrac{3}{8} I\omega^2$

fractional loss $= \dfrac{\Delta KE}{KE_1} = \dfrac{\frac{3}{8}I\omega^2}{\frac{1}{2}I\omega^2} = \dfrac{3}{4}$

18. NTA Ans. (4)

Sol.



$g_1 = \dfrac{GM}{\left(R + \dfrac{R}{2}\right)^2}$ ... (1)

$g_2 = \dfrac{GM(R-d)}{R^3}$ ... (2)

$g_1 = g_2$

$\dfrac{GM}{\left(\dfrac{3R}{2}\right)^2} = \dfrac{GM(R-d)}{R^3}$

$\Rightarrow \dfrac{4}{9} = \dfrac{(R-d)}{R}$

$4R = 9R - 9d$

$5R = 9d \Rightarrow \dfrac{d}{R} = \dfrac{5}{9}$

19. NTA Ans. (2)

Sol. $\Rightarrow E = \vec{E} = 30\hat{j}\sin(1.5\times10^7 t - 5\times10^{-2}x) V/m$

$\Rightarrow B \Rightarrow E/V \Rightarrow \dfrac{30}{1.5\times10^7}\times5\times10^{-2}$

$\Rightarrow 10^{-7}$ Tesla

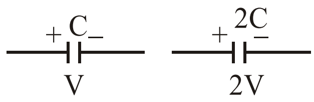
$\Rightarrow F_{mag} = q\left(\vec{V}\times\vec{B}\right) = |qVB|$

$= 1.6 \times 10^{-19} \times 0.1 \times 3 \times 10^8 \times 10^{-7}$

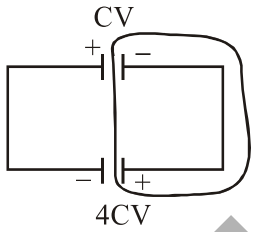
$= 4.8 \times 10^{-19}$ N

20. NTA Ans. (4)

Sol.



$Q_1 = CV$ $\quad Q_2 = 2C \times 2V = 4CV$



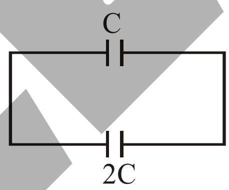
$\Rightarrow$ By conservation of charge

$q_i = q_f$

$Q_1 + Q_2 = q_1 + q_2$

$4CV - CV = (C + 2C)\, V_C$

$V_C = \dfrac{3CV}{3C} \Rightarrow V$

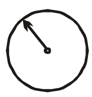


$\Rightarrow \dfrac{1}{2}\times(3C)\times V_C^2$

$= \dfrac{1}{2} \times 3C \times V^2 = \dfrac{3}{2} CV^2$

21. NTA Ans. (5.00)

Sol.



$B = \dfrac{\mu_0 NI}{2R}$

$\phi = \dfrac{\mu_0 NN'I}{2R}\pi r^2$

$\varepsilon = \dfrac{d\phi}{dt} = \dfrac{2\pi\times10^{-7}\times10^5\times\pi\times10^{-4}}{0.2}$

$= 8 \times 10^{-4} = 0.8$ mV

22. NTA Ans. (195)

Sol. $\vec{\tau} = \left(\vec{r}_2 - \vec{r}_1\right) \times \vec{F}$

$= [(4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})] \times \vec{F}$

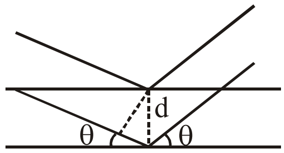
$= \left(3\hat{i} + \hat{j} - 2\hat{k}\right) \times \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$$

$= 7\hat{i} - 11\hat{j} + 5\hat{k}$

$|\vec{\tau}| = \sqrt{195}$

23. NTA Ans. (50.00 to 51.00)

Sol. 

$$2d\sin\theta = \lambda = \frac{h}{\sqrt{2mE}}$$

$$2 \times 10^{-10} \times \frac{\sqrt{3}}{2} = \frac{6.6 \times 10^{-34}}{\sqrt{2mE}}$$

$$E = \frac{1}{2} \times \frac{6.64^2 \times 10^{-48}}{9.1 \times 10^{-31} \times 3 \times 1.6 \times 10^{-19}} = 50.47$$

24. NTA Ans. (51.00)

Sol. $mV_0 = MV = p$

$$10.2 = \frac{p^2}{2m} - \frac{p^2}{2M} = \frac{p^2}{2m}\left(1 - \frac{m}{M}\right)$$

$$= \frac{p^2}{2m}(1 - 0.2)$$

$$\Rightarrow \frac{p^2}{2m} = K = \frac{10.2}{0.8}$$

25. NTA Ans. (50.00)

Sol. Final image at ∞

$\Rightarrow$ obj. for eye piece at 5cm

$\Rightarrow$ image for objective at 5 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{5} + \frac{1}{x} = 1$$

$$\frac{1}{x} = 1 - \frac{1}{5} = \frac{4}{5} \Rightarrow x = \frac{5}{4}$$

CHEMISTRY

1. NTA Ans. (1)

Sol. (1) Water gas shift reaction

$$CO_{(g)} + H_2O_{(g)} \xrightarrow[\text{catalyst}]{673K} CO_{2(g)} + H_{2(g)}$$

(2) Water gas is produced by this reaction.

$$CH_{4(g)} + H_2O_{(g)} \xrightarrow[\text{Ni}]{1270K} CO_{(g)} + 3H_{2(g)}$$

(3) Water gas is produced by this reaction

$$C_{(s)} + H_2O_{(g)} \xrightarrow{1270K} CO_{(g)} + H_{2(g)}$$

(4) producer gas is produced by this reaction.

$$2C_{(s)} + O_{2(g)} + 4N_{2(g)} \xrightarrow{1270K} 2CO_{(g)} + 4N_{2(g)}$$

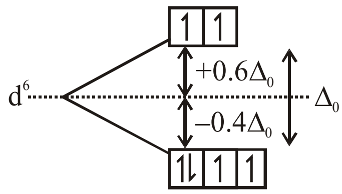
2. NTA Ans. (1)

Sol. $\Delta H^o > 0$ T↓ equation shifts back ward.

N_2 is treated as inert gas in this case hence no effect on equilibrium.

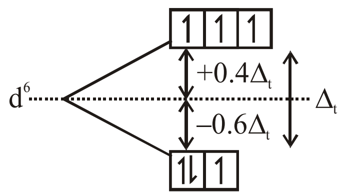
3. NTA Ans. (3)

Sol. For high spin octahedral field



CFSE = (4) $(-0.4\Delta_0)$ + $2(0.6\ \Delta_0)$ = $-\ 0.4\ \Delta_0$

For high spin tetrahedral field



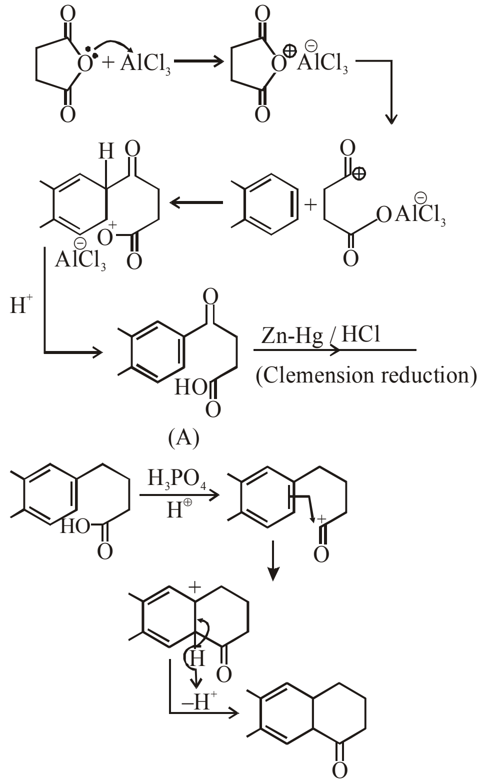
CFSE = $3(-0.6\Delta_t)$ + $3(0.4\ \Delta_t)$ = $-\ 0.6\ \Delta_t$

4. NTA Ans. (4)

Sol. Tyrosine is not an essential amino acid.

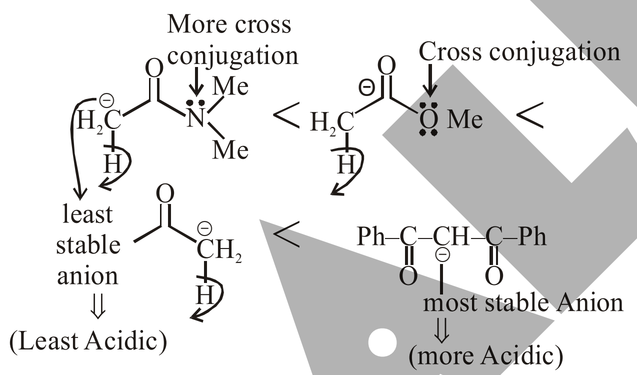
5. NTA Ans. (1)

Sol.



6. NTA Ans. (4)

Sol. $\boxed{D < C < A < B}$

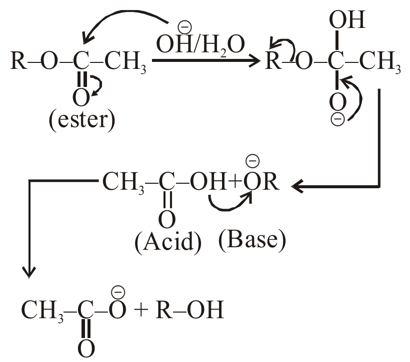


7. NTA Ans. (3)

Sol. Ellingham diagram provides information about temperature dependence of the standard gibbs energies of formation of some metal oxides.

8. NTA Ans. (2)

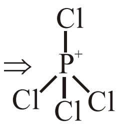
Sol.



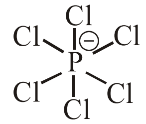
It is a hydrolysis of ester in basic medium.

9. NTA Ans. (2)

Sol. $PCl_{5(s)}$ exist as $[PCl_4]^+$ and $[PCl_6]^-$

$[PCl_4]^+ \Rightarrow$  (sp^3 hybridisation)

Tetrahedral

$[PCl_6]^- \Rightarrow$ 

octahedral

sp^3d^2 hybridization

10. NTA Ans. (2)

Sol.

$$CH_3–CH_2–C\equiv N \xrightarrow{?} CH_3–CH_2–CH_2–NH_2$$

$$CH_3–CH_2–C\equiv N \xrightarrow{LiAlH_4} CH_3–CH_2–CH_2–NH_2$$

11. NTA Ans. (4)

Sol.

$$\frac{\Delta R_1}{\Delta R_2} = \frac{(r_4 - r_3)_{4^{2+}}}{(r_4 - r_3)_{He^+}} = \frac{\frac{4^2}{3} - \frac{3^2}{3}}{\frac{4^2}{2} - \frac{3^2}{2}} = \frac{7/3}{7/2} = \frac{2}{3}$$

12. NTA Ans. (4)

Sol. $[A]_t = 4[B]_t$

$$[A]_0\, e^{-(\ln^2/300)^t} = 4[B]_0 e^{(-\ln 2/180)t}$$

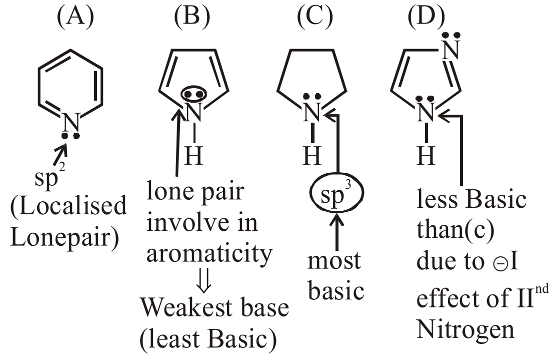
$$e^{\left(\frac{\ln^2}{180} - \frac{\ln^2}{300}\right)} = 4$$

$$\left(\frac{\ln^2}{180} - \frac{\ln^2}{300}\right)t = \ln 4$$

$$\left(\frac{1}{180} - \frac{1}{300}\right)t = 2 \Rightarrow t = \frac{2 \times 180 \times 300}{120} = 900\,\text{sec}.$$

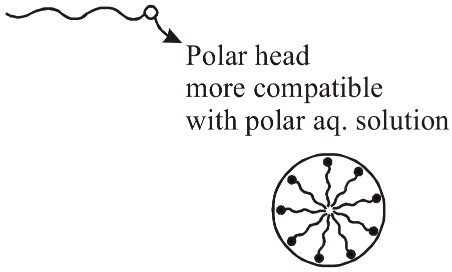
13. NTA Ans. (4)

Sol.



14. NTA Ans. (3)

Sol.



Micelles formed at CMC.

15. NTA Ans. (4)

Sol. Anti depressant → drug which enhance the mood. Non adrenaline is neurotransmitter and its level is low in body due to some reason then person suffers from depression and in that situation anti depressant drug is required.

16. NTA Ans. (2)

Sol. Electronic configuration of Gd^{3+} is

${}_{64}Gd^{3+}$ = [Xe]$4f^7$

[Xe] $\boxed{1|1|1|1|1|1|1}$

Gd^{3+} having 7 unpaired electrons.

Magnetic moment (μ) = $\sqrt{n(n+2)}$B.M.

$\mu = \sqrt{7(7+2)}$B.M.

= 7.9 B.M.

n ⇒ Number of unpaired electrons.

17. NTA Ans. (2)

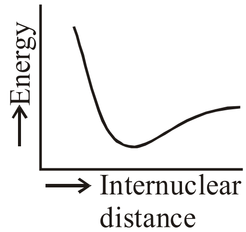
Sol. In Eutrophication nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity. It indicates polluted environment.

18. NTA Ans. (4)

Sol. As per (n + ℓ) rule in 6th period, order of orbitals filling is 6s, 4f, 5d, 6p.

19. NTA Ans. (2)

Sol. Potential energy curve for H_2 molecule is.



20. NTA Ans. (3)

Sol. $p = \frac{2 \times \frac{M}{N_A}}{a^3} \Rightarrow 6.17 = \frac{2 \times \frac{M}{N_A}}{(3 \times 10^{-8} cm)^3}$

$\Rightarrow M \simeq 50$ gm / mol

No = $\frac{w}{M} \times N_A = \frac{200}{50} \times N_A = 4N_A$

21. NTA Ans. (6)

Sol. $\Delta G° = -AFE° = -3 \times 96500 \times E°$

$\Rightarrow E° = -6 \times 10^{-2}$ V

22. NTA Ans. (18)

Sol. $\underset{1mole}{C_3H_8} + \underset{5mole}{5O_2} \rightarrow 3CO_2 + 4H_2O$

For 1 mole propane combustion 5 mole O_2 required

$C_4H_{10} + \frac{13}{2}O_2 \rightarrow 4CO_2 + 5H_2O$

1 mole 6.5 mole

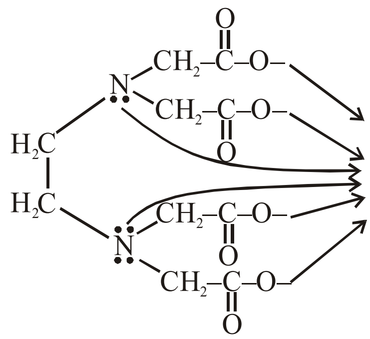
2 mole 13 mole

For 2 moles of butane 13 mole of O_2 is required

total moles = 13 + 5 = 18

23. NTA Ans. (6)

Sol. $EDTA^{4-}$ is hexadentate ligand, so its donation sites are six.



24. NTA Ans. (4)

25. NTA Ans. (37)

Sol. $P_{CO_2} = K_H \times CO_2$

$\frac{3}{30} = \frac{K_H . n_{CO_2}}{K_H 1} \Rightarrow n_{CO_2 = 0.1}$mol

$pH = \frac{1}{2}(pka_1 - \log c) = \frac{1}{2}(6.4 \times 1) = 3.7$

pH = 37 × 10^{-1}

MATHEMATICS

1. NTA Ans. (1)

Sol. Given that

$3^{4-\sin 2\alpha} + 3^{2\sin 2\alpha - 1} = 28$

Let $3^{2\sin 2\alpha} = t$

$\frac{81}{t} + \frac{t}{3} = 28$

$t = 81, 3$

$3^{2\sin 2\alpha} = 3^1, 3^4$

$2\sin 2\alpha = 1, 4$

$\sin 2\alpha = \frac{1}{2}, 2$ (rejected)

First term $a = 3^{2\sin 2\alpha - 1}$

$a = 1$

Second term = 14

$\therefore$ common difference d = 13

$T_6 = a + 5d$

$T_6 = 1 + 5 \times 13$

$T_6 = 66$

2. NTA Ans. (1)

Sol. f(x) is continuous and differentiable

$f(\pi^-) = f(\pi) = f(\pi^+)$

$-1 = -k_2$

$\boxed{k_2 = 1}$

$$f'(x) = \begin{cases} 2k_1(x-\pi) \,;\, x \le \pi \\ -k_2 \sin x \quad ; x > \pi \end{cases}$$

$f'(\pi^-) = f'(\pi^+)$

$0 = 0$

so, differentiable at x = 0

$$f''(x) = \begin{cases} 2k_1 \quad ; x \le \pi \\ -k_2 \cos x \,; x > \pi \end{cases}$$

$f''(\pi^-) = f''(\pi^+)$

$2k_1 = k_2$

$\boxed{k_1 = \frac{1}{2}}$

$(k_1, k_2) = \left(\frac{1}{2}, 1\right)$

3. NTA Ans. (3)

Sol. $y = mx + \frac{1}{m}$ (tangent at $y^2 = 4x$)

$y = mx - m^2$ (tangent at $x^2 = 4y$)

$\frac{1}{m} = -m^2$ (for common tangent)

$m^3 = -1$

$\boxed{m = -1}$

$y = -x - 1$

$x + y + 1 = 0$

This line touches circle

$\therefore$ apply p = r

$c = \left|\frac{0+0+1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

4. NTA Ans. (3)

Sol. $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$x \leftrightarrow \sim y \equiv (x \to \sim y) \wedge (\sim y \to x)$

$\because (p \to q \equiv \sim p \vee q)$

$x \leftrightarrow \sim y \equiv (\sim x \vee \sim y) \wedge (y \vee x)$

$\sim (x \leftrightarrow \sim y) \equiv (x \wedge y) \vee (\sim x \wedge \sim y)$

5. NTA Ans. (2)

Sol. $v = [\vec{a}\,\vec{b}\,\vec{c}]$

$$158 = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix}, \; n \ge 0$$

$158 = 1\,(12 + n^2) - (6 + n) + n(2n - 4)$

$158 = n^2 + 12 - 6 - n + 2n^2 - 4n$

$3n^2 - 5n - 152 = 0$

$n = 8, -\frac{38}{6}$ (rejected)

$\vec{a} \cdot \vec{c} = 1 + n + 3n = 1 + 4n = 33$

$\vec{b} \cdot \vec{c} = 2 + 4n - 3n = 2 + n = 10$

6. NTA Ans. (2)

Sol. $\frac{(5+e^x)}{2+y}\frac{dy}{dx} = -e^x$

$\int \frac{dy}{2+y} = \int \frac{-e^x}{e^x+5}\,dx$

$\ln(y + 2) = -\ln(e^x + 5) + k$

$(y + 2)(e^x + 5) = C$

$\because y(0) = 1$

$\Rightarrow C = 18$

$y + 2 = \frac{18}{e^x+5}$

at $x = \ln 13$

$y + 2 = \frac{18}{13+5} = 1$

$\boxed{y = -1}$

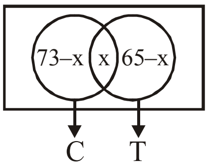
7. NTA Ans. (4)

Sol. C $\rightarrow$ person like coffee

T $\rightarrow$ person like Tea

$n(C) = 73$

$n(T) = 65$



$n(C \cup T) \le 100$

$n(C) + n(T) - n(C \cap T) \le 100$

$73 + 65 - x \le 100$

$x \ge 38$

$73 - x \ge 0 \Rightarrow x \le 73$

$65 - x \ge 0 \Rightarrow x \le 65$

$\boxed{38 \le x \le 65}$

8. NTA Ans. (2)

Sol. $9x^2 - 18|x| + 5 = 0$

$9|x|^2 - 15|x| - 3|x| + 5 = 0$ ($\because x^2 = |x|^2$)

$3|x|(3|x| - 5) - (3|x| - 5) = 0$

$|x| = \frac{1}{3}, \frac{5}{3}$

$x = \pm\frac{1}{3}, \pm\frac{5}{3}$

Product of roots $= \frac{25}{81}$

9. NTA Ans. (1)

Sol. $e^{2x} + 2e^x - e^{-x} - 1$

$= e^x(e^x + 1) - e^{-x}(e^x + 1) + e^x$

$= [(e^x + 1)(e^x - e^{-x}) + e^x]$

so $I = \int (e^x + 1)(e^x - e^{-x})e^{e^x + e^{-x}} + \int e^x \cdot e^{e^x + e^{-x}} dx$

$= (e^x + 1)e^{e^x + e^{-x}} - \int e^x . e^{e^x + e^{-x}} dx + \int e^x . e^{e^x + e^{-x}} dx$

$= (e^x + 1)e^{e^x + e^{-x}} + C$

$\therefore g(x) = e^x + 1 \Rightarrow g(0) = 2$

10. NTA Ans. (4)

Sol. $C_3 \rightarrow C_3 - (C_1 - C_2)$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 0 \\ -\cos^2\theta & -1-\cos^2\theta & 0 \\ 12 & 10 & -4 \end{vmatrix}$$

$= -4[(1 + \cos^2\theta)\sin^2\theta - \cos^2\theta(1 + \sin^2\theta)]$

$= -4[\sin^2\theta + \sin^2\theta\cos^2\theta - \cos^2\theta - \cos^2\theta\sin^2\theta]$

$f(\theta) = 4\cos 2\theta$

$\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$2\theta \in \left[\frac{\pi}{2}, \pi\right]$

$f(\theta) \in [-4, 0]$

$(m, M) = (-4, 0)$

11. NTA Ans. (1)

Sol. $D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix}$

$= 2(3\lambda + 2)(\lambda - 3)$

$D_1 = -2(\lambda - 3)$

$D_2 = -2(\lambda + 1)(\lambda - 3)$

$D_3 = -2(\lambda - 3)$

When $\boxed{\lambda = 3}$, then

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow$ Infinite many solution

when $\boxed{\lambda = -\frac{2}{3}}$ then D_1, D_2, D_3 none of them is zero so equations are inconsistant

$\boxed{\therefore \lambda = -\frac{2}{3}}$

12. NTA Ans. (4)

Sol. $S = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \ldots$

$S = \tan^{-1}\left(\frac{2-1}{1+1.2}\right) + \tan^{-1}\left(\frac{3-2}{1+2\times 3}\right) + \tan^{-1}\left(\frac{4-3}{1+3\times 4}\right) + \ldots + \tan^{-1}\left(\frac{11-10}{1+10\times 11}\right)$

$S = (\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + (\tan^{-1}4 - \tan^{-1}3) + \ldots + (\tan^{-1}(11) - \tan^{-1}(10))$

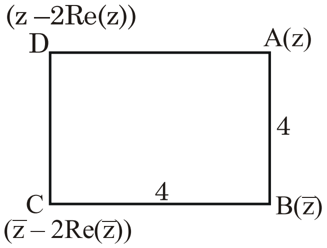
$S = \tan^{-1}11 - \tan^{-1}1 = \tan^{-1}\left(\frac{11-1}{1+11}\right)$

$\tan(S) = \frac{11-1}{1+11\times 1} = \frac{10}{12} = \frac{5}{6}$

13. NTA Ans. (4)

Sol. Let z = x + iy

Length of side = 4



AB = 4

$|z - \overline{z}| = 4$

$|2y| = 4$; $|y| = 2$

BC = 4

$\left|\overline{z} - (\overline{z} - 2\,\text{Re}(\overline{z})\right| = 4$

$|2x| = 4$; $|x| = 2$

$|z| = \sqrt{x^2 + y^2} = \sqrt{4+4} = 2\sqrt{2}$

14. NTA Ans. (2)

Sol. Given ellipse is $\frac{x^2}{5} + \frac{y^2}{4} = 1$

Let point P is $(\sqrt{5}\cos\theta, 2\sin\theta)$

$(PQ)^2 = 5\cos^2\theta + 4(\sin\theta + 2)^2$

$(PQ)^2 = \cos^2\theta + 16\sin\theta + 20$

$(PQ)^2 = -\sin^2\theta + 16\sin\theta + 21$

$= 85 - (\sin\theta - 8)^2$

will be maximum when $\sin\theta = 1$

$\Rightarrow (PQ)^2_{max} = 85 - 49 = 36$

15. NTA Ans. (1)

Sol. $\overline{x} = \frac{2+4+10+12+14+x+y}{7} = 8$

$x + y = 14$ .....(i)

$(\sigma)^2 = \frac{\sum (x_i)^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$16 = \frac{4+16+100+144+196+x^2+y^2}{7} - 8^2$

$16 + 64 = \frac{460 + x^2 + y^2}{7}$

$560 = 460 + x^2 + y^2$

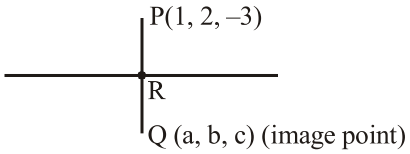
$x^2 + y^2 = 100$ ....(ii)

Clearly by (i) and (ii), $|x - y| = 2$

Ans. 1

16. NTA Ans. (2)

Sol.



Line is $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$: Let point R is $(2\lambda - 1, -2\lambda + 3, -\lambda)$

Direction ratio of PQ$\equiv(2\lambda - 2, -2\lambda + 1, 3 - \lambda)$

PQ is $\perp^r$ to line

$\Rightarrow 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(3 - \lambda) = 0$

$4\lambda - 4 + 4\lambda - 2 - 3 + \lambda = 0$

$9\lambda = 9 \Rightarrow \lambda = 1$

$\Rightarrow$ Point R is (1, 1, –1)

$\frac{a+1}{2} = 1$	$\frac{b+2}{2} = 1$	$\frac{c-3}{2} = -1$
a = 1	b = 0	c = 1

$\Rightarrow a + b + c = 2$

17. NTA Ans. (4)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx$ ....(1)

Apply King property

$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{-\sin x}}dx = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}}dx$ ...(2)

Add (1) & (2)

$2I = \int_{-\pi/2}^{\pi/2} dx = \pi$

$I = \frac{\pi}{2}$

18. NTA Ans. (3)

Sol. $a = 2^{10}$; $r = \frac{3}{2}$; n = 11 (G.P.)

$S' = (2^{10})\frac{\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\frac{3}{2} - 1} = 2^{11}\left(\frac{3^{11}}{2^{11}} - 1\right)$

$S' = 3^{11} - 2^{11} = S - 2^{11}$ (Given)

$\therefore S = 3^{11}$

19. NTA Ans. (1)

Sol. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$a = 4$; $b = 3$; $e = \sqrt{\frac{16-9}{16}} = \frac{\sqrt{7}}{4}$

A and B are foci

$\Rightarrow PA + PB = 2a = 2 \times 4 = 8$

20. NTA Ans. (1)

Sol. $x^2 - x - 2 = 0$

roots are 2 & –1

$$\Rightarrow \lim_{x\to 2^+} \frac{\sqrt{1-\cos(x^2-x-2)}}{(x-2)}$$

$$= \lim_{x\to 2^+} \frac{\sqrt{2\sin^2\frac{(x^2-x-2)}{2}}}{(x-2)}$$

$$= \lim_{x\to 2^+} \frac{\sqrt{2}\sin\left(\frac{(x-2)(x+1)}{2}\right)}{(x-2)}$$

$$= \frac{3}{\sqrt{2}}$$

21. NTA Ans. (11)

Sol. 4 dice are independently thrown. Each die has probability to show 3 or 5 is

$$p = \frac{2}{6} = \frac{1}{3}$$

$\therefore \quad q = 1 - \frac{1}{3} = \frac{2}{3}$ (not showing 3 or 5)

Experiment is performed with 4 dices independently.

$\therefore$ Their binomial distribution is

$(q + p)^4 = (q)^4 + {}^4C_1\, q^3p + {}^4C_2\, q^2p^2 + {}^4C_3\, qp^3 + {}^4C_4 p^4$

$\therefore$ In one throw of each dice probability of showing 3 or 5 at least twice is

$= p^4 + {}^4C_3\, qp^3 + {}^4C_2 q^2p^2$

$$= \frac{33}{81}$$

$\therefore$ Such experiment performed 27 times

$\therefore$ so expected out comes = np

$$= \frac{33}{81}\times 27 = 11$$

22. NTA Ans. (30)

Sol. Apply distance between parallel line formula

$4x - 2y + \alpha = 0$

$4x - 2y + 6 = 0$

$$\left|\frac{\alpha - 6}{255}\right| = \frac{1}{55}$$

$|\alpha - 6| = 2 \Rightarrow \alpha = 8, 4$

sum = 12

again

$6x - 3y + \beta = 0$

$6x - 3y + 9 = 0$

$$\left|\frac{\beta - 9}{3\sqrt{5}}\right| = \frac{2}{\sqrt{5}}$$

$|\beta - 9| = 6 \Rightarrow \beta = 15, 3$

sum = 18

sum of all values of α and β is = 30

23. NTA Ans. (13)

Sol. $T_{r+1} = {}^{22}C_r (x^m)^{22-r}\left(\frac{1}{x^2}\right)^r = {}^{22}C_r x^{22m-mr-2r}$

$= {}^{22}C_r\ x$

$\because\ {}^{22}C_3 = {}^{22}C_{19} = 1540$

$\therefore$ r = 3 or 19

$22m - mr - 2r = 1$

$$m = \frac{2r+1}{22-5}$$

$r = 3,\ m = \frac{7}{19} \notin N$

$r = 19,\ m = \frac{38+1}{22-19} = \frac{39}{3} = 13$

m = 13

24. NTA Ans. (240)

Sol. S_2YL_2ABU

ABCC type words

$$= \underbrace{{}^2C_1}_{\text{selection of two alike letters}} \times \underbrace{{}^5C_2}_{\text{selection of of two distinct letters}} \times \underbrace{\frac{\lfloor 4}{\lfloor 2}}_{\text{arrangement of selected letters}}$$

= 240

25. NTA Ans. (8)

Sol. $x \in (-10, 10)$

$\frac{x}{2} \in (-5, 5) \to 9$ integers

check continuity at x = 0

$$\left.\begin{array}{l} f(0) = 0 \\ f(0^+) = 0 \\ f(0^-) = 0 \end{array}\right\} \quad \text{continuous at } x = 0$$

function will be distcontinuous when

$$\frac{x}{2} = \pm 4,\ \pm 3,\ \pm 2,\ \pm 1$$

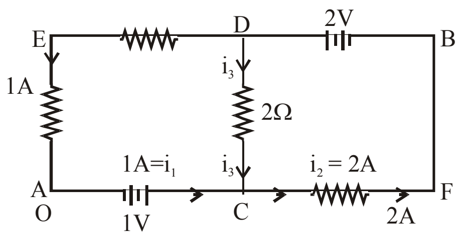
8 points of discontinuity

SET # 08

PHYSICS

1. NTA Ans. (1)

Sol.



Let us asssume the potential at A = $V_A = 0$

Now at junction C, According to KCL

$i_1 + i_3 = i_2$

$1A + i_3 = 2A$

$i_3 = 2A$

Now Analyse potential along ACDB

$v_A + 1 + i_3(2) - 2 = v_B$

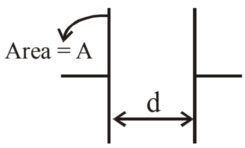
$0 + 1 + 2(1) - 2 = v_B$

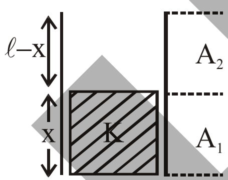
$v_B = 3 - 2$

$v_B = 1$ Volt

2. NTA Ans. (3)

Sol.





Before inserting slab

After inserting dielectric slab

$$C_i = \frac{\varepsilon_0 A}{d}$$

$$C_f = C_1 + C_2$$

$$C_i = \frac{\varepsilon_0 \ell w}{d}$$

$$C_f = \frac{K\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d}$$

$$C_f = \frac{K\varepsilon_0 wx}{d} + \frac{\varepsilon_0 w(\ell - x)}{d}$$

$$C_f = 2C_i \Rightarrow \frac{K\varepsilon_0 wx}{x} + \frac{\varepsilon_0 w(\ell - x)}{d} = \frac{2\varepsilon_0 \ell w}{d}$$

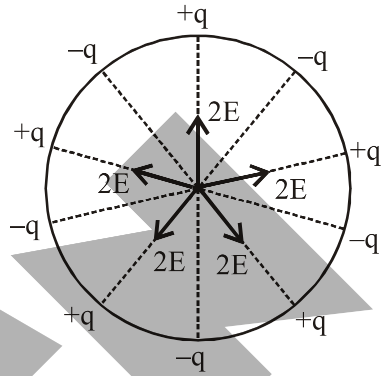
$$4x + \ell - x = 2\ell$$

$$\boxed{x = \frac{\ell}{3}}$$

3. NTA Ans. (3)

Sol. Potential of centre = V = $\Sigma\left(\frac{kq}{R}\right)$

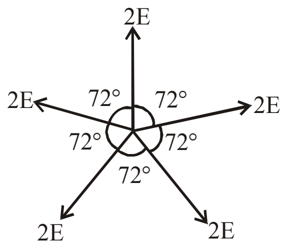
$$V_C = \frac{K(\Sigma q)}{R}$$



$$V_C = \frac{K(0)}{R} = 0$$

Electric field at centre $\vec{E}_B = \Sigma\vec{E}$

Let E be electric field produced by each charge at the centre, then resultant electric field will be



$E_C = 0$, Since equal electric field vectors are acting at equal angle so their resultant is equal to zero.

4. NTA Ans. (4)

Sol. $M = \mu_r NiA$

Here

μ_r = Relative permeability

N = No. of turns

i = Current

A = Aea of cross section

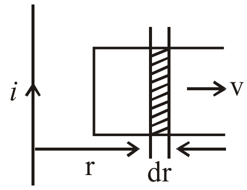
$M = \mu_r NiA = \mu_r n\ell iA$

$M = \mu_r niV = 1000(1000)\ 0.5\ (10^{-3})$

$= 500 = 5 \times 10^2\ Am^2$

5. NTA Ans. (2)

Sol. $B = \dfrac{\mu_0 i}{2\pi r}$



$\phi = \dfrac{\mu_0 i}{2\pi r}\,\ell dr$

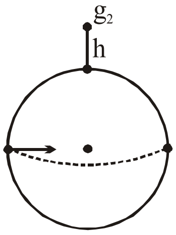
$\Rightarrow \dfrac{d\phi}{dt} = \dfrac{\mu_0 i \ell}{2\pi r}.\dfrac{dr}{dt}$

$\Rightarrow e = \dfrac{\mu_0}{2\pi}.\dfrac{iv\ell}{r}$

$i = \dfrac{e}{R} = \dfrac{\mu_0}{2\pi}.\dfrac{iv\ell}{Rr}$

6. NTA Ans. (4)

Sol. $g_e = g - R\omega^2$



$g_2 = g\left(1 - \dfrac{2h}{R}\right) \qquad g_1 = ge$

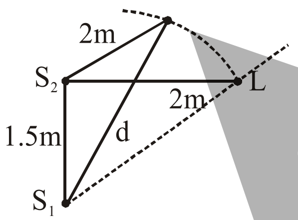
$g_2 = g - \dfrac{2gh}{R}$

Now $R\omega^2 = \dfrac{2gh}{R}$

$h = \dfrac{R^2\omega^2}{2g}$

7. NTA Ans. (2)

Sol.



Initially $S_2L = 2m$

$S_1L = \sqrt{2^2 + (3/2)^2}$

$S_1L = \dfrac{5}{2} = 2.5$ m

$\Delta x = S_1L - S_2L = 0.5$ m

So since $\lambda = 1m \quad \therefore \Delta x = \dfrac{\lambda}{2}$

So while listener moves away from S_1

Then, Δx (= $S_1L - S_2L$) increases

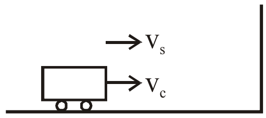
and hence, at $\Delta x = \lambda$ first maxima will appear.

$\Delta x = \lambda = S_1L - S_2L$

$1 = d - 2 \Rightarrow d = 3m$

8. NTA Ans. (4)

Sol.



f_1 = frequency heard by wall = $f_s = \left(\dfrac{v_s}{v_s - v_c}\right)$

f_2 = frequency heard by driver after reflection from wall

$f_2 = \left(\dfrac{v_s + v_c}{v_s}\right) f_1 = \left(\dfrac{v_s + v_c}{v_s - v_c}\right) f_0$

$\dfrac{f_2}{f_0} = \dfrac{v_s - v_c}{v_s + v_c}$

$\dfrac{48}{44} = \dfrac{v_s - v_c}{v_s + v_c}$

$12(v_s + v_c) = 11(v_s - v_c)$

$23v_c = v_s$

$v_c = \dfrac{v_s}{23} = \dfrac{345}{23} = 15 m/s$

$= \dfrac{15 \times 18}{5}$ = 54 km/hr

9. NTA Ans. (4)

Sol. In adiabatic process

PV^γ = constant

$P\left(\dfrac{m}{\rho}\right)^\gamma = \text{constant}$

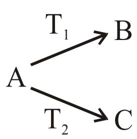
as mass is constant

$P \propto \rho^\gamma$

$\dfrac{P_f}{P_i} = \left(\dfrac{\rho_f}{\rho_i}\right)^\gamma = (32)^{7/5} = 2^7 = 128$

10. NTA Ans. (1)

Sol.



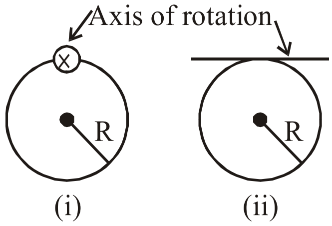
$\dfrac{1}{T_{eff}} = \dfrac{1}{T_1} + \dfrac{1}{T_2}$

$T_{eff} = \dfrac{T_1T_2}{T_1 + T_2} = \dfrac{1000}{110} = \dfrac{100}{11} = 9.09$

$T_{eff} \cong 9$

11. NTA Ans. (1)

Sol.



Moment of inertia in case (i) is I_1

Moment of inertia in case (ii) is I_2

$I_1 = 2MR^2$

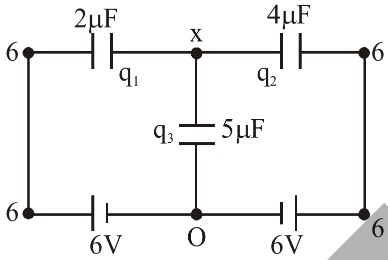
$I_2 = \frac{3}{2}MR^2$

$T_1 = 2\pi\sqrt{\frac{I_1}{Mgd}}$; $T_2 = 2\pi\sqrt{\frac{I_2}{Mgd}}$

$$\frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}} = \frac{2}{\sqrt{3}}$$

12. NTA Ans. (2)

Sol.



Let potential of point O $v_0 = 0$

Now, using junction analysis

We can say, $q_1 + q_2 + q_3 = 0$

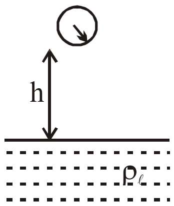
$2(x - 6) + 4(x - 6) + 5(x) = 0$

$x = \frac{36}{11}$ $\quad q_3 = \frac{36(5)}{11} = \frac{180}{11}$

$q_3 = 16.36\ \mu C$

13. NTA Ans. (2)

Sol.



After falling through h, the velocity be equal to terminal velocity

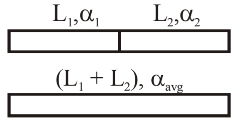
$$\sqrt{2gh} = \frac{2}{9}\frac{r^2g}{\eta}(\rho_\ell - \rho)$$

$$\Rightarrow h = \frac{2}{81}\frac{r^4g\,(\rho_\ell - \rho)^2}{\eta^2}$$

$\Rightarrow h \propto r^4$

14. NTA Ans. (4)

Sol. At T°C $\quad L = L_1 + L_2$



At T + ΔT $\quad L'_{eq} = L'_1 + L'_2$

where $L'_1 = L_1(1 + \alpha_1\Delta T)$

$L'_2 = L_2(1 + \alpha_2\Delta T)$

$L'_{eq} = (L_1 + L_2)\,(1 + \alpha_{avg}\Delta T)$

$\Rightarrow (L_1 + L_2)(1 + \alpha_{avg}\Delta T) = L_1 + L_2 + L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$

$\Rightarrow (L_1 + L_2)\,\alpha_{avg} = L_1\alpha_1 + L_2\alpha_2$

$$\Rightarrow \alpha_{avg} = \frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$$

15. NTA Ans. (2)

Sol. $x = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \text{speed} \Rightarrow [x] = [L^1T^{-1}]$

$y = \frac{E}{B} = \text{speed} \Rightarrow [y] = [L^1T^{-1}]$

$z = \frac{\ell}{RC} = \frac{\ell}{\tau} \Rightarrow [z] = [L^1\,T^{-1}]$

So, x, y, z all have the same dimensions.

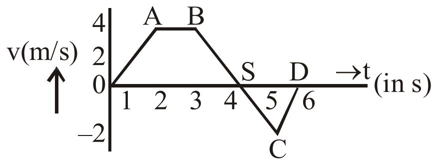
16. NTA Ans. (1)

Sol. Figure of Merit = $C = \frac{i}{\theta}$

$= C = \frac{6 \times 10^{-3}}{2} = 3 \times 10^{-3}\ \text{Am}^2$

17. NTA Ans. (4)

Sol.



$OS = 4 + \frac{1}{3} = \frac{13}{3}$

$SD = 2 - \frac{1}{3} = \frac{5}{3}$

Area of OABS is A_1

Area of SCD is A_2

Distance = $|A_1| + |A_2|$

$A_1 = \frac{1}{2}\left[\frac{13}{3} + 1\right]4 = \frac{32}{3}$

$A_2 = \frac{1}{2} \times \frac{5}{3} \times 2 = \frac{5}{3}$

Distance = $|A_1| + |A_2|$

$= \frac{32}{3} + \frac{5}{3} = \frac{37}{3}$

18. NTA Ans. (4)

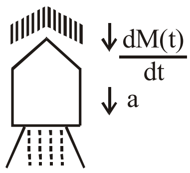
Sol. $\frac{dm(t)}{dt} = bv^2$

$F_{thast} = v\frac{dm}{dt}$

Force on statellile = $-\vec{v}\frac{dm(t)}{dt}$

M(t) a = – v (bv²)

$\boxed{a = a\frac{bv^3}{M(t)}}$

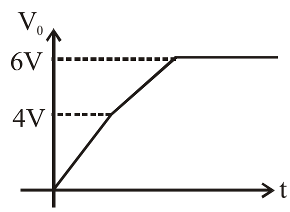


19. NTA Ans. (4)

Sol. Till input voltage Reaches 4V No zener is in Breakdown Region So $V_0 = V_i$ Then Now when V_i changes between 4V to 6V One Zener with 4V will Breakdown are P.D. across This zener will become constant and Remaining Potential will drop. acro

Resistance in series with 4V Zener.

Now current in circuit increases Abruptly and source must have an internal resistance due to which. Some potential will get drop across the source also so correct graph between V_0 and t. will be



We have to Assume some resistance in series with source.

20. NTA Ans. (4)

Sol. Energes of given Radiation can have The following relation

$E_{\gamma\text{-Rays}} > E_{X\text{-Rays}} > E_{microwave} > E_{AM\ Radiowaves}$

$\therefore \lambda_{\gamma\text{-Rays}} < \lambda_{X\text{-Rays}} < \lambda_{microwave} < \lambda_{AM\ Radiowaves}$

According To tres.

(a) Microwave → 10^{-3}m (iv)

(b) Gamma Rays → 10^{-15}m (ii)

(c) AM Radio wave → 100 m (i)

(d) X-Rays → 10^{-10}m (iii)

21. NTA Ans. (2.00)

Sol. $E_1 = \phi + K_1$...(1)

$E_2 = \phi + K_2$ ...(2)

$E_1 - E_2 = K_1 - K_2$

Now $\frac{V_1}{V_2} = 2$

$\frac{K_1}{K_2} = 4$

$K_1 = 4K_2$

Now from equation (2)

$\Rightarrow 4 - 2.5 = 4K_2 - K_2$

$1.5 = 3K_2$

$K_2 = 0.5eV$

Now putting This

Value in equation (2)

$2.5 = \phi + 0.5eV$

$\boxed{\phi = 2ev}$

22. NTA Ans. (40.00 to 41.00)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

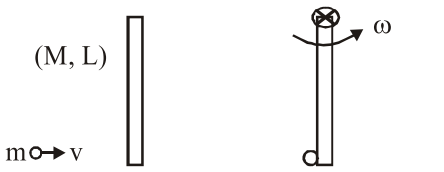
$V_{N_2} = V_{H_2}$

$\sqrt{\frac{3RT_{N_2}}{M_{N_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$

$\frac{573}{28} = \frac{T_{H_2}}{2} \Rightarrow T_{H_2} = 40.928$

23. NTA Ans. (20.00)

Sol.



Before collision After collision

$\vec{L}_i = \vec{L}_f$

mvL = Iω

$$mvL = \left(\frac{ML^2}{3} + mL^2\right)\omega$$

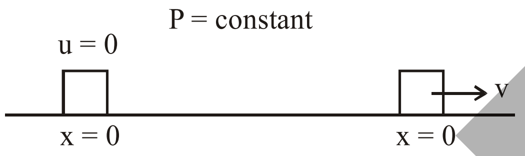
$$0.1 \times 80 \times 1 = \left(\frac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right)\omega$$

$$8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega = \frac{4}{10}\omega$$

$$\omega = 20 \text{ rad}\frac{\text{rad}}{\text{sec}}$$

24. NTA Ans. (18.00)

Sol.



P = mav

$$m\frac{dv}{dt}v = P$$

$$\int_0^v v\,dv = \frac{P}{m}\int_0^t dt$$

$$\frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$\frac{dx}{dt} = \sqrt{\frac{2P}{m}}\,t^{1/2}$$

$$\int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt$$

$$x = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{3/2} = \sqrt{\frac{2P}{m}} \times \frac{2}{3}t^{3/2} = \sqrt{\frac{2 \times 1}{2}} \times \frac{2}{3} \times 9^{3/2}$$

$$= \frac{2}{3} \times 27 = 18$$

25. NTA Ans. (5.00)

Sol. $\delta_{min} = (\mu - 1)\,A = (1.5 - 1)1 = 0.5$

$$\delta_{min} = \frac{5}{10}$$

N = 5

CHEMISTRY

1. NTA Ans. (1)

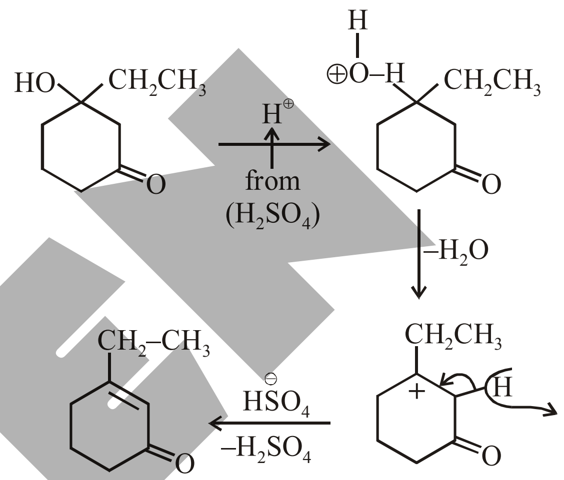
$$\text{Slope} = -\frac{E_a}{R}$$

$$-\frac{10}{5} = -\frac{E_a}{R}$$

$E_a = 2R$

2. NTA Ans. (2)

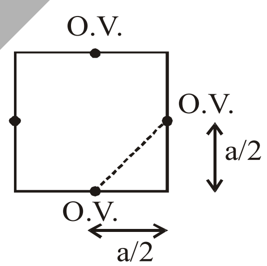
Sol.



3. NTA Ans. (3)

4. NTA Ans. (3)

Sol.



distance between nearest octahedral voids(O.V.)

$$= \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} \Rightarrow = \frac{a}{\sqrt{2}}$$

5. NTA Ans. (1)

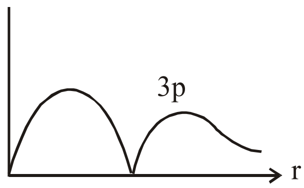
Sol. Its a weak electrolyte hence : CH_3COOH

6. NTA Ans. (3)

Sol. Temporary hardness of water is removed by <u>clark method</u> and boiling. While permanent hardness of water is removed by treatment with sodium carbonate (Na_2CO_3), <u>calgons method</u> and <u>ion-exchange method</u>

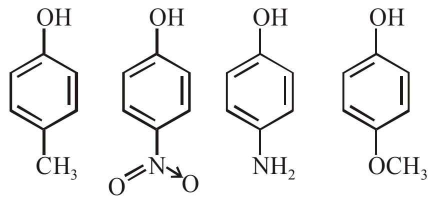
7. NTA Ans. (2)

Sol. $[R(r)]^2$



8. NTA Ans. (1)

Sol.

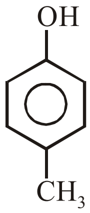


	I	II	III	IV
BP value from net	$\simeq$202°C	$\simeq$279°	$\simeq$284°C	$\simeq$243°C

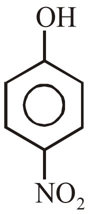
$\boxed{\text{BP} \propto \text{dipolemoment } (\mu)}$

Alter

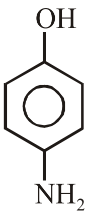
Increasing order of boiling point is :

I. 

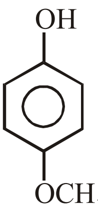
$\Rightarrow$ Shows hydrogen bonding from –O–H group only

II. 

$\Rightarrow$ Shows strongest hydrogen bonding from both sides of –OH group as well as $-NO_2$ group.

III. 

$\Rightarrow$ Shows stronger hydrogen from both side of –OH group as well as $-NH_2$ group.

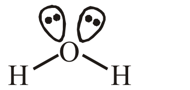
IV. 

$\Rightarrow$ Shows stronger hydrogen bonding from one side –OH–group and another side of $-OCH_3$ group shows only dipole-dipole interaction.

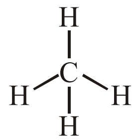
$\Rightarrow$ Hence correct order of boiling point is:

(I) < (IV) < (III) < (II)

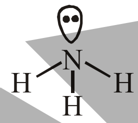
9. NTA Ans. (2)

Sol. (1) 

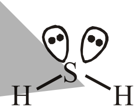
sp^3, 104°5'

(2) 

sp^3, 109°28'

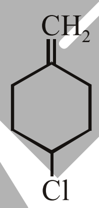
(3) 

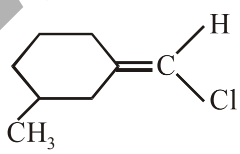
sp^3, 107°

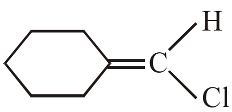
(4) 

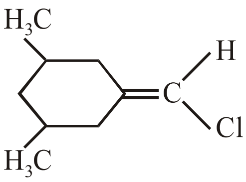
92°

10. NTA Ans. (2)

Sol. (1)  Not show GI

(2)  Show GI

(3)  Not show GI

(4)  Show GI

11. NTA Ans. (1)

Sol. $\boxed{\text{BuNa–N}}$ is an addition polymer

$\downarrow$

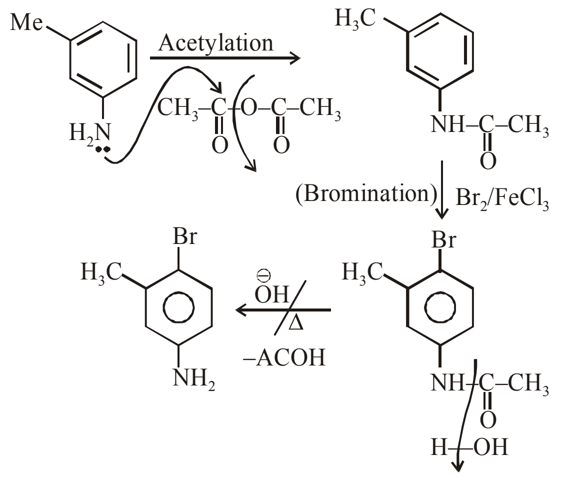
Buta-1, 3-diene + Acrylonitrile

$CH_2=CH-CH=CH_2$ $\quad$ $CH_2=CH-C\equiv N$

$\xrightarrow{Na}$ [BuNa-N]

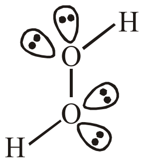
12. NTA Ans. (1)

Sol.



13. NTA Ans. (1)

Sol.



hydrogen peroxide, in the pure state, is non-planar and almost colourless (very pale blue) liquid.

14. NTA Ans. (4)

Sol. "Boron" and "Silicon" of very high purity can be obtained through :-

zone refining method only.

While other methods are used for other metals/ elements i.e.

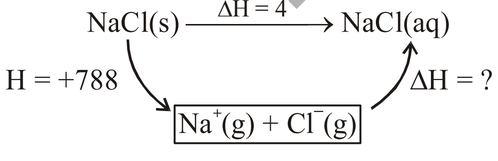
(i) Vapour phase refining

(ii) electrolytic refining

(iii) liquation etc.

15. NTA Ans. (2)

Sol.



$4 = 788 + \Delta H$

$\Delta H = -784$ kJ

16. NTA Ans. (4)

Sol. $NH_3 + \underset{\text{excess}}{3Cl_2} \longrightarrow NCl_3 + 3HCl$

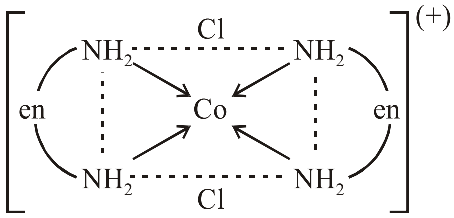
17. NTA Ans. (3)

Sol. Correct order of size for isoelectronic species.

$Al^{3+} < Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

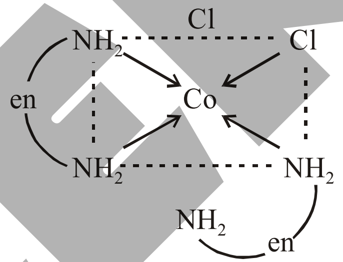
18. NTA Ans. (4)

Sol. (A) *trans*-$[Co(en)_2Cl_2]^+$



⇒ (A) is trans form and shows plane of symmetry which is optically inactive (not optically active)

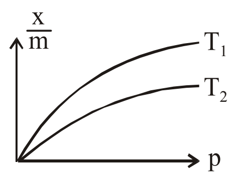
(B) *cis*-$[Co(en)_2Cl_2]^+$



⇒ (B) is cis form and does not shows plane of symmetry, hence it is optically active.

19. NTA Ans. (3)

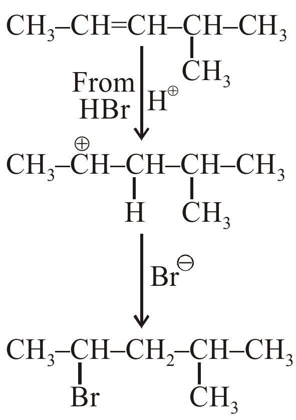
Sol. $\frac{x}{m} = K.P^{1/n}$



$(T_2 > T_1)$

20. NTA Ans. (1)

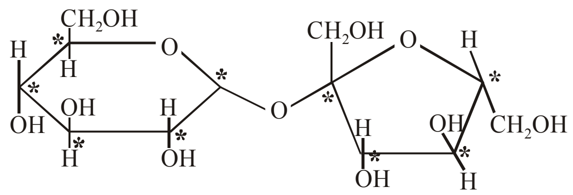
Sol.



Addition of HBr according to M.R.

21. NTA Ans. (9)

Sol.



[α-D-Glucose] [β-D-Fructose]

Total no. of chiral carbon in sucrose = 9

22. NTA Ans. (–13540.00 to –13537.00)

Sol. $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$= (\Delta U^\circ + \Delta n_g RT) - T\Delta S^\circ$

$$= \left[\left\{-20+(-1))\frac{8.314}{1000}\times 298\right\}-\frac{298}{1000}\times(-30)\right]\text{kJ}$$

$= -13.537572$ kJ

$= -13537.57$ Joule

23. NTA Ans. (16)

Sol.

	X	+ Y	= 2Z
t = 0	1	1.5	0.5
At eq.	0.75	1.25	1

;

$$K_{eq.} = \frac{1^2}{\frac{3}{4}\times\frac{5}{4}} = \frac{16}{15}$$

24. NTA Ans. (50.00)

Sol. $K_2Cr_2O_7 + FeC_2O_4 \longrightarrow Cr^{+3} + Fe^{+3} + CO_2$

n = 6 n = 3

$$\frac{0.02\times 6\times V(mL)}{1000} = \frac{0.288}{144}\times 3$$

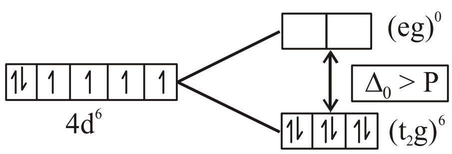
$\Rightarrow \boxed{V = 50mL}$

25. NTA Ans. (00)

Sol. Magnetic moment (in B.M.) of $[Ru(H_2O)_6]^{2+}$ would be; while considering that $\Delta_0 > P$,

$Ru_{(44)}$; $[Kr]4d^7 5s^1$ (in ground state)

$\Rightarrow$ In $Ru^{2+} \Rightarrow 4d^6 \Rightarrow (t_2g)^6(eg)^0$



$\Rightarrow$ Here number of unpaired electrons in

$Ru^{2+} = (t_2g)^6\ (eg)^0 = 0$ and Hence

$\mu_m = \sqrt{n(n+2)}\text{B.M.} = \boxed{0\ \text{B.M.}}$

MATHEMATICS

1. NTA Ans. (2)

Sol. $x + y + 3z = 0$ .....(i)

$x + 3y + k^2z = 0$ .....(ii)

$3x + y + 3z = 0$ .....(iii)

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 9 + 3 + 3k^2 - 27 - k^2 - 3 = 0$

$\Rightarrow k^2 = 9$

(i) – (iii) $\Rightarrow -2x = 0 \Rightarrow x = 0$

Now from (i) $\Rightarrow y + 3z = 0$

$\Rightarrow \frac{y}{z} = -3$

$x + \frac{y}{z} = -3$

2. NTA Ans. (1)

Sol. $7x^2 - 3x - 2 = 0$

$\alpha + \beta = \frac{3}{7}$

$\alpha\beta = \frac{-2}{7}$

$$\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2} = \frac{\alpha+\beta-\alpha\beta(\alpha+\beta)}{1-\alpha^2-\beta^2+\alpha^2\beta^2}$$

$$= \frac{\frac{3}{7}+\frac{2}{7}\left(\frac{3}{7}\right)}{1-(\alpha+\beta)^2+2\alpha\beta+\alpha^2\beta^2} = \frac{27}{16}$$

3. NTA Ans. (4)

Sol. $460 = \log_7 x \cdot (2 + 3 + 4 + ... + 20 + 21)$

$\Rightarrow 460 = \log_7 x \cdot \left(\frac{21\times 22}{2}-1\right)$

$\Rightarrow 460 = 230 \cdot \log_7 x$

$\Rightarrow \log_7 x = 2 \Rightarrow x = 49$

4. NTA Ans. (4)

Sol. $\lim\limits_{x\to 0}\dfrac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$

$\because \lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2+x^4}-1}{x}$ $\left(\dfrac{0}{0}\text{ from}\right)$

$\lim\limits_{x\to 0}\dfrac{(1+x^2+x^4)-1}{x(\sqrt{1+x^2+x^4}+1}$

$\lim\limits_{x\to 0}\dfrac{x(1+x^2)}{\left(\sqrt{1+x^2+x^4}+1\right)} = 0$

So $\lim\limits_{x\to 0}\dfrac{x\left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)}-1\right)}{\sqrt{1+x^2+x^4}-1}$ $\left(\dfrac{0}{0}\text{ from}\right)$

$\lim\limits_{x\to 0}\dfrac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}}-1}{\left(\dfrac{\sqrt{1+x^2+x^4}-1}{x}\right)} = 1$

5. NTA Ans. (2)

Sol. Let first term = a > 0

Common ratio = r > 0

$ar + ar^2 + ar^3 = 3$ ....(i)

$ar^5 + ar^6 + ar^7 = 243$ ....(ii)

$r^4(ar + ar^2 + ar^3) = 243$

$r^4(3) = 243 \Rightarrow r = 3$ as $r > 0$

from (1)

$3a + 9a + 27a = 3$

$a = \dfrac{1}{13}$

$S_{50} = \dfrac{a(r^{50}-1)}{(r-1)} = \dfrac{1}{26}(3^{50}-1)$

6. NTA Ans. (3)

Sol. $\left(\dfrac{-1+i\sqrt{3}}{1-i}\right)^{30} = \left(\dfrac{2\omega}{1-i}\right)^{30}$

$= \dfrac{2^{30}\cdot\omega^{30}}{\left((1-i)^2\right)^{30}}$

$= \dfrac{2^{30}\cdot 1}{(1+i^2-2i)^{15}}$

$= \dfrac{2^{30}}{-2^{15}\cdot i^{15}}$

$= -2^{15}i$

7. NTA Ans. (2)

Sol. Let $f = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$f = \tan^{-1}\left(\dfrac{\sec\theta-1}{\tan\theta}\right)$

$f = \tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right) = \dfrac{\theta}{2}$

$f = \dfrac{\tan^{-1}x}{2} \Rightarrow \dfrac{df}{dx} = \dfrac{1}{2(1+x^2)}$ ....(i)

Let $g = \tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$g = \tan^{-1}\left(\dfrac{2\sin\theta\cos\theta}{1-2\sin^2\theta}\right)$

$g = \tan^{-1}(\tan 2\theta) = 2\theta$

$g = 2\sin^{-1}x$

$\dfrac{dg}{dx} = \dfrac{2}{\sqrt{1-x^2}}$ ....(ii)

$\dfrac{df}{dg} = \dfrac{1}{2(1+x^2)}\;\dfrac{\sqrt{1-x^2}}{2}$

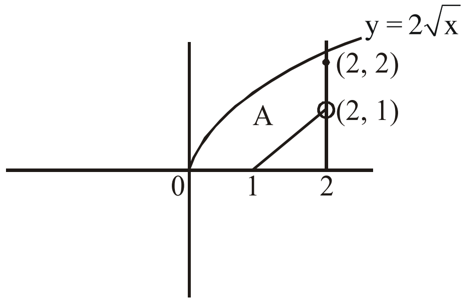
at $x = \dfrac{1}{2}$ $\left(\dfrac{df}{dg}\right)_{x=\frac{1}{2}} = \dfrac{\sqrt{3}}{10}$

8. NTA Ans. (1)

Sol. $(x-1)[x] \le y \le 2\sqrt{x}$, $\boxed{0\le x\le 2}$

Draw $y = 2\sqrt{x} \Rightarrow y^2 = 4x$ $\boxed{x\ge 0}$

$y = (x-1)[x] = \begin{cases} 0 & , 0\le x<1 \\ x-1 & , 1\le x<2 \\ 2 & , x=2 \end{cases}$



$A = \int_0^2 2\sqrt{x}\,dx - \dfrac{1}{2}1\cdot 1$

$A = 2\cdot\left[\dfrac{x^{3/2}}{(3/2)}\right]_0^2 - \dfrac{1}{2} = \dfrac{8\sqrt{2}}{3} - \dfrac{1}{2}$

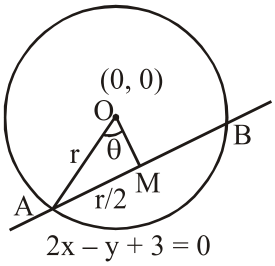
9. NTA Ans. (2)

Sol. Let chord

AB = r

$\because$ ΔAOM is right angled triangle

$\therefore$ $OM = \dfrac{r\sqrt{3}}{2}$ = perpendicular distance of line AB from (0,0)



$\dfrac{r\sqrt{3}}{2} = \left|\dfrac{3}{\sqrt{5}}\right|$

$r^2 = \dfrac{12}{5}$

10. NTA Ans. (1)

Sol. $f(x) = (3x^2 + ax - 2 - a)e^x$

$f'(x) = (3x^2 + ax - 2 - a)e^x + e^x(6x + a)$

$= e^x(3x^2 + x(6 + a) - 2)$

$f'(x) = 0$ at $x = 1$

$\Rightarrow 3 + (6 + a) - 2 = 0$

$a = -7$

$f'(x) = e^x(3x^2 - x - 2)$

$= e^x(x - 1)(3x + 2)$

$\begin{array}{ccccc} + & & - & & + \\ & -2/3 & & 1 & \end{array}$

x = 1 is point of local minima

$x = \dfrac{-2}{3}$ is point of local maxima

11. NTA Ans. (2)

Sol. Mean = 5

$\dfrac{3+5+7+a+b}{5} = 5$

$a + b = 10$ ....(i)

S.d. $= 2 \Rightarrow \sqrt{\dfrac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{5}} = 2$

$(3-5)^2 + (5-5)^2 + (7-5)^2 + (a-5)^2 + (b-5)^2 = 20$

$\Rightarrow 4 + 0 + 4 + (a - 5)^2 + (b - 5)^2 = 20$

$a^2 + b^2 - 10(a + b) + 50 = 12$

$(a + b)^2 - 2ab - 100 + 50 = 12$

$ab = 19$ ....(ii)

Equation is $x^2 - 10x + 19 = 0$

12. NTA Ans. (2)

Sol. $a + x = b + y = c + z + 1$

$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ $\quad C_3 \to C_3 - C_1$

$\begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix}$ $\quad C_2 \to C_2 - C_3$

$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix}$ $\quad R_3 \to R_3 - R_1,\ R_2 \to R_2 - R_1$

$\begin{vmatrix} x & y & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$

$= (-y)[(y - x)(c - a) - (b - a)(z - x)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c - 1)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c) + b - a)$

$= -y(b - a) = y(a - b)$

13. NTA Ans. (4)

Sol. $\displaystyle\int \frac{\cos\theta\, d\theta}{5 + 7\sin\theta - 2\cos^2\theta}$

$\displaystyle\int \frac{\cos\theta\, d\theta}{3 + 7\sin\theta + 2\sin^2\theta}$ $\quad \boxed{\begin{array}{l} \sin\theta = t \\ \cos\theta d\theta = dt \end{array}}$

$\displaystyle\int \frac{dt}{2t^2 + 7t + 3} = \int \frac{dt}{(2t+1)(t+3)}$

$= \dfrac{1}{5}\displaystyle\int\left(\frac{2}{2t+1} - \frac{1}{t+3}\right)dt$

$= \dfrac{1}{5} ln\left|\dfrac{2t+1}{t+3}\right| + C$

$= \dfrac{1}{5} ln\left|\dfrac{2\sin\theta + 1}{\sin\theta + 3}\right| + C$

$A = \dfrac{1}{5}$ and $B(\theta) = \dfrac{2\sin\theta + 1}{\sin\theta + 3}$

14. NTA Ans. (2)

Sol. y = mx + c is tangent to

$\frac{x^2}{100}-\frac{y^2}{64}=1$ and $x^2 + y^2 = 36$

$c^2 = 100m^2 - 64$ | $c^2 = 36(1 + m^2)$

$\Rightarrow 100m^2 - 64 = 36 + 36m^2$

$m^2 = \frac{100}{64} \Rightarrow m = \pm\frac{10}{8}$

$c^2 = 36\left(1+\frac{100}{64}\right) = \frac{36\times164}{64}$

$4c^2 = 369$

15. NTA Ans. (4)

Sol.

A	B	C
$\boxed{5}$	$\boxed{5}$	$\boxed{5}$
1	2	2
2	1	2
2	2	1
1	1	3
1	3	1
3	1	1

Total number of selection

$= ({}^5C_1\ {}^5C_2\ {}^5C_2)\cdot3 + ({}^5C_1\ {}^5C_1\ {}^5C_3)\cdot3$

$= 5 \cdot 10 \cdot 10 \cdot 3 + 5 \cdot 5 \cdot 10 \cdot 3$

$= 2250$

16. NTA Ans. (4)

Sol. $L_1 \equiv \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$

$L_2 \equiv \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$

Point A(–1, 2, 1) B(–2, –1, –1)

$\because$ L_1 and L_2 are coplanar

$$\Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$$

$\alpha = -4$

$L_2 \equiv \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Check options (2, –10, –2) lies on L_2

17. NTA Ans. (1)

Sol. $\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x$

$\frac{dy}{dx} + \frac{2\sin x}{\cos x}y = 2\sin x$

$\text{I.F.} = e^{\int 2\frac{\sin x}{\cos x}dx}$

$= e^{2\ln\sec x} = \sec^2 x$

$y \cdot \sec^2 x = \int 2\sin x\cdot\sec^2 x\,dx$

$y\sec^2 x = 2\int \tan x\sec x\,dx$

$y\sec^2 x = 2\sec x + c$

At $x = \frac{\pi}{3}$, $y = 0$

$\Rightarrow 0 = 2\sec\frac{\pi}{3} + C \Rightarrow C = -4$

$\boxed{y\sec^2 x = 2\sec x - 4}$

Put $x = \frac{\pi}{4}$

$y \cdot 2 = 2\sqrt{2} - 4$

$y = \sqrt{2} - 2$

18. NTA Ans. (2)

Sol. $x^4e^y + 2\sqrt{y+1} = 3$

d.w.r. to x

$x^4e^y y' + e^y 4x^3 + \frac{2y'}{2\sqrt{y+1}} = 0$

at P(1, 0)

$y'_P + 4 + y'_P = 0$

$\Rightarrow y'_P = -2$

Tangent at P(1, 0) is

$y - 0 = -2(x - 1)$

$2x + y = 2$

(–2, 6) lies on it

19. NTA Ans. (3)

Sol.

p	q	$q\to p$	$p\to(q\to p)$	$p\vee q$	$p\to p\vee q$	$(p\to(q\to p))\to(p\to(p\vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

20. NTA Ans. (1)

Sol. $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$\left(\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$

$\Rightarrow L = \left(\frac{1-\cos(\pi/8)}{2}\right) - \left(\frac{1-\cos(\pi/4)}{2}\right)$

$L = \frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{8}\right)\right]$

$L = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\left(\frac{\pi}{8}\right)$

$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$M = \frac{1+\cos(\pi/8)}{2} - \frac{1-\cos(\pi/4)}{2}$

$M = \frac{1}{2}\cos\left(\frac{\pi}{8}\right) + \frac{1}{2\sqrt{2}}$

21. NTA Ans. (11.00)

Sol. $P(H) = \frac{1}{2}$

$P(\bar{H}) = \frac{1}{2}$

Let total 'n' bomb are required to destroy the target

$1 - {}^nC_n\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^n \geq \frac{99}{100}$

$1 - \frac{1}{2^n} - \frac{n}{2^n} \geq \frac{99}{100}$

$\frac{1}{100} \geq \frac{n+1}{2^n}$

Now check for value of n

$\boxed{n = 11}$

22. NTA Ans. (19.00)

Sol. $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one$\}$

Case-I : If $f(x) = 2\ \forall\ x \in A$ then number of function = 1

Case-II : If $f(x) = 2$ for exactly two elements then total number of many-one function = ${}^3C_2\ {}^3C_1 = 9$

Case-III : If $f(x) = 2$ for exactly one element then total number of many-one functions = ${}^3C_1\ {}^3C_1 = 9$

Total = 19

23. NTA Ans. (120.00)

Sol. $(1 + x + x^2 + x^3)^6 = \left((1+x)(1+x^2)\right)^6$

$= (1 + x)^6 (1 + x^2)^6$

$= \sum_{r=0}^{6} {}^6C_r x^r \sum_{r=0}^{6} {}^6C_t x^{2t}$

$= \sum_{r=0}^{6}\sum_{t=0}^{6} {}^6C_r\,{}^6C_t\, x^{r+2t}$

For coefficient of $x^4 \Rightarrow r + 2t = 4$

r	t
0	2
2	1
4	0

Coefficient of x^4

$= {}^6C_0\,{}^6C_2 + {}^6C_2\,{}^6C_1 + {}^6C_4\,{}^6C_0$

$= 120$

24. NTA Ans. (0.50)

Sol. $y = x^2 - 3x + 2$

At x-axis $y = 0 = x^2 - 3x + 2$

$x = 1, 2$

$\frac{dy}{dx} = 2x - 3$

A(1, 0) B(2, 0)

$\left(\frac{dy}{dx}\right)_{x=1} = -1$ and $\left(\frac{dy}{dx}\right)_{x=2} = 1$

$x + y = a \Rightarrow \frac{dy}{dx} = -1$ So A(1, 0) lies on it

$\Rightarrow 1 + 0 = a \Rightarrow \boxed{a = 1}$

$x - y = b \Rightarrow \frac{dy}{dx} = 1$ So B(2, 0) lies on it

$2 - 0 = b \Rightarrow \boxed{b = 2}$

$\frac{a}{b} = 0.50$

25. NTA Ans. (6.00)

Sol. Projection of $\vec{b}$ on $\vec{a}$ = projection of $\vec{c}$ on $\vec{a}$

$\Rightarrow \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|} \Rightarrow \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}$

$\because \vec{b}$ is perpendicular to $\vec{c} \Rightarrow \vec{b}\cdot\vec{c} = 0$

Let $|\vec{a} + \vec{b} - \vec{c}| = k$

Square both sides

$k^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}\cdot\vec{b} - 2\vec{a}\cdot\vec{c} - 2\vec{b}\cdot\vec{c}$

$k^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 = 36$

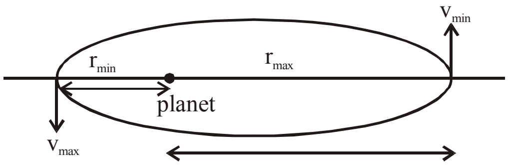
$k = 6 = |\vec{a} + \vec{b} - \vec{c}|$

SET # 09

PHYSICS

1. NTA Ans. (1)

Sol.



By angular momentum conservation

$r_{min}v_{max} = r_{max}v_{min}$ ... (i)

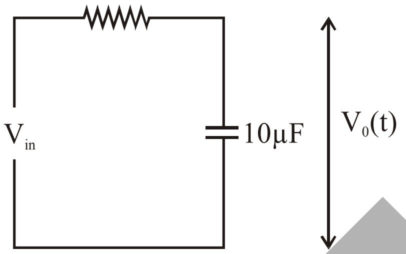
Given $v_{min} = \dfrac{v_{max}}{6}$

from equation (i)

$$\frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$$

2. NTA Ans. (1)

Sol.



$$V_0(t) = V_{in}\left(1 - e^{-\frac{t}{RC}}\right)$$

at t = 5μs

$$V_0(t) = 5\left(1 - e^{-\frac{5\times10^{-6}}{10^3\times10\times10^{-9}}}\right)$$

= 5 (1 – $e^{-0.5}$) = 2V

Now V_{in} = 0 means discharging

$$V_0(t) = 2e^{-\frac{t}{RC}} = 2e^{-0.5}$$

= 1.21 V

Now for next 5 μs

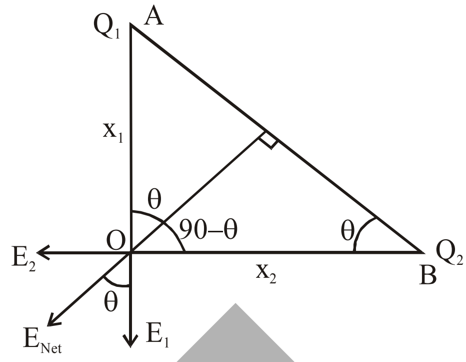
$$V_0(t) = 5 - 3.79e^{-\frac{t}{RC}}$$

after 5 μs again

$V_0(t)$ = 2.79 Volt ≈ 3V

3. NTA Ans. (3)

Sol.



E_2 = electric field due to Q_2

$$= \frac{kQ_2}{x_2^2}$$

$$E_1 = \frac{kQ_1}{x_1^2}$$

From diagram

$$\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2}$$

$$\frac{kQ_2}{x_2^2 \times \dfrac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$$

$$\frac{Q_2x_1^2}{Q_1x_2^2} = \frac{x_1}{x_2}$$

$$\frac{Q_2}{Q_1} = \frac{x_2}{x_1}$$

$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

Ans. (3)

4. NTA Ans. (2)

Sol. Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{no. of division on circular scale}}$$

$$= \frac{0.5}{50}\text{mm} = 1\times10^{-5}\text{m}$$

= 10 μm

Zero error in positive

5. NTA Ans. (1)

Sol. An elastic wire can be treated as a spring with

$$k = \frac{YA}{\ell}$$

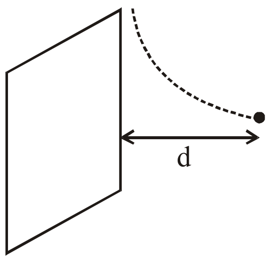
$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{m\ell}}$$

Ans. (1)

6. NTA Ans. (2)

Sol.



In uniform magnetic field particle moves in a circular path, if the radius of the circular path is 'd', particle will not hit the screen.

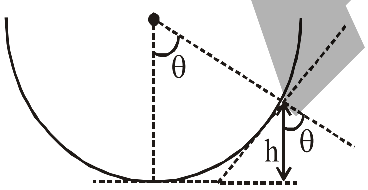
$$d = \frac{mv}{qB_0}$$

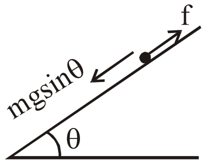
$$v = \frac{qB_0 d}{m}$$

$\therefore$ correct option is (2)

7. NTA Ans. (4)

Sol.



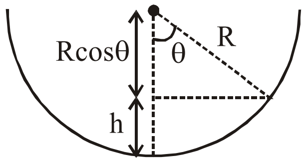


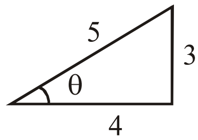
For balancing mgsinθ = f

$mg\sin\theta = \mu mg\cos\theta$

$\tan\theta = \mu$

$$\tan\theta = \frac{3}{4}$$





$h = R - R\cos\theta$

$$= R - R\left(\frac{4}{5}\right) = \frac{R}{5}$$

$$h = \frac{R}{5} = 0.2\text{m}$$

$\therefore$ correct option is (4)

8. NTA Ans. (1)

Sol. R = 0.1 m

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105 \text{ rad/sec}$$

$a = \omega^2 R$

$= (0.105)^2\ (0.1)$

$= 0.0011$

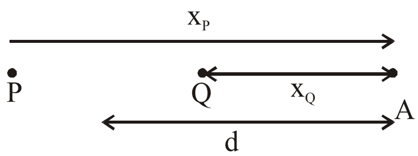
$= 1.1 \times 10^{-3}$

Average acceleration is of the order of 10^{-3}

$\therefore$ correct option is (1)

9. NTA Ans. (4)

Sol. For (A)



$x_P - x_Q = (d + 2.5) - (d - 2.5) = 5\text{m}$

$$\Delta\phi \text{ due to path difference } = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{20}(5)$$

$$= \frac{\pi}{2}$$

At A, Q is ahead of P by path, as wave emitted by Q reaches before wave emitted by P.

Total phase difference at A

$= \frac{\pi}{2} - \frac{\pi}{2}$ (due to P being ahead of Q by 90°)

= 0

$I_A = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$

$= I + I + 2\sqrt{I}\sqrt{I}\cos(0)$

$= 4I$

For C

$x_Q - x_P = 5$ m

$\Delta\phi$ due to path difference $= \frac{2\pi}{\lambda}(\Delta x)$

$= \frac{2\pi}{20}(5) = \frac{\pi}{2}$

Total phase difference at C = $\frac{\pi}{2} + \frac{\pi}{2} = \pi$

$I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\Delta\phi)$

$= I + I + 2\sqrt{I}\sqrt{I}\cos(\pi) = 0$

For B

$x_P - x_Q = 0$,

$\Delta\phi = \frac{\pi}{2}$ (Due to P being ahead of Q by 90°)

$I_B = I + I + 2\sqrt{I}\sqrt{I}\cos\frac{\pi}{2} = 2I$

$I_A : I_B : I_C = 4I : 2I : 0$

$= 2 : 1 : 0$

$\therefore$ correct option is (4)

10. NTA Ans. (4)

Sol. $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2m(KE)}}$

$\lambda \propto \frac{1}{\sqrt{m}} \Rightarrow \lambda = \frac{C}{\sqrt{m}}$

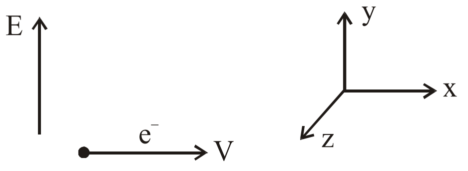
$m_{He^{++}} > m_P > m_e$

$\therefore \lambda_{He^{++}} < \lambda_P < \lambda_e$

$\therefore$ correct option is (4)

11. NTA Ans. (1)

Sol.



$\vec{B}$ must be in +z axis.

$\vec{V} = 6\times10^6\hat{i}$

$\vec{E} = 300\hat{j}$ V/cm = 3×10^4 V/m

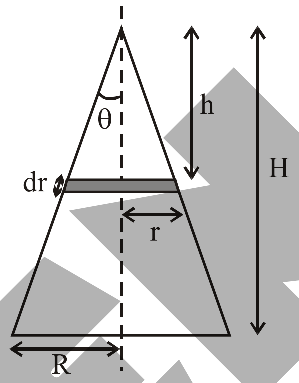
$q\vec{E} + q\vec{V}\times\vec{B} = 0$

E = VB

$B = \frac{E}{V} = \frac{3\times10^4}{6\times10^6} = 5\times10^{-3}T$

12. NTA Ans. (1)

Sol.



Area = $\pi R\ell = \pi R\left(\sqrt{H^2+R^2}\right)$

Area of element dA = $2\pi r d\ell$ = $= 2\pi r\frac{dh}{\cos\theta}$

mass of element $dm = \frac{M}{\pi R\sqrt{H^2+R^2}} \times \frac{2\pi r dh}{\cos\theta}$

$dm = \frac{2Mh\tan\theta dh}{R\sqrt{H^2+R^2}\cos\theta}$ (here r = h tanθ)

$I = \int(dm)r^2 = \int\frac{h^2\tan^2\theta}{\cos\theta}\left(\frac{2m}{R}\frac{h\tan\theta}{\sqrt{R^2+H^2}}\right)dh$

$= \frac{2M}{\cos\theta R}\frac{\tan^3\theta}{\sqrt{R^2+H^2}}$

$\int_0^H h^3 dh = \frac{MR^2H^4}{2RH^3\sqrt{R^2+H^2}\cos\theta}$

$= \frac{MR^2H\sqrt{R^2+H^2}}{2\sqrt{R^2+H^2}\times H}$

$= \frac{MR^2}{2}$

13. NTA Ans. (2)

Sol. $Q=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{100}\sqrt{\frac{80\times10^{-3}}{2\times10^{-6}}}$

$=\frac{1}{100}\sqrt{40\times10^{3}}$

$=\frac{200}{100}=2$

14. NTA Ans. (3)

Sol. $U=\frac{-A}{r^6}+\frac{B}{r^{12}}$

$F=-\frac{dU}{dr}=-\left(A\left(-6r^{-7}\right)\right)+B\left(-12r^{-13}\right)$

$0=\frac{6A}{r^7}-\frac{12B}{r^{13}}$

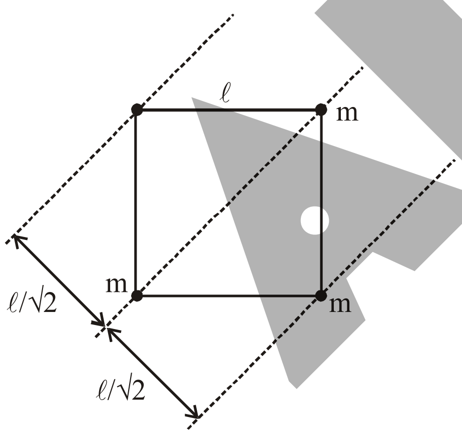
$\frac{6A}{12B}=\frac{1}{r^6}\Rightarrow r=\left(\frac{2B}{A}\right)^{1/6}$

$U\left(r=\left(\frac{2B}{A}\right)^{1/6}\right)=-\frac{A}{2B/A}+\frac{B}{4B^2/A^2}$

$=\frac{-A^2}{2B}+\frac{A^2}{4B}=\frac{-A^2}{4B}$

15. NTA Ans. (2)

Sol.



$I=m(0)^2+m\left(\frac{\ell}{\sqrt{2}}\right)^2\times2+m\left(\sqrt{2}\ell\right)^2$

$=\frac{2m\ell^2}{2}+2m\ell^2=3m\ell^2$

Angular momentum L = Iω

$=3m\ell^2\omega$

16. NTA Ans. (2)

Sol. ${}^{7}_{3}Li+{}^{1}_{1}H\rightarrow2\left({}^{4}_{2}He\right)$

$\Delta m\Rightarrow\left[m_{Li}+m_H\right]-2\left[M_{He}\right]$

Energy released in 1 reaction $\Rightarrow\Delta mc^2$.

In use of 7.016 u Li energy is Δmc^2

In use of 1gm Li energy is $\frac{\Delta mc^2}{m_{Li}}$

In use of 20 gm energy is $\Rightarrow\frac{\Delta mc^2}{m_{Li}}\times20gm$

$\Rightarrow\frac{\left[(7.016+1.0079)-2\times4.0026\right]u\times c^2}{7.016\times1.6\times10^{-24}\,gm}\times20gm$

$\Rightarrow\left(\frac{0.0187\times1.6\times10^{-19}\times10^{9}}{7.016\times1.6\times10^{-24}\,gm}\times20gm\right)$ Joule

$\Rightarrow0.05\times10^{+14}$ J

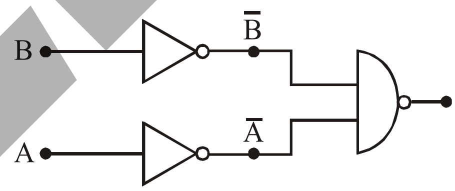
$\Rightarrow1.4\times10^{+6}$ kwh

[1 J $\Rightarrow2.778\times10^{-7}$ kwh]

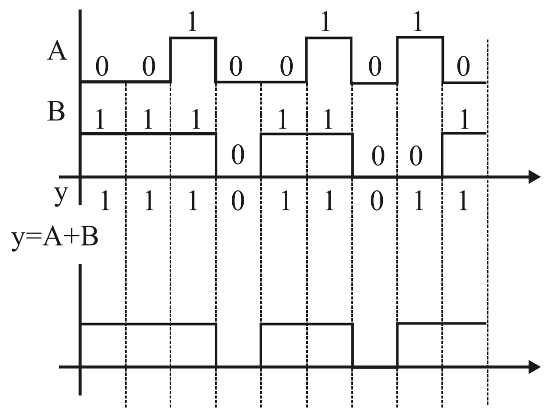
Ans. (2)

17. NTA Ans. (3)

Sol.



$y=\overline{\overline{A}.\overline{B}}=\overline{\overline{A}}+\overline{\overline{B}}=A+B$



18. NTA Ans. (4)

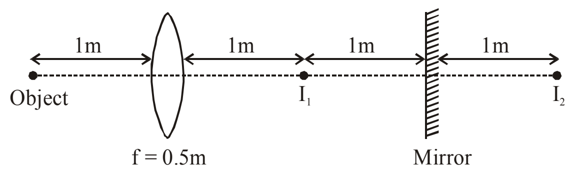
Sol. Total degree of freedom = 3 + 2 = 5

$$U = \frac{nfRT}{2} \Rightarrow \frac{5RT}{2}$$

$$\gamma \Rightarrow \frac{C_P}{C_V} \Rightarrow 1 + \frac{2}{f} \Rightarrow 1 + \frac{2}{5} \Rightarrow \frac{7}{5}$$

19. NTA Ans. (1,4)

Sol.



Object is at 2f. So image will also be at '2f'. (I_1).

Image of I_1 will be 1m behind mirror.

i.e. $\Rightarrow I_2$

Now I_2 will be object for lens.

$\therefore$ u $\Rightarrow$ – 3m

f $\Rightarrow$ +0.5 m

$$\frac{1}{v} \Rightarrow \frac{1}{f} + \frac{1}{u}$$

$$\Rightarrow \frac{1}{+0.5} + \frac{1}{-3}$$

$$v \Rightarrow \frac{3}{5} \Rightarrow 0.6m$$

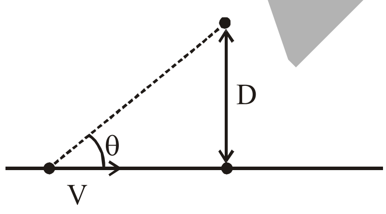
So total distance from mirror $\Rightarrow$ 2 + 0.6

$\Rightarrow$ 2.6 m and real image

Ans. (3)

20. NTA Ans. (4)

Sol.



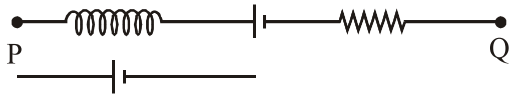
$$f_{observed} \Rightarrow \left(\frac{v_{sound}}{v_{sound} - v\cos\theta}\right) f_0$$

initially θ will be less $\Rightarrow$ cosθ more

$\therefore$ $f_{observed}$ more, then it will decrease.

21. NTA Ans. (33.00)

Sol.



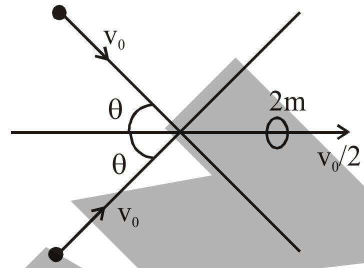
$$\frac{Ldi}{dt} = 5$$

V_P – 5 – 30 + 2 × 1 = VQ

$V_P - V_Q$ = 33 volt

22. NTA Ans. (120.00)

Sol.



Momentum conservation along x

$$2mv_0\cos\theta = 2m\frac{v_0}{2}$$

$$\cos\theta = \frac{1}{2}$$

θ = 60

Angle is 2θ = 120

Ans. 120.00

23. NTA Ans. (275.00)

Allen Ans. (194.00)

Sol. I = $\in_0 E_{rms}^2 C$

$$E_{rms}^2 = \frac{I}{\in_0 C}$$

$$= \frac{315}{\pi \in_0} \times \frac{1}{C}$$

$$= \frac{4\times 315}{4\pi\in_0} \times \frac{1}{3\times 10^8}$$

$$= \frac{4\times 315\times 9\times 10^9}{3\times 10^8}$$

E_{rms}^2 = 4 × 315 × 30

$E_{rms} = 2\sqrt{315\times 30}$

= 194.42

Ans. 194.00

24. NTA Ans. (1050.00)

Sol. $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$

$\rho = \frac{6}{\pi} M\ D^{-3}$

taking log

$\ell n\rho = \ell n\left(\frac{6}{\pi}\right) + \ell nM - 3\ell mD$

Differentiates

$\frac{d\rho}{\rho} = 0 + \frac{dM}{M} - 3\frac{d(D)}{D}$

for maximum error

$100 \times \frac{d\rho}{\rho} = \frac{dM}{M} \times 100 + \frac{3dD}{D} \times 100$

= 6 + 3 × 1.5

= 10.5 %

$= \frac{1050}{100}\%$ so x = 1050.00

25. NTA Ans. (5.00)

Sol. PV = nRT

P_1V_1 = nR 250

$P_2(2V_1) = \frac{5n}{4} R \times 2000$

Divide

$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000}$

$\frac{P_1}{P_2} = \frac{1}{5}$

$\frac{P_2}{P_1} = 5$

Ans. 5.00

CHEMISTRY

1. NTA Ans. (2)

Sol. d-block elements are considered as transition elements except Zn, Cd, Hg.

Here Sc(Z = 21), Mn (Z = 25), Mo (Z = 42), Hf(Z = 72) are transition elements.

2. NTA Ans. (2)

Sol. For lanthanoids +3 oxidation state is common but Ce, Pr, Dy, Tb exhibit higher oxidation state (+4) also.

Eu does not show (+4) higher oxidation state due to stable $4f^7 5d^0$ configuration.

$_{63}Eu$ – [Xe] $4f^7\ 5d^0\ 6s^2$

Eu^{+2} – [Xe] $4f^7\ 5d^0$

3. NTA Ans. (2)

Sol. Bronze = Cu(88 – 96%) + Sn(4 – 12%)

Brass = Cu(70%) + Zn(30%)

German silver = Cu(50%) + Zn(30%) + Ni(20%)

Wrought iron is the purest form of iron. It is manufactured by cast iron.

4. NTA Ans. (2)

Sol.
1. Liquid nitrogen is used in cryosurgery.
2. Dinitrogen can be used as an inert diluent for reactive chemicals because due to high bond energy (N ≡ N), it is very less reactive in nature.
3. At 25ºC (room temperature), dinitrogen does not react with dioxygen.
4. N_2 is diamagnetic in nature due to absence of unparied e^- in its M.O. configuration.

5. NTA Ans. (4)

Sol. n_1 moles → 1^{st} component (Molecular weight) = m_1
n_1 moles → 2^{nd} component (Molecular weight) = m_2

$x_2 \rightarrow$ mole fraction of component(2)
$C_2 \rightarrow$ molarity of component(2)
Let total moles of two components=1
Moles of component - (2) = x_2
Moles of component - (1) = $(1 - x_2)$
Mass of component - (2) = $x_2 M_2$
Mass of component - (1) = $(1-x_2)M_1$
Total mass = $(x_2M_2 + (1 - x_2)M_1)$ gm

Total volume = $\left(\dfrac{x_2M_2+(1-x_2)M_1}{d}\right)$ml

Molarity (C_2)

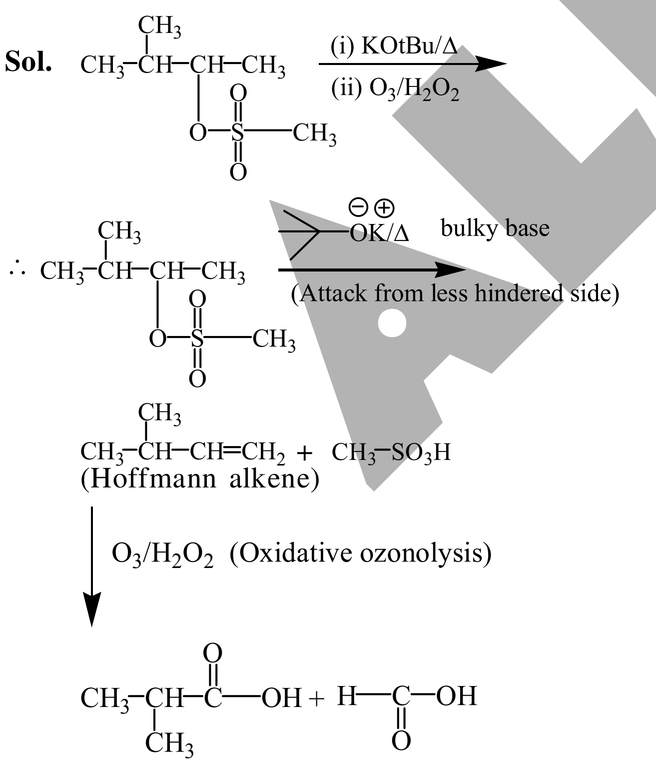
$$= \frac{\text{Moles of component(2)}}{\text{Volume of solution (in ml)}} \times 1000$$

$$= \frac{x_2}{\dfrac{x_2M_2+(1-x_2)M_1}{d}} \times 1000$$

$$C_2 = \frac{1000.d.x_2}{M_1 + x_2(M_2 - M_1)}$$

6. NTA Ans. (1)

Sol.



7. NTA Ans. (3)

Sol. Kraft temperature is temperature above which formation of micelles takes place.

8. NTA Ans. (3)

Sol. Assertion(A)

$$\underset{\text{(Ethene)}}{nCH_2{=}CH_2} \xrightarrow[\text{Zeiglar Natta Catalyst}]{TiCl_4 + Al(Et)_3} \underset{\text{Polythene (High density)}}{-\!\!(CH_2{-}CH_2)_n\!\!-}$$

(333K to 343K) / 6 – 7 atm

∴ High density polythene is formed which is used to make buckets and dustbins.

Reason :(R) High density polythene are closely packed and are chemically inert.

Option(3) Both (A) and (R) are correct and (R) is correct explanation of (A).

9. NTA Ans. (2)

Sol. 1. $[Ni(CO)_4]$

Ni → [Ar] $3d^8$ $4s^2$

due to presence of strong field ligand back pairing of electrons will take place, so

Ni → [Ar] $3d^{10}$ $4s^0$ ⇅ ⇅ ⇅ ⇅ ⇅ 3d

Hybridisation = sp^3, tetrahedral

Number of unpaired e^- = 0

Hence μ_m = 0

2. $[MnBr_4]^{2-}$

Mn^{+2} → [Ar] $3d^5$ $4s^0$ ↑ ↑ ↑ ↑ ↑ 3d

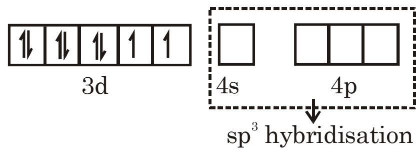
Since Br^- is weak field ligand, so hybridisation = sp^3, tetrahedral

number of unpaired e^- = 5

$\mu_m = \sqrt{n(n+2)} = \sqrt{35}\,BM = 5.91\,BM$

3. $[NiCl_4]^{2-}$

Ni^{+2} → [Ar] $3d^8$ $4s^0$



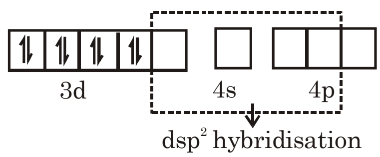
No. of unparied electron = 2

$\mu_m = \sqrt{8}\ BM = 2.83\,BM$

4. $[Ni(CN)_4]^{2-}$

Ni^{+2} → [Ar] $3d^8$ $4s^0$

CN^- is strong field ligand so rearrangement of electrons takes place here.

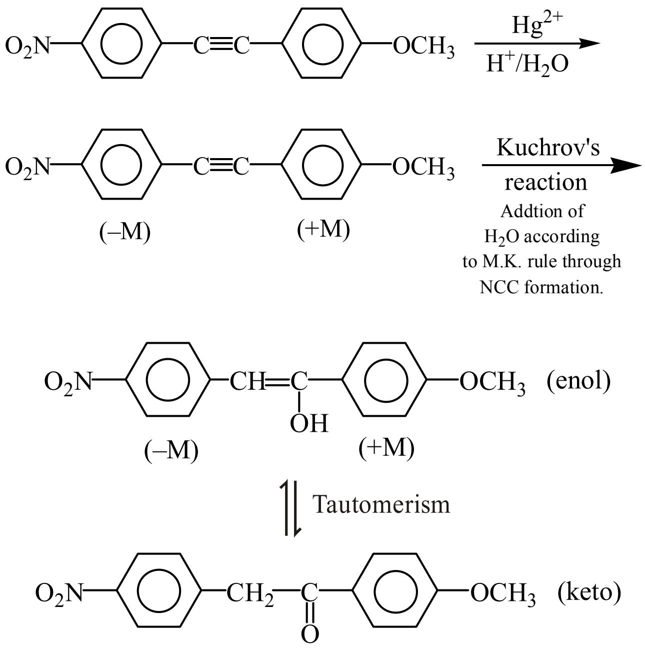


number of unpaired electron = 0

μ_m = 0

10. NTA Ans. (3)

Sol.



11. NTA Ans. (4)

Sol. For reaction

$$Fe_2N(s) + \frac{3}{2}H_2(g) \rightleftharpoons 2Fe(s) + NH_3(g)$$

$$\Delta ng = \left(1-\frac{3}{2}\right) = -\frac{1}{2}$$

$K_p = K_C(RT)^{-1/2}$

$K_C = K_P(RT)^{1/2}$

12. NTA Ans. (4)

Sol. (1) HCl $\rightarrow$ Strong acid

(2) NaOH $\rightarrow$ Strong acid

(3) $CH_3COONa \rightarrow$ Salt of strong base and weak acid

(4) NaCl $\rightarrow$ Salt of strong acid and strong base

pH order (B) > (C) > (D) > (A)

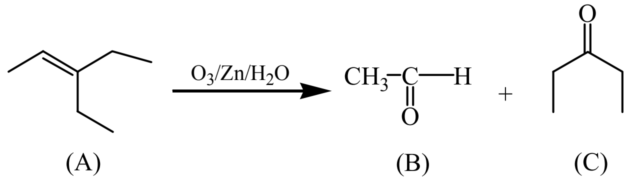
pOH order (A) > (D) > (C) > (B)

13. NTA Ans. (3)

Sol. F^- ion conentration up to 1 ppm in drinking water is safe for teeth, but F^- ion concentration above 2 ppm is harmfull for teeth and above 10 ppm is harmfull for bones.

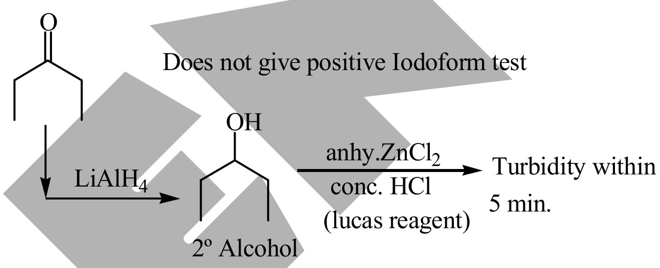
14. NTA Ans. (2)

Sol. 'A' C_7H_{14} ; DU = 1



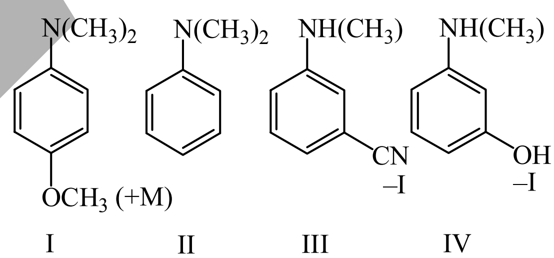
(B) $CH_3-\overset{O}{\overset{\|}{C}}-H$ gives positive iodoform and tollens test

(C)



15. NTA Ans. (1)

Sol.



Basic strength α K_b α $\frac{1}{pK_b}$

B.S. α + M, + H, + I α $\frac{1}{-M, -H, -I}$

B.S. order $\Rightarrow$ I > II > IV > III

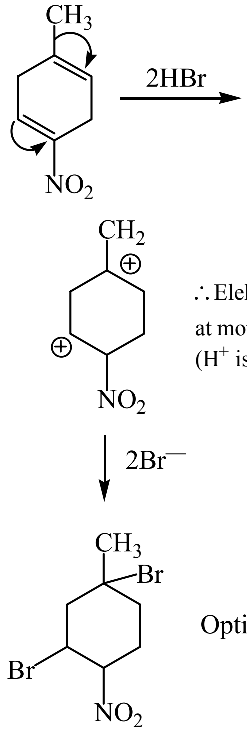
pK_b order $\Rightarrow$ III > IV > II > I

16. NTA Ans. (1)

Sol. Due to high hydration energy of $BeSO_4$ and $MgSO_4$, their solubility in water will be high.

17. NTA Ans. (2)

Sol.



∴ Elelectrophile attacks at more electron rich position (H^+ is electrophile)

Option (2)

∴ According to electrophile addition reaction

18. NTA Ans. (3)

Sol. $T_1 = 298$ K ; $K_1 = 10$

$T_2 = 373$ K ; $K_2 = 100$

For ΔH calculation :

$$\log\left(\frac{K_2}{K_1}\right) = \frac{\Delta H}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\log\left(\frac{100}{10}\right) = \frac{\Delta H}{2.303\times\left(\frac{8.314}{1000}\right)}\left(\frac{1}{298} - \frac{1}{373}\right)$$

$\Delta H = 28.4$ kJ /mole

for (ΔG_1^0) and (ΔG_2^0) calculation.

$\Delta G_1^0 = -2.303RT \log K$

$\Delta G_1^0 = -2.303 \times \dfrac{8.314}{1000} \times 298 \times \log 10$

$= -5.71$ kJ/mole

$\Delta G_2^0 = -2.303 \times \dfrac{8.314}{1000} \times 373 \times \log 100$

$= -14.29$ kJ/mole

19. NTA Ans. (3)

Sol. Rate law R = K [conc.]^n [n $\rightarrow$ order]

log(Rate) = log K + n log [conc.]

For log(rate) v/s log [conc.] graph

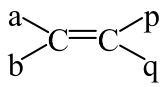
[Slope = n]

As slope increases order (n) increases

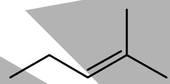
$\therefore$ Order d > b > a > C

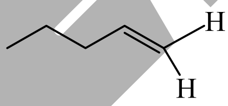
20. NTA Ans. (3)

Sol. $\therefore$ Condition of G.I.

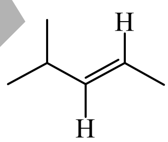


p ≠ q
a ≠ b G.I.(✓)

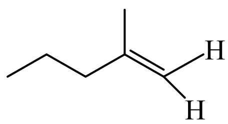
(1)

G.I.(×)
2-methylpent-2-ene

(2)


G.I.(×)
4–methyl pent–1–ene

(3)


G.I.(✓) Different group at each terminal
4-methylpent-2-ene

(4)


G.I.(×)
2-methylpent-1-ene

21. NTA Ans. (50.00)

Sol. $\therefore$ % of Br in compound is

mass of organic compound = 1.6 gm

mass of AgBr = 1.88 gm

Molecular mass of AgBr = 188 gmol^{-1}

188 gm of AgBr contains 80 gm of bromine

1.88 gm of AgBr will contain

$\dfrac{80}{188} \times 1.88 = 0.8\text{gm bromine}$

% of bromine = $\dfrac{0.8}{1.6} \times 100 = 50\%$

22. NTA Ans. (5.00)

Sol. $\dfrac{(\Delta T_b)_{complex}}{(\Delta T_b)_{CaCl_2}} = 2$

$\dfrac{i \times K_b \times 0.1}{3 \times K_b \times 0.05} = 2$

As of $CaCl_2$; i = 3

$\therefore$ i for complex = 3

$\Rightarrow$ As co-ordination number is 6, and for complex i = 3

$[Cr(NH_3)_5 Cl] Cl_2$ is structure

x = 5

23. NTA Ans. (750.00)

Sol. As in both baloon amount of gas taken and temperature is same.

$\therefore$ $P_1V_1 = P_2V_2$ can be applied

$48 \times 10^{-3} \times \frac{4}{3}\pi(3)^3 = P_2 \times \frac{4}{3}\pi(12)^3$

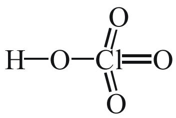
$48 \times 10^{-3} \times 3^3 = P_2 \times (3^3 \times 4^3)$

$\frac{48}{64} \times 10^{-3}\,\text{bar} = P_2$

$\left(\frac{48}{64} \times 1000\right) \times 10^{-6}\,\text{bar} = P_2$

24. NTA Ans. (3.00)

Sol. Structure of perchloric acid ($HClO_4$)



Hence number of Cl = O bonds in perchloric acid is 3.00.

25. NTA Ans. (11.00)

Sol. $Cl^- + 6OH^- \rightarrow ClO_3^- + 3H_2O + 6e^-$

Equivalent of $KClO_3 = \dfrac{i \times t}{96500} \times \left(\dfrac{\eta}{100}\right)$

where (η = 60%)

$\frac{10}{122} \times 6 = \frac{2 \times t}{96500} \times \frac{60}{1000}$

For [$KClO_3 \rightarrow Cl^-$; nf = 6]

$t = \dfrac{10 \times 96500 \times 100 \times 6}{122 \times 2 \times 60}$ sec.

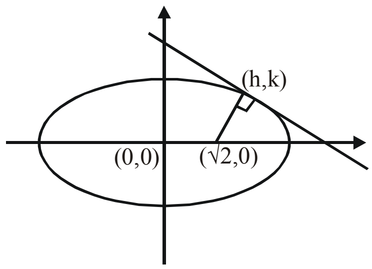
$t = \left(\dfrac{10 \times 96500 \times 100 \times 6}{122 \times 2 \times 60}\right) \times \dfrac{1}{3600}$ hr

$t \simeq 11$hr

MATHEMATICS

1. NTA Ans. (1)

Sol. Let foot of perpendicular is (h,k)



$\frac{x^2}{4} + \frac{y^2}{2} = 1$ (Given)

$a = 2$, $b = \sqrt{2}$, $e = \sqrt{1 - \frac{2}{4}} = \frac{1}{\sqrt{2}}$

$\therefore$ Focus (ae,0) $= \left(\sqrt{2}, 0\right)$

Equation of tangent

$y = mx + \sqrt{a^2m^2 + b^2}$

$y = mx + \sqrt{4m^2 + 2}$

Passes throguh (h,k)

$(k - mh)^2 = 4m^2 + 2$ ...(1)

line perpendicular to tangent will have slope $-\frac{1}{m}$

$y - 0 = -\frac{1}{m}\left(x - \sqrt{2}\right)$

$my = -x + \sqrt{2}$

$(h + mk)^2 = 2$ ...(2)

Add equaiton (1) and (2)

$k^2(1 + m^2) + h^2(1 + m^2) = 4(1 + m^2)$

$h^2 + k^2 = 4$

$x^2 + y^2 = 4$ (Auxilary circle)

$\therefore$ $\left(-1, \sqrt{3}\right)$ lies on the locus.

2. NTA Ans. (2)

Sol.

Family 1	Family 2	Family 3
3	3	4

$$= \underset{\text{Arrangment of 3 Families}}{3!} \quad \underset{\text{Interval Arrangment of families members}}{3! \times 3! \times 4!}$$

so option(2) is correct.

3. NTA Ans. (Bonus)

Sol. $$\lim_{x\to 1} \frac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)\sin(x-1)} \left(\frac{0}{0}\right)$$

Apply L Hopital Rule

$$= \lim_{x\to 1} \frac{2(x-1).(x-1)^2\cos(x-1)^4 - 0}{(x-1).\cos(x-1)+\sin(x-1)} \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 1} \frac{2(x-1)^3.\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\frac{\sin(x-1)}{(x-1)}\right]}$$

$$= \lim_{x\to 1} \frac{2(x-1)^2\cos(x-1)^4}{\left[\cos(x-1)+\frac{\sin(x-1)}{(x-1)}\right]}$$

$$= \lim_{x\to 1} \frac{2(x-1)^2\cos(x-1)^4}{\cos(x-1)+\frac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$= \frac{0}{1+1} = 0$$

4. NTA Ans. (1)

Sol. $$\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{(3^2)^{100}}{8}\right\}$$

$$= \left\{\frac{(1+8)^{100}}{8}\right\}$$

$$= \left\{\frac{1+{}^{100}C_1.8+{}^{100}C_2.8^2+...+{}^{100}C_{100}8^{100}}{8}\right\}$$

$$= \left\{\frac{1+8m}{8}\right\}$$

$$= \frac{1}{8}$$

5. NTA Ans. (4)

Sol. For infinite many solutions

$D = D_1 = D_2 = D_3 = 0$

$$\text{Now } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$1.(2\lambda - 9) - 1.(\lambda - 3) + 1.(3 - 2) = 0$

$\therefore \lambda = 5$

$$\text{Now } D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0$$

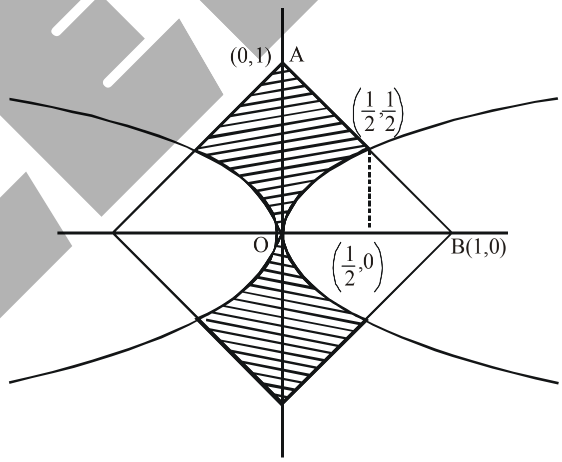
$2(10 - 9) - 1(25 - 3\mu) + 1(15 - 2\mu) = 0$

$\mu = 8$

6. NTA Ans. (4)

Sol. $|x| + |y| \le 1$

$2y^2 \ge |x|$



For point of intersection

$x + y = 1 \Rightarrow x = 1 - y$

$$y^2 = \frac{x}{2} \Rightarrow 2y^2 = x$$

$2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$$y = \frac{1}{2} \text{ or } -1$$

$$\text{Now Area of } \Delta OAB = \frac{1}{2}\times 1\times 1 = \frac{1}{2}$$

$$\text{Area of Region } R_1 = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$\text{Area of Region } R_2 = \frac{1}{\sqrt{2}}\int_0^{\frac{1}{2}} \sqrt{x}\,dx = \frac{1}{6}$$

Now area of shaded region in first quadrant = Area of $\Delta OAB - R_1 - R_2$

$$=\frac{1}{2}-\left(\frac{1}{6}\right)-\left(\frac{1}{8}\right)=\frac{5}{24}$$

So required area $=4\left(\frac{5}{24}\right)=\frac{5}{6}$

so option (4) is correct.

7. NTA Ans. (3)

Sol. Out of 11 consecutive natural numbers either 6 even and 5 odd numbers or 5 even and 6 odd numbers

when 3 numbers are selected at random then total cases = ${}^{11}C_3$

Since these 3 numbers are in A.P. Let no's are a,b,c

2b $\Rightarrow$ even number

$$a + c \Rightarrow \begin{pmatrix} \text{even} + \text{even} \\ \text{odd} + \text{odd} \end{pmatrix}$$

so favourable cases = ${}^6C_2 + {}^5C_2$

= 15 + 10 = 25

P(3 numbers are in A.P. $=\frac{25}{{}^{11}C_3}=\frac{25}{165}=\frac{5}{33}$)

8. NTA Ans. (2)

Sol. $$S.D = \sqrt{\frac{\sum_{i=1}^{n}(x_i-a)}{n}-\left(\frac{\sum_{i=1}^{n}(x_i-a)}{n}\right)^2}$$

$$=\sqrt{\frac{na}{n}-\left(\frac{n}{n}\right)^2}$$

{Given $\sum_{i=1}^{n}(x_i-a)=n \quad \sum_{i=1}^{n}(x_i-a)^2=na$ }

$$=\sqrt{a-1}$$

9. NTA Ans. (1)

Sol. $y^2 = 4(x + 1)$

equation of tangent $y = m(x + 1) + \frac{1}{m}$

$y = mx + m + \frac{1}{m}$

$y^2 = 8(x + 2)$

equation of tangent $y=m'(x+2)+\frac{2}{m'}$

$$y=m'x+2\left(m'+\frac{1}{m'}\right)$$

since lines intersect at right angles

$\therefore$ mm' = –1

Now $y = mx + m + \frac{1}{m}$ ...(1)

$$y=m'x+2\left(m'+\frac{1}{m'}\right)$$

$$y=-\frac{1}{m}x+2\left(-\frac{1}{m}-m\right)$$

$$y=-\frac{1}{m}x-2\left(m+\frac{1}{m}\right) \quad \text{...(2)}$$

From equation (1) and (2)

$$mx+m+\frac{1}{m}=-\frac{1}{m}x-2\left(m+\frac{1}{m}\right)$$

$$\left(m+\frac{1}{m}\right)x+3\left(m+\frac{1}{m}\right)=0$$

$\therefore$ x + 3 = 0

10. NTA Ans. (3)

Sol. Negation of $\phi\vee(\sim p\wedge q)$

$p\vee(\sim p\wedge q) = (p\vee\sim p)\wedge(p\vee q)$

$=(T)\wedge(p\vee q)$

$=(p\vee q)$

now negation of $(p\vee q)$ is

$\sim(p\vee q)=\sim p\wedge\sim q$

11. NTA Ans. (2)

Sol. $f(x + y) = f(x).\ f(y)$

$\sum_{x=1}^{\infty}f(x)=2$ where x,y $\in$ N

$f(1) + f(2) + f(3) + ... \infty = 2$ ...(1) (Given)

Now for $f(2)$ put $x = y = 1$

$f(2) = f(1+1) = f(1).\ f(1) = (f(1))^2$

$f(3) = f(2+1) = f(2).\ f(1) = (f(1))^3$

Now put these values in equation (1)

$f(1) + (f(1))^2 + [f(1)^2 + ...\infty = 2]$

$$\frac{f(1)}{1-f(1)} = 2$$

$$\Rightarrow f(1) = \frac{2}{3}$$

Now $f(2) = \left(\frac{2}{3}\right)^2$

$$f(4) = \left(\frac{2}{3}\right)^4$$

then the value of $\frac{f(4)}{f(2)} = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2} = \frac{4}{9}$

12. NTA Ans. (1)

Sol. $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$

$$\Rightarrow \sqrt{(1+x)^2(1+y^2)} + xy\frac{dy}{dx} = 0$$

$$\Rightarrow \sqrt{1+x^2}\sqrt{1+y^2} = -xy\frac{dy}{dx}$$

$$\Rightarrow \int\frac{ydy}{\sqrt{1+y^2}} = -\int\frac{\sqrt{1+x^2}}{x}dx \quad ...(1)$$

Now put $1 + x^2 = u^2$ and $1 + y^2 = v^2$

$2xdx = 2udu$ and $2ydy = 2vdv$

$\Rightarrow xdx = udu$ and $ydy = vdv$

substitude these values in equation (1)

$$\int\frac{vdv}{v} = -\int\frac{u^2.du}{u^2-1}$$

$$\Rightarrow \int dv = -\int\frac{u^2-1+1}{u^2-1}du$$

$$\Rightarrow v = -\int\left(1+\frac{1}{u^2-1}\right)du$$

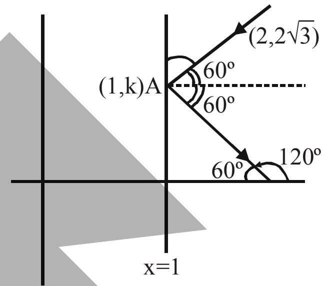
$$\Rightarrow v = -u - \frac{1}{2}\log_e\left|\frac{u-1}{u+1}\right| + c$$

$$\Rightarrow \sqrt{1+y^2} = -\sqrt{1+x^2} + \frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right| + c$$

$$\Rightarrow \sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right| + c$$

13. NTA Ans. (2)

Sol. For point A



$$\tan 60° = \frac{2\sqrt{3}-k}{2-1}$$

$$\sqrt{3} = 2\sqrt{3} - k$$

$$\therefore\ k = \sqrt{3}$$

so point $A(1,\sqrt{3})$

Now slope of line AB is $m_{AB} = \tan 120°$

m $m_{AB} = -\sqrt{3}$

Now equation of line AB is

$$y - \sqrt{3} = -\sqrt{3}(x-1)$$

$$\sqrt{3}x + y = 2\sqrt{3}$$

Now satisfy options

14. NTA Ans. (3)

Sol. $(a^2 + b^2 + c^2)p^2 + 2(ab + bc + cd)p + b^2 + c^2 + d^2 = 0$

$\Rightarrow (a^2p^2 + 2abp + b^2) + (b^2p^2 + 2bcp + c^2) + (c^2p^2 + 2cdp + d^2) = 0$

$\Rightarrow (ab + b)^2 + (bp + c)^2 + (cp + d)^2 = 0$

This is possible only when

$ap + b = 0$ and $bp + c = 0$ and $cp + d = 0$

$$p = -\frac{b}{a} = -\frac{c}{b} = -\frac{d}{c}$$

or $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$\therefore$ a,b,c,d are in G.P.

15. NTA Ans. (1)

Sol. $I_1 = \int_0^1 (1-x^{50})^{100}dx$ and $I_2 = \int_0^1 (1-x^{50})^{101}dx$

and $I_1 = \lambda I_2$

$$I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$$

$$I_2 = \int_0^1 \left(1-x^{50}\right)\left(1-x^{50}\right)^{100} dx$$

$$I_2 = \int_0^1 \left(1-x^{50}\right) dx - \int_0^1 x^{50}.\left(1-x^{50}\right)^{100} dx$$

$$I_2 = I_1 - \int_0^1 \underset{I}{\underbrace{x}}.\underset{II}{\underbrace{x^{49}.\left(1-x^{50}\right)^{100} dx}}$$

Now apply IBP

$$I_2 = I_1 - \left[x\int x^{49}.\left(1-x^{50}\right)^{100} dx - \int \frac{d(x)}{dx}.\int \frac{d(x)}{dx}.\int x^{49}.\left(1-x^{50}\right)^{100} dx\right]$$

Let $(1 - x^{50}) = t$

$-50x^{49}dx = dt$

$$I_2 = I_1 - \left[x.\left(-\frac{1}{50}\right)\frac{\left(1-x^{50}\right)^{101}}{101}\Bigg|_{x=0}^{x=1} - \int_0^1 \left(-\frac{1}{50}\right)\frac{\left(1-x^{50}\right)^{101}}{101}dx\right]$$

$$I_2 = I_1 - 0 - \frac{1}{50}.\frac{1}{101}.I_2 = I_1 - \frac{1}{5050}I_2$$

$$I_2 + \frac{1}{5050}I_2 = I_1 \Rightarrow \frac{5051}{5050}I_2 = I_1$$

$$\therefore\ \alpha = \frac{5050}{5051}$$

$$I_2 = \frac{5050}{5051}I_1$$

$\because I_2 = \alpha.I_1$

16. NTA Ans. (4)

Sol. $\dfrac{f(t_2)-f(t_1)}{t_2-t_1} = 2at+b$

$$\frac{a\left(t_2^2-t_1^2\right)+b\left(t_2-t_1\right)}{t_2-t_1} = 2at+b$$

$\Rightarrow a(t_2 + t_1) + b = 2at + b$

$$\Rightarrow\ t = \frac{t_1+t_2}{2}$$

17. NTA Ans. (4)

Sol. $z = x + iy$

$|z| - ke(z) \le 1$

$$\Rightarrow \sqrt{x^2+y^2} - x \le 1$$

$$\Rightarrow \sqrt{x^2+y^2} \le 1 + x$$

$$\Rightarrow x^2 + y^2 \le 1 + 2x + x^2$$

$$\Rightarrow y^2 \le 2x + 1$$

$$\Rightarrow y^2 \le 2\left(x + \frac{1}{2}\right)$$

18. NTA Ans. (4)

Sol. $x^2 - 64x + 256 = 0$

$\alpha + \beta = 64,\ \alpha\beta = 256$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$$

$$= \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$= \frac{\alpha+\beta}{(\alpha\beta)^{5/8}}$$

$$= \frac{64}{(256)^{5/8}}$$

$= 2$

19. NTA Ans. (4)

Sol. Line of intersection of planes

$x + y + z + 1 = 0$ ...(1)

$2x - y + z + 3 = 0$ ...(2)

eliminate y

$3x + 2z + 4 = 0$

$$x = \frac{-2z-4}{3} \quad ...(3)$$

put in equaiton (1)

$z = -3y + 1$ ...(4)

from (3) and (4)

$$\frac{3x+4}{-2} = -3y + 1 = z$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\frac{1}{3}}{-\frac{1}{3}} = \frac{z-0}{1}$$

now shortest distance between skew lines

$$\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\left(\frac{1}{3}\right)}{-\frac{1}{3}} = \frac{z-0}{1}$$

$$\text{S.D.} = \left|\frac{(\vec{b}-\vec{a}).(\vec{c}\times\vec{d})}{|\vec{c}\times\vec{d}|}\right|$$

where $\vec{a} = (1,-1,0)$

$$\vec{b} = \left(-\frac{4}{3},\frac{1}{3},0\right)$$

$$\vec{c} = (0,-1,1)$$

$$\vec{d} = \left(-\frac{2}{3},-\frac{1}{3},1\right)$$

$$\Rightarrow \text{S.D} = \frac{1}{\sqrt{3}}$$

20. NTA Ans. (1)

Sol. $$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

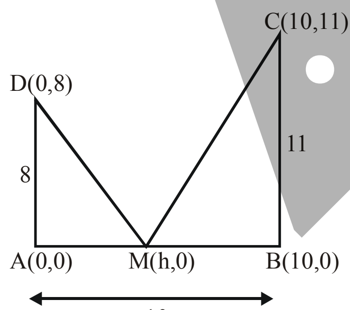
$$= -1(\sin^2 x) - 1(1+\sin 2x+\cos^2 x)$$

$$= -\sin 2x - 2$$

$m = -3,\ M = -1$

21. NTA Ans. (5.00)

Sol.



$(MD)^2 + (MC)^2 = h^2 + 64 + (h - 10)^2 + 121$

$= 2h^2 - 20h + 64 + 100 + 121$

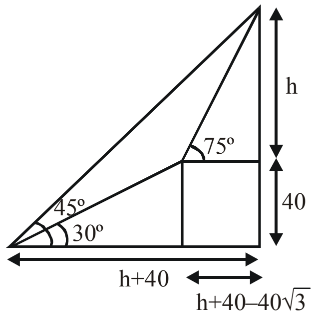
$= 2(h^2 - 10h) + 285$

$= 2(h - 5)^2 + 235$

it is minimum if $h = 5$

22. NTA Ans. (80.00)

Sol.



$$\tan 75^\circ = \frac{h}{h+40-40\sqrt{3}}$$

$$\frac{2+\sqrt{3}}{1} = \frac{h}{h+40-40\sqrt{3}}$$

$$\Rightarrow 2h+80-80\sqrt{3}+\sqrt{3}h+40\sqrt{3}-120 = h$$

$$\Rightarrow h(\sqrt{3}+1) = 40+40\sqrt{3}$$

$$\Rightarrow h = 40$$

$\therefore$ Height of hill = 40 + 40 = 80m

23. NTA Ans. (28.00)

Sol. $2^m - 2^n = 112$

$m = 7,\ n = 4$

$(2^7 - 2^4 = 112)$

$m \times n = 7 \times 4 = 28$

24. NTA Ans. (4.00)

Sol. $\sqrt{3}|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}|$

$$= \sqrt{3}\left(\sqrt{2+2\cos\theta}\right) + \sqrt{2-2\cos\theta}$$

$$= \sqrt{6}\left(\sqrt{1+\cos\theta}\right) + \sqrt{2}\left(\sqrt{1-\cos\theta}\right)$$

$$= 2\sqrt{3}\left|\cos\frac{\theta}{2}\right| + 2\left|\sin\frac{\theta}{2}\right|$$

$$\le \sqrt{(2\sqrt{3})^2 + (2)^2} = 4$$

25. NTA Ans. (5.00)

Sol. $f(x) = x^5.\sin\frac{1}{x} + 5x^2$ if $x < 0$

$f(x) = 0$ if $x = 0$

$f(x) = x^5.\cos\frac{1}{x} + \lambda x^2$ if $x > 0$

LHD of $f'(x)$ at $x = 0$ is 10

RHD of $f'(x)$ at $x = 0$ is 2λ

if $f''(0)$ exists then

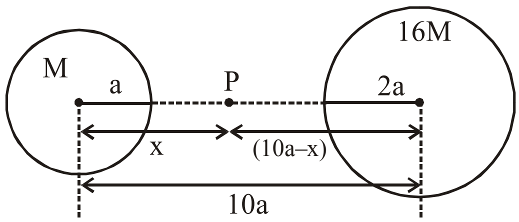
$2\lambda = 10 \Rightarrow \lambda = 5$

SET # 10

PHYSICS

1. NTA Ans. (2)

Sol.



$$\frac{GM}{x^2}=\frac{G(16M)}{(10a-x)^2}$$

$$\frac{1}{x}=\frac{4}{(10a-x)} \qquad \Rightarrow 4x = 10a - x$$

$x = 2a$ ....(i)

COME

$$-\frac{GMm}{8a}-\frac{G(16M)m}{2a}+KE$$

$$= -\frac{GMm}{2a}-\frac{G(16M)m}{8a}$$

$$KE = GMm\left[\frac{1}{8a}+\frac{16}{2a}-\frac{1}{2a}-\frac{16}{8a}\right]$$

$$KE = GMm\left[\frac{1+64-4-16}{8a}\right]$$

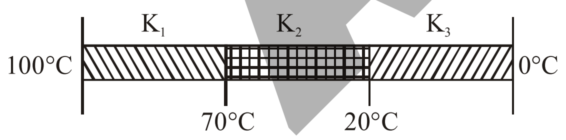
$$\frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right]$$

$$V=\sqrt{\frac{90GM}{8a}}$$

$$V=\frac{3}{2}\sqrt{\frac{5GM}{a}}$$

2. NTA Ans. (1)

Sol.



Rods are identical have same length (ℓ) and area of cross-section (A)

Combination are in series, so heat current is same for all Rods

$$\left(\frac{\Delta Q}{\Delta t}\right)_{AB}=\left(\frac{\Delta Q}{\Delta t}\right)_{BC}=\left(\frac{\Delta Q}{\Delta t}\right)_{CD} = \text{Heat current}$$

$$\frac{(100-70)K_1A}{\ell}=\frac{(70-20)K_2A}{\ell}=\frac{(20-0)K_3A}{\ell}$$

$30K_1 = 50K_2 = 20K_3$

$3K_1 = 2K_3$

$$\frac{K_1}{K_3}=\frac{2}{3}=2:3$$

$5K_2 = 2K_3$

$$\frac{K_2}{K_3}=\frac{2}{5}=2:5$$

3. NTA Ans. (2)

Sol. $\vec{E}$ and $\vec{B}$ are perpendicular for EM wave

$E_0 = CB_0$

$= 3 \times 10^8 \times 1.2 \times 10^{-7} = 36$

Having same phase

Propagation is along –x-axis, $\vec{B}$ is along z-axis hence $\vec{E}$ must be along y-axis.

So, option (2) is correct

4. NTA Ans. (4)

Sol.

$R_1 = R_2 = R$

Power (P)

Refractive index is assume (μ_ℓ)

$$P=\frac{1}{f}=(\mu_\ell-1)\left(\frac{2}{R}\right) \qquad ....(i)$$

$$P'=\frac{1}{f'}=(\mu_\ell-1)\left(\frac{1}{R'}\right) \qquad ....(ii)$$

$$P'=\frac{3}{2}P$$

$$(\mu_\ell-1)\left(\frac{1}{R'}\right)=\mu\frac{3}{2}(\mu_\ell-1)\left(\frac{2}{R}\right)$$

$$\therefore \; R'=\frac{R}{3}$$

5. NTA Ans. (1)

Sol. Conceptual

Option (1) is correct

Ammeter :- In series connection, the same current flows through all the components. It aims at measuring the curent flowing through the circuit and hence, it is connected in series.

Voltmeter :- A voltmeter measures voltage change between two points in a circuit, So we have to place the voltmeter in parallal with the circuit component.

6. NTA Ans. (3)

Sol. $\frac{dv_x}{dt} = \frac{k}{m}v_y$

$\frac{dv_y}{dt} = \frac{k}{m}v_x$

$\frac{dv_y}{dv_x} = \frac{v_x}{v_y} \Rightarrow \int v_y dv_y = \int v_x dv_x$

$v_y^2 = v_x^2 + C$

$v_y^2 - v_x^2 = \text{constant}$

Option (3)

$\vec{v} \times \vec{a} = (v_x\hat{i} + v_y\hat{j}) \times \frac{k}{m}(v_y\hat{i} + v_x\hat{j})$

$= (v_x^2\hat{k} - v_y^2\hat{k})\frac{k}{m}$

$= (v_x^2 - v_y^2)\frac{k}{m}\hat{k}$

= Constant

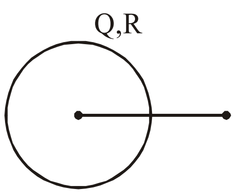
7. NTA Ans. (1)

Sol. Inside the shell

E = 0

hence F = 0

Oustside the sheell



$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

hence $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ for r > R

8. NTA Ans. (1)

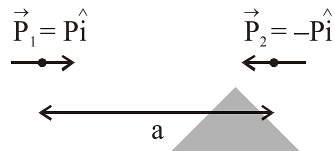
Sol. Only in case-I, $M_{LHS} > M_{RHS}$ i.e.

total mass on reactant side is greater then that on the product side. Hence it will only be allowed.

9. NTA Ans. (3)

Sol. Using energy conservation:

$KE_i + PE_i = KE_f + PE_f$



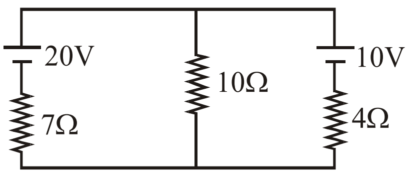
$O + \frac{2KP}{a^3} \times P = \frac{1}{2}mv^2 \times 2 + 0$

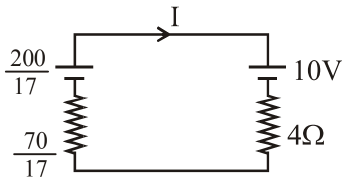
$V = \sqrt{\frac{2P^2}{4\pi\varepsilon_0 a^3 m}} = \frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 am}}$

10. NTA Ans. (3)

Sol. $E_{eq} = \frac{20 \times 10}{17} = \frac{200}{17}$

and $R_{eq} = \frac{7 \times 10}{17} = \frac{70}{17}$



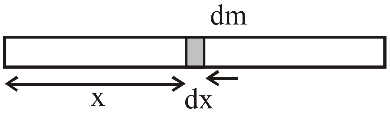


$\therefore \quad I = \frac{\frac{20}{17} - 10}{4 + \frac{70}{17}} = 0.21 \text{ A}$

from +ve to –ve terminal

11. NTA Ans. (4)

Sol.



$$I = \int r^2 dm = \int x^2 \lambda dx$$

$$I = \int_0^L x^2 \lambda_0 \left(1 + \frac{x}{L}\right) dx$$

$$I = \lambda_0 \int_0^L \left(x^2 + \frac{x^3}{L}\right) dx$$

$$I = \lambda \left[\frac{L^3}{3} + \frac{L^3}{4}\right]$$

$$I = \frac{7L^3\lambda_0}{12} \qquad \text{....(i)}$$

$$M = \int_0^L \lambda dx = \int_0^L \lambda_0 \left(1 + \frac{x}{L}\right) dx$$

$$M = \lambda_0 \left(L + \frac{L}{2}\right) = \lambda_0 \frac{3L}{2}$$

$$\frac{2}{3} M = (\lambda_0 L) \qquad \text{....(ii)}$$

From (i) & (ii)

$$I = \frac{7}{12}\left(\frac{2}{3}M\right)L^2 = \frac{7ML^2}{18}$$

12. NTA Ans. (3)

Sol. Use significant figures. Answer must be upto three significant figures.

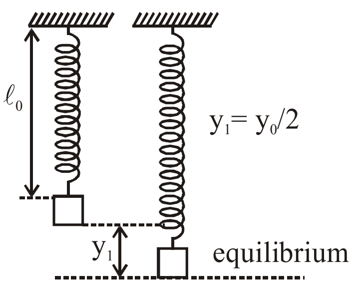
13. NTA Ans. (2)

Sol. $y = y_0 \sin^2 \omega t$

$$y = \frac{y_0}{2}(1 - \cos 2\omega t)$$

$$y - \frac{y_0}{2} = -\frac{y_0}{2}\cos 2\omega t$$

Amplitude : $\frac{y_0}{2}$



$$\frac{y_0}{2} = \frac{mg}{K}$$

$$2\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{2g}{y_0}}$$

$$\omega = \sqrt{\frac{g}{2y_0}}$$

14. NTA Ans. (3)

Sol. $v_{avg} \propto \sqrt{T}$

t_0 : mean time

λ : mean free path

$$t_0 = \frac{\lambda}{v_{avg}} \propto \frac{1}{\sqrt{T}}$$

15. NTA Ans. (2)

Sol. Applying Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P + \frac{1}{2}\rho v^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$$

$$\frac{2P}{2\rho} + \frac{1}{2}\frac{\rho v^2}{\rho} \times 2 = V^2$$

$$\sqrt{\frac{P}{\rho} + v^2} = V$$

16. NTA Ans. (2)

Sol. $v_{rms} = \sqrt{\frac{3KT}{m}}$

m$\rightarrow$mass of one molecule (in kg) $= \frac{\text{molar mass}}{\text{NA}}$

de-Broglie wavelenth,

$$\lambda = \frac{h}{mv}$$

given, $v = v_{rms}$

$$\lambda = \frac{h}{m\sqrt{\frac{3KT}{m}}} = \frac{h}{\sqrt{3KTm}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.38 \times 10^{-23} \times 400 \times \left(\frac{28 \times 10^{-3}}{6.023 \times 10^{-23}}\right)}}$$

$$\lambda = \frac{6.63 \times 10^{-11}}{2.77} = 2.39 \times 10^{-11} \text{m}$$

$\lambda = 0.24$ Å

17. NTA Ans. (4)

Sol. $\vec{v}_{01} = (\sqrt{3}\hat{i} + \hat{j})\ \text{m/s}$

$\vec{v}_{02} = \vec{0}$

$m_1 = 2m_2$

After collision, $\vec{v}_1 = (\hat{i} + \sqrt{3}\hat{j})\ \text{m/s}$

$\vec{v}_2 = ?$

Applying conservation of linear momentum,

$$m_1\vec{v}_{01} + m_2\vec{v}_{02} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$2m_2(\sqrt{3}\hat{i} + \hat{j}) + 0 = 2m_2(\hat{i} + \sqrt{3}\hat{j}) + m_2\vec{v}_2$$

$$\vec{v}_2 = 2(\sqrt{3}\hat{i} + \hat{j}) - 2(\hat{i} + \sqrt{3}\hat{j})$$

$$= 2(\sqrt{3}\hat{i} - \hat{j}) + 2(\hat{i} - \sqrt{3}\hat{j})$$

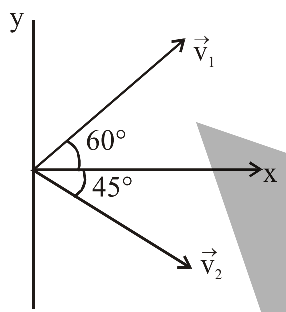
$$\vec{v}_2 = 2(\sqrt{3} - 1)(\hat{i} - \hat{j})$$

for angle between $\vec{v}_1$ & $\vec{v}_2$,

$$\cos\theta = \frac{\vec{v}_1 . \vec{v}_2}{v_1 v_2} = \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{2 \times 2\sqrt{2}(\sqrt{3}-1)}$$

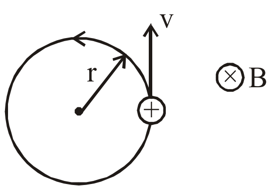
$$\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}} \Rightarrow \theta = 105°$$

or



18. NTA Ans. (4)

Sol.



Magnetic moment

M = iA

$$M = \left(\frac{q}{T}\right) \times \pi r^2 = \frac{q\pi r^2}{\left(\frac{2\pi r}{v}\right)} = \frac{qvr}{2}$$

$$M = \frac{qv}{2} \times \frac{vm}{qB}$$

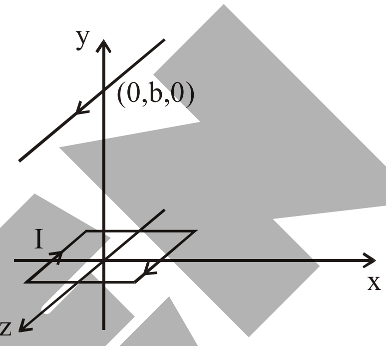
$$M = \frac{mv^2}{2B}$$

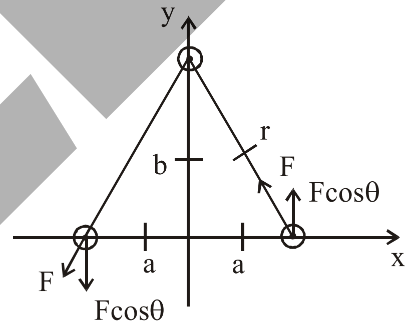
As we can see from the figure, direction of magnetic moment (M) is opposite to magnetic field.

$$\vec{M} = -\frac{mv^2}{2B}\hat{B} = -\frac{mv^2}{2B^2}\vec{B}$$

19. NTA Ans. (1)

Sol.





$r = \sqrt{b^2 + a^2}$

$$F = BI2a = \frac{\mu_0 I}{2\pi r} I \times 2a$$

$$F = \frac{\mu_0 I^2 a}{\pi\sqrt{b^2 + a^2}}$$

$$\tau = F\cos\theta \times 2a$$

$$= \frac{\mu_0 I^2 a}{\pi\sqrt{b^2 + a^2}} \times \frac{b}{\sqrt{b^2 + a^2}} \times 2a$$

$$\tau = \frac{2\mu_0 I^2 a^2 b}{\pi(a^2 + b^2)}$$

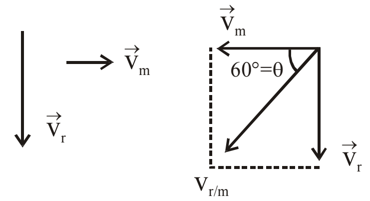
If b >> a then $\tau = \dfrac{2\mu_0 I^2 a^2}{\pi b}$

But among the given options (1) is most appropriate

20. NTA Ans. (4)

Sol. Rain is falling vertically downwards.

$$\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$$

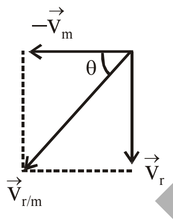


$$\tan 60° = \frac{v_r}{v_m} = \sqrt{3}$$

$$v_r = v_m\sqrt{3} = v\sqrt{3}$$

Now, $v_m = (1 + B)v$

and $\theta = 45°$



$$\tan 45 = \frac{v_r}{v_m} = 1$$

$v_r = v_m$

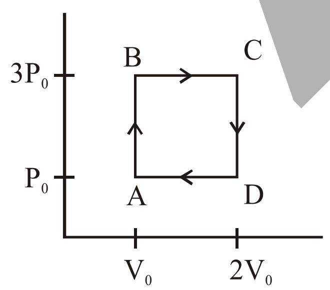
$$v\sqrt{3} = (1+\beta)v$$

$$\sqrt{3} = 1+\beta$$

$$\Rightarrow \beta = \sqrt{3} - 1 = 0.73$$

21. NTA Ans. (19.00 to 19.10)

Sol.



$W_{ABCDA} = 2P_0V_0$

$Q_{in} = Q_{AB} + Q_{BC}$

$Q_{AB} = nC(T_B - T_A)$

$$= \frac{n3R}{2}(T_B - T_A)$$

$$= \frac{3}{2}(P_BV_B - P_AV_A)$$

$$= \frac{3}{2}(3P_BV_0 = P_0V_0) = 3P_0V_0$$

$Q_{BC} = nC_P(T_C - T_B)$

$$= \frac{n5R}{2}(T_C - T_B)$$

$$= \frac{5}{2}(P_CV_C - P_BV_B)$$

$$= \frac{5}{2}(6P_0V_0 - 3P_0V_0) = \frac{15}{2}P_0V_0$$

$$\eta = \frac{W}{Q_{in}} \times 100 = \frac{2P_0V_0}{3P_0V_0 + \frac{15}{2}P_0V_0} \times 100$$

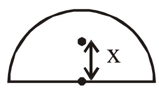
$$\eta = \frac{400}{21} = 19.04 \approx 19$$

$\eta = 19$

22. NTA Ans. (3.00)

Sol. $x = \frac{3R}{8} = 3cm$

$x = 3$



23. NTA Ans. (150.00)

Sol. $\Delta I_B = (30 - 20) = 10\mu A$

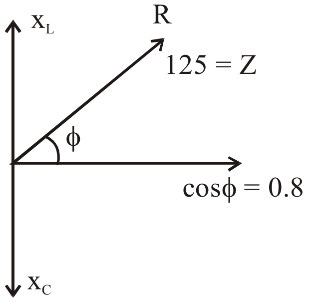
$\Delta I_C = (4.5 - 3)\ mA = 1.5mA$

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{1.5mA}{10\mu A} = 150$$

$\beta_{ac} = 150$

24. NTA Ans. (400.00)

Sol.



$$P = \frac{E_{rms}^2}{Z}\cos\phi$$

$$400 = \frac{(250)^2 \times 0.8}{Z}$$

$Z = 25 \times 5 = 125$

$X_L = 125 \sin\phi = 125 \times 0.6 = 75$

25. NTA Ans. (9.00)

Sol. $I_{max} = k$

$I_1 = I_2 = K/4$

$\Delta x = \lambda/6 \quad \Rightarrow \Delta\phi = \pi/3$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$I = \frac{K}{4} + \frac{K}{4} + 2 \times \frac{K}{4}\frac{1}{2}$$

$$= \frac{K}{2} + \frac{K}{4} = \frac{3K}{4} = \frac{9K}{12}$$

n = 9

CHEMISTRY

1. NTA Ans. (2)

Sol. $N_2 + 3H_2 \rightleftharpoons 2NH_3 \rightarrow K_C = 64$

$$2NH_3 \rightleftharpoons N_2 + 3H_2 \rightarrow K_C = \frac{1}{64}$$

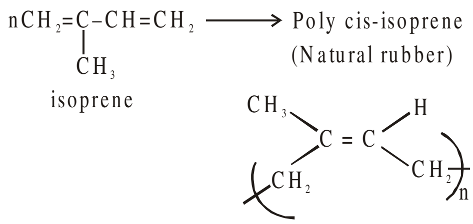
$$NH_3 \rightleftharpoons \frac{1}{2}N_2 + \frac{3}{2}H_2 \rightarrow K_C = \left(\frac{1}{64}\right)^{1/2} = \frac{1}{8}$$

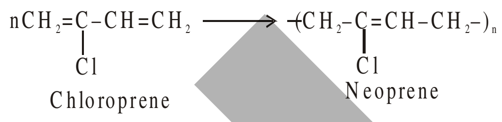
2. NTA Ans. (2)

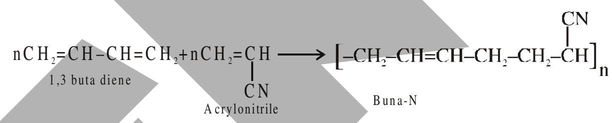
Sol. Impure zinc is refined by distillation method.

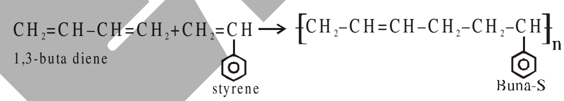
3. NTA Ans. (2)

Sol.

(a) 

(b) 

(c) 

(d) 

4. NTA Ans. (1)

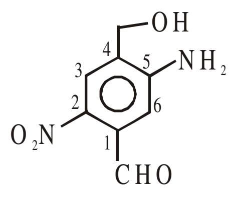
Sol. Alloys of lanthanides with Fe are called Misch metal, which consists of a lanthanoid metal (~95%) and iron (~5%) and traces of S, C, Ca and Al.

5. NTA Ans. (2)

Sol. $\underset{(X)}{Na_2SO_3} \xrightarrow{\text{dil. } H_2SO_4} \underset{(Y)}{SO_2} \xrightarrow{NaOH} \underset{(X)}{Na_2SO_3} \xrightarrow{SO_2 + H_2O} \underset{(Z)}{NaHSO_3}$

6. NTA Ans. (4)

Sol.



5-amino-4-hydroxymethyl-2-nitrobenzaldehyde

7. NTA Ans. (1)

Sol. High purity (>99.95%) dihydrogen is obtained by electrolysing warm aqueous barium hydroxide solution between nickel electrodes.

8. NTA Ans. (1)

Sol. **Test** **Correct reagent**

(i) Lucas test $\longrightarrow$ conc. HCl + $ZnCl_2$

(ii) Dumas method $\longrightarrow$ CuO / CO_2

(iii) Kjeldahl's method $\longrightarrow$ H_2SO_4

(iv) Hinsberg Test $\longrightarrow$ $C_6H_5SO_2Cl$ + aq. KOH

9. NTA Ans. (4)

Sol. $2NO + N_2O_4 \xrightarrow{250K} \underset{\text{blue solid}}{2N_2O_3}$

10. NTA Ans. (4)

Sol. $\Delta G = -n\,F\,E_{cell}$

ΔG is negative, if E_{cell} is positive

Anode : $Cu(s) \longrightarrow Cu^{+2}(C_1) + 2e^- : E^\circ$

Cathode : $Cu^{+2}(C_2) + 2e^- \longrightarrow Cu(S) : -E^\circ$

Cell reaction : $Cu^{+2}(C_2) \longrightarrow Cu^{+2}(C_1)\ E^\circ_{cell} = 0$

$$E_{cell} = E^\circ_{cell} - \frac{2.303RT}{nF}\log Q$$

$$E_{cell} = 0 - \frac{2.303RT}{nF}\log\left(\frac{C_1}{C_2}\right)$$

$$E_{cell} > 0:\ \text{if}\ \frac{C_1}{C_2} < 1 \Rightarrow C_1 < C_2$$

11. NTA Ans. (1)

Sol. O^{-2} ions form ccp. $\underset{(-8\ \text{charge})}{O_4}$

M_1 = 50% of O.V. $\Rightarrow \frac{50}{100} \times 4 = 2 : (M_1)_2$

M_2 = 12.5% of T.V. $\Rightarrow \frac{12.5}{100} \times 8 = 1 : (M_2)_1$

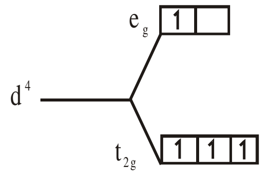
So formula is : $(M_1)_2\ (M_2)_1O_4$

This must be neutral. Both metals must have +8 charge in total.

From given options : $\left\{ \begin{array}{r} \text{O.N. of } M_1 = +2 \\ M_2 = +4 \end{array} \right\}$

12. NTA Ans. (3)

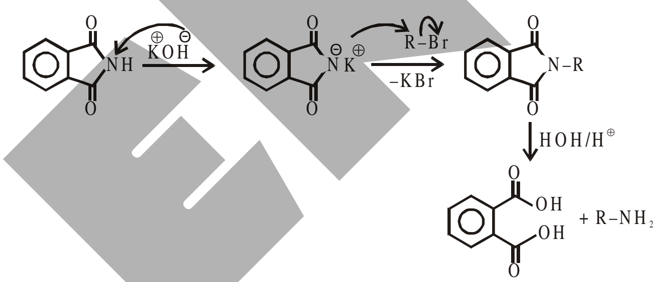
Sol.

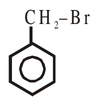


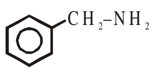
back pairing is not possible because pairing energy > Δ_O.

13. NTA Ans. (1)

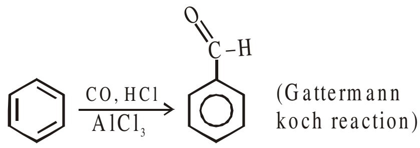
Sol. Gabriel phthalimide synthesis is used for preparation of 1° Aliphatic amine

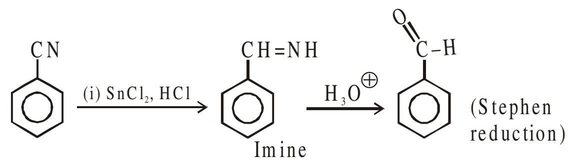


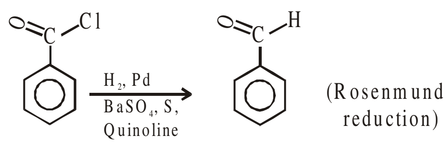
Here R–Br = 

R–NH_2 = 

14. NTA Ans. (3)

Sol. (i) 

(ii) 

(iii) 

15. NTA Ans. (4)

	^{35}Cl	^{37}Cl	Av. molar mass = 35.5
Sol. let mole ratio	x	: 1	

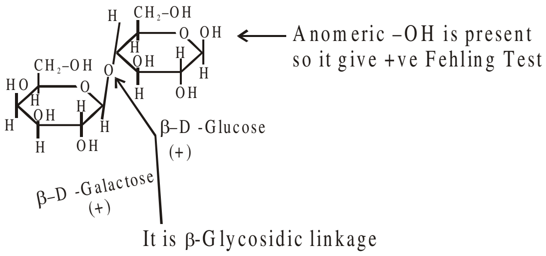
Av. molar mass = $\dfrac{n_1M_1 + n_2M_2}{(n_1 + n_2)}$

$$35.5 = \frac{x \times 35 + 1 \times 37}{x + 1}$$

x = 3

16. NTA Ans. (1)

Sol.



Structure of Lactose

structure of lactose

17. NTA Ans. (1)

Sol. Relative lowering of V.P. = $\dfrac{\Delta P}{P^0} = x_{solute}$

$$\left(\frac{\Delta P}{P^0}\right)_A = \frac{\frac{10}{100}}{\frac{10}{100} + \frac{180}{18}} : \left(\frac{\Delta P}{P^0}\right)_B = \frac{\frac{10}{200}}{\frac{10}{200} + \frac{180}{18}}$$

$$\left(\frac{\Delta P}{P^0}\right)_C = \frac{\frac{10}{10,000}}{\frac{10}{10,000} + \frac{180}{18}} : \left(\frac{\Delta P}{P^0}\right)_A > \left(\frac{\Delta P}{P^0}\right)_B > \left(\frac{\Delta P}{P^0}\right)_C$$

18. NTA Ans. (2)

19. NTA Ans. (4)

Sol. (I) $Ca(OH)_2$ is used in white wash

(II) NaCl is used in preparation of washing soda

$2NH_3 + H_2O + CO_2 \longrightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + CO_2 \longrightarrow 2NH_4HCO_3$

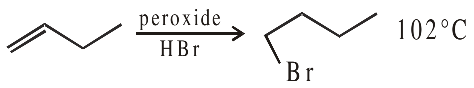
$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + NaHCO_3(s)$

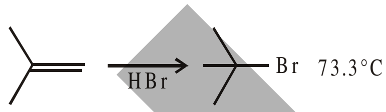
$2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + CO_2 + H_2O$

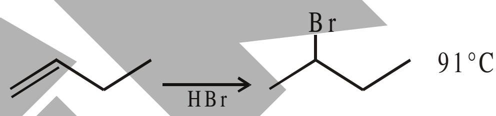
(III) $CaSO_4.\frac{1}{2}H_2O$ (Plaster of Paris) is used for making casts of statues

(IV) $CaCO_3$ is used as an antacid

20. NTA Ans. (2)

Sol. (a) 

(b) 

(c) 

$$B.P. \propto \frac{1}{\text{Branching}} \qquad \therefore\ a > c > b \text{ (order of B.P.)}$$

21. NTA Ans. (48.00)

Sol. $\dfrac{x}{m} = KP^{1/n}$

$$\log\left(\frac{x}{m}\right) = \frac{1}{n}\log P + \log K$$

$$\text{slope} = \frac{1}{n} = 2$$

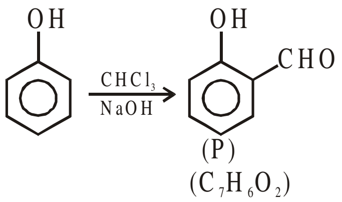
intercept = log K = 0.4771

K = 3

mass of gas adsorbed per gm of adsorbent = $\dfrac{x}{m}$

$$\frac{x}{m} = 3 \times (0.04)^2 = 48 \times 10^{-4}$$

22. NTA Ans. (69.00)

Sol. 

(Reimer Tiemann reaction)

Molecular weight of $C_7H_6O_2$ = 122

$$\%C = \frac{12\times7\times100}{122} = 68.85 \approx 69$$

23. NTA Ans. (2.00)

Sol. $AB_2(s) \rightleftharpoons \underset{S}{A^{+2}_{(aq.)}} + \underset{2S}{2B^{-}_{(aq.)}} : K_{sp}$

$K_{SP} = S^1 \times (2s)^2 = 4s^3$

$3.2 \times 10^{-11} = 4 \times S^3$

$S = 2 \times 10^{-4}$ M/L

24. NTA Ans. (99.90 to 100.10)

Sol. $\ell n\left(\frac{K_{T_2}}{K_{T_1}}\right) = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

$T_1 = 303$ K ; $T_2 = 313$ K

$\frac{K_{T_2}}{K_{T_1}} = 3.555$

$\ell n(3.555) = \frac{E_a}{8.314}\left[\frac{1}{303} - \frac{1}{313}\right]$

$E_a = 99980.715$

$E_a = 99.98 \ \frac{kJ}{mole}$

25. NTA Ans. (101.00)

Sol. Unnilunium $\Rightarrow$ 101

MATHEMATICS

1. NTA Ans. (4)

Sol. $f(x) = (1 - \cos^2 x)(\lambda + \sin x)$

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$f(x) = \lambda\sin^2 x + \sin^3 x$

$f'(x) = 2\lambda\sin x\cos x + 3\sin^2 x\cos x$

$f'(x) = \sin x\cos x(2\lambda + 3\sin x)$

$\sin x = 0, \ \frac{-2\lambda}{3}$, $(\lambda \neq 0)$

for exactly one maxima & minima

$\frac{-2\lambda}{3} \in (-1, 1) \Rightarrow \lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right)$

$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

2. NTA Ans. (1)

Sol. $f(0) = f(1) = f'(0) = 0$

Apply Rolles theorem on $y = f(x)$ in $x \in [0, 1]$

$f(0) = f(1) = 0$

$\Rightarrow f'(\alpha) = 0$ where $\alpha \in (0, 1)$

Now apply Rolles theorem on $y = f'(x)$

in $x \in [0, \alpha]$

$f'(0) = f'(\alpha) = 0$ and $f'(x)$ is continuous and differentiable

$\Rightarrow f''(\beta) = 0$ for some , $\beta \in (0, \alpha) \in (0, 1)$

$\Rightarrow f''(x) = 0$ for some $x \in (0, 1)$

3. NTA Ans. (3)

Sol. $f(x) = x\log_e x$

$$f'(x)\Big|_{(c,f(c))} = \frac{e-0}{e-1}$$

$f'(x) = 1 + \log_e x$

$$f'(x)\Big|_{(c,f(c))} = 1 + \log_e c = \frac{e}{e-1}$$

$$\log_e c = \frac{e-(e-1)}{e-1} = \frac{1}{e-1} \Rightarrow c = e^{\frac{1}{e-1}}$$

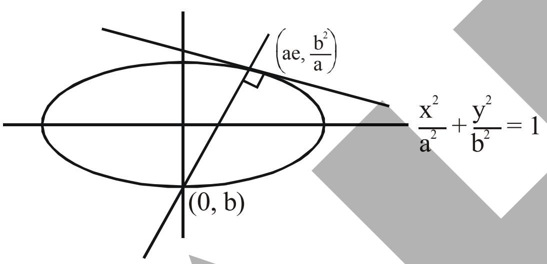
4. NTA Ans. (2)

Sol. Contrapositive of $(p \to q)$ is $\sim q \to \sim p$

For an integer n, if n is even then $(n^3 - 1)$ is odd

5. NTA Ans. (4)

Sol.



$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2e^2$$

$$\frac{a^2x}{ae} - \frac{b^2y}{b^2}.a = a^2e^2$$

$$\frac{ax}{e} - ay = a^2e^2 \Rightarrow \frac{x}{e} - y = ae^2$$

passes through (0, b)

$-b = ae^2 \Rightarrow b^2 = a^2e^4$

$a^2(1 - e^2) = a^2e^4 \Rightarrow e^4 + e^2 = 1$

6. NTA Ans. (2)

Sol. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$A \equiv (a, 0, 0)$, $B \equiv (0, b, 0)$, $C \equiv (0, 0, c)$

Centroid $\equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, 1, 2)$

$a = 3$, $b = 3$, $c = 6$

Plane : $\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$2x + 2y + z = 6$

line $\perp$ to the plane (DR of line = $2\hat{i} + 2\hat{j} + \hat{k}$)

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$$

7. NTA Ans. (3)

Sol. α and β are the roots of the equation $4x^2 + 2x - 1 = 0$

$4\alpha^2 + 2\alpha = 1 \Rightarrow \frac{1}{2} = 2\alpha^2 + \alpha$ ...(1)

$$\beta = \frac{-1}{2} - \alpha$$

using equation (1)

$\beta = -(2\alpha^2 + \alpha) - \alpha$

$\beta = -2\alpha^2 - 2\alpha$

$\beta = -2\alpha(\alpha + 1)$

8. NTA Ans. (3)

Sol. $z = x + iy$

$z^2 = i|z|^2$

$(x + iy)^2 = i(x^2 + y^2)$

$(x^2 - y^2) - i(x^2 + y^2 - 2xy) = 0$

$(x - y)(x + y) - i(x - y)^2 = 0$

$(x - y)((x + y) - i(x - y)) = 0$

$\Rightarrow x = y$

z lies on $y = x$

9. NTA Ans. (2)

Sol. $a_1, a_2, ..., a_n \rightarrow (CD = d)$

$b_1, b_2, ..., b_m \rightarrow (CD = d + 2)$

$a_{40} = a + 39d = -159$ ...(1)

$a_{100} = a + 99d = -399$ ...(2)

Subtract : $60d = -240 \Rightarrow d = -4$

using equation (1)

$a + 39(-4) = -159$

$a = 156 - 159 = -3$

$a_{70} = a + 69d = -3 + 69(-4) = -279 = b_{100}$

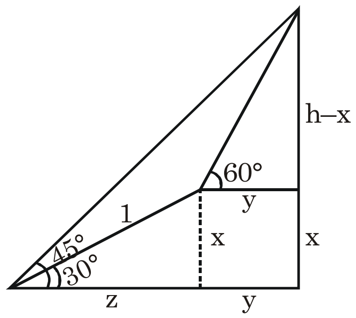
$b_{100} = -279$

$b_1 + 99(d + 2) = -279$

$b_1 - 198 = -279 \Rightarrow b_1 = -81$

10. NTA Ans. (1)

Sol.



$\sin 30° = x \Rightarrow x = \frac{1}{2}$

$\cos 30° = z \Rightarrow z = \frac{\sqrt{3}}{2}$

$\tan 45° = \frac{h}{y+z} \Rightarrow h = y + z$

$\tan 60° = \frac{h-x}{y} \Rightarrow \tan 60° = \frac{h-x}{h-z}$

$\sqrt{3}(h-z) = h - x$

$(\sqrt{3}-1)h = \sqrt{3}z - x$

$\Rightarrow (\sqrt{3}-1)h = \frac{3}{2} - \frac{1}{2}$

$\Rightarrow (\sqrt{3}-1)h = 1$

$h = \frac{1}{\sqrt{3}-1}$

11. NTA Ans. (2)

Sol. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$B = A + A^4$

$$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\begin{bmatrix}\cos4\theta & \sin4\theta\\ -\sin4\theta & \cos4\theta\end{bmatrix}$$

$$B=\begin{bmatrix}(\cos\theta+\cos4\theta) & (\sin\theta+\sin4\theta)\\ -(\sin\theta+\sin4\theta) & (\cos\theta+\cos4\theta)\end{bmatrix}$$

$|B| = (\cos\theta + \cos4\theta)^2 + (\sin\theta + \sin4\theta)^2$

$|B| = 2 + 2\cos3\theta,$ when $\theta=\frac{\pi}{5}$

$$|B|=2+2\cos\frac{3\pi}{5}=2(1-\sin18)$$

$$|B|=2\left(1-\frac{\sqrt{5}-1}{4}\right)=2\left(\frac{5-\sqrt{5}}{4}\right)=\frac{5-\sqrt{5}}{2}$$

12. NTA Ans. (2)

Sol. $f(x)=\frac{a-x}{a+x}$

$x \in R - \{-a\} \to R$

$$f(f(x))=\frac{a-f(x)}{a+f(x)}=\frac{a-\left(\frac{a-x}{a+x}\right)}{a+\left(\frac{a-x}{a+x}\right)}$$

$$f(f(x))=\frac{(a^2-a)+x(a+1)}{(a^2+a)+x(a-1)}=x$$

$\Rightarrow (a^2 - a) + x(a + 1) = (a^2 + a)x + x^2(a - 1)$

$\Rightarrow a(a - 1) + x(1 - a^2) - x^2(a - 1) = 0$

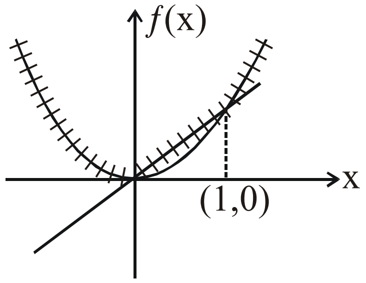
$\Rightarrow a = 1$

$f(x)=\frac{1-x}{1+x}$,

$$f\left(\frac{-1}{2}\right)=\frac{1+\frac{1}{2}}{1-\frac{1}{2}}=3$$

13. NTA Ans. (1)

Sol. $f(x) = \max(x, x^2)$

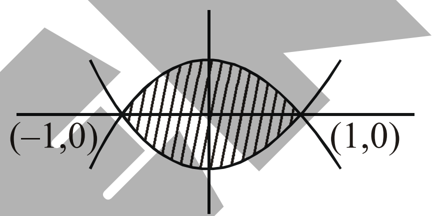


Non-differentiable at x = 0, 1

S = {0, 1}

14. NTA Ans. (2)

Sol. $y = x^2 - 1$ and $y = 1 - x^2$



$$A=\int_{-1}^{1}\left((1-x^2)-(x^2-1)\right)dx$$

$$A=\int_{-1}^{1}(2-2x^2)dx=4\int_{0}^{1}(1-x^2)dx$$

$$A=4\left(x-\frac{x^3}{3}\right)_0^1=4\left(\frac{2}{3}\right)=\frac{8}{3}$$

15. NTA Ans. (3)

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.2$

$P(A\cup B\cup C)=\Sigma P(A)-\Sigma P(A\cap B)+P(A\cap B\cap C)$

$\alpha = 1.5 - (0.2 + 0.3 + \beta) + 0.2$

$\alpha = 1.2 - \beta \in [0.85, 0.95]$

(where $\alpha \in [0.85, 0.95]$)

$\beta \in [0.25, 0.35]$

16. NTA Ans. (3)

Sol. $\left(\sqrt{x}-\dfrac{k}{x^2}\right)^{10}$

$$T_{r+1} = {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r$$

$$T_{r+1} = {}^{10}C_r.x^{\frac{10-r}{2}}.(-k)^r.x^{-2r}$$

$$T_{r+1} = {}^{10}C_r x^{\frac{10-5r}{2}}(-k)^r$$

Constant term : $\dfrac{10-5r}{2} = 0 \Rightarrow r = 2$

$T_3 = {}^{10}C_2.(-k)^2 = 405$

$k^2 = \dfrac{405}{45} = 9$

$k = \pm 3 \Rightarrow |k| = 3$

17. NTA Ans. (4)

Sol. $\displaystyle\int_1^2 e^x.x^x\left(2+\log_e x\right)dx$

$$\int_1^2 e^x\left(2x^x + x^x\log_e x\right)dx$$

$$\int_1^2 e^x\left(\underbrace{x^x}_{f(x)} + \underbrace{x^x\left(1+\log_e x\right)}_{f'(x)}\right)dx$$

$$\left(e^x.x^x\right)_1^2 = 4e^2 - e$$

18. NTA Ans. (3)

Sol. L : $\dfrac{x}{3}+\dfrac{y}{1}=1 \Rightarrow x + 3y - 3 = 0$

Image of point $(-1, -4)$

$$\frac{x+1}{1} = \frac{y+4}{3} = -2\left(\frac{-1-12-3}{10}\right)$$

$$\frac{x+1}{1} = \frac{y+4}{3} = \frac{16}{5}$$

$$(x,y) \equiv \left(\frac{11}{5}, \frac{28}{5}\right)$$

19. NTA Ans. (1)

Sol. $y = \left(\dfrac{2x}{\pi}-1\right)\operatorname{cosec} x$ ...(1)

$$\frac{dy}{dx} = \frac{2}{\pi}\operatorname{cosec}x - \left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x\cot x$$

$$\frac{dy}{dx} = \frac{2\operatorname{cosec}x}{\pi} - y\cot x$$

using equation (1)

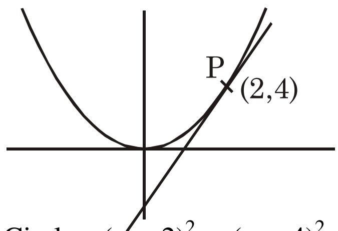
$$\frac{dy}{dx} + y\cot x = \frac{2\operatorname{cosec}x}{\pi}$$

$$\frac{dy}{dx} + p(x).y = \frac{2\operatorname{cosec}x}{\pi} \qquad x \in \left(0, \frac{\pi}{2}\right)$$

Compare : $p(x) = \cot x$

20. NTA Ans. (2)

Sol.



$y = x^2$

$\left.\dfrac{dy}{dx}\right|_P = 4$

$(y-4) = 4(x-2)$

$4x - y - 4 = 0$

Circle : $(x-2)^2 + (y-4)^2 + \lambda(4x - y - 4) = 0$

passes through (0, 1)

$4 + 9 + \lambda(-5) = 0 \Rightarrow \lambda = \dfrac{13}{5}$

Circle : $x^2 + y^2 + x(4\lambda - 4) + y(-\lambda - 8) + (20 - 4\lambda) = 0$

Centre : $\left(2-2\lambda, \dfrac{\lambda+8}{2}\right) \equiv \left(\dfrac{-16}{5}, \dfrac{53}{10}\right)$

21. NTA Ans. (3.00)

Sol. $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + (3\lambda - 3)z = 0$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & 3-\lambda & \lambda-3 \\ \lambda-3 & \lambda-3 & -2(\lambda-3) \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$$(\lambda-3)^2 \begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$(\lambda - 3)^2 [(3\lambda + 1) + (3\lambda - 1)] = 0$

$6\lambda(\lambda - 3)^2 = 0 \Rightarrow \lambda = 0, 3$

Sum = 3

22. NTA Ans. (5.00)

Sol. $f(x + y) = f(x)\, f(y)$

put x = y = 1 $\quad f(2) = (f(1))^2 = 3^2$

put x = 2, y = 1 $\quad f(3) = (f(1))^3 = 3^3$

$\vdots$

Similarly $f(x) = 3^x$

$$\sum_{i=1}^{n} f(i) = 363 \Rightarrow \sum_{i=1}^{n} 3^i = 363$$

$(3 + 3^2 + ... + 3^n) = 363$

$$\frac{3(3^n - 1)}{2} = 363$$

$3^n - 1 = 242 \Rightarrow 3^n = 243$

$\Rightarrow n = 5$

23. NTA Ans. (120.00)

Sol. LETTER

vowels = EE, consonant = LTTR

_ L _ T _ T _ R _

$$\frac{4!}{2!} \times {}^5C_2 \times \frac{2!}{2!} = 12 \times 10 = 120$$

24. NTA Ans. (6.00)

Sol.

x	0	2	4	8		2^n
f	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$		${}^nC_n$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{\sum_{r=1}^{n} 2^r\ {}^nC_r}{\sum_{r=0}^{n} {}^nC_r}$$

$$\text{Mean} = \frac{(1+2)^n - {}^nC_0}{2^n} = \frac{728}{2^n}$$

$$\Rightarrow \frac{3^n - 1}{2^n} = \frac{728}{2^n}$$

$\Rightarrow 3^n = 729 \Rightarrow n = 6$

25. NTA Ans. (1.00)

Sol. $|\vec{x} + \vec{y}| = |\vec{x}|$

$$\sqrt{|\vec{x}|^2 + |\vec{y}|^2 + 2\vec{x}.\vec{y}} = |\vec{x}|$$

$|\vec{y}|^2 + 2\vec{x}.\vec{y} = 0 \qquad .... (1)$

Now $(2\vec{x} + \lambda\vec{y}) . \vec{y} = 0$

$2\vec{x} . \vec{y} + \lambda|\vec{y}|^2 = 0$

from (1)

$-|\vec{y}|^2 + \lambda|\vec{y}|^2 = 0$

$(\lambda - 1)|\vec{y}|^2 = 0$

given $|\vec{y}| \neq 0 \qquad \Rightarrow \lambda = 1$