ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

Chapter Contents 01

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & APRIL 2019

PHYSICS

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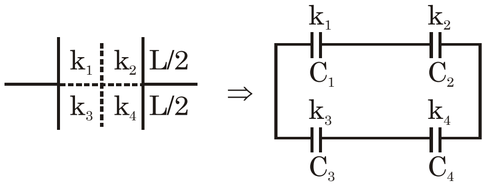
JEE (MAIN)
TOPICWISE SOLUTION OF TEST PAPERS
JANUARY & APRIL 2019

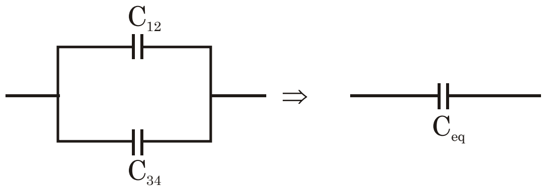
PHYSICS

JANUARY 2019 ATTEMPT (PHYSICS)

CAPACITOR

1. Ans. (Bonus)





$$C_{12} = \frac{C_1C_2}{C_1+C_2} = \frac{\dfrac{k_1 \epsilon_0 \frac{L}{2}\times L}{d/2} \cdot \dfrac{k_2\left[\epsilon_0 \frac{L}{2}\times L\right]}{d/2}}{(k_1+k_2)\left[\dfrac{\epsilon_0 \cdot \frac{L}{2}\times L}{d/2}\right]}$$

$$C_{12} = \frac{k_1k_2}{k_1+k_2}\frac{\epsilon_0 L^2}{d}$$

in the same way we get, $C_{34} = \dfrac{k_3k_4}{k_3+k_4}\dfrac{\epsilon_0 L^2}{d}$

$$\therefore C_{eq} = C_{12} + C_{34} = \left[\frac{k_1k_2}{k_1+k_2}+\frac{k_3k_4}{k_3+k_4}\right]\frac{\epsilon_0 L^2}{d} \quad ..(i)$$

Now if $k_{eq} = k$, $C_{eq} = \dfrac{k \epsilon_0 L^2}{d}$ .....(ii)

on comparing equation (i) to equation (ii), we get

$$k_{eq} = \frac{k_1k_2(k_3+k_4)+k_3k_4(k_1+k_2)}{(k_1+k_2)(k_3+k_4)}$$

This does not match with any of the options so probably they have assumed the wrong combination

$$C_{13} = \frac{k_1 \epsilon_0 L\frac{L}{2}}{d/2} + k_3 \epsilon_0 \frac{L.\frac{L}{2}}{d/2}$$

$$= (k_1 + k_3)\frac{\epsilon_0 L^2}{d}$$

$$C_{24} = (k_2 + k_4)\frac{\epsilon_0 L^2}{d}$$

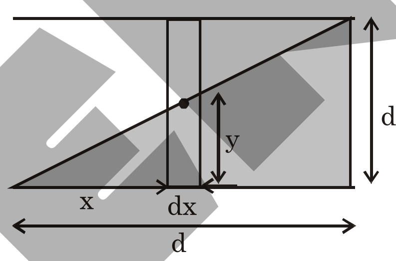
$$C_{eq} = \frac{C_{13}C_{24}}{C_{13}C_{24}} = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}\frac{\epsilon_0 L^2}{d}$$

$$= \frac{k \epsilon_0 L^2}{d}$$

$$k = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}$$

However this is one of the four options.
It must be a "Bonus" logically but of the given options probably they might go with (4)

2. Ans. (3)



$$\frac{y}{x} = \frac{d}{a}$$

$$y = \frac{d}{a}x$$

$$dy = \frac{d}{a}(dx)$$

$$\frac{1}{dc} = \frac{y}{KE \cdot adx} + \frac{(d-y)}{\epsilon_0\ adx}$$

$$\frac{1}{dc} = \frac{1}{\epsilon_0\ adx}\left(\frac{y}{k}+d-y\right)$$

$$\int dc = \int \frac{\epsilon_0\ adx}{\frac{y}{k}+d-y}$$

$$c = \epsilon_0\ a \cdot \frac{a}{d}\int_0^d \frac{dy}{d+y\left(\frac{1}{k}-1\right)}$$

$$= \frac{\epsilon_0 a^2}{\left(\frac{1}{k}-1\right)d}\left[\ell n\left(d+y\left(\frac{1}{k}-1\right)\right)\right]_0^d$$

$$= \frac{k\epsilon_0 a^2}{(1-k)d}\ell n\left(\frac{d+d\left(\frac{1}{k}-1\right)}{d}\right)$$

$$= \frac{k\epsilon_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right) = \frac{k\epsilon_0 a^2 \ell nk}{(k-1)d}$$

3. Ans. (3)

Initial energy of capacitor

$$U_i = \frac{1}{2}\frac{v^2}{c}$$

$$= \frac{1}{2}\times\frac{120\times120}{12} = 600 \text{ J}$$

Since battery is disconnected so charge remain same.

Final energy of capacitor

$$U_f = \frac{1}{2}\frac{v^2}{c}$$

$$= \frac{1}{2}\times\frac{120\times120}{12\times6.5} = 92$$

$W + U_f = U_i$

$W = 508$ J

4. Ans. (1)

Let dielectric constant of material used be K.

$$\therefore \frac{10\epsilon_0 A/3}{d} + \frac{12\epsilon_0 A/3}{d} + \frac{14\epsilon_0 A/3}{d} = \frac{K\epsilon_0 A}{d}$$

$\Rightarrow K = 12$

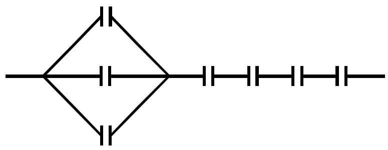
5. Ans. (4)

$$C_{eq} = \frac{6}{13}\mu F$$

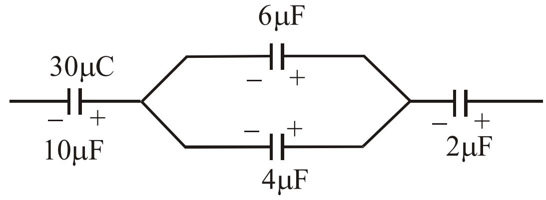
Therefore three capacitors most be in parallel to get 6 in

$$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$C_{eq} = \frac{3C}{13} = \frac{6}{13}\mu F$$



6. Ans. (4)



6μF & 4μF are in parallel & total charge on this combination is 30 μC

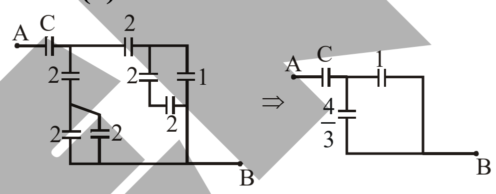
$\therefore$ Charge on 6μF capacitor = $\frac{6}{6+4}\times 30$

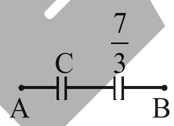
= 18 μC

Since charge is asked on right plate therefore is +18μC

Correct answer is (4)

7. Ans. (2)



$\Rightarrow$ 

From equs.

$$\frac{\frac{7C}{3}}{\frac{7}{3}+C} = \frac{1}{2}$$

$\Rightarrow 14\ C = 7 + 3\ C$

$\Rightarrow C = \frac{7}{11}$

8. Ans. (3)

9. Ans. (4)

$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$Q = AE\epsilon_0$

$Q = (1)(100)(8.85\times10^{-12})$

$Q = 8.85 \times 10^{-10}C$

10. Ans. (1)

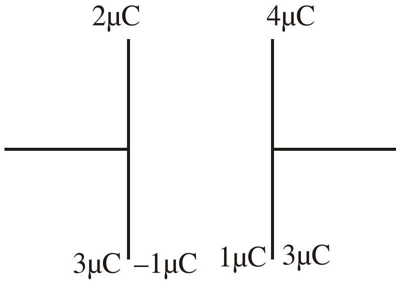
$$V_i = \frac{1}{2}CE^2$$

$$V_f = \frac{(CE)^2}{2\times4c} = \frac{1}{2}\frac{CE^2}{4}$$

$$\Delta E = \frac{1}{2}CE^2\times\frac{3}{4} = \frac{3}{8}CE^2$$

11. Ans. (4)

Sol.



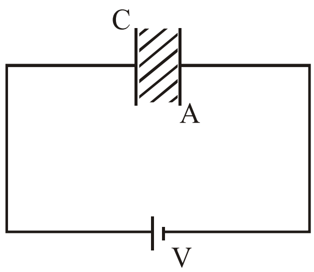
Charges at inner plates are 1μC and –1μC

$\therefore$ Potential difference across capacitor

$$= \frac{q}{c} = \frac{1\mu C}{1\mu F} = \frac{1\times10^{-6}C}{1\times10^{-6}\,\text{Farad}} = 1V$$

12. Ans. (4)

Sol.



$A = 10^{-4}\ m^2$

$E_{max} = 10^6\ V/m$

$C = 15\ \mu F$

$$C = \frac{k\varepsilon_0 A}{d}$$

$$\frac{Cd}{\varepsilon_0 A} = k$$

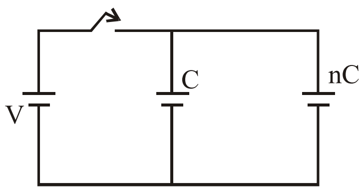
$$k = \frac{15\times10^{-12}\times500\times10^{-6}}{8.86\times10^{-12}\times10^{4}}$$

$$= \frac{15\times5}{8.86} = 8.465$$

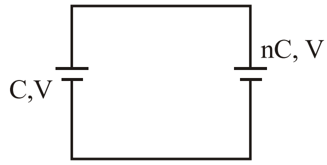
$k \approx 8.5$

13. Ans. (3)

Sol.



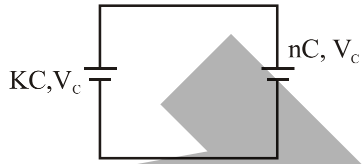
After fully charging, battery is disconnected



Total charge of the system = CV + nCV

= (n + 1)CV

After the insertion of dielectric of constant K



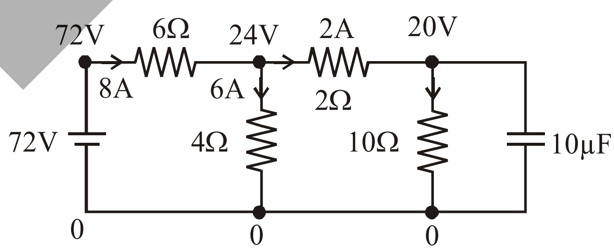
New potential (common)

$$V_C = \frac{\text{total charge}}{\text{total capacitance}}$$

$$= \frac{(n+1)CV}{KC+nC} = \frac{(n+1)V}{K+n}.$$

14. Ans. (3)

Sol. Applying point potential method



$q = cV$

$q = 10\mu F \times 20 = 200\mu C$

Option (3)

15. Ans. (1)

Sol. Work done = ΔU

$= U_f - U_i$

$$= \frac{q^2}{2C_f} - \frac{q^2}{2C_i}$$

$$= \frac{(5\times10^{-6})^2}{2}\left(\frac{1}{2\times10^{-6}} - \frac{1}{5\times10^{-6}}\right)$$

$= \frac{15}{4} \times 10^{-6}$

$= 3.75 \times 10^{-6}$ J

Option (1)

16. Ans. (4)

Sol. As q = CV

Hence slope of graph will give capacitance. Slope will be more in parallel combination. Hence capacitance in parallel should be 50μF & in series combination must be 8μF.

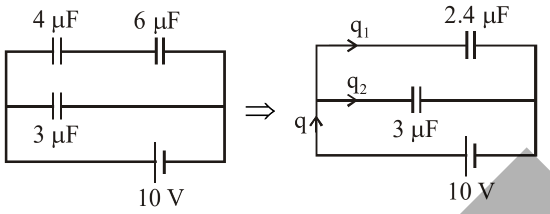
Only in option 40μF & 10 μF

$C_{parallel} = 40 + 10 = 50\mu F$

$C_{series} = \frac{40 \times 10}{40 + 10} = 8\ \mu F$

17. Ans. (2)

Sol.



So total charge flow = q = 5.4 μF × 10V

= 54 μC

The charge will be distributed in the ratio of capacitance

$\Rightarrow \frac{q_1}{q_2} = \frac{2.4}{3} = \frac{4}{5}$

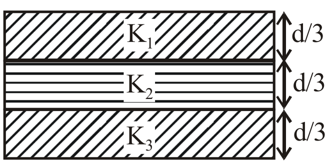
∴ 9X = 54 μC ⇒ X = 6 μC

∴ charge on 4 μF capacitor will be = 4X = 4 × 6 μC

= 24 μC

18. Ans. (1)

Sol. I.



$C_1 = \frac{3\varepsilon_0 AK_1}{d}$

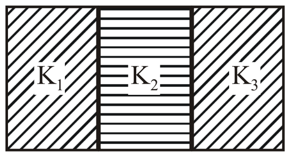
$C_2 = \frac{3\varepsilon_0 AK_2}{d}$

$C_3 = \frac{3\varepsilon_0 AK_3}{d}$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow C_{eq} = \frac{3\varepsilon_0 AK_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)} \quad ....(1)$$

II.



$C_1 = \frac{\varepsilon_0 K_1 A}{3d}$

$C_2 = \frac{\varepsilon_0 K_2 A}{3d}$

$C_3 = \frac{\varepsilon_0 K_3 A}{3d}$

$C'_{eq} = C_1 + C_2 + C_3$

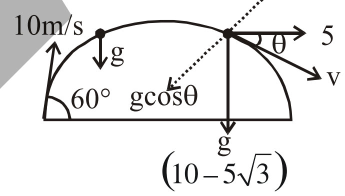
$$= \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \quad ....(2)$$

Now,

$$\boxed{\frac{E_1}{E_2} = \frac{\frac{1}{2}C_{eq}.V^2}{\frac{1}{2}C'_{eq}V^2} = \frac{9K_1K_2K_3}{(K_1 + K_2 + K_3)(K_1K_2 + K_2K_3 + K_3K_1)}}$$

CIRCULAR MOTION

1. Ans. (3)



$v_x = 10\cos 60° = 5$ m/s

$v_y = 10\cos 30° = 5\sqrt{3}$ m/s

velocity after t = 1 sec.

$v_x = 5$ m/s

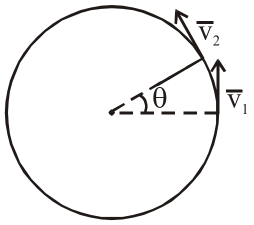
$v_y = \left|(5\sqrt{3} - 10)\right|$ m/s $= 10 - 5\sqrt{3}$

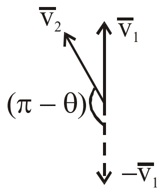
$$a_n = \frac{v^2}{R} \Rightarrow R = \frac{v_x^2 + v_y^2}{a_n} = \frac{25 + 100 + 75 - 100\sqrt{3}}{10\cos\theta}$$

$$\tan\theta = \frac{10 - 5\sqrt{3}}{5} = 2 - \sqrt{3} \Rightarrow \theta = 15°$$

$$R = \frac{100(2 - \sqrt{3})}{10\cos 15} = 2.8\text{m}$$

2. Ans. (2)





$$|\Delta\overline{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\pi - \theta)}$$

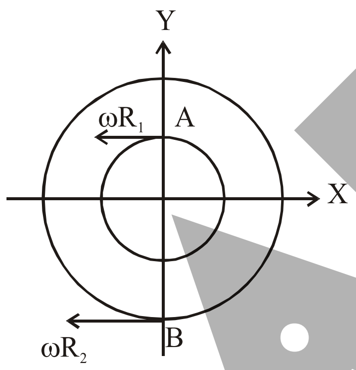
$$= 2v\sin\frac{\theta}{2} \quad \text{since}\left[|\overline{v}_1| = |\overline{v}_2|\right]$$

$= (2 \times 10) \times \sin(30°)$

$= 10$ m/s

3. Ans. (4)

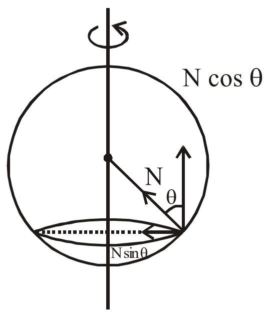
$$\theta = \omega t = \omega\frac{\pi}{2\omega} = \frac{\pi}{2}$$



$$\vec{V}_A - \vec{V}_S = \omega R_1\left(-\hat{i}\right) - \omega R_2\left(-i\right)$$

4. Ans. (4)

Sol.



$$N\sin\theta = m\frac{r}{2}\omega^2 \qquad ......(1)$$

$$N\cos\theta = mg \qquad ....(2)$$

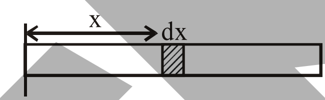
$$\tan\theta = \frac{r\omega^2}{2g}$$

$$\frac{r}{2\frac{\sqrt{3}\,r}{2}} = \frac{r\omega^2}{2g}$$

$$\omega^2 = \frac{2g}{\sqrt{3}\,r}$$

5. Ans. (4)

Sol.



$$T = \int_{x=x}^{x=\ell} dm\omega^2 x = \int_{x=x}^{x=\ell}\frac{m}{\ell}dx\,\omega^2 x$$

$$= \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)$$

$$\boxed{T = \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)}$$

COM & COLLISION

1. Ans. (1)

$$k_i = \frac{1}{2}mv_0^2$$

From linear momentum conservation

$mv_0 = (2m + M)\,v_f$

$$\Rightarrow v_f = \frac{mv_0}{2m + M}$$

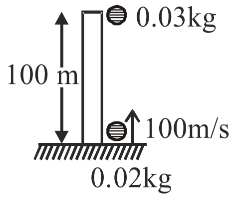
$$\frac{k_i}{k_f} = 6$$

$$\Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m + M)\left(\frac{mv_0}{2m + M}\right)^2} = 6$$

$$\Rightarrow \frac{2m + M}{m} = 6$$

$\Rightarrow \dfrac{M}{m} = 4$

2. Ans. (3)



Time taken for the particles to collide,

$t = \dfrac{d}{V_{rel}} = \dfrac{100}{100} = 1\,\text{sec}$

Speed of wood just before collision = gt = 10 m/s

& speed of bullet just before collision v-gt

= 100 – 10 = 90 m/s

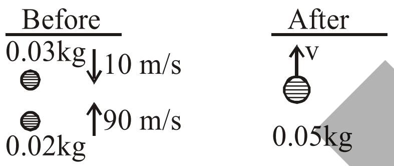
Now, conservation of linear momentum just before and after the collision -

–(0.02) (1v) + (0.02) (9v) = (0.05)v

$\Rightarrow$ 150 = 5v

$\Rightarrow$ v = 30 m/s

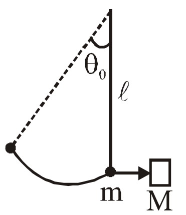
Max. height reached by body h = $\dfrac{v^2}{2g}$



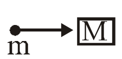
$h = \dfrac{30 \times 30}{2 \times 10} = 45\text{m}$

$\therefore$ Height above tower = 40 m

3. Ans. (3)



Before colision After collision



$v = \sqrt{2g\ell(1-\cos\theta_0)}$ $v_1 = \sqrt{2g\ell(1-\cos\theta_1)}$

By momentum conservation

$$m\sqrt{2g\ell(1-\cos\theta_0)} = MV_m - m\sqrt{2gl(1-\cos\theta}$$

$$\Rightarrow m\sqrt{2g\ell}\left\{\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}\right\} = MV_m$$

and $e = 1 = \dfrac{V_m + \sqrt{2g\ell(1-\cos\theta_1)}}{\sqrt{2g\ell(1-\cos\theta_0)}}$

$$\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}\right) = V_m \quad ..(I)$$

$$m\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}\right) = MV_M \quad ..(II)$$

Dividing

$$\frac{\left(\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}\right)}{\left(\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}\right)} = \frac{M}{m}$$

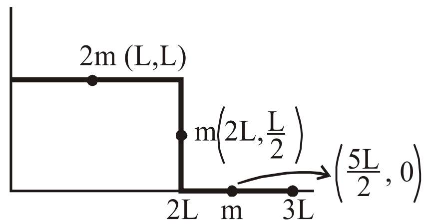
By componendo divided

$$\frac{m-M}{m+M} = \frac{\sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0}} = \frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}$$

$$\Rightarrow \frac{M}{m} = \frac{\theta_0 - \theta_1}{\theta_0 + \theta_1} \Rightarrow M = m\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}$$

4. Ans. (1)

Sol.

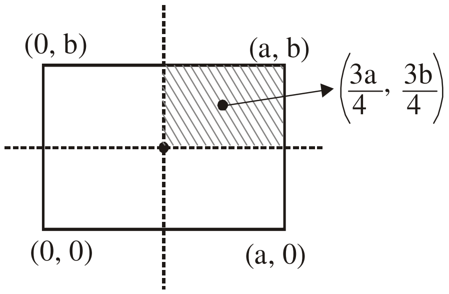


$$X_{cm} = \frac{2mL + 2mL + \frac{5mL}{2}}{4m} = \frac{13}{8}L$$

$$Y_{cm} = \frac{2m \times L + m \times \left(\frac{L}{2}\right) + m \times 0}{4m} = \frac{5L}{8}$$

5. Ans. (4)

Sol.



$$x = \frac{M\frac{a}{2} - \frac{M}{4}\times\frac{3a}{4}}{M - \frac{M}{4}}$$

$$= \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{\frac{5a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

$$y = \frac{M\frac{b}{2} - \frac{M}{4}\times\frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

6. Ans. (3)

Sol. Applying linear momentum conservation

$m_1v_1\hat{i} + m_2v_2\hat{i} = m_1v_3\hat{i} + m_2v_4\hat{i}$

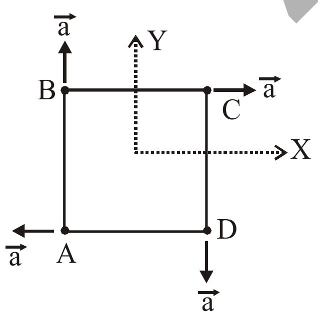
$m_1v_1 + 0.5\, m_1v_2 = m_1(0.5\, v_1) + 0.5\, m_1v_4$

$0.5\, m_1v_1 = 0.5\, m_1(v_4 - v_2)$

$v_1 = v_4 - v_2$

7. Ans. (1)

Sol.



$\vec{a}_A = -a\hat{i}$

$\vec{a}_B = a\hat{j}$

$\vec{a}_C = a\hat{i}$

$\vec{a}_D = -a\hat{j}$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2ma\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

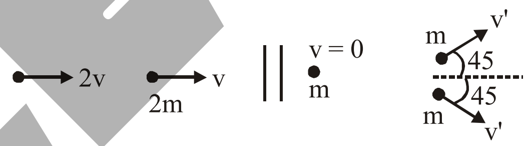
$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m}$$

$$= \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j}$$

$$= \frac{a}{5}\left(\hat{i} - \hat{j}\right)$$

8. Ans. (2)

Sol.



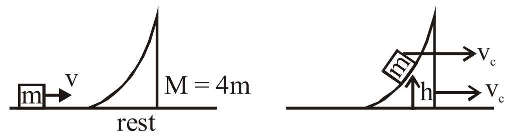
Linear momentum conservation

$$m\,2v + 2m\,v = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$$

$v' = 2\sqrt{2}\, v.$

9. Ans. (3)

Sol.



Applying Linear momentum conservation

$mv = (m + M)v_c$

$$v_c = \frac{v}{5}$$

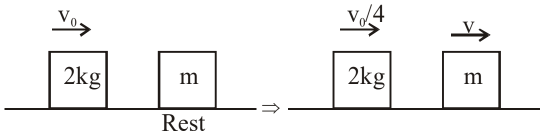
applying work energy theorem

$$-mgh = \frac{1}{2}(m + M)v_c^2 - \frac{1}{2}mv^2$$

$$\text{solve, } h = \frac{2v^2}{5g}$$

10. Ans. (2)

Sol.



By conservation of linear momentum :-

$$2v_0 = 2\left(\frac{v_0}{4}\right) + mv \Rightarrow 2v_0 = \frac{v_0}{2} + mv$$

$$\Rightarrow \frac{3v_0}{2} = mv \text{ ....(1)}$$

Since collision is elastic →

$V_{separation} = v_{approch}$

$$\Rightarrow v - \frac{v_0}{4} = v_0 \Rightarrow \frac{5v_0}{4} = v \text{ .....(2)}$$

equating (2) and (1)

$$\frac{3v_0}{2} = m\left(\frac{5v_0}{4}\right) \Rightarrow m = \frac{6}{5} = 1.2 \text{ kg}$$

Option (2)

11. Ans. (3)

Sol. $M \times 10 \cos 30° + 2M \times 5 \cos 45°$

$= 2M \times v_1 \cos 30° + M\, v_2 \cos 45°$

$$5\sqrt{3} + 5\sqrt{2} = 2v_1\frac{\sqrt{3}}{2} + \frac{v_2}{\sqrt{2}}$$

$10 \times M \sin 30° - 2M \times 5 \sin 45°$

$= M\, v_2 \sin 45° - 2M\, v_1 \sin 30°$

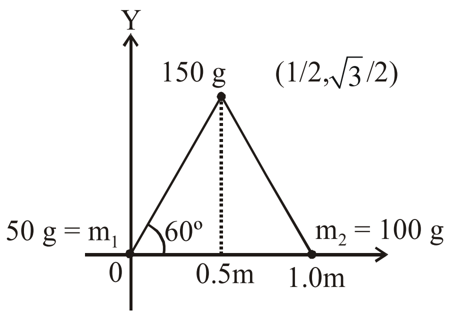
$$5 - 5\sqrt{2} = \frac{v_2}{\sqrt{2}} - v_1$$

Solving $v_1 = \frac{17.5}{2.7} \simeq 6.5 \text{m/s}$

$v_2 \approx 6.3$ m/s

12. Ans. (3)

Sol.



The co-ordinates of the centre of mass

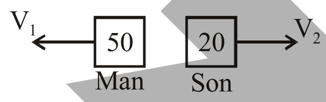
$$\vec{r}_{cm} = \frac{0 + 150 \times \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) + 100 \times \hat{i}}{300}$$

$$\vec{r}_{cm} = \frac{7}{12}\hat{i} + \frac{\sqrt{3}}{4}\hat{j}$$

$$\therefore \text{ Co-ordinate } \left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right) \text{m}$$

13. Ans. (1)

Sol.

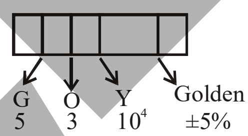


$\Rightarrow 0 = 50V_1 - 20V_2$ and $V_1 + V_2 = 0.7$

$\Rightarrow V_1 = 0.2$

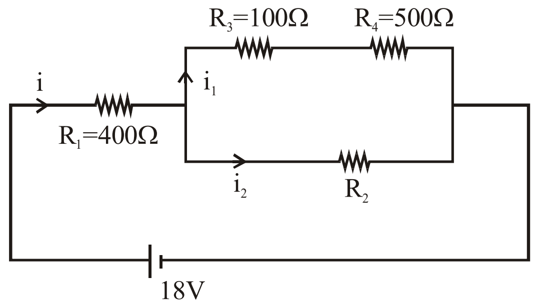
CURRENT ELECTRICITY

1. Ans. (2)



$R = 53 \times 10^4 \pm 5\% = 530 \text{ k}\Omega \pm 5\%$

2. Ans. (1)



$V_4 = 5V$

$$i_1 = \frac{V_4}{R_4} = 0.01 \text{ A}$$

$V_3 = i_1R_3 = 1V$

$V_3 + V_4 = 6V = V_2$

$V_1 + V_3 + V_4 = 18V$

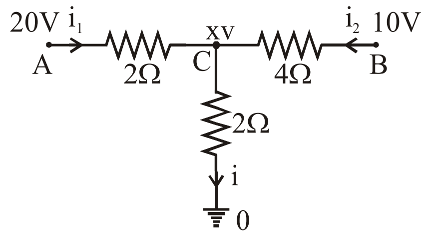
$V_1 = 12$ V

$$i = \frac{V_1}{R_1} = 0.03 \text{Amp.}$$

$i_2 = 0.02$ Amp $\quad V_2 = 6V$

$$R_2 = \frac{V_2}{i_2} = \frac{6}{0.02} = 300\Omega$$

3. Ans. (2)



Let voltage at C = xv

KCL : $i_1 + i_2 = i$

$$\frac{20-x}{2} + \frac{10-x}{4} = \frac{x-0}{2}$$

$\Rightarrow x = 10$

and i = 5 amp.

4. Ans. (4)

Color code :

Red violet orange silver

$R = 27 \times 10^3\ \Omega \pm 10\%$

$= 27\ K\Omega \pm 10\%$

5. Ans. (3)

$R = \frac{\rho\ell}{A}$ and volume (V) = $A\ell$.

$$R = \frac{\rho\ell^2}{V}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell} = 1\%$$

6. Ans. (4)

$I = neAv_d$

$$\Rightarrow v_d = \frac{I}{neA} = \frac{1.5}{9\times10^{28}\times1.6\times10^{-19}\times5\times10^{-6}}$$

= 0.02 m/s

7. Ans. (2)

$$0.95\ R = \frac{R R_\upsilon}{R + R_\upsilon}$$

$0.95 \times 30 = 0.05\ R_\upsilon$

$R_\upsilon = 19 \times 30 = 570\ \Omega$

8. Ans. (1)

$P = I^2R$

$4.4 = 4 \times 10^{-6}\ R$

$R = 1.1 \times 10^6\ \Omega$

$$P' = \frac{11^2}{R} = \frac{11^2}{1.1}\times10^{-6} = 11\times10^{-5}\,W$$

9. Ans. (2)

$R_1 = 32 \times 10 = 320$

for wheat stone bridge

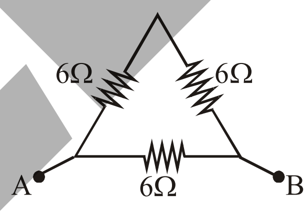
$$\Rightarrow \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

$$\frac{320}{R_3} = \frac{80}{40}$$

$R_3 = 160$

Brown $\quad$ Blue $\quad$ Brown

10. Ans. (3)



R_{eq} between any two vertex will be

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{6} \Rightarrow R_{eq.} = 4\Omega$$

11. Ans. (4)

$P = i^2R$.

$\therefore$ for i_{max}, R must be minimum

from color coding $R = 50\times10^2\Omega$

$\therefore$ $i_{max} = 20mA$

12. Ans. (4)

$$i = \frac{\varepsilon}{13r}$$

$$i\left(\frac{x}{L}12r\right) = \frac{\varepsilon}{2}$$

$$\frac{\varepsilon}{13r}\left[\frac{x}{L}.12r\right] = \frac{\varepsilon}{2} \Rightarrow \boxed{x = \frac{13L}{24}}$$

13. Ans. (4)

$i_1 = \frac{10}{20} = 0.5A$

$i_2 = 0$

14. Ans. (4)

Potential difference across AB will be equal to battery equivalent across CD

$$V_{AB} = V_{CD} = \frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{\frac{1}{1}+\frac{2}{1}+\frac{3}{1}}{\frac{1}{1}+\frac{1}{1}+\frac{1}{1}}$$

$$= \frac{6}{3} = 2V$$

15. Ans. (2)

$\frac{R_1}{R_2} = \frac{2}{3}$ ……(i)

$\frac{R_1+10}{R_2} = 1 \Rightarrow R_1 + 10 = R_2$ ……(ii)

$\frac{2R_2}{3} + 10 = R_2$

$10 = \frac{R_2}{3} \Rightarrow R_2 = 30\Omega$

& $R_1 = 20\Omega$

$$\frac{\frac{30\times R}{30+R}}{30} = \frac{2}{3}$$

$R = 60\ \Omega$

16. Ans. (3)

Potential gradient with $R_h = 2\Omega$

is $\left(\frac{6}{2+4}\right)\times\frac{4}{L} = \frac{dV}{dL}$; $L = 100\,cm$

Let null point be at ℓ cm

thus $\varepsilon_1 = 0.5V = \left(\frac{6}{2+4}\right)\times\frac{4}{L}\times\ell$ ...(1)

Now with $R_h = 6\Omega$ new potential gradient is

$\left(\frac{6}{4+6}\right)\times\frac{4}{L}$ and at null point

$$\left(\frac{6}{4+6}\right)\left(\frac{4}{L}\right)\times\ell = \varepsilon_2 \quad ...(2)$$

dividing equation (1) by (2) we get

$\frac{0.5}{\varepsilon_2} = \frac{10}{6}$ thus $\varepsilon_2 = 0.3$

17. Ans. (2)

In series condition, equivalent resistance is 2R

thus power consumed is $60W = \frac{\varepsilon^2}{2R}$

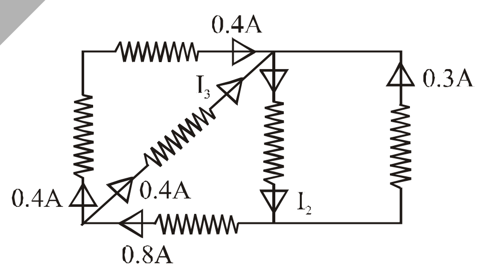
In parallel condition, equivalent resistance is R/2 thus new power is

$$P' = \frac{\varepsilon^2}{(R/2)}$$

or $P' = 4P = 240W$

18. Ans. (4)

19. Ans. (1)



From KCL, $I_3 = 0.8 - 0.4 = 0.4A$

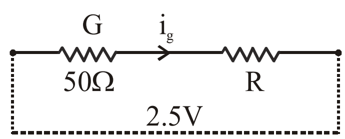
$I_2 = 0.4 + 0.4 + 0.3$

$= 1.1\ A$

$I_6 = 0.4A$

20. Ans. (3)

$I_g = 4 \times 10^{-4} \times 25 = 10^{-2}\ A$



$2.5 = (50 + R)\ 10^{-2}$ $\therefore$ $R = 200\ \Omega$

21. Ans. (3)

$$R_1 = \frac{220^2}{25}$$

$$R_2 = \frac{220^2}{100}$$

$$L = \frac{220}{R_1 + R_2}$$

$P_1 = i^2\ R_1$

$P_2 = i^2\ (R_2 = 4W)$

$$= \frac{220^2}{\left(\frac{220^2}{25} + \frac{220^2}{100}\right)} \times \frac{220^2}{25}$$

$$= \frac{400}{25} = 16W$$

22. Ans. (4)

case I $\quad i_g = \dfrac{E}{220 + R_g} = C\theta_0 \quad$ ..(i)

Case II

$$i_g = \left(\frac{E}{220 + \frac{5R_g}{5 + R_g}}\right) \times \frac{5}{(R_g + 5)} = \frac{C\theta_0}{5} \quad ..(ii)$$

$$\Rightarrow \frac{5E}{225R_g + 1100} = \frac{C\theta_0}{5} \quad ..(ii)$$

$$\frac{E}{220 + R_g} = C\theta \quad ...(i)$$

$$\Rightarrow \frac{225R_g + 1100}{1100 + 5R_g} = 5$$

$\Rightarrow 5500 + 25R_g = 225R_g + 1100$

$200R_g = 4400$

$R_g = 22\Omega$

Ans. – 4

23. Ans. (1)

For the given wire : $dR = C\dfrac{d\ell}{\sqrt{\ell}}$,

where C = constant.

Let resistance of part AP is R_1 and PB is R_2

$\therefore \quad \dfrac{R'}{R'} = \dfrac{R_1}{R_2}$ or $R_1 = R_2$ By balanced WSB concept.

Now $\int dR = c\int \dfrac{d\ell}{\sqrt{\ell}}$

$$\therefore R_1 = C\int_0^{\ell} \ell^{-1/2} d\ell = C.2.\sqrt{\ell}$$

$$R_2 = C\int_{\ell}^{1} \ell^{-1/2} d\ell = C.(2 - 2\sqrt{\ell})$$

Putting $R_1 = R_2$

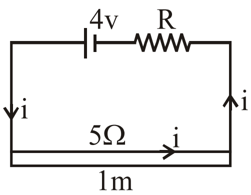
$C2\sqrt{\ell} = C\,(2 - 2\sqrt{\ell})$

$\therefore 2\sqrt{\ell} = 1$

$\sqrt{\ell} = \dfrac{1}{2}$

i.e. $\ell = \dfrac{1}{4}$ m $\quad \Rightarrow \quad 0.25$ m

24. Ans. (3)



Let current flowing in the wire is i.

$\therefore \quad i = \left(\dfrac{4}{R+5}\right)A$

If resistance of 10 m length of wire is x

then $x = 0.5\ \Omega = 5 \times \dfrac{0.1}{1}\Omega$

$\therefore \quad \Delta V$ = P. d. on wire = i. x

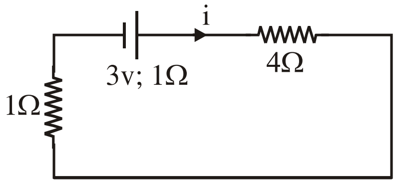
$5 \times 10^{-3} = \left(\dfrac{4}{R+5}\right)\cdot(0.5)$

$\therefore \quad \dfrac{4}{R+5} = 10^{-2}$ or $R + 5 = 400\ \Omega$

$\therefore R = 395\ \Omega$

25. Ans. (2)

Sol.



Resistance of wire AB = 400 × 0.01 = 4Ω

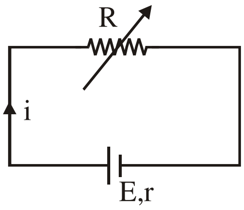
$i = \frac{3}{6} = 0.5A$

Now voltmeter reading = i (Resistance of 50 cm length)

= (0.5A) (0.01 × 50) = 0.25 volt

26. Ans. (4)

Sol.



Current $i = \frac{E}{r+R}$

Power generated in R

$P = i^2R$

$P = \frac{E^2R}{(r+R)^2}$

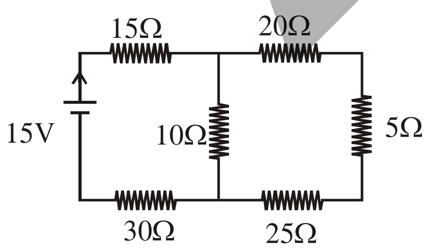
for maximum power $\frac{dP}{dR} = 0$

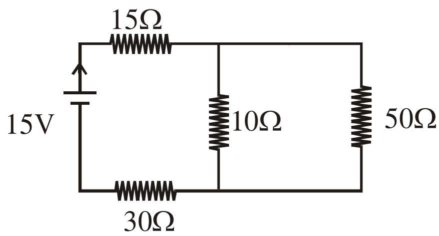
$$E^2\left[\frac{(r+R)^2 \times 1 - R \times 2(r+R)}{(r+R)^4}\right] = 0$$

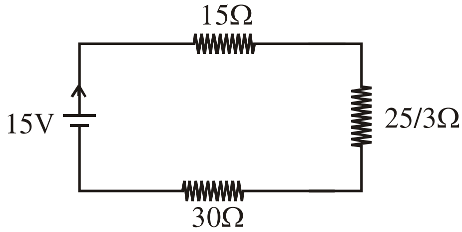
$\Rightarrow r = R$

27. Ans. (3)

Sol.







$R_{eq} = 15 + \frac{25}{3} + 30 = \frac{45+25+90}{3} = \frac{160}{3}$

$I = \frac{E}{R_{eq}} = \frac{15 \times 3}{160} = \frac{9}{32}$ amp.

28. Ans. (2)

Sol. $E_{eq} = \dfrac{\frac{E_1}{2R_1} + \frac{E_2}{R_2} + \frac{E_3}{2R_1}}{\frac{1}{2R_1} + \frac{1}{R_2} + \frac{1}{2R_1}}$

$= \dfrac{\frac{2}{2} + \frac{4}{2} + \frac{4}{2}}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}}$

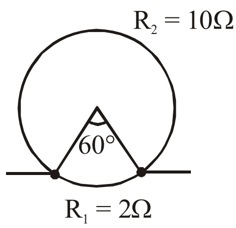
$= \dfrac{5}{\frac{3}{2}} = \frac{10}{3} = 3.3$

29. Ans. (3)

Sol. When red is replaced with green 1st digit changes to 5 so new resistance will be 500Ω

30. Ans. (2)

Sol.



$R = \frac{\rho \ell^2}{A\ell D} d = \frac{\rho d \ell^2}{m}$

$R \propto \ell^2$

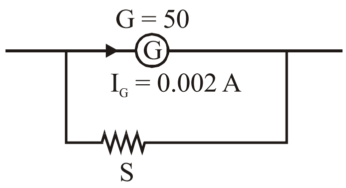
R = 12Ω (new resistance of wire)

$R_1 = 2\Omega$ $R_2 = 10\Omega$

$R_{eq} = \frac{10 \times 2}{10+2} = \frac{5}{3}\Omega$.

31. Ans. (1)

Sol.



$S(0.5 - 0.002) = 50 \times 0.002$

$S = \frac{50 \times 0.002}{(0.5 - 0.002)} = \frac{0.1}{0.498} = 0.2$

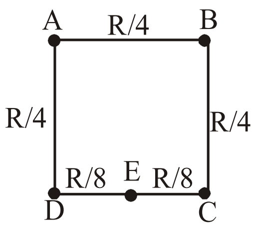
32. Ans. (4)

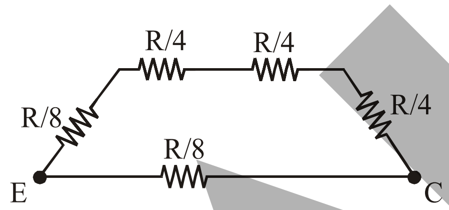
Sol. $\rho = \frac{m}{ne^2\tau}$

$= 1.67 \times 10^{-8}\Omega m$

33. Ans. (3)

Sol.





$\frac{1}{R_{eq}} = \frac{8}{7R} + \frac{8}{R}$

$\frac{1}{R_{eq}} = \frac{8 + 56}{7R}$

$R_{eq} = \frac{7R}{64}$

Option (3)

34. Ans. (2)

Sol. $G = 50\ \Omega$

$S = 5000\ \Omega$

$i_g = 4 \times 10^{-3}$

$V = i_g (G + S)$

$V = 4 \times 10^{-3} (50 + 5000)$

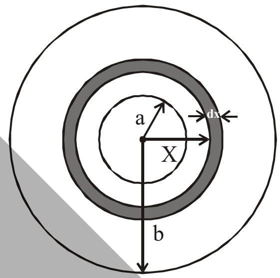
$= 4 \times 10^{-3}(5050)$

$= 20.2$ volt

Option (2)

35. Ans. (1)

Sol. $dR = \rho . \frac{dx}{4\pi x^2}$



$\int dR = \rho . \int_a^b \frac{dx}{4\pi x^2}$

$R = \frac{\rho}{4\pi}\left[-\frac{1}{x}\right]_a^b$

$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$

36. Ans. (4)

Sol. $\mu = \frac{V_d}{E}$ $\quad E = \rho J$

$= \frac{1.1 \times 10^{-3}}{1.7 \times 10^{-8} \times \frac{5}{\pi \times 25 \times 10^{-6}}}$

$= \frac{1.1 \times 10^{-3} \times \pi \times 25 \times 10^{-6}}{1.7 \times 10^{-8} \times 5} \approx 1.01 m^2 / Vs$

37. Ans. (1)

Sol. as $x = \frac{R(100 - \ell)}{\ell}$

for (1) $\quad x = \frac{1000 \times (100 - 60)}{40} \approx 667$

for (2) $\quad x = \frac{100 \times (100 - 13)}{13} \approx 669$

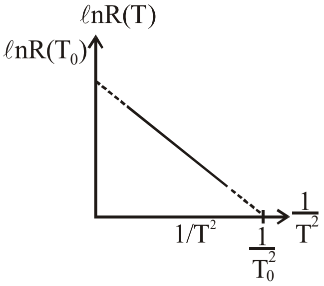
for (3) $\quad x = \frac{10 \times (100 - 1.5)}{98.5} \approx 656$

for (4) $\quad x = \frac{1 \times (100 - 1)}{1} \approx 95$

So option (4) is completely different hence correct Ans. (4)

38. Ans. (3)

Sol. $\dfrac{\frac{1}{T^2}}{\frac{1}{T_0^2}} + \dfrac{\ell n(T)}{\ell nR(T_0)} = 1$



$\Rightarrow \ \ell nR(T) = [\ell nR(T_0)]\left(1 - \dfrac{T_0^2}{T^2}\right)$

$\Rightarrow$ R(T) = $R_0 e^{\left(-\frac{T_0^2}{T^2}\right)}$

39. Ans. (Bonus)

Sol. $200 + 10^{-4}$ G = 5

G = –ve

So answer is Bonus

40. Ans. (4)

Sol. $R_{eq} = \dfrac{15\times10}{25} + 2 + 2r$

$= 8 + 2r$

$i = \dfrac{3}{8+2r}$

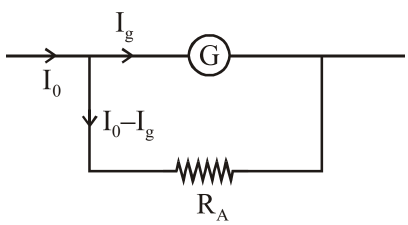
$2 = i\,R_{eq} = \dfrac{3}{8+2r}\times 6$

16 + 4r = 18

$\Rightarrow$ r = 0.5 Ω

41. Ans. (2)

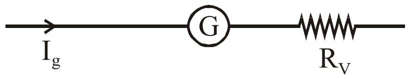
Sol. When galvanometer is used as an ammeter shunt is used in parallel with galvanometer.



$\therefore$ $I_g G = (I_0 - I_g)R_A$

$\therefore$ $R_A = \left(\dfrac{I_g}{I_0 - I_g}\right)G$

When galvanometer is used as a voltmeter, resistance is used in series with galvanometer.



$I_g(G + R_V) = V = GI_0$ (given $V = GI_0$)

$\therefore$ $R_V = \dfrac{(I_0 - I_g)G}{I_g}$

$\therefore$ $R_A R_V = G^2$ & $\dfrac{R_A}{R_V} = \left(\dfrac{I_g}{I_0 - I_g}\right)^2$

42. Ans. (2)

Sol. $q = \int I\,dt$

$q = \int_0^{L/R} \dfrac{E}{R}\left[1 - e^{\frac{-Rt}{L}}\right]dt$

$q = \dfrac{EL}{R^2}\dfrac{1}{e}$

$q = \dfrac{EL}{2.7R^2}$

43. Ans. (4)

Sol. $20 \times 50 \times 10^{-6} = 10^{-3}$ Amp.

$V_1 = \dfrac{2}{10^{-3}} = 100 + R_1$

$1900 = R_1$

$V_2 = \dfrac{10}{10^{-3}} = (2000 + R_2)$

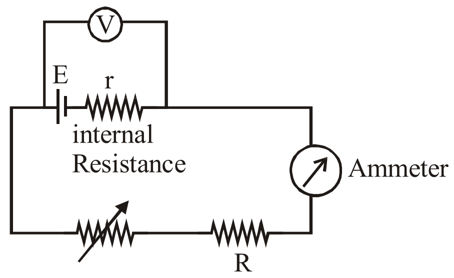
$R_2 = 8000$

$V_3 = \dfrac{20}{10^{-3}} = 10 \times 10^3 + R_3$

$10 \times 10^3 = R_3$

44. Ans. (3)

Sol.



V = E – Ir

when $V = V_0 = 0 \Rightarrow 0 = E - Ir$

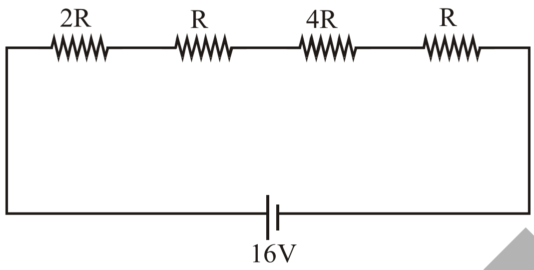
$\therefore$ E = r

when I = 0, V = E = 1.5V

$\therefore$ r = 1.5Ω.

45. Ans. (1)

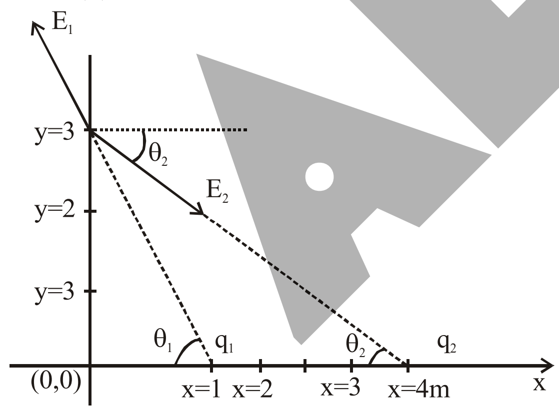
Sol.



$P = \frac{16^2}{8R} = 4 \quad \therefore R = 8\Omega$

ELECTROSTATICS

1. Ans. (3)



Let $\vec{E}_1$ & $\vec{E}_2$ are the vaues of electric field due to q_1 & q_2 respectively magnitude of

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}$$

$$E_2 = \frac{9\times10^9\times(25)\times10^{-6}}{(4^2+3^2)} \text{V/m}$$

$E_2 = 9 \times 10^3$ V/m

$$\therefore \vec{E}_2 = 9\times10^3\left(\cos\theta_2\hat{i} - \sin\theta_2\hat{j}\right)$$

$$\because \tan\theta_2 = \frac{3}{4}$$

$$\therefore \vec{E}_2 = 9\times10^3\left(\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right) = \left(72\hat{i} - 54\hat{j}\right)\times10^2$$

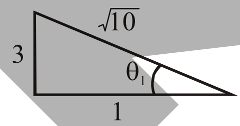
Magnitude of $E_1 = \frac{1}{4\pi\epsilon_0}\frac{\sqrt{10}\times10^{-6}}{(1^2+3^2)}$

$$= \left(9\times10^9\right)\times\sqrt{10}\times10^{-7}$$

$$= 9\sqrt{10}\times10^2$$

$$\therefore \vec{E}_1 = 9\sqrt{10}\times10^2\left[\cos\theta_1\left(-\hat{i}\right) + \sin\theta_1\hat{j}\right]$$

$\therefore$ $\tan\theta_1 = 3$



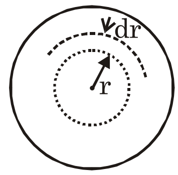
$$E_1 = 9\times\sqrt{10}\times10^2\left[\frac{1}{\sqrt{10}}\left(-\hat{i}\right) + \frac{3}{\sqrt{10}}\hat{j}\right]$$

$$E_1 = 9\times10^2\left[-\hat{i}+3\hat{j}\right] = \left[-9\hat{i}+27\hat{j}\right]10^2$$

$$\therefore \vec{E} = \vec{E}_1 + \vec{E}_2 = \left(63\hat{i} - 27\hat{j}\right)\times10^2 \text{ V/m}$$

$\therefore$ correct answer is (3)

2. Ans. (4)



$Q = \int \rho dv$

$$= \int_0^R \frac{A}{r^2}e^{-2r/a}\left(4\pi r^2 dr\right)$$

$$= \int_0^R \frac{A}{r^2}e^{-2r/a}\left(4\pi r^2 dr\right)$$

$$= 4\pi A\int_0^R e^{-2r/a}dr$$

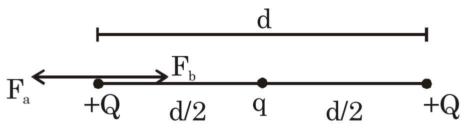
$$= 4\pi A\left(\frac{e^{-2r/a}}{-\frac{2}{a}}\right)_0^R$$

$$= 4\pi A\left(-\frac{a}{2}\right)\left(e^{-2R/a}-1\right)$$

$Q = 2\pi aA(1 - e^{-2R/a})$

$$R = \frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$$

3. Ans. (3)



For equilibrium,

$\vec{F}_a + \vec{F}_B = 0$

$\vec{F}_a = -\vec{F}_B$

$$\frac{kQQ}{d^2} = -\frac{kQq}{(d/2)^2}$$

$$\Rightarrow q = -\frac{Q}{4}$$

4. Ans. (3)

Electric field on axis of ring

$$E = \frac{kQh}{\left(h^2+R^2\right)^{3/2}}$$

for maximum electric field

$$\frac{dE}{dh} = 0$$

$$\Rightarrow h = \frac{R}{\sqrt{2}}$$

5. Ans. (4)

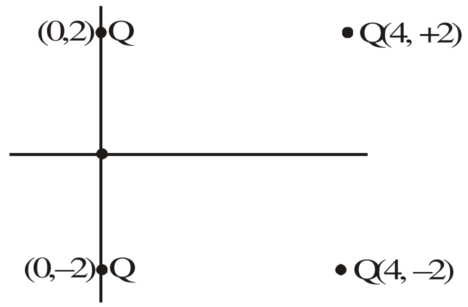
Electric field of equitorial plane of dipole

$$= -\frac{K\vec{P}}{r^3}$$

$\therefore$ At P, $F = -\frac{K\vec{P}}{r^3}Q.$

At P^1, $F^1 = -\frac{K\vec{P}Q}{(r/3)^3} = 27F$.

6. Ans. (2)



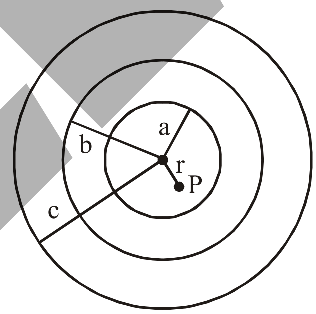
Potential at origin $= \frac{KQ}{2} + \frac{KQ}{2} + \frac{KQ}{\sqrt{20}} + \frac{KQ}{\sqrt{20}}$

(Potential at $\infty = 0$)

$$= KQ\left(1+\frac{1}{\sqrt{5}}\right)$$

$\therefore$ Work required to put a fifth charge Q at origin is equal to $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

7. Ans. (2)



Potential at point P, $V = \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$

$\because Q_a : Q_b : Q_c :: a^2 : b^2 : c^2$

$\{\text{since } \sigma_a = \sigma_b = \sigma_c\}$

$$\therefore Q_a = \left[\frac{a^2}{a^2+b^2+c^2}\right]Q$$

$$Q_b = \left[\frac{b^2}{a^2+b^2+c^2}\right]Q$$

$$Q_c = \left[\frac{c^2}{a^2+b^2+c^2}\right]Q$$

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{(a+b+c)}{a^2+b^2+c^2}\right]$$

$\therefore$ correct answer is (2)

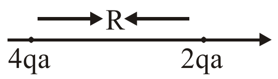
8. Correct Ans. (3)

According to JEE-Mains Ans. key (1 or 3)

$$V = \frac{4qa}{(R+x)} = \frac{2qa}{(x^2)}$$

$$\sqrt{2}x = R + x$$

$$x = \frac{R}{\sqrt{2}-1}$$



$$\text{dist} = \frac{R}{\sqrt{2}-1} + R = \frac{\sqrt{2}R}{\sqrt{2}-1}$$

9. Ans. (2)

$$U = -\vec{P}.\vec{E}$$

$= -PE \cos\theta$

$= -(10^{-29})(10^{3}) \cos 45^\circ$

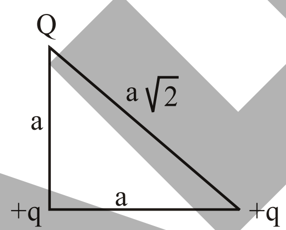
$= -0.707 \times 10^{-26}$ J

$= -7 \times 10^{-27}$ J.

10. Ans. (1)

$$U = K\left[\frac{q^2}{a} + \frac{Qq}{a} + \frac{Qq}{a\sqrt{2}}\right] = 0$$

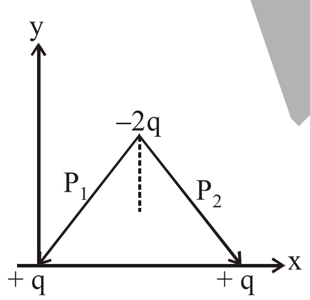
$$\Rightarrow q = -Q\left[1 + \frac{1}{\sqrt{2}}\right]$$



$$\Rightarrow Q = \frac{-q\sqrt{2}}{\sqrt{2}+1}$$

11. Ans. (2)

12. Ans. (3)



$|P_1| = q(d)$

$|P_2| = qd$

$|\text{Resultant}| = 2\,P\cos 30^\circ$

$$2\,qd\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,qd$$

13. Ans. (1)

At any instant 't'

Total energy of charge distribution is constant

$$\text{i.e. } \frac{1}{2}mV^2 + \frac{KQ^2}{2R} = 0 + \frac{KQ^2}{2R_0}$$

$$\therefore \frac{1}{2}mV^2 = \frac{KQ^2}{2R_0} - \frac{KQ^2}{2R}$$

$$\therefore V = \sqrt{\frac{2}{m}\frac{KQ^2}{2}.\left(\frac{1}{R_0} - \frac{1}{R}\right)}$$

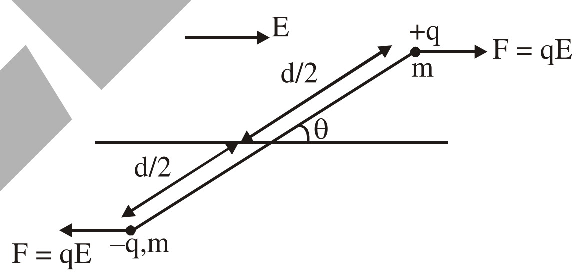
$$\therefore V = \sqrt{\frac{KQ^2}{m}\left(\frac{1}{R_0} - \frac{1}{R}\right)} = C\sqrt{\frac{1}{R_0} - \frac{1}{R}}$$

Also the slope of v-s curve will go on decreasing

$\therefore$ Graph is correctly shown by option(1)

14. Ans. (3)

Sol.



$$\text{moment of inertia (I)} = m\left(\frac{d}{2}\right)^2 \times 2 = \frac{md^2}{2}$$

Now by $\tau = I\alpha$

$$(qE)(d\sin\theta) = \frac{md^2}{2}.\alpha$$

$$\alpha = \left(\frac{2qE}{md}\right)\sin\theta$$

for small θ

$$\Rightarrow \alpha = \left(\frac{2qE}{md}\right)\theta$$

$$\Rightarrow \text{Angular frequency } \omega = \sqrt{\frac{2qE}{md}}$$

15. Ans. (4)

Sol. $\frac{1}{2}mV^2 = -q\left(V_f - V_i\right)$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_0}{r}\right)$$

$$\frac{1}{2}mv^2 = \frac{-q\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_0}{r}\right)$$

$$v \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$$

16. Ans. (3)

Sol. $\vec{E} = \left(20x + 10\right)\hat{i}$

$$V_1 - V_2 - \int_{-5}^{1}\left(20x + 10\right)dx$$

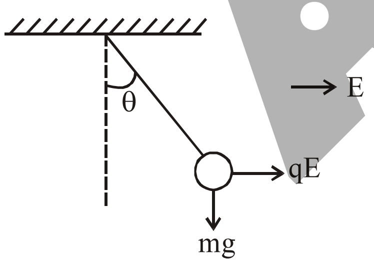
$$V_1 - V_2 = -\left(10x^2 + 10x\right)_{-5}^{1}$$

$$V_1 - V_2 = 10(25 - 5 - 1 - 1)$$

$$V_1 - V_2 = 180\text{ V}$$

17. Ans. (3)

Sol.

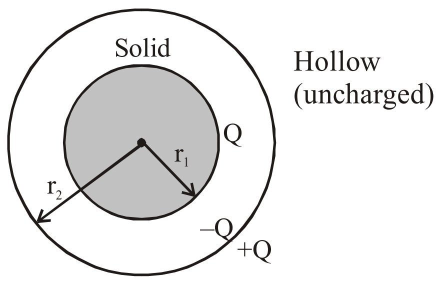


$$\tan\theta = \frac{qE}{mg} = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10}$$

$$\tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}(0.5)$$

18. Ans. (1)

Sol. As given in the first condition :

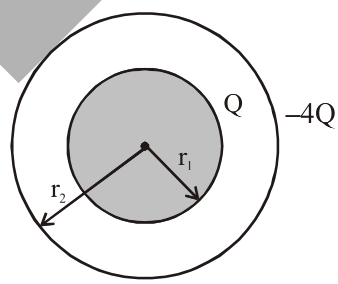


Both conducting spheres are shown.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1}\right) - \left(\frac{kQ}{r_2}\right)$$

$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

In the second condition :



Shell is now given charge –4Q.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1} - \frac{4kQ}{r_2}\right) - \left(\frac{kQ}{r_2} - \frac{4kQ}{r_2}\right)$$

$$= \frac{kQ}{r_1} - \frac{kQ}{r_2}$$

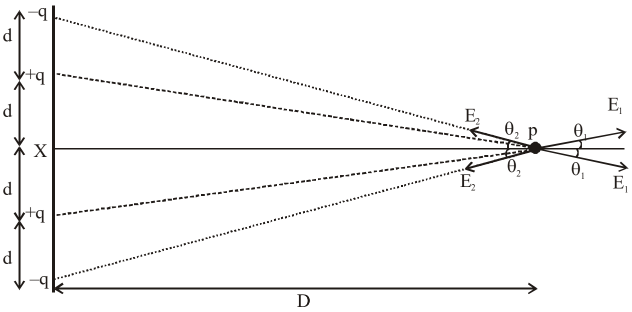
$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

Hence, we also obtain that potential difference does not depend on charge of outer sphere.

$\therefore$ P.d. remains same

19. Ans. (4)

Sol.



Electric field at p = $2E_1\cos\theta_1 - 2E_1\cos\theta_2$

$$= \frac{2Kq}{(d^2+D^2)} \times \frac{D}{(d^2+D^2)^{1/2}} - \frac{2Kq}{[(2d)^2+D^2]} \times \frac{D}{[(2d)^2+D^2]^{1/2}}$$

$$= 2KqD\left[(d^2+D^2)^{-3/2} - (4d^2+D^2)^{-3/2}\right]$$

$$= \frac{2KqD}{D^3}\left[\left(1+\frac{d^2}{D^2}\right)^{-3/2} - \left(1+\frac{4d^2}{D^2}\right)^{-3/2}\right]$$

Applying binomial approximation $\because d << D$

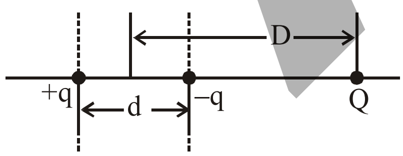
$$= \frac{2KqD}{D^3}\left[1-\frac{3}{2}\frac{d^2}{D^2} - \left(1-\frac{3\times 4d^2}{2D^2}\right)\right]$$

$$= \frac{2KqD}{D^3}\left[\frac{12}{2}\frac{d^2}{D^2} - \frac{3}{2}\frac{d^2}{D^2}\right]$$

$$= \frac{9kqd^2}{D^4}$$

20. Ans. (4)

Sol.



D >> d

$U_{total} = U_{self\ of\ dipole} + U_{interaction}$

$$= -\frac{kq^2}{d} - \left(\frac{kQ}{D^2}\right)qd$$

$$= -k\left[\frac{q^2}{d} + \frac{qQd}{D^2}\right]$$

Option (4)

21. Ans. (1)

Sol. $g_{eff} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

$$= 2\pi\sqrt{\frac{\ell}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$$

22. Ans. (1)

Sol. $W_E = -[\Delta U] = U_i - U_F = \frac{1}{2}mv^2$

$$U = \frac{kq_1q_2}{r}$$

$$\frac{(9\times10^9)\times10^{-12}}{10^{-3}} - \frac{(9\times10^9)\times10^{-12}}{9\times10^{-3}} = \frac{1}{2}\times(4\times10^{-6})v^2$$

$v^2 = 4 \times 10^6$

$v = 2 \times 10^3$ m/s

23. Ans. (2)

Sol. $U_i + K_i = U_f + K_f$

$$\frac{kq^2}{\sqrt{16a^2+9a^2}} + \frac{1}{2}mv^2 = \frac{kq^2}{3a}$$

$$\frac{1}{2}mv^2 = \frac{kq^2}{a}\left(\frac{1}{3}-\frac{1}{5}\right) = \frac{2kq^2}{15a}$$

$$v = \sqrt{\frac{4kq^2}{15ma}}$$

24. Ans. (3)

Sol. $E\,4\pi a^2 = \dfrac{\int_0^a kr\,4\pi r^2\,dr}{\varepsilon_0}$

$$E = \frac{k\,4\pi a^4}{4\times 4\pi\varepsilon_0}$$

$$2Q = \int_0^R kr\,4\pi r^2\,dr$$

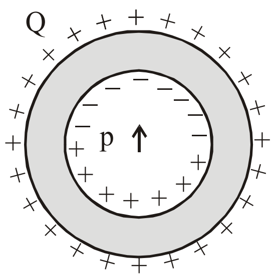
$$k = \frac{2Q}{\pi R^4}$$

$$QE = \frac{1}{4\pi\varepsilon_0}\frac{QQ}{(2a)^2}$$

$R = a8^{1/4}$

25. Ans. (1)

Sol.



Total charge of dipole = 0, so charge induced on outside surface = 0.

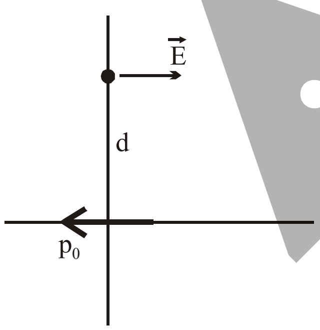
But due to non uniform electric field of dipole, the charge induced on inner surface is non zero and non uniform.

So, for any abserver outside the shell, the resultant electric field is due to Q uniformly distributed on outer surface only and it is equal to.

$$\boxed{E = \frac{KQ}{r^2}}$$

26. Ans. (4)

Sol.



$V = 0$

$$E = -\frac{K\vec{P}}{r^3}$$

$$= -\frac{\vec{p}}{4\pi\varepsilon_0 d^3}$$

EMI & AC

1. Ans. (4)

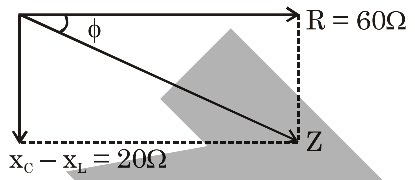
$R = 60\Omega \quad f = 50Hz,\ \omega = 2\pi f = 100\ \pi$

$$x_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 120 \times 10^{-6}}$$

$x_C = 26.52\ \Omega$

$x_L = \omega L = 100\pi \times 20 \times 10^{-3} = 2\pi\Omega$

$x_C - x_L = 20.24 \approx 20$



$$z = \sqrt{R^2 + (x_C - x_L)^2}$$

$z = 20\sqrt{10}\Omega$

$$\cos\phi = \frac{R}{z} = \frac{3}{\sqrt{10}}$$

$$P_{avg} = VI\cos\phi,\ I = \frac{v}{z}$$

$$= \frac{v^2}{z}\cos\phi$$

$= 8.64$ watt

$Q = P.t = 8.64 \times 60 = 5.18 \times 10^2$

2. Ans. (4)

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_S I_S}{V_P I_P}$$

$$\Rightarrow 0.9 = \frac{23 \times I_S}{230 \times 5}$$

$$\Rightarrow I_S = 45A$$

3. Ans. (1)

$$L\frac{di}{dt} = 25$$

$$L \times \frac{15}{1} = 25$$

$$L = \frac{5}{3}H$$

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2) = \frac{5}{6} \times 525 = 437.5\ J$$

4. Ans. (3)

Potential difference between two faces perpendicular to x-axis will be

$$\ell.(\vec{V}\times\vec{B}) = 12\text{mV}$$

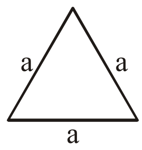
5. Ans. (2)

Total length L will remain constant

L = (3a) N (N = total turns)

and length of winding = (d) N

(d = diameter of wire)



self inductance = $\mu_0 n^2 A\ell$

$$= \mu_0 n^2\left(\frac{\sqrt{3}\,a^2}{4}\right) dN$$

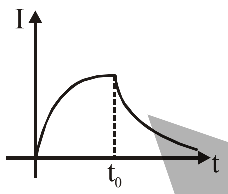
$\propto a^2\ N \propto a$

So self inductance will become 3 times

6. Correct Ans. (2)

According to JEE-Mains Ans. key (1 or 3 or 4)

From time $t = 0$ to $t = t_0$, growth of current takes place and after that decay of current takes place.

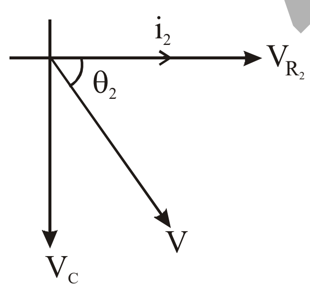


most appropriate is (2)

7. Ans. (3)

According to JEE-Mains Ans. key (Bonus)

For L-C circuit :

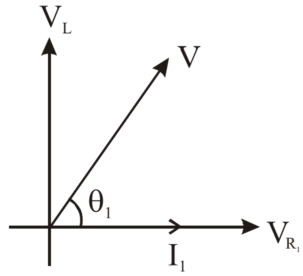


$$\tan\theta_2 = \frac{x_C}{R_2} = \frac{10^3}{\sqrt{3}}$$

θ_2 is close to 90°

For L–R circuit :

$$x_L = \omega L = 100\times\frac{\sqrt{3}}{10} = 10\sqrt{3}$$



R_1 = 10

$$\tan\theta_1 = \frac{x_e}{R}$$

$\tan\theta_1 = \sqrt{3}$

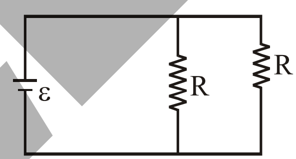
θ_1 = 60

So phase difference comes out 90 + 60 = 150.

Therefore Ans. is Bonus

8. Ans. (1)

Ideal inductor will behave like zero resistance long time after switch is closed



$$I = \frac{2\varepsilon}{R} = \frac{2\times 15}{5} = 6\text{A}$$

9. Ans. (4)

Sol. Given phase difference = $\frac{\pi}{4}$

and ω = 100 rad/s

$\Rightarrow$ Reactance (X) = Resistance (R)

now by checking options

Option (1)

$$R = 1000\ \Omega \text{ and } X_C = \frac{1}{10^{-6}\times 100} = 10^4\,\Omega$$

Option (2)

$R = 10^3\ \Omega$ and $X_L = 10^{-3}\times 100 = 10^{-1}\Omega$

Option (3)

$R = 10^3\ \Omega$ and $X_L = 10\times 10^{-3}\times 100 = 1\Omega$

Option (4)

$$R = 10^3\ \Omega \text{ and } X_C = \frac{1}{10\times 10^{-6}\times 100} = 10^3\,\Omega$$

Clear option (4) matches the given condition

10. Ans. (3)

Sol. LIdI = I^2R

$$L \times \frac{E}{10}\left(-e^{-t/2}\right) \times \frac{-1}{2} = \frac{E}{10}\left(1 - e^{-t/2}\right) \times 10$$

$$e^{-t/2} = 1 - e^{-t/2}$$

$t = 2\ell n2$

11. Ans. (2)

Sol. $T_0 = 2\pi\sqrt{\frac{m}{k}}$

$$= \frac{2\pi}{\sqrt{10}}$$

$$A = A_0 e^{-t/\gamma}$$

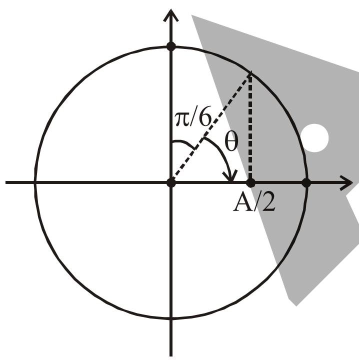
$\therefore$ for $A = \frac{A_0}{e}$, $t = \gamma$

$$t = \gamma = \frac{2m}{b} = \frac{2m}{\frac{B^2\ell^2}{R}} = 10^4 s$$

$\therefore$ No of oscillation $\frac{t}{T_0} = \frac{10^4}{2\pi/\sqrt{10}} \approx 5000$.

12. Ans. (3)

Sol.



V(t) = 220 sin(100πt) volt

time taken,

$$t = \frac{\theta}{\omega} = \frac{\frac{\pi}{3}}{100\pi} = \frac{1}{300}\text{sec}$$

= 3.3 ms

13. Official Ans. by NTA (2)

Final Ans. by NTA (4)

Sol. $\phi_{outer} = (\mu_0 nKte^{-\alpha t})4\pi R^2$

$$\varepsilon = \frac{-d\phi}{dt} = -Ce^{-\alpha t}[1 - \alpha t]$$

$$i_{induced} = \frac{-Ce^{-\alpha t}\ [1 - at]}{(\text{Resistance})}$$

At t = 0 $i_{induced}$ = –ve.

14. Ans. (2)

Sol. ϕ = NBA = LI

$N\mu_0 nI\pi R^2 = LI$

$$N\mu_0 \frac{N}{\ell} I\pi R^2 = LI$$

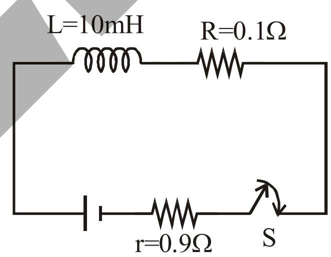
N & R constant

self inductance (L) $\propto \frac{1}{\ell} \propto \frac{1}{\text{length}}$

Option (2)

15. Ans. (2)

Sol.



$i = i_0(1 - e^{-t/\tau})$

$$\frac{80}{100} i_0 = i_0(1 - e^{-t/\tau})$$

$0.8 = 1 - e^{-t/\tau}$

$$e^{-t/\tau} = 0.2 = \frac{1}{5}$$

$$-\frac{t}{\tau} = \ln\left(\frac{1}{5}\right)$$

$$-\frac{t}{\tau} = -\ln(5)$$

$t = \tau.\ln(5)$

$$= \frac{L}{R_{eq}}.\ln(5)$$

$= \dfrac{10\times10^{-3}}{(0.1+0.9)}\times1.6$

$t = 1.6 \times 10^{-2}$

$t = 0.016$ s

16. Ans. (2)

Sol. Given $N_P = 300$, $N_s = 150$, $P_0 = 2200W$

$I_s = 10A$

$P_0 = V_0I_0 \Rightarrow 2200 = V_0 \times 10 \Rightarrow V_0 = 220V$

$\because \dfrac{V_i}{V_0} = \dfrac{N_p}{N_s} \Rightarrow V_i = 2\times220 = 440V$

Also $P_0 = V_iI_i$

$\Rightarrow I_i = \dfrac{2200}{440} = 5A$

17. Ans. (2)

Sol. $A = A_0e^{-0.1t} = \dfrac{A_0}{2}$

$ln2 = 0.1t$

$t = 10\, ln2 = 6.93 \approx 7$ sec

18. Ans. (2)

Sol. Since it is a balanced wheatstone bridge, its equivalent resistance $= \dfrac{4}{3}\Omega$

$\varepsilon = B\ell v = 5 \times 10^{-4}$ V

So total resistance

$R = \dfrac{4}{3} + 1.7 \approx 3\Omega$

$\therefore i = \dfrac{\varepsilon}{R} \approx 166\mu A \approx 170\mu A$

EMW

1. Ans. (3)

Average energy density of magnetic field,

$u_B = \dfrac{B_0^2}{2\mu_0}$, B_0 is maximum value of magnetic field.

Average energy density of electric field,

$u_E = \dfrac{\varepsilon_0 \in_0^2}{2}$

now, $\in_0 = CB_0$, $C^2 = \dfrac{1}{\mu_0 \in_0}$

$u_E = \dfrac{\in_0}{2}\times C^2B_0^2$

$= \dfrac{\in_0}{2}\times\dfrac{1}{\mu_0 \in_0}\times B_0^2 = \dfrac{B_0^2}{2\mu_0} = u_B$

$u_E = u_B$

since energy density of electric & magnetic field is same, energy associated with equal volume will be equal.

$u_E = u_B$

2. Ans. (3)

$|B| = \dfrac{|E|}{C} = \dfrac{6.3}{3\times10^8} = 2.1\times10^{-8}T$

and $\hat{E}\times\hat{B} = \hat{C}$

$\hat{j}\times\hat{B} = \hat{i}$

$\hat{B} = \hat{k}$

$\vec{B} = |B|\hat{B} = 2.1\times10^8\hat{k}T$

3. Ans. (1)

$Q = \dfrac{\Delta\phi}{R} = \dfrac{1}{10}A(B_f - B_i) = \dfrac{1}{10}\times3.5\times10^{-3}\left(0.4\sin\dfrac{\pi}{2} - 0\right)$

$= \dfrac{1}{10}\left(3.5\times10^{-3}\right)(0.4-0)$

$= 1.4 \times 10^{-4} = 0.14$ mC

No answer matching but NTA answer is 14 mC

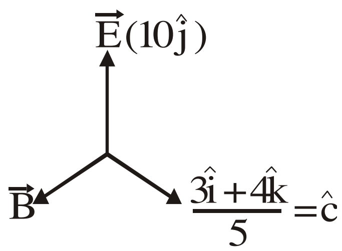
4. Ans. (2)

$\vec{E} = 10\hat{j}\cos\left[\left(6\hat{i}+8\hat{k}\right)\cdot\left(x\hat{i}+z\hat{k}\right)\right]$

$= 10\hat{j}\cos[\vec{K}\cdot\vec{r}]$

$\therefore \vec{K} = 6\hat{i}+8\hat{k}$; direction of waves travel.

i.e. direction of 'c'.



$\therefore$ Direction of $\hat{B}$ will be along

$$\hat{C}\times\hat{E}=\frac{-4\hat{i}+3\hat{k}}{5}$$

Mag. of $\vec{B}$ will be along $\hat{C}\times\hat{E}=\frac{-4\hat{i}+3\hat{k}}{5}$

Mag. of $\vec{B}$ = $\frac{E}{C}=\frac{10}{C}$

$$\therefore\ \vec{B}=\frac{10}{C}\left(\frac{-4\hat{i}+3\hat{k}}{5}\right)=\frac{\left(-8\hat{i}+6\hat{k}\right)}{C}$$

5. Ans. (4)

$E_0 = B_0 \times C$

$= 100 \times 10^{-6} \times 3 \times 10^8$

$= 3 \times 10^4$ N/C

$\therefore$ correct answer is 3×10^4 N/C

6. Ans. (3)

Intensity of EM wave is given by

$$I=\frac{\text{Power}}{\text{Area}}=\frac{1}{2}\varepsilon_0E_0^2C$$

$$=\frac{27\times10^{-3}}{10\times10^{-6}}=\frac{1}{2}\times9\times10^{-12}\times E^2\times3\times10^8$$

$E=\sqrt{2}\times10^3$ kv/m

$= 1.4$ kv/m

7. Ans. (2)

$$C=\frac{1}{\sqrt{\mu_0\in_0}}$$

$V=\frac{1}{\sqrt{k\in_0\mu_0}}$ [For transparent medium $\mu_r\approx\mu_0$]

$$\therefore\ \frac{C}{V}=\sqrt{k}=n$$

$$\frac{1}{2}\in_0E_0^2C = \text{intensity} = \frac{1}{2}\in_0 kE^2v$$

$$\therefore\ E_0^2C=kE^2v$$

$$\Rightarrow \frac{E_0^2}{E^2}=\frac{kV}{C}=\frac{n^2}{n}\Rightarrow\frac{E_0}{E}=\sqrt{n}$$

similarly

$$\frac{B_0^2C}{2\mu_0}=\frac{B^2v}{2\mu_0}\Rightarrow\frac{B_0}{B}=\frac{1}{\sqrt{n}}$$

8. Ans. (2)

$I = \varepsilon_0\, C\, E_{rms}^2$

$\&E_{rms} = cB_{rms}$

$I = \varepsilon_0\, C^3\, B_{rms}^2$

$$B_{rms}=\sqrt{\frac{I}{\in_0C^3}}$$

$B_{rms} \approx 10^{-4}$

9. Ans. (4)

Sol. If we use that direction of light propagation will be along $\vec{E}\times\vec{B}$. Then (4) option is correct.

Detailed solution is as following.

magnitude of E = CB

$E = 3\times10^8\times1.6\times10^{-6}\times\sqrt{5}$

$E = 4.8\times10^2\sqrt{5}$

$\vec{E}$ and $\vec{B}$ are perpendicular to each other

$\Rightarrow \vec{E}.\vec{B}=0$

$\Rightarrow$ either direction of $\vec{E}$ is $\hat{i}-2\hat{j}$ or $-\hat{i}+2\hat{j}$ from given option

Also wave propagation direction is parallel to $\vec{E}\times\vec{B}$ which is $-\hat{k}$

$\Rightarrow\vec{E}$ is along $\left(-\hat{i}+2\hat{j}\right)$

10. Ans. (3)

Sol. The direction of propogation of an EM wave is direction of $\vec{E}\times\vec{B}$.

$\hat{i}=\hat{j}\times\hat{B}$

$\Rightarrow\hat{B}=\hat{k}$

$$C=\frac{E}{B}\Rightarrow B=\frac{E}{C}=\frac{6}{3\times10^8}$$

B = 2×10^{-8} T along z direction.

11. Ans. (4)

Sol. Maximum Electric field E = (B) (c)

$\vec{E}_0 = (3 \times 10^{-5})c\left(-\hat{j}\right)$

$\vec{E}_1 = (2 \times 10^{-6})c\left(-\hat{i}\right)$

Maximum force

$\vec{F}_{net} = q\vec{E} = qc\left(-3\times10^{-5}\hat{j} - 2\times10^{-6}\hat{i}\right)$

$\vec{F}_{0max} = 10^{-4} \times 3 \times 10^{8}\sqrt{(3\times10^{-5})^2 + (2\times10^{-6})^2}$

= 0.9N

$F_{rms} = \frac{F_0}{\sqrt{2}} = 0.6$ N (approx)

Option (4)

12. Ans. (2)

Sol. $\because \vec{E}\times\vec{B} \parallel \vec{v}$

Given that wave is propagating along positive z-axis and $\vec{E}$ along positive x-axis. Hence $\vec{B}$ along y-axis.

From Maxwell equation

$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$

i.e. $\frac{\partial E}{\partial Z} = -\frac{\partial B}{dt}$ and $B_0 = \frac{E_0}{C}$

13. Ans. (3)

Sol. Magnetic field when electromagnetic wave propagates in +z direction

$B = B_0 \sin(kz - \omega t)$

where

$B_0 = \frac{60}{3\times10^8} = 2 \times 10^{-7}$

$k = \frac{2\pi}{\lambda} = 0.5\times10^3$

$\omega = 2\pi f = 1.5 \times 10^{11}$

14. Ans. (3)

Sol. $\vec{E} = E_0\hat{n}\sin(\omega t + (6y - 8z))$

$= E_0\hat{n}\sin(\omega t + \vec{k}.\vec{r})$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

and $\vec{k}.\vec{r} = 6y - 8z$

$\Rightarrow \vec{k} = 6\hat{j} - 8\hat{k}$

direction of propagation

$\hat{s} = -\hat{k}$

$= \left(\frac{-3\hat{j} + 4\hat{k}}{5}\right)$

ERROR & MEASUREMENT

1. Ans. (2)

$LC = \frac{\text{Pitch}}{\text{No. of division}}$

LC = 0.5×10^{-2} mm

+ve error = $3 \times 0.5 \times 10^{-2}$ mm

= 1.5×10^{-2} mm = 0.015 mm

Reading = MSR + CSR – (+ve error)

= 5.5 mm + ($48 \times 0.5 \times 10^{-2}$) – 0.015

= 5.5 + 0.24 – 0.015 = 5.725 mm

2. Ans. (1)

$\frac{\Delta V}{V} = 2\frac{\Delta d}{d} + \frac{\Delta h}{h} = 2\left(\frac{0.1}{12.6}\right) + \frac{0.1}{34.2}$

$V = 12.6 \times \frac{\pi}{4} \times 314.2$

3. Ans. (3)

$\text{Least count} = \frac{\text{Pitch}}{\text{Number of division on circular scale}}$

$5\times10^{-6} = \frac{10^{-3}}{N}$

N = 200

4. Ans. (4)

Sol. $T = \frac{30\,\text{sec}}{20}$ $\Delta T = \frac{1}{20}$ sec.

L = 55 cm ΔL = 1mm = 0.1 cm

$$g = \frac{4\pi^2 L}{T^2}$$

percentage error in g is

$$\frac{\Delta g}{g}\times 100\% = \left(\frac{\Delta L}{L}+\frac{2\Delta T}{T}\right)100\%$$

$$= \left(\frac{0.1}{55}+\frac{2\left(\frac{1}{20}\right)}{\frac{30}{20}}\right)100\% \simeq 6.8\%$$

5. Ans. (2)

Sol. Total Area = $A_1 + A_2 + \ldots\ldots A_7$

= A + A + ... 7 times

= 37.03 m^2.

Addition of 7 terms all having 2 terms beyond decimal, so final answer must have 2 terms beyond decimal (as per rules of significant digits.)

6. Allen Answer (Bonus)

Final Ans. by NTA (2)

Sol. $\rho = \frac{m}{v}$

maximum % error in S will be given by

$$\frac{\Delta\rho}{\rho}\times 100\% = \left(\frac{\Delta m}{m}\right)\times 100\% + 3\left(\frac{\Delta L}{L}\right)\times 100\% \quad \ldots(i)$$

which is only possible when error is small which is not the case in this question.

Yet if we apply equation (i), we get

$\Delta\rho = 3100\ kg/m^3$

Now, we will calculate error, without using approximation.

$$\rho_{min} = \frac{m_{min}}{v_{max}} = \frac{9.9}{(0.11)^3} = 7438 kg/m^3$$

$$\&\ \rho_{max} = \frac{m_{max}}{v_{min}} = \frac{10.1}{(0.09)^3} = 13854.6 kg/m^3$$

$\Delta\rho = 6416.6\ kg/m^3$

No option is matching.

Therefore this question should be awarded bonus

FLUIDS MECHANICS

1. Ans. (2)

In flow volume = outflow volume

$$\Rightarrow \frac{0.74}{60} = \left(\pi\times 4\times 10^{-4}\right)\times\sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74\times 100}{240\pi}$$

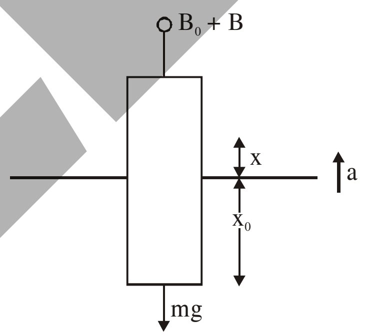
$$\Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740\times 740}{24\times 24\times 10}\left(\pi^2 = 10\right)$$

$$\Rightarrow h = \frac{74\times 74}{2\times 24\times 24}$$

$$\Rightarrow h \approx 4.8m$$

2. Ans. (Bonus)



at equilibrium
$B_0 = mg$

Extra Boyant force = δAxg

$B_0 + B - mg = ma$

$B = ma$

$$a = \left(\frac{\delta Ag}{m}\right)x$$

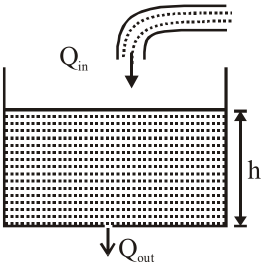
$$w^2 = \frac{\delta Ag}{m}$$

$$w = \sqrt{\frac{10^3\times\pi(2.5)^2\times 10^{-4}\times 10}{310\times 10^{-6}\times 10^3}}$$

$= \sqrt{63.30}$ = 7.95

No answer matches so Bonus by NTA

3. Ans. (4)



Since height of water column is constant therefore, water inflow rate (Q_{in})
= water outflow rate

$Q_{in} = 10^{-4}\ m^3s^{-1}$

$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$

$10^{-4} = 10^{-4}\ \sqrt{20 \times h}$

$h = \frac{1}{20} m$

h = 5cm

∴ correct answer is (4)

4. Ans. (2)

Momentum per second carried by liquid per second is $\rho a v^2$

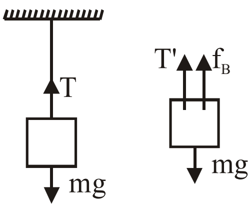
net force due to reflected liquid $= 2 \times \left[\frac{1}{4}\rho a v^2\right]$

net force due to stopped liquid $= \frac{1}{4}\rho a v^2$

Total force $= \frac{3}{4}\rho a v^2$

net pressure $= \frac{3}{4}\rho v^2$

5. Ans. (2)



$\frac{F}{A} = y.\frac{\Delta \ell}{\ell}$

$\Delta \ell \propto F$ .....(i)

$T = mg$

$T = mg - f_B = mg - \frac{m}{\rho_b}.\rho_\ell.g$

$= \left(1 - \frac{\rho_\ell}{\rho_b}\right) mg$

$= \left(1 - \frac{2}{8}\right) mg$

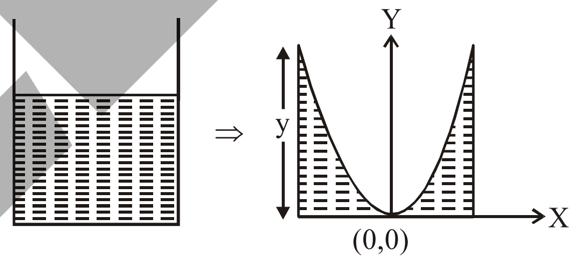
$T' = \frac{3}{4} mg$

From (i)

$\frac{\Delta \ell'}{\Delta \ell} = \frac{T'}{T} = \frac{3}{4}$

$\Delta \ell' = \frac{3}{4} \cdot \Delta \ell = 3\ mm$

6. Ans. (3)



$y = \frac{\omega^2 x^2}{2g} = \frac{(2 \times 2\pi)^2 \times (0.05)^2}{20} \simeq 2\ cm$

7. Ans. (Bonus)

8. Ans. (3)

Sol. Reynolds Number $= \frac{\rho v d}{\eta}$

Volume flow rate = $v \times \pi r^2$

$v = \frac{100 \times 10^{-3}}{60} \times \frac{1}{\pi \times 25 \times 10^{-4}}$

$v = \frac{2}{3\pi} m/s$

Reynolds Number $= \frac{10^3 \times 2 \times 10 \times 10^{-2}}{10^{-3} \times 3\pi}$

$\simeq 2 \times 10^4$

order 10^4

9. Ans. (3)

Sol. In 1st situation

$V_b\rho_b g = V_s\rho_w g$

$\frac{V_s}{V_b} = \frac{\rho_b}{\rho_w} = \frac{4}{5}$ ...(i)

here V_b is volume of block

V_s is submerged volume of block

ρ_b is density of block

ρ_w is density of water

& Let ρ_o is density of oil

finally in equilibrium condition

$V_b\rho_b\, g = \frac{V_b}{2}\rho_O g + \frac{V_b}{2}\rho_w g$

$2\rho_b = \rho_0 + \rho_w$

$\Rightarrow \frac{\rho_O}{\rho_w} = \frac{3}{5} = 0.6$

10. Ans. (3)

Sol. Height of liquid rise in capillary tube h = $\frac{2T\cos\theta_C}{\rho rg}$

$\Rightarrow h \propto \frac{1}{r}$

when radius becomes double height become half

$\therefore h' = \frac{h}{2}$

Now, $M = \pi r^2 h \times \rho$

and $M' = \pi\,(2r)^2\,(h/2) \times \rho = 2M$

Option (3)

11. Ans. (3)

Sol. $P_0 + \rho g d_1 = P_1$

$P_0 + \rho g d_2 = P_2$

$\rho g(d_2 - d_1) = P_2 - P_1$

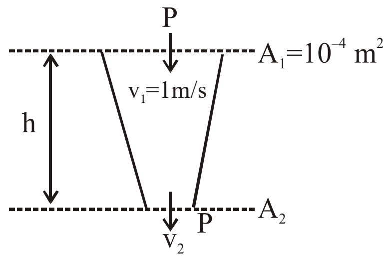
$10^3 \times 10\,(d_2 - d_1) = 3.03 \times 10^6$

$d_2 - d_1 = 303$ m

$\simeq 300$ m

12. Ans. (2)

Sol.



$A_1v_1 = A_2v_2$

$10^{-4} \times 1 = A_2v_2$

$A_2v_2 = 10^{-4}$ ....(1)

$P + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho gh = P$

$v_2^2 = v_1^2 + 2gh$

$v_2 = \sqrt{v_1^2 + 2gh}$

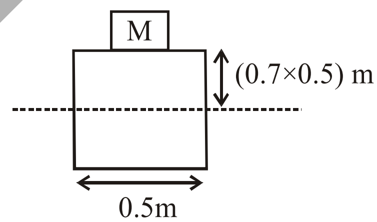
$= \sqrt{1 + 2\times10\times0.15}$

$\frac{10^{-4}}{A_2} = 2$

$A_2 = 5 \times 10^{-5}$ m^2

13. Ans. (2)

Sol.



$M = \rho_L\,[0.5 \times 0.5 \times 0.35]$

$= 10^3\,[0.0875]$

$M = 87.5$ kg

14. Ans. (3)

Sol. $h = \frac{2S_1\cos\theta_1}{r_1\rho_1 g}$

$h = \frac{2S_2\cos\theta_2}{r_2\rho_2 g}$

$\Rightarrow \frac{r_1}{r_2} = \frac{2}{5}$

15. Ans. (4)

Sol. We have

$$V_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_\ell)g \Rightarrow v_T \propto r^2$$

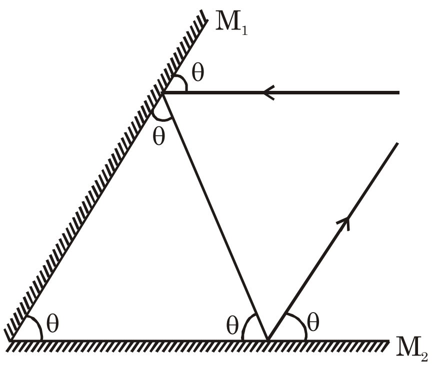
since mass of the sphere will be same

$$\therefore \rho\frac{4}{3}\pi R^3 = 27.\frac{4}{3}\pi r^3\rho \Rightarrow r = \frac{R}{3}$$

$$\therefore \frac{v_1}{v_2} = \frac{R^2}{r^2} = 9$$

GEOMETRICAL OPTICS

1. Ans. (4)



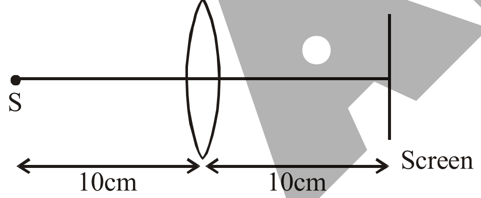
Assuming angles between two mirrors be θ as per geometry,

sum of anlges of Δ

$3\theta = 180°$

$\theta = 60°$

2. Ans. (1)



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{10} - \frac{1}{-10} = \frac{1}{f} \Rightarrow f = 5\text{cm}$$

Shift due to slab $= t\left(1 - \frac{1}{\mu}\right)$ in the direction of incident ray

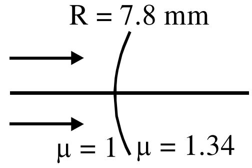
$$= 1.5\left(1 - \frac{2}{3}\right) = 0.5$$

again, $\frac{1}{v} - \frac{1}{-9.5} = \frac{1}{5}$

$$\Rightarrow \frac{1}{u} = \frac{1}{5} - \frac{2}{19} = \frac{9}{95}$$

$$\Rightarrow v = \frac{95}{9} = 10.55\text{cm}$$

3. Ans. (3)



$$\frac{1.34}{V} - \frac{1}{\infty} = \frac{1.34 - 1}{7.8}$$

$\therefore$ V = 30.7 mm

4. Ans. (2)

$$\frac{1}{2f_2} = \frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$\frac{(\mu_1 - 1)}{R} = \frac{(\mu_2 - 1)}{2R}$$

$2\mu_2 - \mu_2 = 1$

5. Ans. (4)

i = e

$$r_1 = r_2 = \frac{A}{2} = 30°$$

by Snell's law

$$1 \times \sin i = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

i = 60

6. Ans. (2)

Since $D_m = (\mu - 1)A$

& on increasing the wavelength, μ decreases & hence D_m decreases. Therefore correct answer is (2)

7. Ans. (3)

From lens equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{(-20)} = \frac{1}{(.3)} = \frac{10}{3}$$

$$\frac{1}{v} = \frac{10}{3} - \frac{1}{20}$$

$\frac{1}{v}=\frac{197}{60}; v=\frac{60}{197}$

$m=\left(\frac{v}{u}\right)=\frac{\left(\frac{60}{197}\right)}{20}$

velocity of image wrt. to lens is given by

$v_{I/L} = m^2 v_{O/L}$

direction of velocity of image is same as that of object

$v_{O/L} = 5$ m/s

$v_{I/L} = \left(\frac{60\times1}{197\times20}\right)^2 (5)$

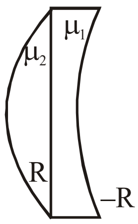
$= 1.16 \times 10^{-3}$ m/s towards the lens

8. Ans. (1)

From $\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

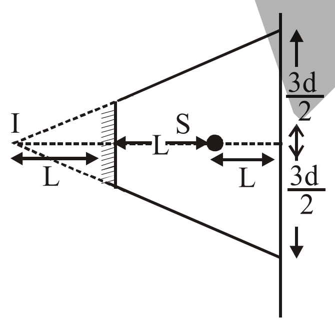
Focal length of lens will change hence image disappears from the screen.

9. Ans. (3)



$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1-\mu_1}{R}+\frac{\mu_2-1}{R}$

10. Ans. (1)



3d

11. Ans. (4)

For first lens

$\frac{1}{V}-\frac{1}{-20}=\frac{1}{5}$

$V=\frac{20}{3}$

For second lens

$V=\frac{20}{3}-2=\frac{14}{3}$

$\frac{1}{V}-\frac{1}{\frac{14}{3}}=\frac{1}{-5}$

V = 70cm

12. Ans. (2)

Sol.

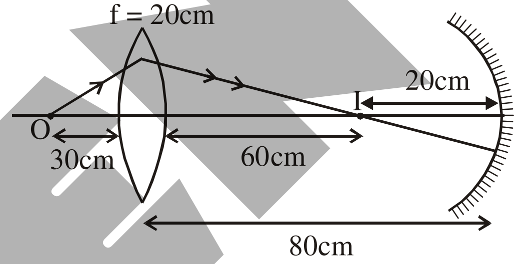


Image formed by lens

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\frac{1}{v}+\frac{1}{30}=\frac{1}{20}$

v = +60 cm

If image position does not change even when mirror is removed it means image formed by lens is formed at centre of curvature of spherical mirror.

Radius of curvature of mirror = 80 – 60 = 20 cm

⇒ focal length of mirror f = 10 cm

for virtual image, object is to be kept between focus and pole.

⇒ maximum distance of object from spherical mirror for which virtual image is formed, is 10cm.

13. Allen Ans. is BONUS

Final Ans. by NTA (2)

Sol. Note :

There will be 3 phenomenon

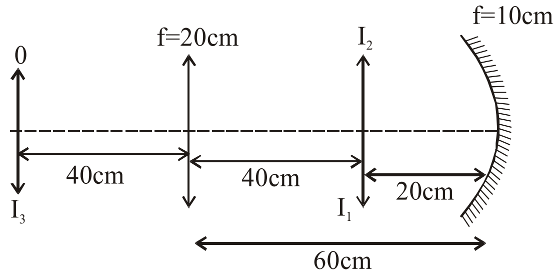
(i) Refraction from lens

(ii) Reflection from mirror

(iii) Refraction from lens

After these phenomena. Image will be on object and will have same size.

None of the option depicts so this question is Bonus



1st refraction $u = -40cm; f = +20cm$

$\Rightarrow v = +40$ cm (image I_1)

and $m_1 = -1$

for reflection

$u = -20cm; f = -10cm$

$\Rightarrow v = -20cm$ (image I_2)

and $m_2 = -1$

2nd refraction

$u = -40cm; f = +20cm$

$\Rightarrow v = +40cm$ (image I_3)

and $m_3 = -1$

Total magnification $= m_1 \times m_2 \times m_3 = -1$

and final image is formed at distance 40cm from convergent lens and is of same size as the object

14. Allen Ans. is 1 or 2

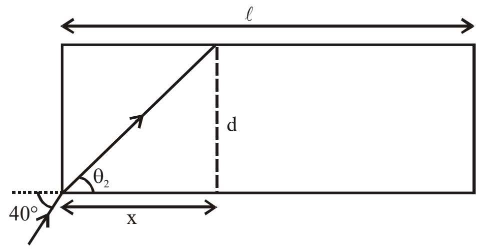
Final Ans. by NTA (2)

Sol. Note :

If we approximate the angle θ_2 as 30° initially then answer will be closer to 57000. But if we solve thorougly, answer will be close to 55000.

So both the answers must be awarded.

Detailed solution is as following.



Exact solution

by Snells' law $1.\sin40° = (1.31)\sin\theta_2$

$$\sin\theta_2 = \frac{.64}{1.31} = \frac{64}{131} \approx .49$$

Now $\tan\theta_2 = \frac{64}{\sqrt{(131)^2-(64)^2}} = \frac{64}{\sqrt{13065}} \approx \frac{64}{114.3} = \frac{d}{x}$

Now no. of reflections

$$= \frac{2\times64}{114.3\times20\times10^{-6}} = \frac{64\times10^5}{114.3}$$

$\approx 55991 \approx 55000$

Approximate solution

By Snells' law $1.\sin40° = (1.31)\sin\theta_2$

$$\sin\theta_2 = \frac{.64}{1.31} = \frac{64}{131} \approx .49$$

If assume $\Rightarrow \theta_2 \approx 30°$

$$\tan 30° = \frac{d}{x} \Rightarrow x = \sqrt{3}d$$

Now number of reflections

$$= \frac{\ell}{\sqrt{3}d} = \frac{2}{\sqrt{3}\times20\times10^{-6}} = \frac{10^5}{\sqrt{3}}$$

$\approx 57735 \approx 57000$

15. Ans. (2)

Sol. $\frac{1}{f_1} = \frac{1}{2}\times\frac{2}{18} = \frac{1}{18}$

$$\frac{1}{f_2} = \frac{(\mu_1-1)}{-18}$$

when μ_1 is filled between lens and mirror

$$P = \frac{2}{18} - \frac{2}{18}(\mu_1-1) = \frac{2-2\mu_1+2}{18}$$

$= F_m = -\left(\frac{18}{2-\mu_1}\right)$

$2 = 6 - 3\mu_1$

$3\mu_1 = 4$

$\mu_1 = 4/3.$

16. Ans. (4)

Sol. Magnification is 2

If image is real, $x_1 = \frac{3f}{2}$

If image is virtual, $x_2 = \frac{f}{2}$

$\frac{x_1}{x_2} = 3:1.$

17. Ans. (4)

Sol. $m = \frac{f}{f-u}$

$5 = \frac{-40}{-40-u}$

$u = -32$ cm

Option (4)

18. Ans. (4)

Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$1 - \frac{v}{u} = \frac{v}{f}$

$1 - m = \frac{v}{f}$

$m = 1 - \frac{v}{f}$

At $v = a$, $m_1 = 1 - \frac{a}{f}$

At $v = a + b$, $m_2 = 1 - \frac{a+b}{f}$

$m_2 - m_1 = c = \left[1 - \frac{a+b}{f}\right] - \left[1 - \frac{a}{f}\right]$

$c = \frac{b}{f}$

$f = \frac{b}{c}$

19. Ans. (4)

Sol. For 1st lens $\frac{1}{f_1} = \left(\frac{\mu_1 - 1}{1}\right)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R}$

for 2nd lens $\frac{1}{f_2} = \left(\frac{\mu_2 - 1}{1}\right)\left(\frac{1}{-R} - 0\right) = -\frac{\mu_2 - 1}{R}$

$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$

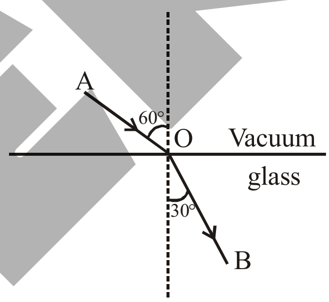
$\frac{1}{f_{eq}} = \frac{R}{\mu_1 - 1} + \frac{R}{-(\mu_2 - 1)} \Rightarrow \frac{1}{f_{eq}} = \frac{R}{\mu_1 - \mu_2}$

Hence $f_{eq} = \frac{\mu_1 - \mu_2}{R}$

20. Ans. (1)

Sol. From Snell's law

$1.\sin 60° = \mu \sin 30°$



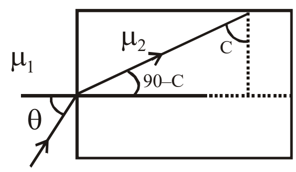
$\Rightarrow \mu = \sqrt{3}$

Optical path = AO + μ(OB)

$= \frac{a}{\cos 60°} + \sqrt{3}\frac{b}{\cos 30°}$

21. Ans. (2)

Sol.



$\sin c = \frac{\mu_1}{\mu_2}$

$\mu_1 \sin\theta = \mu_2 \sin(90° - C)$

$\sin\theta = \frac{\mu_2\sqrt{1 - \frac{\mu_1^2}{\mu_2^2}}}{\mu_1}$

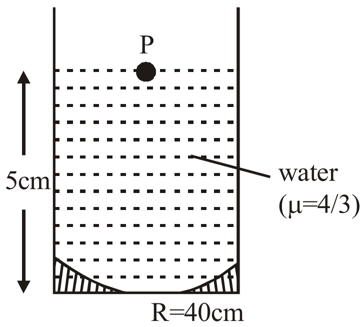
$\theta = \sin^{-1}\sqrt{\frac{\mu_2^2 - \mu_1^2}{\mu_1^2}}$

For TIR

$$\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$$

22. Ans. (1)

Sol. Light incident from particle P will be reflected at mirror



$$u=-5\text{cm},\ f=-\frac{R}{2}=-20\text{cm}$$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\boxed{v_1=+\frac{20}{3}\text{cm}}$$

This image will act as object for light getting refracted at water surface

So, object distance $d=5+\frac{20}{3}=\frac{35}{3}$cm

below water surface.

After refraction, final image is at

$$d'=d\left(\frac{\mu_2}{\mu_1}\right)$$

$$=\left(\frac{35}{3}\right)\left(\frac{1}{4/3}\right)$$

$$=\frac{35}{4}=8.75\text{cm}$$

$$\approx 8.8\text{cm}$$

GRAVITATION

1. Ans. (3)

$U_{surface} + E_1 = U_h$

KE of satelite is zero at earth surface & at height h

$$-\frac{GM_em}{R_e}+E_1=-\frac{GM_em}{(Re+h)}$$

$$E_1 = GM_em\left(\frac{1}{R_e}-\frac{1}{R_e+h}\right)$$

$$E_1 = \frac{GM_em}{(R_e+h)}\times\frac{h}{R_e}$$

Gravitational attraction $F_G = ma_C = \frac{mv^2}{(R_e+h)}$

$$E_2 \Rightarrow \frac{mv^2}{(R_e+h)}=\frac{GM_em}{(R_e+h)^2}$$

$$mv^2 = \frac{GM_em}{(R_e+h)}$$

$$E_2 = \frac{mv^2}{2}=\frac{GM_em}{2(R_e+h)}$$

$E_1 = E_2$

$$\frac{h}{R_e}=\frac{1}{2} \Rightarrow h = \frac{R_e}{2}=3200\text{km}$$

2. Ans. (3)

$$\frac{dA}{dt}=\frac{L}{2m}$$

3. Ans. (4)

By energy convervation between 0 & ∞.

$$-\frac{GMm}{r}+\frac{-GMm}{r}+\frac{1}{2}mV^2=0+0$$

[M is mass of star m is mass of meteroite)

$$\Rightarrow\ v=\sqrt{\frac{4GM}{r}}=2.8\times10^5\,\text{m/s}$$

4. Ans. (2)

At height r from center of earth. orbital velocity

$$=\sqrt{\frac{GM}{r}}$$

$\therefore$ By energy conservation

$$\text{KE of 'm'} + \left(-\frac{GMm}{r}\right) = 0+0$$

(At infinity, PE = KE = 0)

$$\Rightarrow \text{KE of 'm'} = \frac{GMm}{r} = \left(\sqrt{\frac{GM}{r}}\right)^2 m = mv^2$$

5. Ans. (2)

$$\because g = \frac{GM}{R^2}$$

$$\frac{g_p}{g_e} = \frac{M_e}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

Also $T \propto \frac{1}{\sqrt{g}}$

$$\Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$$

$$\Rightarrow T_p = 2\sqrt{3}s$$

6. Ans. (1)

$$v_0 = \sqrt{g(R+h)} \approx \sqrt{gR}$$

$$v_e = \sqrt{2g(R+h)} \approx \sqrt{2gR}$$

$$\Delta v = v_e - v_0 = \left(\sqrt{2}-1\right)\sqrt{gR}$$

7. Ans. (3)

Orbital velocity $V = \sqrt{\frac{GMe}{r}}$

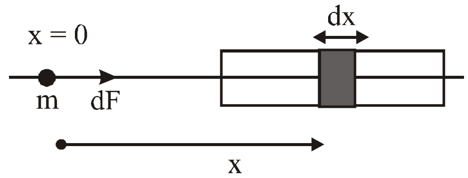
$$T_A = \frac{1}{2}m_A V_A^2$$

$$T_B = \frac{1}{2}m_B V_B^2$$

$$\Rightarrow \frac{T_A}{T_B} = \frac{m \times \frac{Gm}{R}}{2m \times \frac{Gm}{2R}}$$

$$\Rightarrow \frac{T_A}{T_B} = 1$$

8. Ans. (2)



$dm = (A + Bx^2)dx$

$$dF = \frac{GM\,dm}{x^2}$$

$$= F = \int_a^{a+L} \frac{GM}{x^2}\left(A+Bx^2\right)dx$$

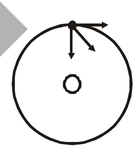
$$= GM\left[-\frac{A}{x}+Bx\right]_a^{a+L}$$

$$= GM\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$$

9. Ans. (3)

$$mv\hat{i} + mv\hat{j}$$

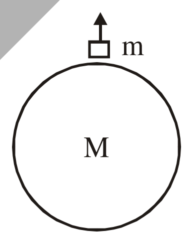
$$= 2m\vec{v}^1$$



$$\vec{v} = \frac{1}{\sqrt{2}} \times \sqrt{\frac{GM}{R}}$$

10. Ans. (2)

Sol.



minimum energy required (E) = – (Potential energy of object at surface of earth)

$$= -\left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

Now $M_{earth} = 64\ M_{moon}$

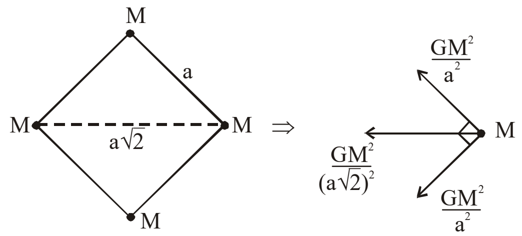
$$\rho.\frac{4}{3}\pi R_e^3 = 64.\frac{4}{3}\pi R_m^3 \Rightarrow R_e = 4R_m$$

$$\text{Now } \frac{E_{moon}}{E_{earth}} = \frac{M_{moon}}{M_{earth}}.\frac{R_{earth}}{R_{moon}} = \frac{1}{64}\times\frac{4}{1}$$

$$\Rightarrow E_{moon} = \frac{E}{16}$$

11. Ans. (3)

Sol.



Net force on particle towards centre of circle

is $F_C = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\sqrt{2}$

$= \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$

This force will act as centripetal force. Distance of particle from centre of circle is

$\frac{a}{\sqrt{2}}$.

$r = \frac{a}{\sqrt{2}}$, $F_C = \frac{mv^2}{r}$

$$\frac{mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$v^2 = \frac{GM}{a}\left(\frac{1}{2\sqrt{2}} + 1\right)$$

$$v^2 = \frac{GM}{a}(1.35)$$

$$v = 1.16\sqrt{\frac{GM}{a}}$$

12. Ans. (4)

Sol. $m = \int_0^R \rho\, 4\pi r^2 dr$

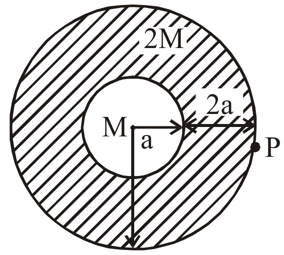
$m = 4\pi KR$

$v \propto \sqrt{4\pi K}$

$\frac{T}{R} = \frac{2\pi}{\sqrt{4\pi K}}$.

13. Ans. (2)

Sol.



We use gauss's Law for gravitation

$g \cdot 4\pi r^2 =$ (Mass enclosed) $4\pi G$

$$g = \frac{3M4\pi G}{4\pi(3a)^2}$$

$$= \frac{MG}{3a^2}$$

Option (2)

14. Ans. (2)

Sol. $F_g = \frac{mv^2}{r}$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$V = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11})(8\times10^{22})}{2.02\times10^6}}$$

$V = 1.625 \times 10^3$

$$T = \frac{2\pi r}{V}$$

$n \times T = 24 \times 60 \times 60$

$$n\left[\frac{2\pi(2.02\times10^6)}{1.625\times10^3}\right] = 24\times3600$$

$$n = \frac{24\times3600\times1.625\times10^3}{2\pi(2.02\times10^6)}$$

$n = 11$

15. Ans. (4)

Sol. $\frac{GM}{(R+h)^2} = \frac{GM}{2R^2}$

$R + h = \sqrt{2}R$

$h = (\sqrt{2} - 1)R$

$\simeq 2.6\times10^6$ m

16. Ans. (2)

Sol. Since mass of the object remains same

$\therefore$ Weight of object will be proportional to 'g' (acceleration due to gravity)

Given

$$\frac{W_{earth}}{W_{planet}} = \frac{9}{4} = \frac{g_{earth}}{g_{planet}}$$

Also, $g_{surface} = \frac{GM}{R^2}$ (M is mass planet, G is universal gravitational constant, R is radius of planet)

$$\therefore \frac{9}{4} = \frac{GM_{earth}\,R^2_{planet}}{GM_{planet}\,R^2_{earth}} = \frac{M_{earth}}{M_{planet}} \times \frac{R^2_{Planet}}{R^2_{earth}} = 9\frac{R^2_{Planet}}{R^2_{earth}}$$

$$\therefore R_{planet} = \frac{R_{earth}}{2} = \frac{R}{2}$$

HEAT & THERMODYNAMICS

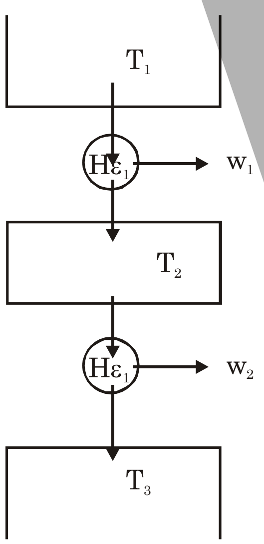
1. Ans. (1)

$Q = nC_v\Delta T$ as gas in closed vessel

$$Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

$Q = 10000\text{ J} = 10\text{ kJ}$

2. Ans. (3)



$w_1 = w_2$

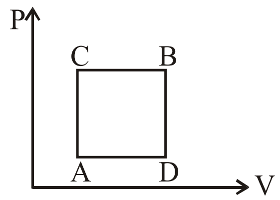
$\Delta u_1 = \Delta u_2$

$T_3 - T_2 = T_2 - T_1$

$2T_2 = T_1 + T_3$

$T_2 = 500\text{ K}$

3. Ans. (4)



$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

$\Rightarrow 60\text{ J} = 30\text{ J} + \Delta U_{ACB}$

$\Rightarrow \Delta U_{ACB} = 30\text{ J}$

$\Rightarrow \Delta U_{ADB} = \Delta U_{ACB} = 30\text{ J}$

$\Delta Q_{ACD} = \Delta U_{ACB} + \Delta W_{ADB}$

$= 10\text{ J} + 30\text{ J} = 40\text{ J}$

4. Ans. (4)

$$\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = 3.16$$

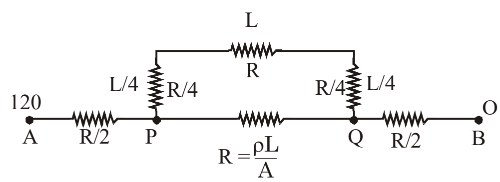
5. Ans. (3)

Young's modulus $y = \frac{\text{Stress}}{\text{Strain}}$

$$= \frac{F/A}{(\Delta\ell/\ell)}$$

$$= \frac{F}{A(\alpha\Delta T)}$$

6. Ans. (4)



$$\frac{\Delta T}{R_{eq.}} = I = \frac{(120)5}{8R} = \frac{120 \times 5}{8R}$$

$$\Delta T_{PQ} = \frac{120 \times 5}{8R} \times \frac{3}{5}R = \frac{360}{8} = 45°C$$

7. Ans. (4)

$192 \times S \times (100 - 21.5)$

$= 128 \times 394 \times (21.5 - 8.4)$

$+ 240 \times 4200 \times (21.5 - 8.4)$

$\Rightarrow S = 916$

8. Ans. (2)

$$WD = P\Delta V = nR\Delta T = \frac{1}{2} \times 8.31 \times 70$$

9. Ans. (4)

Thermal energy of N molecule

$$= N\left(\frac{3}{2}kT\right)$$

$$= \frac{N}{N_A}\frac{3}{2}RT$$

$$= \frac{3}{2}(nRT)$$

$$= \frac{3}{2}PV$$

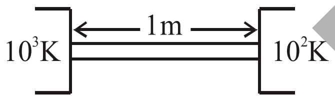
$$= \frac{3}{2}P\left(\frac{m}{8}\right)$$

$$= \frac{3}{2} \times 4 \times 10^4 \times \frac{2}{8}$$

$= 1.5 \times 10^4$

order will 10^4

10. Ans. (1)



$$\left(\frac{dQ}{dt}\right) = \frac{kA\Delta T}{\ell}$$

$$\Rightarrow \frac{1}{A}\left(\frac{dQ}{dt}\right) = \frac{(0.1)(900)}{1} = 90 W/m^2$$

11. Ans. (1)

$$t_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_4}{T_3}$$

$$\Rightarrow T_2 = \sqrt{T_1T_3} = \sqrt{T_1\sqrt{T_2T_4}}$$

$$T_3 = \sqrt{T_2T_4}$$

$$T_2^{3/4} = \sqrt{T_1^{1/2}T_4^{1/4}}$$

$$T_2 = T_1^{2/3}T_4^{1/3}$$

12. Ans. (2)

$$\Delta\ell_1 = \Delta\ell_2$$

$$\ell\alpha_1\Delta T_1 = \ell\alpha_2\Delta T_2$$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta T_1}{\Delta T_2}$$

$$\frac{4}{3} = \frac{T-30}{180-30}$$

$$\boxed{T = 230^\circ C}$$

13. Ans. (1)

$100 \times S_A \times [100 - 90] = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5\ S_B$

$$S_A = \frac{3}{4}S_B$$

Now, $100 \times S_A \times [100 - T] = 50 \times S_B(T - 50)$

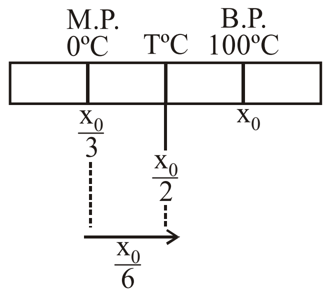
$$2 \times \left(\frac{3}{4}\right)(100 - T) = (T - 50)$$

$300 - 3T = 2T - 100$

$400 = 5T$

$T = 80$

14. Ans. (2)



$$\Rightarrow T^\circ C = \frac{x_0}{6} \ \& \ \left(x_0 - \frac{x_0}{3}\right) = (100 - 0^\circ C)$$

$$x_0 = \frac{300}{2}$$

$$\Rightarrow T^\circ C = \frac{150}{6} = 25^\circ C$$

15. Ans. (1)

VT = K

$\Rightarrow V\left(\frac{PV}{nR}\right) = k \Rightarrow PV^2 = K$

$\because \; C = \frac{R}{1-x} + C_V$ (For polytropic process)

$C = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2}$

$\therefore \; \Delta Q = nC\,\Delta T$

$= \frac{R}{2} \times \Delta T$

16. Ans. (2)

$0.1 \times 400 \times (500 - T) = 0.5 \times 4200 \times (T - 30)$
$+ 800\,(T - 30)$

$\Rightarrow 40(500 - T) = (T - 30)\,(2100 + 800)$

$\Rightarrow 20000 - 40T = 2900\,T - 30 \times 2900$

$\Rightarrow 20000 + 30 \times 2900 = T(2940)$

$T = 30.4°C$

$\frac{\Delta T}{T} \times 100 = \frac{6.4}{30} \times 100$

$\simeq 20\%$

17. Ans. (2)

For adiabatic process : $TV^{\gamma-1}$ = constant

For diatomic process : $\gamma - 1 = \frac{7}{5} - 1$

$\therefore x = \frac{2}{5}$

18. Ans. (3)

$U = \frac{f_1}{2} n_1 RT + \frac{f_2}{2} n_2 RT$

$= \frac{5}{2}(3RT) + \frac{3}{2} \times 5RT$

U = 15RT

19. Ans. (2)

Let amount of ice is m gm.
According to principal of calorimeter
heat taken by ice = heat given by water

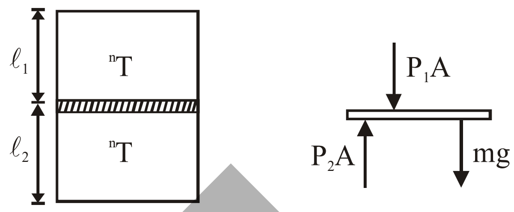
$\therefore \; 20 \times 2.1 \times m + (m - 20) \times 334$
$= 50 \times 4.2 \times 40$

376 m = 8400 + 6680

m = 40.1

$\therefore$ correct answer is (2)

20. Ans. (2)



$P_2A = P_1A + mg$

$\frac{nRT.A}{A\ell_2} = \frac{nRT.A}{A\ell_1} + mg$

$nRT\left(\frac{1}{\ell_2} - \frac{1}{\ell_1}\right) = mg$

$m = \frac{nRT}{g}\left(\frac{\ell_1 - \ell_2}{\ell_1 . \ell_2}\right)$

21. Ans. (1)

$t \propto \frac{\text{Volume}}{\text{velocity}}$

volume $\propto \frac{T}{P}$

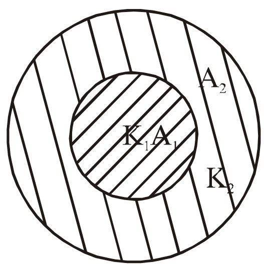
$\therefore \; t \propto \frac{\sqrt{T}}{P}$

$\frac{t_1}{6 \times 10^{-8}} = \frac{\sqrt{500}}{2P} \times \frac{P}{\sqrt{300}}$

$t_1 = 3.8 \times 10^{-8}$

$\approx 4 \times 10^{-8}$

22. Ans. (4)



$K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$

$= \frac{K_1(\pi R^2) + K_2(3\pi R^2)}{4\pi R^2} = \frac{K_1 + 3K_2}{4}$

23. Ans. (4)

Energy = $\frac{1}{2}nRT = \frac{f}{2}PV$

$= \frac{f}{2}(3\times10^6)(2)$

$= f \times 3 \times 10^6$

Considering gas is monoatomic i.e. f = 3

E. = 9×10^6 J

Option-(4)

24. Ans. (3)

Since P–V indicator diagram is given, so work done by gas is area under the cyclic diagram.

$\therefore$ ΔW = Work done by gas = $\frac{1}{2}\times 4\times 5$ J

= 10 J

25. Ans. (4)

Sol. isochoric $\rightarrow$ Process d

isobaric $\rightarrow$ Process a

Adiabatic slope will be more than isothermal so

Isothermal $\rightarrow$ Process b

Adiabatic $\rightarrow$ Process c

order $\rightarrow$ d a b c

26. Ans. (3)

Sol. $v_{rms} = \sqrt{\frac{3RT}{m}}$ $\qquad v_{escape} = \sqrt{2gR_e}$

$v_{rms} = v_{escape}$

$\frac{3RT}{m} = 2gR_e$

$\frac{3\times1.38\times10^{-23}\times6.02\times10^{26}}{2}\times T$

$= 2 \times 10 \times 6.4 \times 10^6$

$T = \frac{4\times10\times6.4\times10^6}{3\times1.38\times6.02\times10^3} = 10\times10^3 = 10^4$k

Note : Question gives avogadro Number $N_A = 6.02 \times 10^{26}$ /kg but we take $N_A = 6.02 \times 10^{26}$/kmol.

27. Ans. (3)

Sol. Energy of catapult $= \frac{1}{2}\times\left(\frac{\Delta\ell}{\ell}\right)^2\times Y\times A\times\ell$

= Kinetic energy of the ball $= \frac{1}{2}mv^2$

therefore,

$\frac{1}{2}\times\left(\frac{20}{42}\right)^2\times Y\times\pi\times3^2\times10^{-6}\times42\times10^{-2} = \frac{1}{2}\times2\times10^{-2}\times(20)^2$

$Y \simeq 3\times10^6\ Nm^{-2}$

28. Ans. (1)

Sol. $-ms\frac{dT}{dt} = e\sigma A\left(T^4 - T_0^4\right)$

$-\frac{dT}{dt} = \frac{e\sigma A}{ms}\left(T^4 - T_0^4\right)$

$-\frac{dT}{dt} = \frac{4e\sigma AT_0^3}{ms}(\Delta T)$

$T = T_0 + (T_i - T_0)e^{-kt}$

where $k = \frac{4e\sigma AT_0^3}{ms}$

$k = \frac{4e\sigma AT_0^3}{\rho vs}$

$\left|\frac{dT}{dt}\right| \propto k$

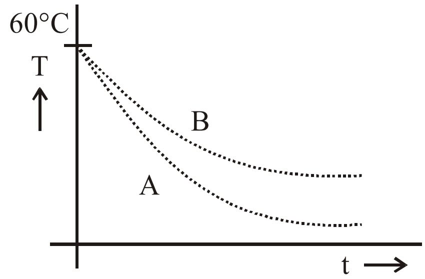
$\therefore \left|\frac{dT}{dt}\right| \propto \frac{1}{\rho s}$

$\rho_A s_A = 2000 \times 8 \times 10^2 = 16 \times 10^5$

$\rho_B s_B = 4000 \times 10^3 = 4 \times 10^6$

$\rho_A s_A < \rho_B s_B$

$\left|\frac{dT}{dt}\right|_A > \left|\frac{dT}{dt}\right|_B$



29. Ans. (4)

Sol. Tensile stresss in wire will be

$= \frac{\text{Tensile force}}{\text{Cross section Area}}$

$= \frac{mg}{\pi R^2} = \frac{4\times3.1\pi}{\pi\times4\times10^{-6}}Nm^{-2} = 3.1\times10^6\ Nm^{-2}$

30. Ans. (3)

Sol. Suppose 'm' gram of water evaporates then, heat required

$\Delta Q_{req} = mL_v$

Mass that converts into ice = (150 – m)

So, heat released in this process

$\Delta Q_{rel} = (150 - m)\, L_f$

Now,

$\Delta Q_{rel} = \Delta Q_{req}$

$(150 - m)\, L_f = mL_V$

$m(L_f + L_v) = 150\, L_f$

$$m = \frac{150L_f}{L_f + L_v}$$

m = 20g

31. Allen Ans. is BONUS

Final Ans. by NTA (3)

Sol. Note :

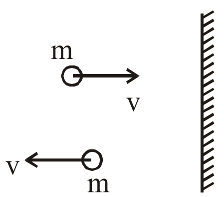
Pressure is defined as normal force per unit area.

Force is calculated as change in momentum/ time

By this answer is 2N/m²

None of the option matches so this question must be Bonus

Detailed solution is as following.



Magnitude of change in momentum per collision = 2mv

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{N(2mv)}{1}$$

$$= \frac{10^{22} \times 2 \times 10^{-26} \times 10^{4}}{1}$$

$= 2N/m^2$

32. Ans. (4)

Sol. By law of conservation of energy

$$\frac{1}{2}kx^2 = (m_1s_1 + m_2s_2)\,\Delta T$$

$$\Delta T = \frac{16 \times 10^{-2}}{4384} = 3.65 \times 10^{-5}.$$

33. Ans. (4)

Sol. For A

$R = C_p - C_v = 7$

$$C_v = \frac{fR}{2} = 22 \Rightarrow f = \frac{44}{7} = 6.3$$

$f \simeq 6$ ↗ 5 (Rotation + Translational)
↘ 1 (Vibration)

For B

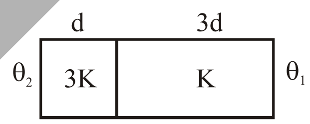
$R = C_p - C_v = 9$

$$C_v = \frac{fR}{2} = 21 \Rightarrow f = \frac{42}{9}$$

$f \simeq 5$ ↗ 5 (Rotation + Translational)
↘ 0 (Vibration)

34. Ans. (2)

Sol.



Let the temperature of interface be "θ"

$i_1 = i_2$ {Steady state conduction}

$$\frac{3KA(\theta_2 - \theta)}{d} = \frac{KA(\theta - \theta_1)}{3d}$$

$$\theta = \frac{9\theta_2}{10} + \frac{\theta_1}{10}$$

35. Ans. (3)

Sol. According to equipartion energy theorem

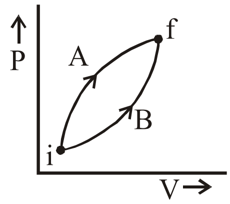
$$\frac{1}{2}m\left(v_{rms}^2\right) = 3 \times \frac{1}{2}K_bT$$

$$T = \frac{m\overline{v}_{rms}^2}{3k}$$

∴ correct option should be (3)

36. Ans. (2)

Sol.



Initial and final states for both the processes are same.

$\therefore \Delta U_A = \Delta U_B$

Work done during process A is greater than in process B.

By First Law of thermodynamics

$\Delta Q = \Delta U + W$

$\Rightarrow \Delta Q_A > \Delta Q_B$

Option (2)

37. Ans. (2)

Sol. $V_{rms} = \sqrt{\dfrac{3RT}{M_w}} \Rightarrow v_{rms} \propto \sqrt{T}$

Now, $\dfrac{v}{200} = \sqrt{\dfrac{500}{400}} \Rightarrow \dfrac{v}{200} = \dfrac{\sqrt{5}}{2}$

$\Rightarrow v = 100\sqrt{5}$ m/s

Option (2)

38. Ans. (1)

Sol. $\dfrac{F}{A} = \text{stress}$

$$\frac{400 \times 4}{\pi d^2} = 379 \times 10^6$$

$$d^2 = \frac{1600}{\pi \times 379 \times 10^6} = 1.34 \times 10^{-6}$$

$$d = \sqrt{1.34} \times 10^{-3} = 1.15 \times 10^{-3} \text{ m}$$

39. Ans. (1)

Sol. $Q = nC_v \Delta T$

$Q' = nC_p \Delta T$

$\therefore \dfrac{Q'}{Q} = \dfrac{C_p}{C_v}$

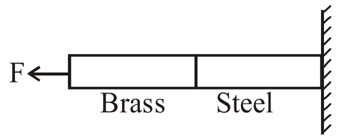
For diatomic gas : $\dfrac{C_p}{C_v} = \gamma = \dfrac{7}{5}$

$Q' = \dfrac{7}{5}Q$

40. Allen Answer is BONUS

Final Ans. by NTA (2)

Sol.



$$k_1 = \frac{y_1 A_1}{\ell_1} = \frac{120 \times 10^9 \times A}{1}$$

$$k_2 = \frac{y_2 A_2}{\ell_2} = \frac{60 \times 10^9 \times A}{1}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 \times k_2} = \frac{120 \times 60}{180} \times 10^9 \times A$$

$k_{eq} = 40 \times 10^9 \times A$

$F = k_{eq}(x)$

$F = (40 \times 10^9)A \,.\, (0.2 \times 10^{-3})$

$\dfrac{F}{A} = 8 \times 10^6 \text{ N/m}^2$

No option is matching. Hence question must be bonus.

41. Ans. (3)

Sol. $P = P_0\left[1 - \dfrac{1}{2}\left(\dfrac{V_0}{V}\right)^2\right]$

Pressure at $V_0 = P_0\left(1 - \dfrac{1}{2}\right) = \dfrac{P_0}{2}$

Pressure at $2V_0 = P_0\left(1 - \dfrac{1}{2} \times \dfrac{1}{4}\right) = \dfrac{7}{8}P_0$

Temperature at $V_0 = \dfrac{\dfrac{P_0}{2}V_0}{nR} = \dfrac{P_0V_0}{2nR}$

Temperature at $2V_0 = \dfrac{\left(\dfrac{7}{8}P_0\right)(2V_0)}{nR} = \dfrac{\dfrac{7}{4}P_0V_0}{nR}$

Change in temperature $= \left(\dfrac{7}{4} - \dfrac{1}{2}\right)\dfrac{P_0V_0}{nR}$

$= \dfrac{5}{4}\dfrac{P_0V_0}{nR} = \dfrac{5P_0V_0}{4R}$

42. Ans. (1)

Sol. $\Delta Q = nC_V\,\Delta T = n\frac{3}{2}R\Delta T$

$$= \left(\frac{67.2}{22.4}\right)\left(\frac{3}{2}\times 8.31\right)(20)$$

≈ 748 J

43. Allen Ans. (3)

Final Ans. by NTA (4)

Sol. $\nu = \frac{V_{av}}{\lambda}$

$$\lambda = \frac{RT}{\sqrt{2}\pi\sigma^2 N_A P}$$

$\sigma = 2 \times .3 \times 10^{-9}$

$$P = \frac{RT}{V}$$

$$\Rightarrow \lambda = \frac{V}{\sqrt{2}\pi\sigma^2 N_A}$$

$$V_{av} = \sqrt{\frac{8}{3\pi}} \times V_{rms}$$

$$\therefore\ \nu = \frac{200\times\sqrt{2}\pi\times\sigma^2 N_A}{25\times 10^{-3}}\times\sqrt{\frac{8}{3\pi}}$$

$= 17.68 \times 10^8$/sec.

$= .1768 \times 10^{10}$/sec. $\sim 10^{10}$

This answer does not match with JEE-Answer key

44. Ans. (3)

Sol. $w = nR\Delta T$

$\Delta H = (C_v + nR)\,\Delta T$

$$\frac{w}{\Delta H} = \frac{nR}{C_v + nR}$$

45. Ans. (2)

Sol. $Q = P \times t$

$Q = mc\Delta T + mL$

$$P = \frac{V_{rms}^2}{R}$$

$$4200 \times 80 + 2260 \times 10^3 = \frac{(200)^2}{20}\times t$$

$t = 1298$ sec

$t \simeq 22$ min

46. Ans. (3)

Sol. Given number density of molecules of gas as a function of r is

$$n(r) = n_0 e^{-\alpha r^4}$$

$\therefore$ Total number of molecule $= \int_0^\infty n(r)\,dV$

$$= \int_0^\infty n_0 e^{-\alpha r^4} 4\pi r^2\,dr$$

$\therefore$ Number of molecules is proportional to $n_0\alpha^{-3/4}$

47. Official Ans. by NTA (2)

Final Ans. by NTA (4)

Sol. Efficiency of Carnot engine $= 1 - \frac{T_{sink}}{T_{source}}$

Given,

$$\frac{1}{6} = 1 - \frac{T_{sink}}{T_{source}} \quad \Rightarrow \quad \frac{T_{sink}}{T_{source}} = \frac{5}{6} \quad .....(1)$$

Also,

$$\frac{2}{6} = 1 - \frac{T_{sink} - 62}{T_{source}} \quad \Rightarrow \quad \frac{62}{T_{source}} = \frac{1}{6} \quad .....(2)$$

$\therefore\ T_{source} = 372\ K = 99ºC$

Also, $T_{sink} = \frac{5}{6} \times 372 = 310\ K = 37ºC$

(**Note :-** Temperature of source is more than temperature of sink)

48. Ans. (1)

Sol. For a diatomic gas, $C_p = \frac{7}{2}R$

Since gas undergoes isobaric process

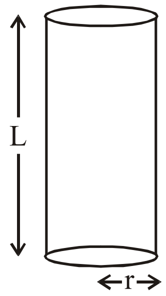
$\Rightarrow\ \Delta Q = nC_P\Delta T$

Also, $\Delta W = nR\Delta T = 10J$(given)

$$\therefore\ \Delta Q = n\frac{7}{2}R\Delta T = \frac{7}{2}(nR\Delta T) = 35\ J$$

49. Ans. (2)

Sol.



$\because$ Length of cylinder remains unchanged

so $\left(\frac{F}{A}\right)_{Compressive} = \left(\frac{F}{A}\right)_{Thermal}$

$\frac{F}{\pi r^2} = Y\alpha T$

(α is linear coefficient of expansion)

$\therefore\ \alpha = \frac{F}{YT\pi r^2}$

$\therefore$ The coefficient of volume expansion $\gamma = 3\alpha$

$\therefore\ \gamma = 3\frac{F}{YT\pi r^2}$

50. Ans. (3)

Sol. Heat lost = Heat gain

$\Rightarrow M_2 \times 1 \times 50 = M_1 \times 0.5 \times 10 + M_1.L_f$

$\Rightarrow L_f = \frac{50M_2 - 5M_1}{M_1}$

$= \frac{50M_2}{M_1} - 5$

51. Ans. (3)

Sol. $f_{mix} = \frac{n_1f_1 + n_2f_2}{n_1 + n_2} = \frac{2\times3+3\times5}{5} = \frac{21}{5}$

$C_V = \frac{fR}{2} = \frac{21}{5} \times \frac{R}{2} = 17.4\,J/mol\,K$

52. Ans. (2)

Sol. $Mg = \left(\frac{Ay}{\ell}\right)\Delta\ell$

$\frac{\Delta\ell}{\ell} = \alpha\Delta T$

$Mg = (Ay)\alpha\Delta T = 2\pi$

It is closest to 9.

53. Ans. (4)

Sol.

	ΔE	ΔW	ΔQ
ab			250
bc		0	60
ca	–180		

	ΔE	ΔW	ΔQ
ab	120	130	250
bc	60	0	60
ca	–180		

KINEMATICS

1. Ans. (3)

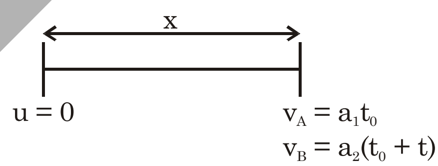
$v_x = -a\omega\sin\omega t \Rightarrow v_y = a\omega\cos\omega t$

$v_z = a\omega \Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$v = \sqrt{2}a\omega$

2. Ans. (4)

For A & B let time taken by A is t_0



from ques.

$v_A - v_B = v = (a_1 - a_2)t_0 - a_2t$ ....(i)

$x_B = x_A = \frac{1}{2}a_1t_0^2 = \frac{1}{2}a_2(t_0+t)^2$

$\Rightarrow \sqrt{a_1}t_0 = \sqrt{a_2}(t_0+t)$

$\Rightarrow \left(\sqrt{a_2} - \sqrt{a_2}\right)t_0 = \sqrt{a_2}t$ ....(ii)

putting t_0 in equation

$v = (a_1 - a_2)\frac{\sqrt{a_2}t}{\sqrt{a_1} - \sqrt{a_2}} - a_2t$

$= \left(\sqrt{a_1} + \sqrt{a_2}\right)\sqrt{a_2}t - a_2t \Rightarrow v = \sqrt{a_1a_2}t$

$\Rightarrow \sqrt{a_1a_2}t + a_2t - a_2t$

3. Ans. (2)

$\frac{dx}{dt} = ky, \frac{dy}{dt} = kx$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$

$\Rightarrow ydy = xdx$

Integrating both side

$y^2 = x^2 + c$

4. Ans. (2)

S = Area under graph

$\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9$ m

5. Ans. (4)

$R = \frac{u^2 \sin 2\theta}{g}$

$A = \pi R^2$

$A \propto R^2$

$A \propto u^4$

$\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$

6. Ans. (1)

$\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{i} + 4\hat{j})4$

$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$

$\vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$

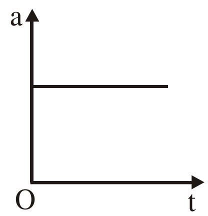
$\vec{r}_f = 20\hat{i} + 20\hat{j}$

$|\vec{r}_f| = 20\sqrt{2}$

7. Ans. (4)

8. Ans. (3)

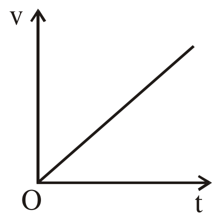
Sol. Given initial velocity u = 0 and acceleration is constant

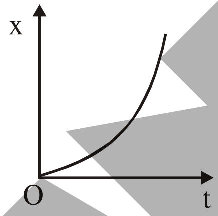


At time t

$v = 0 + at \Rightarrow v = at$

also $x = 0(t) + \frac{1}{2}at^2 \Rightarrow x = \frac{1}{2}at^2$

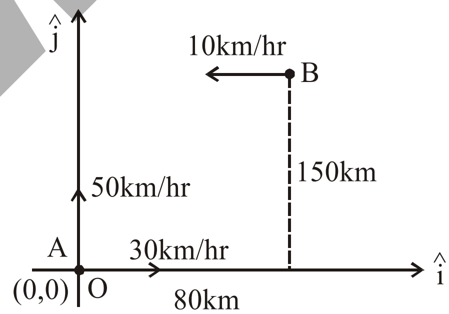




Graph (A) ; (B) and (D) are correct.

9. Ans. (4)

Sol. If we take the position of ship 'A' as origin then positions and velocities of both ships can be given as :



$\vec{v}_A = (30\hat{i} + 50\hat{j})$ km / hr

$\vec{v}_B = -10\hat{i}$ km / hr

$\vec{r}_A = 0\hat{i} + 0\hat{j}$

$\vec{r}_B = (80\hat{i} + 150\hat{j})$ km

Time after which distance between them will be minimum

$t = -\frac{\vec{r}_{BA} \cdot \vec{v}_{BA}}{|\vec{v}_{BA}|^2}$;

where $\vec{r}_{BA} = (80\hat{i} + 150\hat{j})$ km

$\vec{v}_{BA} = -10\hat{i} - \left(30\hat{i} + 50\hat{j}\right)$

$\left(-40\hat{i} - 50\hat{j}\right)\text{km / hr}$

$$\therefore t = -\frac{\left(80\hat{i} + 150\hat{j}\right)\cdot\left(-40\hat{i} - 50\hat{j}\right)}{\left|-40\hat{i} - 50\hat{j}\right|^2}$$

$$= \frac{3200 + 7500}{4100}\text{hr} = \frac{10700}{4100}\text{hr} = 2.6\text{hrs}$$

10. Ans. (4)

Sol. $x = at + bt^2 - ct^3$

$$v = \frac{dx}{dt} = a + 2bt - 3ct^2$$

$$a = \frac{dv}{dt} = 2b - 6ct = 0 \Rightarrow t = \frac{b}{3c}$$

$$v_{\left(\text{at } t = \frac{b}{3c}\right)} = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)$$

$$= a + \frac{b^2}{3c}.$$

11. Ans. (4)

Sol. $\vec{r} = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} + (-40t)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$$

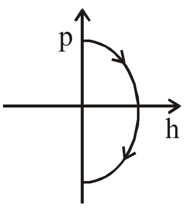
$|\vec{a}| = 50\text{m / s}^2.$

12. Ans. (1)

Sol. Momentum $p = mv$ .....(1)

and for motion under gravity $h = \dfrac{u^2 - v^2}{2g}$ ...(2)

$$h = \frac{u^2 - p^2/m^2}{2g}$$



Option (1)

13. Ans. (2)

Sol. $t = \dfrac{2 \times 2 \times \sin 15°}{g\cos 30°}$

$$S = 2\cos 15° \times t - \frac{1}{2}g\sin 30° t^2$$

Put values and solve

$S \simeq 20\text{cm}$

14. Ans. (2)

Sol. $\nu = b\sqrt{x}$

$$\frac{d\nu}{dt} = \frac{b}{2\sqrt{x}}\frac{dx}{dt}$$

$$a = \frac{b\nu}{2\sqrt{x}}$$

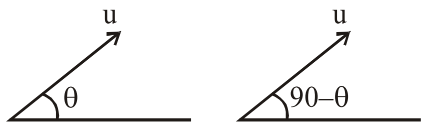
$$a = \frac{b\left(b\sqrt{x}\right)}{2\sqrt{x}}$$

$$\frac{d\nu}{dt} = a = \frac{b^2}{2}$$

$$\nu = \frac{b^2}{2}\tau$$

15. Ans. (2)

Sol.



For same range angle of projection will be θ & 90 – θ

$$R = \frac{u^2 2\sin\theta\cos\theta}{g}$$

$$h_1 = \frac{u^2\sin^2\theta}{g}$$

$$h_2 = \frac{u^2\sin^2(90 - \theta)}{g}$$

$$\frac{R^2}{h_1 h_2} = 16$$

16. Ans. (1)

Sol. Equation of trajectory is given as

$y = 2x - 9x^2$ .... (1)

Comparing with equation :

$y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}.x^2$ .... (2)

We get;

$\tan\theta = 2$

$\therefore \boxed{\cos\theta = \dfrac{1}{\sqrt{5}}}$

Also, $\dfrac{g}{2u^2\cos^2\theta} = 9$

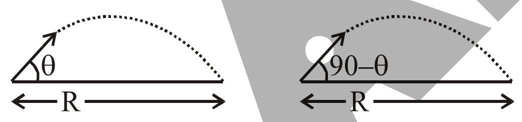
$\Rightarrow \dfrac{10}{2\times 9\times\left(\dfrac{1}{\sqrt{5}}\right)^2} = u^2$

$\Rightarrow u^2 = \dfrac{25}{9}$

$\Rightarrow \boxed{u = \dfrac{5}{3}\,m/s}$

17. Ans. (3)

Sol. Range will be same for time t_1 & t_2, so angles of projection will be 'θ' & '90° – θ'



$t_1 = \dfrac{2u\sin\theta}{g}$ $t_2 = \dfrac{2u\sin(90°-\theta)}{g}$

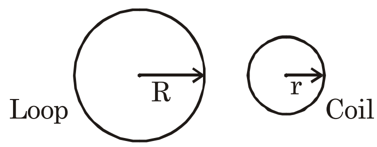
and $R = \dfrac{u^2\sin 2\theta}{g}$

$t_1t_2 = \dfrac{4u^2\sin\theta\cos\theta}{g^2} = \dfrac{2}{g}\left[\dfrac{2u^2\sin\theta\cos\theta}{g}\right]$

$= \dfrac{2R}{g}$

MEC

1. Ans. (3)



$L = 2\pi R$ $L = N\times 2\pi r$

$R = Nr$

$B_L = \dfrac{\mu_0 i}{2R}$ $B_C = \dfrac{\mu_0 Ni}{2r}$

$B_C = \dfrac{\mu_0 N^2 i}{2R}$

$\dfrac{B_L}{B_C} = \dfrac{1}{N^2}$

2. Ans. (1)

$\dfrac{mv^2}{R} = qvB$

$mv = qBR$ ....(i)

Path is straight line

it $qE = qvB$

$E = vB$ ....(ii)

From equation (i) & (ii)

$m = \dfrac{qB^2R}{E}$

$m = 2.0\times 10^{-24}$ kg

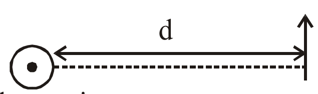
3. Ans. (2)

Coercivity = $H = \dfrac{B}{\mu_0}$

$= ni = \dfrac{N}{\ell}i = \dfrac{100}{0.2}\times 5.2$

$= 2600$ A/m

4. Ans. (3)



∞ long wire

Eqvilent dipole of given loop

$F = m\cdot\dfrac{dB}{dr}$

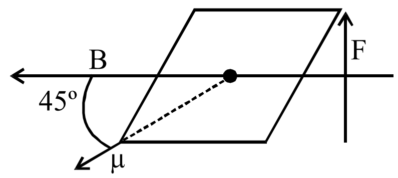
Now $\frac{dB}{dx} = \frac{d}{dx}\left(\frac{\mu_0 I}{2\pi x}\right)$

$\propto \frac{1}{x^2}$

$\Rightarrow$ So $F \propto \frac{M}{x^2} [\because M = NIA]$

$\therefore F \propto \frac{a^2}{d^2}$

5. Ans. (2)



$\mu B \sin 45° = F\frac{\ell}{2}\sin 45°$

$F = 2\mu B$

6. Ans. (4)

$T = 2\pi\sqrt{\frac{I}{\mu B}}$

$T_h = 2\pi\sqrt{\frac{mR^2}{(2\mu)B}}$

$T_C = 2\pi\sqrt{\frac{1/2mR^2}{\mu B}}$

7. Ans. (2)

According to JEE-Mains Ans. key (Bonus)

Work done, $W = (\Delta\vec{\mu})\cdot\vec{B}$

$= 2 \times 10^{-2} \times 1 \cos(0.125)$

$= 0.02$ J

8. Ans. (1)

$\because M = NIA$

$dq = \lambda dx$ & $A = \pi x^2$

$\int dm = \int (x)\frac{\rho_0 x}{\ell} dx.\pi x^2$

$M = \frac{n\rho_0\pi}{\ell}.\int_0^{\ell} x^3.dx = \frac{n\rho_0\pi}{\ell}.\left[\frac{L^4}{4}\right]$

$M = \frac{n\rho_0\pi\ell^3}{4}$ or $\frac{\pi}{4}n\rho\ell^3$

9. Ans. (3)

$\chi = \frac{I}{H}$

$I = \frac{\text{Magnetic moment}}{\text{Volume}}$

$I = \frac{20 \times 10^{-6}}{10^{-6}} = 20 \text{ N/m}^2$

$\chi = \frac{20}{60 \times 10^{+3}} = \frac{1}{3} \times 10^{-3}$

$= 0.33 \times 10^{-3} = 3.3 \times 10^{-4}$

10. Ans. (1)

$R_g = 20\Omega$

$N_L = N_R = N = 30$

FOM $= \frac{I}{\phi} = 0.005$ A/Div.

Current sentivity = CS = $\left(\frac{1}{0.005}\right) = \frac{\phi}{I}$

$Ig_{max} = 0.005 \times 30$

$= 15 \times 10^{-2} = 0.15$

$15 = 0.15 [20 + R]$

$100 = 20 + R$

$R = 80$

11. Ans. (BONUS)

In question its is not given from which point on y-axis charge particle is given velocity

12. Ans. (4)

$\vec{F}_{net} = q\vec{E} + q(\vec{v} \times \vec{B})$

$= (2q\hat{i} + 3q\hat{j}) + q(\vec{v} \times \vec{B})$

$W = \vec{F}_{net}.\vec{S}$

$= 2q + 3q$

$= 5q$

13. Ans. (4)

$M = \mu_0 n_1 n_2 \pi r_1^2$

$L = \mu_0 n_1^2 \pi r_1^2$

$\Rightarrow \frac{M}{L} = \frac{n_2}{n_1}$

14. Ans. (1)

$$r = \frac{\sqrt{2mk}}{eB} = \frac{\sqrt{2me\Delta v}}{eB}$$

$$r = \frac{\sqrt{\frac{2m}{e}.\Delta v}}{B} = \frac{\sqrt{\frac{2\times 9.1\times 10^{-31}}{1.6\times 10^{-19}}(500)}}{100\times 10^{-3}}$$

$$r = \frac{\sqrt{\frac{9.1}{0.16}\times 10^{-10}}}{10^{-1}} = \frac{3}{.4}\times 10^{-4} = 7.5\times 10^{-4}$$

15. Ans. (3)

$x \ \alpha \ \frac{1}{T_C}$

curie law for paramagnetic substane

$$\frac{x_1}{x_2} = \frac{T_{C_2}}{T_{C_1}}$$

$$\frac{2.8\times 10^{-4}}{x_2} = \frac{300}{350}$$

$$x_2 = \frac{2.8\times 350\times 10^{-4}}{300}$$

$= 3.266 \times 10^{-4}$

16. Ans. (2)

Induied emf $= Bv\ell$

$= 0.3 \times 10^{-4} \times 5 \times 10$

$= 1.5 \times 10^{-3}$ V

17. Ans. (1)

$KE = q\Delta V$

$$r = \frac{\sqrt{2mq\Delta V}}{qB}$$

$$r \propto \sqrt{\frac{m}{q}}$$

$$\frac{r_p}{r_\alpha} = \frac{1}{\sqrt{2}}$$

18. Ans. (4)

Magnetic field at 'O' will be done to 'PS' and 'QN' only

i.e. $B_0 = B_{PS} + B_{QN} \rightarrow$ Both inwards

Let current in each wire = i

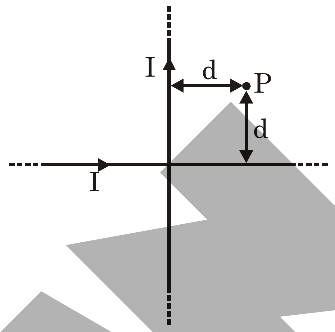
$$\therefore \quad B_0 = \frac{\mu_0 i}{4\pi d} + \frac{\mu_0 i}{4\pi d}$$

$$\text{or} \quad 10^{-4} = \frac{\mu_0 i}{2\pi d} = \frac{2\times 10^{-7}\times i}{4\times 10^{-2}}$$

$\therefore \quad i = 20$ A

19. Ans. (1)

Sol.

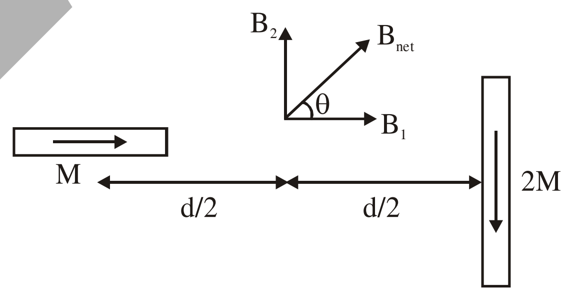


Magnetic field at point P

$$\vec{B}_{net} = \frac{\mu_0 i}{2\pi d}\left(-\hat{k}\right) + \frac{\mu_0 i}{2\pi d}\left(\hat{k}\right) = 0$$

20. Ans. (4)

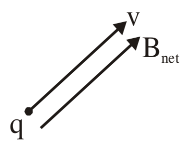
Sol.



$$B_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}; \quad B_2 = \left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}$$

$B_1 = B_2$

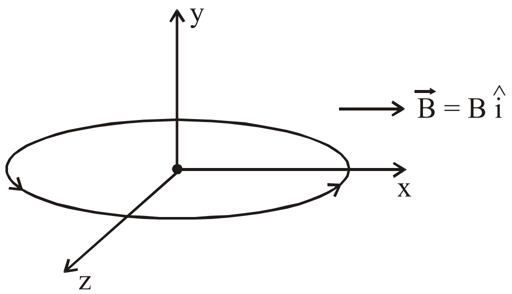
$\Rightarrow B_{net}$ is at 45° ($\theta = 45°$)



velocity of charge and B_{net} are parallel so by $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$ force on charge particle is zero.

21. Ans. (1)

Sol.



Magnetic moment of coil = NIA $\hat{j}$

$= NI\left(\pi r^2\right)\hat{j}$

Torque on loop (coil) = $\vec{M}\times\vec{B}$

$= NI\left(\pi r^2\right)B\sin 90°\left(-\hat{k}\right)$

$= NI\pi r^2 B\left(-\hat{k}\right)$

22. Ans. (2)

Sol. $\phi_q = \dfrac{\mu_0 i_1 R^2}{2(R^2+x^2)^{\frac{3}{2}}}\times \pi r^2 = 10^{-3}$

$\phi_p = \dfrac{\mu_0 i_2 r^2}{2(r^2+x^2)^{\frac{3}{2}}}\times \pi R^2$

$\dfrac{\phi_P}{\phi_Q} = \dfrac{i_2}{i_1}\cdot\dfrac{(R^2+x^2)^{\frac{3}{2}}}{(r^2+x^2)^{\frac{3}{2}}} = \dfrac{\phi_P}{10^{-3}}$

$\dfrac{2}{3} = \dfrac{\phi_P}{10^{-3}}$

$\phi_P = 6.67 \times 10^{-4}$.

23. Ans. (1)

Sol. $\tau = \vec{M}\times\vec{B}$

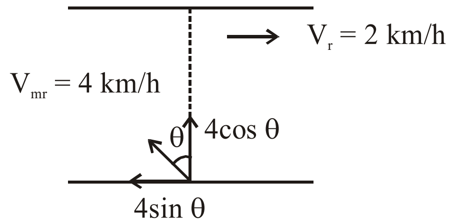
$C\theta = i\,N\,A\,B$

$10^{-6}\times\dfrac{\pi}{180} = 10^{-3}\times 10^{-4}\times 175\times B$

$B = 10^{-3}$ Tesla.

24. Ans. (4)

Sol.



For swimmer to cross the river straight

$\Rightarrow\ 4\sin\theta = 2$

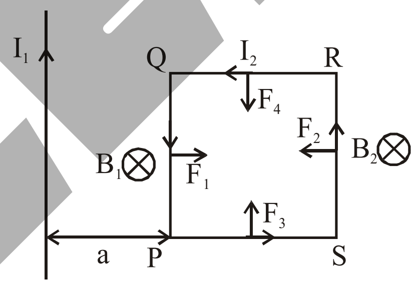
$\Rightarrow \sin\theta = \dfrac{1}{2} \Rightarrow \theta = 30°$

So, angle with direction of river flow = 90° + θ = 120°

Option (4)

25. Ans. (2)

Sol.



F_3 & F_4 cancel each other

Force on PQ will be $F_1 = I_2 B_1 a$

$= I_2 \dfrac{\mu_0 I_1}{2\pi a} a$

$= \dfrac{\mu_0 I_1 I_2}{2\pi}$

Force on RS will be $F_2 = I_2 B_2 a$

$= I_2 \dfrac{\mu_0 I_1}{2\pi 2a} a$

$= \dfrac{\mu_0 I_1 I_2}{4\pi}$

Net force = $F_1 - F_2 = \dfrac{\mu_0 I_1 I_2}{4\pi}$ repulsion

Option (2)

26. Ans. (2)

Sol. $|\vec{\tau}| = |\vec{M} \times \vec{B}|$

$\tau = NI \times A \times B \times \sin 45°$

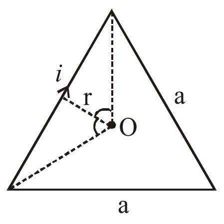
$\tau = 0.27$ Nm

Option (2)

27. Ans. (1)

Sol. $B = 3\left[\frac{\mu_0 i}{4\pi r}(\sin 60° + \sin 60°)\right]$

Here, $r = \frac{a}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$



$B = 3\left[\frac{4\pi \times 10^{-7} \times 10 \times 2\sqrt{3}}{4\pi \times 1}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right]\right]$

$B = 18 \times 10^{-6} = 18\ \mu T$

28. Ans. (2)

Sol. $m = NIA = 1 \times I \times a^2$

here a = side of square

Now,

$4a = 2\pi r$

$r = \frac{2a}{\pi}$

For circular loop

$m' = 1 \times I \times \pi r^2$

$= 1 \times I \times \pi \times \left(\frac{2a}{\pi}\right)^2$

$m' = \frac{4m}{\pi}$

29. Ans. (3)

Sol. $X = 5\ YZ^2$

$Y = \frac{X}{5Z^2}$

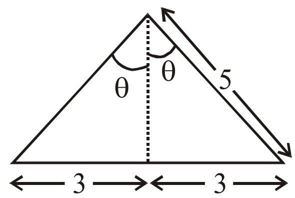
$[Y] = \frac{[X]}{[Z^2]}$

$= \frac{A^2.M^{-1}L^{-2}.T^4}{(MA^{-1}T^{-2})^2}$

$= M^{-3} . L^{-2} . T^8 . A^4$

30. Ans. (4)

Sol.



$B = \frac{\mu_0 I}{4\pi d}\ 2\sin\theta$

$d = 4$ cm

$\sin\theta = \frac{3}{5}$

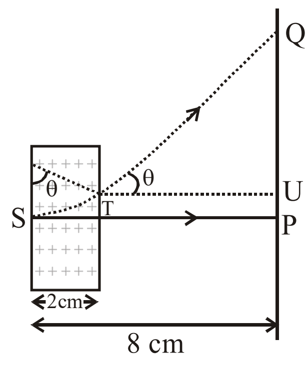
31. Ans. (1)

Sol. $R = \frac{mv}{qB}$

$= \frac{\sqrt{2m(K.E.)}}{qB}$

$R = \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times (100 \times 1.6 \times 10^{-19})}}{1.6 \times 10^{-19} \times 1.5 \times 10^{-3}}$

$R = 2.248$ cm



$\sin\theta = \frac{2}{2.248}$

$\tan\theta = \frac{QU}{TU}$

$$\frac{2}{1.026} = \frac{QU}{6}$$

QU = 11.69

PU = R(1 – cos θ)

= 1.22

d = QU + PU

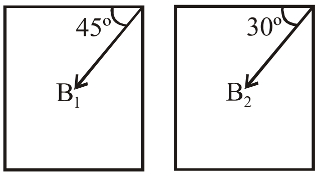
32. Ans. (2)

Sol. $B = \frac{\mu_0 i}{2R} = \frac{\mu_0 q\omega}{2R\,2\pi}$

$\Rightarrow q = 3 \times 10^{-5}$ C

33. Ans. (3)

Sol.



$$f_1 = \frac{1}{2\pi}\sqrt{\frac{\mu B_1\cos 45^\circ}{I}} \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{\mu B_2\cos 30^\circ}{I}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{B_1\cos 45^\circ}{B_2\cos 30^\circ}} \qquad \therefore \frac{B_1}{B_2} \simeq 0.7$$

MODERN PHYSICS

1. Ans. (3)

Half life of A = $\ell n2$

$t_{1/2} = \frac{\ell n2}{\lambda}$

$\lambda_A = 1$

at t = 0 $R_A = R_B$

$N_A e^{-\lambda AT} = N_B e^{-\lambda BT}$

$N_A = N_B$ at t = 0

at t = t $\quad \frac{R_B}{R_A} = \frac{N_0 e^{-\lambda_B t}}{N_0 e^{-\lambda_A t}}$

$e^{-(\lambda_B - \lambda_A)t} = e^{-t}$

$\lambda_B - \lambda_A = 3$

$\lambda_B = 3 + \lambda_A = 4$

$t_{1/2} = \frac{\ell n2}{\lambda_B} = \frac{\ell n2}{4}$

2. Ans. (1)

B = $B_0\sin(\pi \times 10^7C)t + B_0\sin(2\pi \times 10^7C)t$

since there are two EM waves with different frequency, to get maximum kinetic energy we take the photon with higher frequency

$B_1 = B_0\sin(\pi \times 10^7C)t \quad \nu_1 = \frac{10^7}{2}\times C$

$B_2 = B_0\sin(2\pi \times 10^7C)t$

$\nu_2 = 10^7C$

where C is speed of light C = 3×10^8 m/s

$\nu_2 > \nu_1$

so KE of photoelectron will be maximum for photon of higher energy.

$\nu_2 = 10^7C$ Hz

$h\nu = \phi + KE_{max}$

energy of photon

$E_{ph} = h\nu = 6.6 \times 10^{-34} \times 10^7 \times 3 \times 10^9$

$E_{ph} = 6.6 \times 3 \times 10^{-19}$J

$= \frac{6.6\times3\times10^{-19}}{1.6\times10^{-19}}eV = 12.375eV$

$KE_{max} = E_{ph} - \phi$

= 12.375 – 4.7 = 7.675 eV ≈ 7.7 eV

3. Ans. (1)

Activity A = λN

For A $\quad 10 = (2N_0)\lambda_A$

For B $\quad 20 = N_0\lambda_B$

$\therefore \lambda_B = 4\lambda_A \Rightarrow (T_{1/2})_A = 4(T_{1/2})_B$

4. Ans. (1)

$$\frac{hc}{\lambda_1} = \phi + \frac{1}{2}m(2v)^2$$

$$\frac{hc}{\lambda_2} = \phi + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = 4 \Rightarrow \frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$$

$$\Rightarrow \frac{4hc}{\lambda_2} - \frac{hc}{\lambda_1} = 3\phi$$

$$\Rightarrow \phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= \frac{1}{3}\times1240\left(\frac{4\times350-540}{350\times540}\right)$$

= 1.8 eV

5. Ans. (3)

$Ne^{20} \rightarrow 2He^4 + C^{12}$

$8.03 \times 20 \quad 2 \times 7.07 \times 4 + 7.86 \times 12$

$\because$ value $= (BE)_{product} - (BE)_{react} = -9.72$ MeV

So 9.72 MeV will have to be supplied.

No answer matching but closest answer is option (3).

6. Ans. (4)

$$I = \frac{nE}{At}$$

$$16 \times 10^{-3} = \left(\frac{n}{t}\right)_{Photon} \frac{10 \times 1.6 \times 10^{-19}}{10^{-4}} = 10^{12}$$

7. Ans. (3)

at t = 0, $A_0 = \frac{dN}{dt} = 1600 C/s$

at t = 8s, A = 100 C/s

$\frac{A}{A_0} = \frac{1}{16}$ in 8 sec

Therefor half life is $t_{1/2} = 2$ sec

$\therefore$ Activity at t = 6 will be $1600\left(\frac{1}{2}\right)^3$

$= 200$ C/s

$\therefore$ correct answer is (3)

8. Ans. (3)

$\lambda = \frac{h}{p}$ $\quad \{\lambda = 7.5 \times 10^{-12}\}$

$$P = \frac{h}{\lambda}$$

$$KE = \frac{P^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{\left\{\frac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right\}^2}{2 \times 9.1 \times 10^{-31}} J$$

KE = 25 Kev

9. Ans. (3)

For M $\rightarrow$ L steel

$$\frac{1}{\lambda} = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{K \times 5}{36}$$

for N $\rightarrow$ L

$$\frac{1}{\lambda'} = K\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{K \times 3}{16}$$

$$\lambda' = \frac{20}{27}\lambda$$

10. Ans. (2)

$$\frac{hc}{\lambda_1} = \phi + eV_1 \quad \text{....... (i)}$$

$$\frac{hc}{\lambda_2} = \phi + eV_2 \quad \text{....... (ii)}$$

(i) – (ii)

$$hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = e(V_1 - V_2)$$

$$\Rightarrow V_1 - V_2 = \frac{hc}{e}\left(\frac{\lambda_2 - \lambda_1}{\lambda_1 - \lambda_2}\right)$$

$$= (1240nm - V)\frac{100nm}{300nm \times 400nm}$$

$= 1V$

11. Ans. (4)

Energy of photon $= \frac{12500}{980} = 12.75 eV$

$\therefore$ Electron will excite to n= 4

Since 'R' $\propto n^2$

$\therefore$ Radius of atom will be $16a_0$

12. Ans. (1)

$$\frac{h}{mv} = 10^{-3}\left(\frac{3 \times 10^8}{6 \times 10^{14}}\right)$$

$$v = \frac{6.63 \times 10^{-34} \times 6 \times 10^{14}}{9.1 \times 10^{-31} \times 3 \times 10^5}$$

$v = 1.45 \times 10^6$ m/s

13. Ans. (Bonus)

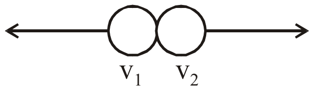
According to JEE-Mains Ans. key (1)

14. Ans. (4)

$${}^{232}_{90}Th \longrightarrow {}^{208}_{78}Y + {}^{4}_{2}He$$

$${}^{208}_{78}Y \longrightarrow {}^{208}_{82}X + 4\beta \text{ praticle}$$

15. Ans. (1)



$mv_0 = mv_2 - mv_1$

$\frac{1}{2}mV_1^2 = 0.36 \times \frac{1}{2}mV_0^2$

$v_1 = 0.6v_0$

$\frac{1}{2}MV_2^2 = 0.64 \times \frac{1}{2}mV_0^2$

$V_2 = \sqrt{\frac{m}{M}} \times 0.8V_0$

$mV_0 = \sqrt{mM} \times 0.8V_0 - m \times 0.6V_0$

$\Rightarrow 1.6m = 0.8\sqrt{mM}$

$4m^2 = mM$

16. Ans. (3)

$\lambda = \frac{1240}{5.6-0.7}$ nm

17. Ans. (4)

$F = \frac{dV}{dr} = kr = \frac{mv^2}{r}$

$mvr = \frac{nh}{2\pi}$

$r^2 \propto n$

$r^2 \propto \sqrt{n}$

$E = \frac{1}{2}kr^2 + \frac{1}{2}mv^2 \propto r^2$

$\propto n$

18. Ans. (2)

K.E. acquired by charge = K = qV

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$

$\therefore \frac{\lambda_A}{\lambda_B} = \frac{\sqrt{2m_Bq_BV_B}}{\sqrt{2m_Aq_AV_A}} = \sqrt{\frac{4m.q.2500}{m.q.50}} = 2\sqrt{50}$

$= 2 \times 7.07 = 14.14$

19. Ans. (1)

Sol. mass densities of all nuclei are same so their ratio is 1.

20. Ans. (2)

Sol. $A = A_0e^{-\gamma t}$

$A = \frac{A_0}{2}$ after 10 oscillations

$\because$ After 2 seconds

$\frac{A_0}{2} = A_0e^{-\gamma(2)}$

$2 = e^{2\gamma}$

$\ell n2 = 2\gamma$

$\gamma = \frac{\ell n2}{2}$

$\because A = A_0e^{-\gamma t}$

$\ell n\frac{A_0}{A} = \gamma t$

$\ell n1000 = \frac{\ell n2}{2}t$

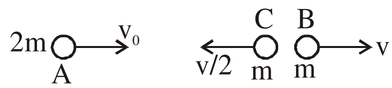
$2\left(\frac{3\ell n10}{\ell n2}\right) = t$

$\frac{6\ell n10}{\ell n2} = t$

$t = 19.931$ sec

$t \approx 20$ sec

21. Ans. (4)

Sol. 

let mass of B and C is m each.

By momentum conservation

$2mv_0 = mv - \frac{mv}{2}$

$v = 4v_0$

$p_A = 2mv_0 \quad p_B = 4mv_0 \quad p_c = 2mv_0$

De-Broglie wavelength $\lambda = \frac{h}{p}$

$\lambda_A = \frac{h}{2mv_0}$; $\lambda_B = \frac{h}{4mv_0}$; $\lambda_C = \frac{h}{2mv_0}$

22. Ans. (2)

Sol. Energy released for transition n = 2 to n = 1 of hydrogen atom

$$E = 13.6 Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$Z = 1$, $n_1 = 1$, $n_2 = 2$

$$E = 13.6\times 1\times\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$E = 13.6\times\frac{3}{4}\text{eV}$$

For He^+ ion z = 2

(1) n = 1 to n = 4

$$E = 13.6\times 2^2\times\left(\frac{1}{1^2}-\frac{1}{4^2}\right) = 13.6\times\frac{15}{4}\text{eV}$$

(2) n = 2 to n = 4

$$E = 13.6\times 2^2\times\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = 13.6\times\frac{3}{4}\text{eV}$$

(3) n = 2 to n = 5

$$E = 13.6\times 2^2\times\left(\frac{1}{2^2}-\frac{1}{5^2}\right) = 13.6\times\frac{21}{25}\text{eV}$$

(4) n = 2 to n = 3

$$E = 13.6\times 2^2\times\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = 13.6\times\frac{5}{9}\text{eV}$$

Energy required for transition of He^+ for n = 2 to n = 4 matches exactly with energy released in transition of H for n = 2 to n = 1.

23. Ans. (4)

Sol. $\circ\!\longrightarrow \quad \frac{h}{\lambda_1} = P_1$ $\qquad \uparrow \quad P_2 = \frac{h}{\lambda_2}$

$$\vec{P}_1 = \frac{h}{\lambda_1}\hat{i}$$

$$\& \ \vec{P}_2 = \frac{h}{\lambda_2}\hat{j}$$

Using momentum conservation

$$\vec{P} = \vec{P}_1 + \vec{P}_2$$

$$= \frac{h}{\lambda_1}\hat{i} + \frac{h}{\lambda_2}\hat{j}$$

$$|\vec{P}| = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{h}{\lambda} = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

24. Ans. (3)

Sol. $(x)\longrightarrow \qquad \longleftarrow(y) \quad = \quad (P)\longrightarrow$

By momentum conservation

$P_x - P_y = P_P$

$$\frac{h}{\lambda_x} - \frac{h}{\lambda_y} = \frac{h}{\lambda_p}$$

$$\lambda_p = \frac{\lambda_x\lambda_y}{|\lambda_y - \lambda_x|}.$$

25. Ans. (4)

Sol. Force on the surface (25% reflecting and rest absorbing)

$$F = \frac{25}{100}\left(\frac{2I}{C}\right) + \frac{75}{100}\left(\frac{I}{C}\right) = \frac{125}{100}\left(\frac{I}{C}\right)$$

$$= \frac{125}{100}\times\left(\frac{50}{3\times 10^8}\right) = 20.83 \times 10^{-8}\ \text{N}.$$

26. Ans. (2)

Sol. Energy levels in Hydrogen like atom is given by

$$E = -13.6\ \frac{z^2}{n^2}\text{eV}$$

As He^+ is 1st excited state

$\therefore$ z = 2, n = 2

E = –13.6 eV

As total energy of He^+ in 1st excited state is –13.6 eV, ionisation energy should be +13.6eV.

27. Ans. (1)

Sol. $\omega = 6 \times 10^{14} \times 2\pi$

$f = 6 \times 10^{14}$

$C = f\lambda$

$$\lambda = \frac{C}{f} = \frac{3\times10^8}{6\times10^{14}} = 5000\text{Å}$$

energy of photon $\Rightarrow \dfrac{12375}{5000}$

$= 2.475$ eV

from Einstein's equation

$KE_{max} = E - \phi$

$eV_s = E - \phi$

$eV_s = 2.475 - 2$

$eV_s = 0.475 - 2$

$eV_s = 0.475$ eV

$V_s = 0.475$ V $= 0.48$ volt

Option (1)

28. Ans. (3)

Sol. $\dfrac{1}{660} = R\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) = \dfrac{5R}{36}$ ....(1)

$\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{4^2}\right) = \dfrac{3R}{16}$ ....(2)

divide equation (1) with (2)

$$\frac{\lambda}{660} = \frac{5\times16}{36\times3}$$

$$\lambda = \frac{4400}{9} = 488.88 = 488.9 \text{ nm}$$

Option (3)

29. Ans. (1)

Sol. Pressure $= \dfrac{I}{C}$

Force = Pressure × Area $= \dfrac{I}{C}$. Area

Momentum transferred = Force . Δt

$= \dfrac{I}{C}$. Area . Δt

$= \dfrac{25\times10^4}{3\times10^8}\times25\times10^{-4}\times40\times60$

$= 5 \times 10^{-3}$ N-s

30. Ans. (3)

Sol. $P = \dfrac{n.\,hc}{\lambda}$

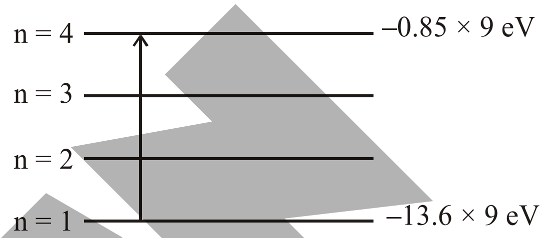
$n = \dfrac{P\lambda}{h.c}$

$= \dfrac{2\times10^{-3}\times500\times10^{-9}}{6.6\times10^{-34}\times3\times10^8}$

$= 1.5 \times 10^{16}$

31. Ans. (3)

Sol.



$\Delta E = \dfrac{hc}{\lambda}$

$13.6 \times 9 - 0.85 \times 9 = \dfrac{hc}{\lambda}$

$\lambda = \dfrac{hc}{9\times(13.6-0.85)\text{eV}}$

$= \dfrac{1240\text{eV.nm}}{9\times12.75\text{eV}}$

$\lambda = 10.8$ nm

32. Ans. (3)

Sol. $N_A = N_0\, e^{-5\lambda t}$

$N_B = N_0\, e^{-\lambda t}$

$\dfrac{N_A}{N_B} = \dfrac{e^{-5\lambda t}}{e^{-\lambda t}} = \dfrac{1}{e^2}$

$\Rightarrow e^{-4\lambda t} = e^{-2}$

$\Rightarrow 4\lambda t = 2$

$\Rightarrow t = \dfrac{1}{2\lambda}$

33. Ans. (3)

Sol. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mK}}{qB}$

$r_{He} = r_p > r_e$

34. Ans. (1)

Sol. $K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$\Rightarrow K_{max} = hc\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$

$\Rightarrow K_{max} = (1237)\left(\frac{380-260}{380\times 260}\right)$

$= 1.5$ eV

35. Ans. (2)

Sol. $N_1 = N_0 e^{-10\lambda t}$

$N_2 = N_0 e^{-\lambda t}$

$\frac{1}{e} = \frac{N_1}{N_2} = e^{-9\lambda t}$

$\Rightarrow 9\lambda t = 1$

$t = \frac{1}{9\lambda}$

36. Ans. (4)

Sol. $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$\frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$\frac{1}{\lambda_1} = R\left(\frac{7}{9\times 16}\right)$

$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$= R\left(\frac{5}{4\times 9}\right)$

$\frac{\lambda_1}{\lambda_2} = \frac{\frac{5}{36}}{\frac{7}{9\times 16}}$

$= \frac{20}{7}$

37. Ans. (3)

Sol. $2\pi r_n = n\lambda_n$

$\lambda_3 = \frac{2\pi(4.65\times 10^{-10})}{3}$

$\lambda_3 = 9.7$ Å

38. Ans. (1)

Sol. $N_A = N_{OA}e^{-\lambda t} = \frac{N_{OA}}{2^{t/t_{1/2}}} = \frac{N_{OA}}{2^6}$

$\therefore$ Number of nuclei decayed

$= N_{OA} - \frac{N_{OA}}{2^6} = \frac{63N_{OA}}{64}$

$N_B = N_{OB}e^{-\lambda t} = \frac{N_{OB}}{2^{t/t_{1/2}}} = \frac{N_{OB}}{2^3}$

$\therefore$ Number of nuclei decayed

$= N_{OB} - \frac{N_{OB}}{2^3} = \frac{7N_{OB}}{8}$

Since $N_{OA} = N_{OB}$

$\therefore$ Ratio of decayed numbers of nuclei

A & B $= \frac{63N_{OA}\times 8}{64\times 7N_{OB}} = \frac{9}{8}$

39. Ans. (3)

Sol. $h\nu = \phi + ev_0$

$v_0 = \frac{h\nu}{e} - \frac{\phi}{e}$

v_0 is zero for $\nu = 4 \times 10^{14}$ Hz

$0 = \frac{h\nu}{e} - \frac{\phi}{e} \Rightarrow \phi = h\nu$

$= \frac{6.63\times 10^{-34}\times 4\times 10^{14}}{1.6\times 10^{-19}} = 1.66$ ev.

40. Ans. (1)

Sol. $\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)z^2$

$\frac{1}{1085} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)2^2$

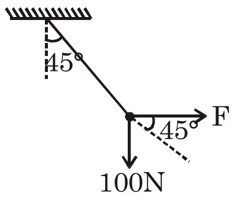
$\frac{1}{304} = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right)2^2$

$\therefore$ m = 2

$\therefore$ n = 5

NLM & FRICTION

1. Ans. (2)

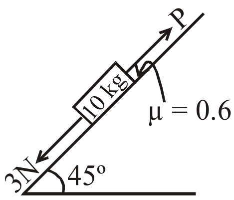


at equation

$\tan 45^\circ = \frac{100}{F}$

F = 100 N

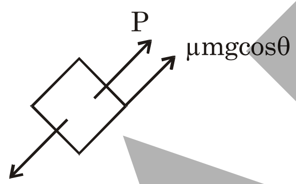
2. Ans. (1)



$mg\sin 45^\circ = \frac{100}{\sqrt{2}} = 50\sqrt{2}$

$\mu mg\cos\theta = 0.6 \times mg \times \frac{1}{\sqrt{2}} = 0.6 \times 50\sqrt{2}$

$P = 31.28 \simeq 32N$



73.7=3+mgsinθ

3. Ans. (1)

$\frac{dp}{dt} = F = kt$

$\int_P^{3P} dP = \int_O^T kt\ dt$

$2p = \frac{KT^2}{2}$

$T = 2\sqrt{\frac{P}{K}}$

4. Ans. (2)

2 + mg sin30 = μmg cos30°

10 = mgsin 30 + μ mg cos30°

= 2μmg cos30 –2

6 = μmg cos 30

4 = mg sin 30

$\frac{3}{2} = \mu \times \sqrt{3}$

$\mu = \frac{\sqrt{3}}{2}$

5. Ans. (4)

Sol. m = 20 g , u = 1 m/s, v = ?

$S = 20 \times 10^{-2}$ m $\qquad a = \frac{-2.5 \times 10^{-2}}{20 \times 10^{-3}} m/s^2$

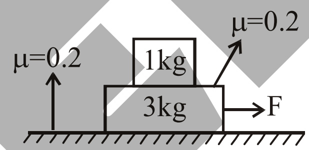
$v^2 = u^2 + 2as$

$v^2 = 1 - 2 \times \frac{2.5 \times 10^{-2}}{20 \times 10^{-3}} \times \frac{20}{100}$

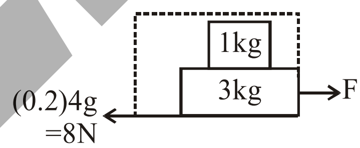
$v = \frac{1}{\sqrt{2}} \simeq 0.7\ m/s$

6. Ans. (1)

Sol.



$a_{Amax} = \mu g = 2\ m/s^2$



F – 8 = 4 × 2

F = 16 N

7. Ans. (2)

Sol. $-(g + \gamma v^2) = \frac{dv}{dt}$

$-gdt = \frac{g}{\gamma}\left(\frac{dv}{\frac{g}{\gamma} + v^2}\right)$

Integrating $0 \to t$ & $V_0 \to 0$:-

$-gt = -\sqrt{\frac{g}{\gamma}} \tan^{-1}\left(\frac{V_0}{\sqrt{\frac{g}{\gamma}}}\right)$

$t = \frac{1}{\sqrt{\gamma g}} \tan^{-1}\left(\sqrt{\frac{\gamma}{g}} V_0\right)$

8. Ans. (3)

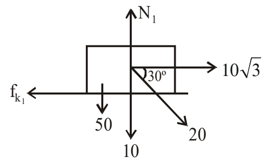
Sol. $k_1 = \dfrac{C}{\ell_1}$

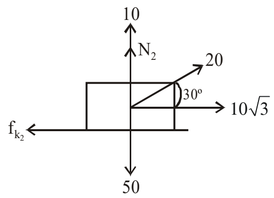
$k_2 = \dfrac{C}{\ell_2}$

$$\frac{k_1}{k_2} = \frac{C\ell_2}{\ell_1 C}\ell_2 = \frac{\ell_2}{n\ell_2} = \frac{1}{n}$$

9. Ans. (2)

Sol.





$N_1 = 60$ $\quad$ $N_2 = 40$

$a_1 = \dfrac{10\sqrt{3} - 0.2\times 60}{5}$ $\quad$ $a_2 = \dfrac{10\sqrt{3} - 0.2\times 40}{5}$

$a_1 - a_2 = 0.8$

POC

1. Ans. (4)

$$f = \frac{3\times 10^8}{8\times 10^{-7}} = \frac{30}{8}\times 10^{14}\,\text{Hz}$$

$= 3.75 \times 10^{14}$ Hz

1% of f = 0.0375×10^{14} Hz

$= 3.75 \times 10^{12}$ Hz $= 3.75 \times 10^6$ MHz

number of channels $= \dfrac{3.75\times 10^6}{6} = 6.25 \times 10^5$

$\therefore$ correct answer is (4)

2. Ans. (2)

$$f_{carrier} = \frac{250}{0.1} = 2500\,\text{KHZ}$$

$\therefore$ Range of signal = 2250 Hz to 2750 Hz

Now check all options : for 2000 KHZ

f_{mod} = 200 Hz

$\therefore$ Range = 1800 KHZ to 2200 KHZ

3. Ans. (4)

Maximum distance upto which signal can be broadcasted is

$$d_{max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where h_T and h_R are heights of transmiter tower and height of receiver respectively.

Putting all values -

$$d_{max} = \sqrt{2\times 6.4\times 106}\left[\sqrt{104} + \sqrt{40}\right]$$

on solving, d_{max} = 65 km

4. Ans. (4)

Analysis of graph says

(1) Amplitude varies as 8 – 10 V or 9 ± 1

(2) Two time period as

100 μs (signal wave) & 8 μs (carrier wave)

Hence signal is $\left[9 \pm 1\sin\left(\dfrac{2\pi t}{T_1}\right)\right]\sin\left(\dfrac{2\pi t}{T_2}\right)$

$= 9 \pm 1\sin\,(2\pi \times 10^4 t)\,\sin 2.5\pi \times 10^5\, t$

5. Ans. (3)

$$V(t) = 10 + \frac{3}{2}[2\cos A\sin B]$$

$$= 10 + \frac{3}{2}[\sin(A+B) - \sin(A-B)]$$

$$= 10 + \frac{3}{2}[\sin(57.2\times 10^4 t) - \sin(52.8\times 10^4 t)]$$

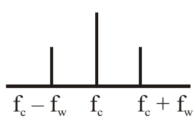
$\omega_1 = 57.2 \times 10^4 = 2\pi f_1$

$$f_1 = \frac{57.2\times 10^4}{2\times\left(\frac{22}{7}\right)} = 9.1\times 10^4$$

$\simeq 91\,\text{KHz}$

$$f_2 = \frac{52.8\times 10^4}{2\times\left(\frac{22}{7}\right)}$$

$\simeq 84\,\text{KHz}$



Side band frequency are

$$f_1 = f_c - f_w = \frac{52.8\times10^4}{2\pi} \approx 85.00 \text{ kHz}$$

$$f_2 = f_c + f_w = \frac{57.2\times10^4}{2\pi} \approx 90.00 \text{ kHz}$$

No answer matching but closest answer is option (3).

6. Ans. (2)

7. Ans. (1)

$E_m + E_c = 160$

$E_m + 100 = 160$

$E_m = 60$

$\mu = \frac{E_m}{E_C} = \frac{60}{100}$

$\mu = 0.6$

8. Ans. (3)

Sol. Range $= \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$50 \times 10^3 = \sqrt{2\times6400\times10^3\times h_T} + \sqrt{2\times6400\times10^3\times70}$$

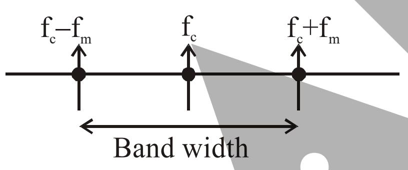
by solving $h_T = 32$ m

9. Ans. (3)

Sol. The physical size of antenna of reciver and transmitter both inversely proportional to carrier frequency.

10. Ans. (2)

Sol.



Option (2)

11. Ans. (4)

Sol. $f_m = 100$ MHz $= 10^8$ Hz, $(V_m)_0 = 100$V

$f_c = 300$ GHz, $(V_c)_0 = 400$V

Modulaton Index $= \frac{(V_m)_0}{(V_c)_0} = \frac{100}{400} = \frac{1}{4} = 0.25$

Upper band frequency (UBF) $= f_c + f_m$

Lower band frequency (LBF) $= f_c - f_m$

$\therefore$ UBF – LBF $= 2f_m = 2 \times 10^8$ Hz

12. Ans. (4)

Sol. Conceptual

13. Ans. (1)

Sol. Modulation index is given by

$$m = \frac{A_m}{A_c} = \frac{2}{4} = 0.5$$

& (a) carrier wave frequency is given by

$= 2\pi f_c = 2 \times 10^4 \pi$

$f_c = 10$ kHz

(b) modulating wave frequency (f_m)

$2\pi f_m = 2000\pi$

$\Rightarrow f_m = 1$ kHz

lower side band frequency $\Rightarrow f_c - f_m$

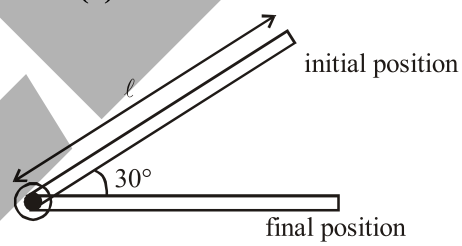
$\Rightarrow$ 10 kHz – 1 kHz = 9 kHz

14. Ans. (4)

Sol. To minimise attenuation, wavelength of carrier waves is close to 1500 nm

ROTATIONAL MECHANICS

1. Ans. (1)



Work done by gravity from initial to final position is,

$$W = mg\frac{\ell}{2}\sin 30°$$

$$= \frac{mg\ell}{4}$$

According to work energy theorem

$$W = \frac{1}{2}I\omega^2$$

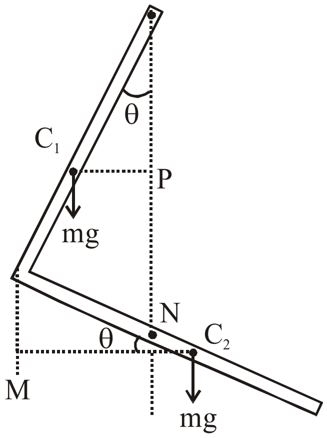
$$\Rightarrow \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = \frac{mg\ell}{4}$$

$$\omega = \sqrt{\frac{3g}{2\ell}} = \sqrt{\frac{3\times10}{2\times0.5}}$$

$$\omega = \sqrt{30} \text{ rad/sec}$$

$\therefore$ correct answer is (1)

2. Ans. (2)



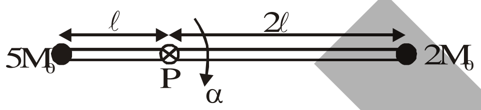
Let mass of one rod is m.
Balancing torque about hinge point.
mg (C_1P) = mg (C_2N)

$$mg\left(\frac{L}{2}\sin\theta\right)=mg\left(\frac{L}{2}\cos\theta-L\sin\theta\right)$$

$$\Rightarrow \frac{3}{2}mgL\sin\theta=\frac{mgL}{2}\cos\theta$$

$$\Rightarrow \tan\theta=\frac{1}{3}$$

3. Ans. (3)



Applying torque equation about point P.
$2M_0\,(2l) - 5\,M_0\,gl = I\alpha$
$I = 2M_0\,(2l)^2 + 5M_0\,l^2 = 13\,M_0l^2d$

$$\therefore\ \alpha=-\frac{M_0g\ell}{13M_0\ell^2}\ \Rightarrow\ \alpha=-\frac{g}{13\ell}$$

$$\therefore\ \alpha=\frac{g}{13\ell}\ \text{anticlockwise}$$

4. Ans. (3)
For Ball
using parallel axis theorem.

$$I_{ball}=\frac{2}{5}MR^2+M(2R)^2$$

$$=\frac{22}{5}\ MR^2$$

2 Balls so $\frac{44}{5}MR^2$

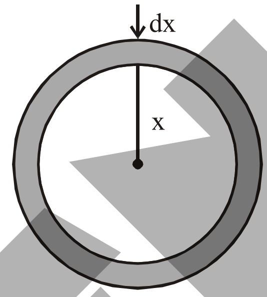
Irod = for rod $\frac{M(2R)^2}{R}=\frac{MR^2}{3}$

$I_{system} = I_{Ball} + I_{rod}$

$$=\frac{44}{5}MR^2+\frac{MR^2}{3}$$

$$=\frac{137}{15}\ MR^2$$

5. Ans. (1)



Consider a strip of radius x & thickness dx,

Torque due to friction on this strip.

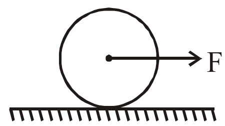
$$\int d\tau=\int_0^R\frac{x\mu F\cdot 2\pi x dx}{\pi R^2}$$

$$\tau=\frac{2\mu F}{R^2}\cdot\frac{R^3}{3}$$

$$\tau=\frac{2\mu FR}{3}$$

∴ correct answer is (1)

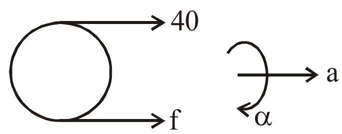
6. Ans. (3)



$$FR=\frac{3}{2}MR^2\alpha$$

$$\alpha=\frac{2F}{3MR}$$

7. Ans. (2)



$40 + f = m(R\alpha)$ .....(i)

$40 \times R - f \times R = mR^2\alpha$

$40 - f = mR\alpha$ ...... (ii)

From (i) and (ii)

$$\alpha = \frac{40}{mR} = 16$$

8. Ans. (2)

$2.5 = 1 \times 5 \sin\theta$

$$\sin\theta = 0.5 = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

9. Ans. (1)

$$I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$$

$$= \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2$$

$= 3\ MR^2$

10. Ans. (4)

Suppose M is mass and a is side of larger triangle, then $\frac{M}{4}$ and $\frac{a}{2}$ will be mass and side length of smaller triangle.

$$\frac{I_{removed}}{I_{original}} = \frac{\frac{M}{4}}{M} \cdot \frac{\left(\frac{a}{2}\right)^2}{(a)^2}$$

$$I_{removed} = \frac{I_0}{16}$$

$$\text{So, } I = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$$

11. Ans. (4)

Torque for F_1 force

$$\vec{F}_1 = \frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})$$

$$\vec{r}_1 = 0\hat{i} + 6\hat{j}$$

$$\vec{\tau}_{F_1} = \vec{r}_1 \times \vec{F}_1 = 3F\hat{k}$$

Torque for F_2 force

$$\vec{F}_2 = F\hat{k}$$

$$\vec{r}_2 = 2\hat{i} + 3\hat{j}$$

$$\vec{\tau}_{F_2} = \vec{r}_2 \times \vec{F}_2 = 3F\hat{i} + 2F(-\hat{j})$$

$$\vec{\tau}_{net} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2}$$

$$= 3F\hat{i} + 2F(-\hat{j}) + 3F(\hat{k})$$

12. Ans. (2)

13. Ans. (2)

$$I = \frac{2}{5}mR^2 + mx^2$$

14. Ans. (3)

15. Ans. (1)

Sol. for solid sphere

$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mR^2.\frac{v^2}{R^2} = mgh_{sph.}$$

for solid cylinder

$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{1}{2}mR^2.\frac{v^2}{R^2} = mgh_{cyl.}$$

$$\Rightarrow \frac{h_{sph.}}{h_{cyl.}} = \frac{7/5}{3/2} = \frac{14}{15}$$

16. Ans. (3)

Sol. Angular impulse = change in angular momentum

$\tau\,\Delta t = \Delta L$

$$mg\frac{\ell}{2} \times .01 = \frac{m\ell^2}{3}\omega$$

$$\omega = \frac{3g \times 0.01}{2\ell}$$

$$= \frac{3 \times 10 \times .01}{2 \times 0.3}$$

$$= \frac{1}{2} = 0.5\ \text{rad/s}$$

time taken by rod to hit the ground

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1\ \text{sec.}$$

in this time angle rotate by rod

$\theta = \omega t = 0.5 \times 1 = 0.5$ radian

17. Ans. (4)

Sol. $M = \int_0^R \rho_0 r(2\pi r dr) = \dfrac{\rho_0 \times 2\pi \times R^3}{3}$

$\underset{\text{(MOI about COM)}}{I_0} = \int_0^R \rho_0 r(2\pi r dr) \times r^2 = \dfrac{\rho_0 \times 2\pi R^5}{5}$

by parallel axis theorem

$I = I_0 + MR^2$

$= \dfrac{\rho_0 \times 2\pi R^5}{5} + \dfrac{\rho_0 \times 2\pi R^3}{3} \times R^2 = \rho_0 2\pi R^5 \times \dfrac{8}{15}$

$= MR^2 \times \dfrac{8}{5}$

18. Ans. (1)

Sol. Given moment of inertia 'I' = 1.5 kgm^2

Angular Acc. "α" = 20 Rad/s^2

$KE = \dfrac{1}{2} I\omega^2$

$1200 = \dfrac{1}{2} 1.5 \times \omega^2$

$\omega^2 = \dfrac{1200 \times 2}{1.5} = 1600$

$\omega = 40$ rad/s^2

$\omega = \omega_0 + \alpha t$

$40 = 0 + 20\,t$

t = 2sec.

19. Ans. (4)

Sol. Applying angular momentum conservation, about axis of rotation

$L_i = L_f$

$\dfrac{ML^2}{12}\omega_0 = \left(\dfrac{ML^2}{12} + m\left(\dfrac{L}{2}\right)^2 \times 2\right)\omega$

$\Rightarrow \omega = \dfrac{M\omega_0}{M + 6m}$.

20. Ans. (2)

Sol. $\dfrac{1}{2}\left(m + \dfrac{I}{R^2}\right)v^2 = mgh$

if radius of gyration is k, then

$h = \dfrac{\left(1 + \dfrac{k^2}{R^2}\right)v^2}{2g}$, $\dfrac{k_{ring}}{R_{ring}} = 1$, $\dfrac{k_{\text{solid cylinder}}}{R_{\text{solid cylinder}}} = \dfrac{1}{\sqrt{2}}$

$\dfrac{k_{\text{solid sphere}}}{R_{\text{solid sphere}}} = \sqrt{\dfrac{2}{5}}$

$h_1 : h_2 : h_3 :: (1 + 1) : \left(1 + \dfrac{1}{2}\right) : \left(1 + \dfrac{2}{5}\right) :: 20 : 15 : 14$

Therefor **most appropriate option is (2)** athough which in not in correct sequence

21. Ans. (4)

Sol. Kinetic energy KE = $\dfrac{1}{2} I\omega^2 = k\theta^2$

$\Rightarrow \omega^2 = \dfrac{2k\theta^2}{I} \Rightarrow \omega = \sqrt{\dfrac{2k}{I}}\,\theta$ ....(1)

Differentiate (1) wrt time →

$\dfrac{d\omega}{dt} = \alpha = \sqrt{\dfrac{2k}{I}}\left(\dfrac{d\theta}{dt}\right)$

$\Rightarrow \alpha = \sqrt{\dfrac{2k}{I}} \cdot \sqrt{\dfrac{2k}{I}}\,\theta$ {by (1)}

$\Rightarrow \alpha = \dfrac{2k}{I}\theta$

Option (4)

22. Ans. (2)

Sol. $\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{25 \times 2\pi}{5} = 10\pi \text{ rad/sec}^2$

$\tau = \left(\dfrac{5}{4}MR^2\right)\alpha$

$= \dfrac{5}{4} \times 5 \times 10^{-3} \times (10^{-2})^2 \times 10\pi$

$= 1.9625 \times 10^{-5}$ Nm

$\simeq 2.0 \times 10^{-5}$ Nm

23. Ans. (4)

Sol. $\vec{L} = m[\vec{r} \times \vec{v}]$

m = 2 kg

$\vec{r} = 2t\,\hat{i} - 3t^2\hat{j}$

$= 4\,\hat{i} - 12\,\hat{j}$ (At t = 2 sec)

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j} = 2\hat{i} - 12\hat{j}$$

$$\vec{r} \times \vec{v} = (4\hat{i} - 12\hat{j}) \times (2\hat{i} - 12\hat{j})$$

$$= -24\hat{k}$$

$$\vec{L} = m(\vec{r} \times \vec{v})$$

$$= -48\hat{k}$$

24. Ans. (4)

Sol. $$I_1 = \frac{\left(\frac{7M}{8}\right)(2R)^2}{2} = \left(\frac{7}{16} \times 4\right)MR^2 = \frac{7}{4}MR^2$$

$$I_2 = \frac{2}{5}\left(\frac{M}{8}\right)R_1^2 = \frac{2}{5}\left(\frac{M}{8}\right)\frac{R^2}{4} = \frac{MR^2}{80}$$

$$\frac{4}{3}\pi R^3 = 8\left(\frac{4}{3}\pi R_1^3\right)$$

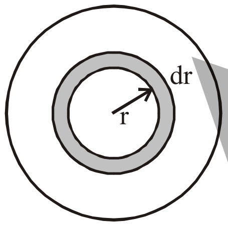
$$R^3 = 8R_1^3$$

$$R = 2R_1$$

$$\therefore \frac{I_1}{I_2} = \frac{7/4\ MR^2}{\frac{MR^2}{80}} = \frac{7}{4} \times 80 = 140$$

25. Ans. (3)

Sol.



$$I_{Disc} = \int_0^R (dm)r^2 \Rightarrow I_{Disc} = \int_0^R (\sigma 2\pi r dr)r^2$$

$$I_{Disc} = \int_0^R (kr^2 2\pi r dr)r^2 \qquad \text{Mass of disc}$$

$$I_{Disc} = 2\pi k \int_0^R r^5 dr \qquad M = \int_0^R 2\pi r dr\ kr^2$$

$$I_{Disc} = 2\pi k \left(\frac{r^6}{6}\right)_0^R \qquad M = 2\pi k \int_0^R r^3 dr$$

$$I_{Disc} = 2\pi k \frac{R^6}{6} \qquad M = 2\pi k \left.\frac{r^4}{4}\right|_0^R$$

$$I_{Disc} = \frac{\pi k R^6}{3} = \left(\frac{\pi k R^4}{2}\right)\frac{R^2 2}{3} \qquad M = 2\pi k \frac{R^4}{4}$$

$$I_{Disc} = \frac{M2R^2}{3}$$

$$I_{Disc} = \frac{2}{3}MR^2$$

26. Ans. (4)

Sol. $$E_i = \frac{1}{2}I_1 \times \omega_1^2 + \frac{1}{2}\frac{I_1}{2} \times \frac{\omega_1^2}{4}$$

$$= \frac{I_1\omega_1^2}{2}\left(\frac{9}{8}\right) = \frac{9}{16}I_1\omega_1^2$$

$$I_1\omega_1 + \frac{I_1\omega_1}{4} = \frac{3I_1}{2}\omega$$

$$\frac{5}{4}I_1\omega_1 = \frac{3I_1}{2}\omega$$

$$\omega = \frac{5}{6}\omega_1$$

$$E_f = \frac{1}{2} \times \frac{3I_1}{2} \times \frac{25}{36}\omega_1^2$$

$$= \frac{25}{48}I_1\omega_1^2$$

$$\Rightarrow E_f - E_i = I_1\omega_1^2\left(\frac{25}{48} - \frac{9}{16}\right) = \frac{-2}{48}I_1\omega_1^2$$

$$= \frac{-I_1\omega_1^2}{24}$$

27. Ans. (2)

Sol. $$F = -m\,(a\omega_1^2 \cos\omega_1 t\hat{i} + b\omega_2^2 \sin\omega_2 t\hat{j})$$

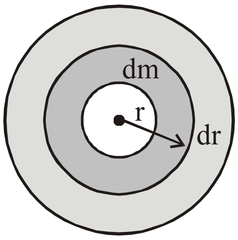
$$\vec{r} = (x_0 + a\cos\omega_1 t)\hat{i} + (y_0 + b\sin\omega_2 t)\hat{j}$$

$$\vec{T} = \vec{r} \times \vec{F} = -m(x_0 + a\cos\omega_1 t)b\omega_2^2 \sin\omega_2 t\,\hat{k}$$

$$+ m(y_0 + b\sin\omega_2 t)a\omega_1^2 \cos\omega_1 t\,\hat{k}$$

$$= ma\omega_1^2 y_0\hat{k}$$

28. Ans. (4)

Sol. 

$dI = (dm)r^2$

$= (\sigma\, dA)r^2$

$$= \left(\frac{\sigma_0}{r}2\pi r dr\right)r^2$$

$= (\sigma_0\, 2\pi)r^2 dr$

$$I = \int dI = \int_a^b \sigma_0 2\pi r^2 dr$$

$$= \sigma_0 2\pi\left(\frac{b^3 - a^3}{3}\right)$$

$$m = \int dm = \int \sigma dA$$

$$= \sigma_0 2\pi \int_a^b dr$$

$m = \sigma_0\, 2\pi\, (b-a)$

Radius of gyration

$$k = \sqrt{\frac{I}{m}} \quad = \sqrt{\frac{(b^3 - a^3)}{3(b-a)}}$$

$$= \sqrt{\left(\frac{a^3 + b^3 + ab}{3}\right)}$$

29. Ans. (2)

Sol. Angular momentum conservation.

$$\cancel{M}V_0 L = \cancel{M}V_1(L - \ell)$$

$$V_1 = V_0\left(\frac{L}{L-\ell}\right)$$

$w_g + w_p = \Delta KE$

$$-mg\ell + w_p = \frac{1}{2}m\left(V_1^2 - V_0^2\right)$$

$$w_p = mg\ell + \frac{1}{2}mV_0^2\left(\left(\frac{L}{L-\ell}\right)^2 - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1 - \frac{\ell}{L}\right)^{-2} - 1\right)$$

Now, $\ell << L$

By, Binomial approximation

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1 + \frac{2\ell}{L}\right) - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\frac{2\ell}{L}\right)$$

$$W_p = mg\ell + mv_0^2\frac{\ell}{L}$$

here, V_0 = maximum velocity

$= \omega \times A$

$$= \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$$

$$V_0 = \theta_0\sqrt{gL}$$

$$\text{so, } w_p = mg\ell + m\left(\theta_0\sqrt{gL}\right)^2\frac{\ell}{L}$$

$$= mg\ell\left(1 + \theta_0^2\right)$$

SEMICONDUCTOR

1. Ans. (3)

Initially Ge & Si are both forward biased so current will effectivily pass through Ge diode with a drop of 0.3 V

if "Ge" is revesed then current will flow through "Si" diode hence an effective drop of (0.7 – 0.3) = 0.4 V is observed.

2. Ans. (2)

$j = \sigma E = nev_d$

$$\sigma = ne\frac{v_d}{E}$$

$= ne\mu$

$$\frac{1}{\sigma} = \rho = \frac{1}{n_e e\mu_e}$$

$$= \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 1.6}$$

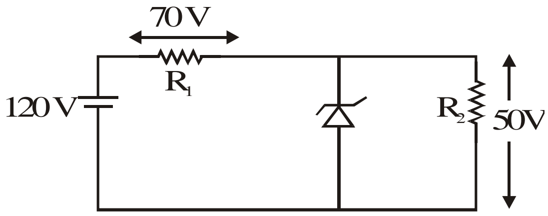
$= 0.4\ \Omega m$

3. Ans. (4)

Assuming zener diode doesnot undergo breakdown, current in circuit $= \frac{120}{15000} = 8\,\text{mA}$

$\therefore$ Voltage drop across diode = 80 V > 50 V.

The diode undergo breakdown.



Current is $R_1 = \frac{70}{5000} = 14\text{mA}$

Current is $R_2 = \frac{50}{10000} = 5\,\text{mA}$

$\therefore$ Current through diode = 9mA

4. Ans. (4)

$p = \overline{x} + y$

$Q = \overline{\overline{y}.x} = y + \overline{x}$

$O/P = \overline{P+Q}$

To make O/P

P + Q must be 'O'

SO, $y = 0$

$x = 1$

5. Ans. (2)

$I = \frac{6}{300} = 0.002$ (D_2 is in reverse bias)

6. Ans. (4)

Since voltage across zener diode must be less than 10V therefore it will not work in breakdown region, & its resistance will be infinite & current through it = 0

$\therefore$ correct answer is (4)

7. Ans (2)

At saturation, $V_{CE} = 0$

$V_{CE} = V_{CC} - I_C R_C$

$\Rightarrow I_C = \frac{V_{CC}}{R_C} = 5 \times 10^{-3}$ A

Given

$\beta_{dc} = \frac{I_C}{I_B}$

$I_B = \frac{5 \times 10^{-3}}{200}$

$I_B = 25\ \mu A$

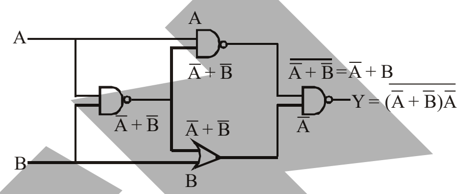
At input side

$V_{BB} = I_B R_B + V_{BE}$

$= (25\text{mA})\ (100\text{k}\Omega) + 1\text{V}$

$V_{BB} = 3.5$ V

8. Ans. (2)



$Y = \overline{\left(\overline{A}+\overline{B}\right)\overline{A}}$

$= \overline{\overline{A} + \overline{A}B}$

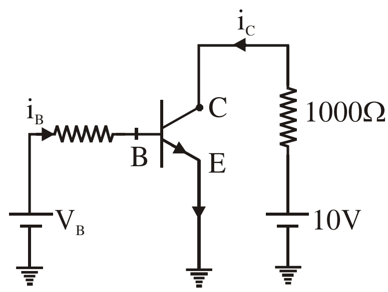
$= A\left(\overline{\overline{A}B}\right)$

$= A\left(A+\overline{B}\right)$

$= A + A\overline{B} \quad = A\overline{B}$

9. Ans. (3)

Sol.



At saturation state, V_{CE} becomes zero

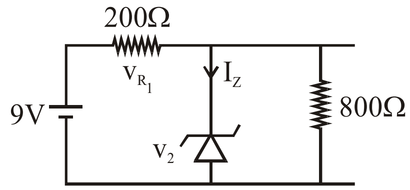
$\Rightarrow i_C = \frac{10V}{1000\Omega} = 10\text{mA}$

now current gain factor $\beta = \frac{i_C}{i_B}$

$\Rightarrow i_B = \frac{10\text{mA}}{250} = 40\mu\text{A}$

10. Ans. (3)

Sol.



$9 = V_Z + V_{R_1}$

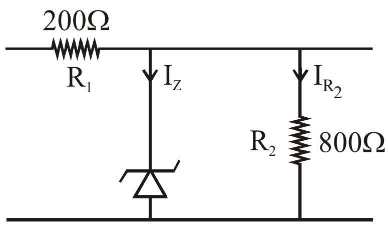
$V_Z = 5.6$ V

$V_{R_1} = 9 - 5.6$

$V_{R_1} = 3.4$

$I_{R_1} = \frac{V_{R_1}}{R} = \frac{3.4}{200}$

$I_{R_1} = 17$ mA



$V_z = V_{R_2} = I_{R_2}(R_2)$

$\frac{5.6}{800} = I_{R_2}$

$I_{R_2} = 7\text{mA}$

$I_z = (17 - 7)$ mA

= 10 mA

11. Ans. (1)

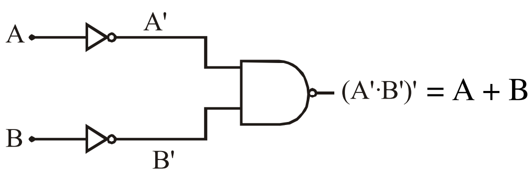
Sol. Method 1

Truth table can be formed as

A	B	Equivalent
0	0	0
0	1	1
1	0	1
1	1	1

Hence the Equivalent is "OR" gate.

Method 2



(OR GATE)

12. Ans. (4)

Sol. input current = 15×10^{-6}

output current = 3×10^{-3}

resistance output = 1000

$V_{input} = 10 \times 10^{-3}$

Now $V_{input} = r_{input} \times i_{input}$

$10 \times 10^{-3} = r_{input} \times 15 \times 10^{-6}$

$r_{input} = \frac{2000}{3} = 0.67$ KΩ.

voltage gain $= \frac{V_{output}}{V_{input}} = \frac{1000 \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$

Option (4)

13. Ans. (2)

Sol. At $V_B = 8$V

$i_L = \frac{6 \times 10^{-3}}{4} = 1.5 \times 10^{-3}$ A

$i_R = \frac{8 - 6 \times 10^{-3}}{1} = 2 \times 10^{-3}$ A

$\therefore\ i_{zener\ diode} = i_R - i_{load}$

$= 0.5 \times 10^{-3}$ A

At $V_B = 16$ V

$i_L = 1.5 \times 10^{-3}$ A

$i_R = \frac{(16-6) \times 10^{-3}}{1} = 10 \times 10^{-3}$ A

$\therefore\ i_{zener\ diode} = i_R - i_L$

$= 8.5 \times 10^{-3}$ A

14. Ans. (4)

Sol. $A_v \times \beta = P_{gain}$

$60 = 10 \log_{10}\left(\frac{P}{P_0}\right)$

$P = 10^6 = \beta^2 \times \frac{R_{out}}{R_{in}}$

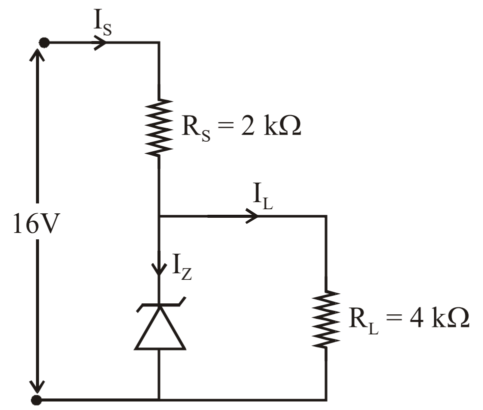
$= \beta^2 \times \frac{10^4}{100}$

$\beta^2 = 10^4$

$\beta = 100$

15. Ans. (2)

Sol. Maximum current will flow from zener if input voltage is maximum.



When zener diode works in breakdown state, voltage across the zener will remain same.

$\therefore$ $V_{across\ 4k\Omega} = 6V$

$\therefore$ Current through 4KΩ = $\frac{6}{4000}A = \frac{6}{4}mA$

Since input voltage = 16V

$\therefore$ Potential difference across 2KΩ = 10V

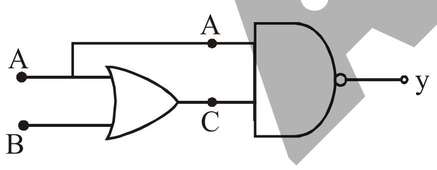
$\therefore$ Current through 2kΩ = $\frac{10}{2000} = 5mA$

$\therefore$ Current through zener diode

$= (I_S - I_L) = 3.5$ mA

16. Ans. (1)

Sol.



C = A + B

and y = $\overline{A.C}$

A	B	$C=(A+B)$	A.C.	$y=\overline{A.C}$
0	0	0	0	1
0	1	1	0	1
1	0	1	1	0
1	1	1	1	0

17. Ans. (3)

Sol. $V_{gain} = \left(\frac{\Delta I_C}{\Delta I_B}\right)\frac{R_{out}}{R_{in}}$

$= \left(\frac{5\times10^{-3}}{100\times10^{-6}}\right) \times 10^3$

$= \frac{1}{20}\times10^6 = 5 \times 10^4$

$P_{gain} = \left(\frac{\Delta I_C}{\Delta I_b}\right)(V_{gain})$

$= \left(\frac{5\times10^{-3}}{100\times10^{-6}}\right)(5\times10^4)$

$= 2.5 \times 10^6$

SHM

1. Ans. (2)

Frequency of torsonal oscillations is given by

$f = \frac{k}{\sqrt{I}}$

$f_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}$

$f_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}}$

$f_2 = 0.8\ f_1$

$\frac{m}{M} = 0.375$

2. Ans. (3)

Potential energy (U) = $\frac{1}{2}kx^2$

Kinetic energy (K) = $\frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to the question, U = k

$\therefore$ $\frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$\mathbf{x} = \pm\frac{A}{\sqrt{2}}$

$\therefore$ Correct answer is (3)

3. Ans. (3)

Maximum speed is at mean position (equilibrium). F = kx

$$x = \frac{F}{k}$$

$$W_F + W_{sp} = \Delta KE$$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

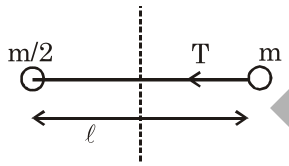
$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v_{max} = \frac{F}{\sqrt{mk}}$$

4. Ans. (4)

$$\omega = \sqrt{\frac{k}{I}}$$

$$\omega = \sqrt{\frac{3k}{m\ell^2}}$$



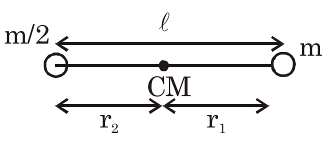
$\Omega = \omega\theta_0$ = average velocity

$T = m\Omega^2 r_1$

$$T = m\Omega^2\frac{\ell}{3} = m\omega^2\theta_0^2\frac{\ell}{3} = m\frac{3k}{m\ell^2}\theta_0^2\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

$$I = \mu\ell^2 = \frac{\frac{m^2}{2}}{\frac{3m}{2}}\ell^2$$

$$= \frac{m\ell^2}{3}$$



$$\frac{r_1}{r_2} = \frac{1}{2} \Rightarrow r_1 = \frac{\ell}{3}$$

5. Ans. (4)

$v = \omega\sqrt{A^2 - x^2}$ ____(1)

$a = -\omega^2 x$ ____(2)

$|v| = |a|$ ____(3)

$$\omega\sqrt{A^2 - x^2} = \omega^2 x$$

$$A^2 - x^2 = \omega^2 x^2$$

$$5^2 - 4^2 = \omega^2(4^2)$$

$$\Rightarrow 3 = \omega \times 4$$

$$T = 2\pi/\omega$$

6. Ans. (1)

Angular frequency of pendulum

$$\omega = \sqrt{\frac{g_{eff}}{\ell}}$$

$$\therefore \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$$

$$\Delta\omega = \frac{1}{2}\frac{\Delta g}{g} \times \omega$$

[ω_s = angular frequency of support]

$$\Delta\omega = \frac{1}{2} \times \frac{2A\omega_s^2}{100} \times 100$$

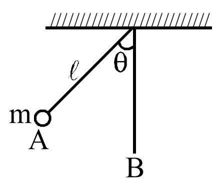
$\Delta\omega = 10^{-3}$ rad/sec.

7. Ans. (2)

Maximum kinetic energy at lowest point B is given by

$K = mgl(1 - \cos\theta)$

where θ = angular amp.



$K_1 = mg\ell(1 - \cos\theta)$

$K_2 = mg(2\ell)(1 - \cos\theta)$

$K_2 = 2K_1$.

8. Ans. (3)

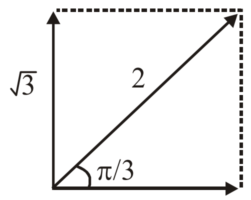
$k = \frac{1}{2}m\omega^2A^2\cos^2\omega t$

$U = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\frac{k}{U} = \cot^2\omega t = \cot^2\frac{\pi}{90}(210) = \frac{1}{3}$

No answer is matching as correct answer is 1/3. Hence ratio is 3 (most appropriate)

9. Ans. (4)



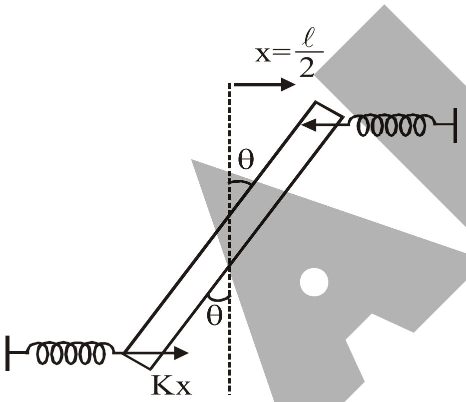
$y = 5\left[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)\right]$

$= 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

Amplitude = 10 cm

$T = \frac{2\pi}{w} = \frac{2\pi}{3\pi} = \frac{2}{3}\text{sec}$

10. Ans. (1)



$\tau = -2Kx\frac{\ell}{2}\cos\theta$

$\Rightarrow \tau = \left(\frac{K\ell^2}{2}\right)\theta = -C\theta$

$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}}$

$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$

11. Ans. (1)

Sol. For a simple pendulum $T = 2\pi\sqrt{\frac{L}{g_{eff}}}$

situation 1 : when pendulum is in air $\rightarrow g_{eff} = g$

situation 2 : when pendulum is in liquid

$\rightarrow g_{eff} = g\left(1 - \frac{\rho_{liquid}}{\rho_{body}}\right) = g\left(1 - \frac{1}{16}\right) = \frac{15g}{16}$

So, $\frac{T'}{T} = \frac{2\pi\sqrt{\frac{L}{15g/16}}}{2\pi\sqrt{\frac{L}{g}}} \Rightarrow T' = \frac{4T}{\sqrt{15}}$

Option (1)

UNIT & DIMENSION

1. Ans. (4)

$F = \frac{GM^2}{R^2} \Rightarrow G = [M^{-1}L^3T^{-2}]$

$E = h\nu \Rightarrow h = [ML^2T^{-1}]$

$C = [LT^{-1}]$

$t \propto G^xh^yC^z$

$[T] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]^z$

$[M^0L^0T^1] = [M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}]$

on comparing the powers of M, L, T

$-x + y = 0 \Rightarrow x = y$

$3x + 2y + z = 0 \Rightarrow 5x + z = 0$ ....(i)

$-2x - y - z = 1 \Rightarrow 3x + z = -1$ ...(ii)

on solving (i) & (ii) $x = y = \frac{1}{2}, z = -\frac{5}{2}$

$t \propto \sqrt{\frac{Gh}{C^5}}$

2. Ans. (4)

$\frac{128\text{kg}}{\text{m}^3} = \frac{125(50g)(20)}{(25\text{cm})^3(4)^3}$

$= \frac{128}{64}(20)\text{units}$

= 40 units

3. Ans. (2)

$$\frac{F}{A} = y.\frac{\Delta\ell}{\ell}$$

$$[Y] = \frac{F}{A}$$

Now from dimension

$$F = \frac{ML}{T^2}$$

$$L = \frac{F}{M}.T^2$$

$$L^2 = \frac{F^2}{M^2}\left(\frac{V}{A}\right)^4 \quad \because \ T = \frac{V}{A}$$

$$L^2 = \frac{F^2}{M^2A^2}\frac{v^4}{A^2} \qquad F = MA$$

$$L^2 = \frac{V^4}{A^2}$$

$$[Y] = \frac{[F]}{[A]} = F^1\ V^{-4}\ A^2$$

4. Ans. (2)

$$F = \alpha\beta e^{\left(\frac{-x^2}{\alpha KT}\right)}$$

$$\left[\frac{x^2}{\alpha KT}\right] = M°L°T°$$

$$\frac{L^2}{[\alpha]ML^2T^{-2}} = M°L°T°$$

$\Rightarrow [\alpha] = M^{-1}T^2$

$[F] = [\alpha]\ [\beta]$

$MLT^{-2} = M^{-1}T^2[\beta]$

$\Rightarrow [\beta] = M^2LT^{-4}$

5. Ans. (3)

$$\left[\frac{\ell}{r}\right] = T$$

$[CV] = AT$

So, $\left[\frac{\ell}{rCV}\right] = \frac{T}{AT} = A^{-1}$

6. Ans. (2)

Sol. $p = k\ s^aI^bh^c$

where k is dimensionless constant

$MLT^{-1} = (MT^{-2})^a(ML^2)^b(ML^2T^{-1})^c$

$a + b + c = 1$

$2b + 2c = 1$

$-2a - c = -1$

$a = \frac{1}{2} \qquad b = \frac{1}{2} \qquad c = 0$

$s^{1/2}I^{1/2}h^0$

7. Ans. (2)

Sol. dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

$[\mu_0] = [MLT^{-2}A^{-2}]$

dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}} = \left[\frac{M^{-1}L^{-3}T^4A^2}{MLT^{-2}A^{-2}}\right]^{\frac{1}{2}}$

$= [M^{-2}L^{-4}T^6A^4]^{1/2}$

$= [M^{-1}L^{-2}T^3A^2]$

8. Ans. (3)

Sol. $[\varepsilon_0] = M^{-1}\ L^{-3}\ T^4\ A^2$

$[\mu_0] = M\ L\ T^{-2}\ A^{-2}$

$[R] = M\ L^2\ T^{-3}\ A^{-2}$

$$[R] = \left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$$

VECTOR

1. Ans. (4)

2. Ans. (3)

$|\vec{A} + \vec{B}| = 2a\cos\theta/2$ ____(1)

$|\vec{A} - \vec{B}| = 2a\cos\frac{(\pi - \theta)}{2} = 2a\sin\theta/2$ ____(2)

$\Rightarrow n\left(2a\cos\frac{\theta}{2}\right) = 2a\frac{\sin\theta}{2}$

$\Rightarrow \tan\frac{\theta}{2} = n$

3. Ans. (2)

$$\vec{r}_g = \frac{a}{2}\hat{i} + \frac{a}{2}\hat{k}$$

$$\vec{r}_H = \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$$

$$\vec{r}_H - \vec{r}_g = \frac{a}{2}\left(\hat{j} - \hat{i}\right)$$

4. Ans. (3)

Sol. $\left|\vec{A}_1\right| = 3 \qquad \left|\vec{A}_2\right| = 5 \qquad \left|\vec{A}_1 + \vec{A}_2\right| = 5$

$$\left|\vec{A}_1 + \vec{A}_2\right| = \sqrt{\left|\vec{A}_1\right|^2 + \left|\vec{A}_2\right|^2 + 2\left|\vec{A}_1\right|\left|\vec{A}_2\right|\cos\theta}$$

$$5 = \sqrt{9 + 25 + 2\times3\times5\cos\theta}$$

$$\cos\theta = -\frac{9}{2\times3\times5} = -\frac{3}{10}$$

$$\left(2\vec{A}_1 + 3\vec{A}_2\right).\left(3\vec{A}_1 - 2\vec{A}_2\right)$$

$$= 6\left|\vec{A}_1\right|^2 + 9\vec{A}_1.\vec{A}_2 - 4\vec{A}_1\vec{A}_2 - 6\left|\vec{A}_2\right|^2$$

$$54 + 5\times3\times5\left(-\frac{3}{10}\right) - 6\times25$$

$$= 54 - 150 - \frac{45}{2} = -118.5$$

WAVE MOTION

1. Ans. (4)

Frequency of the sound produced by flute,

$$f = 2\left(\frac{v}{2\ell}\right) = \frac{2\times330}{2\times0.5} = 660\text{Hz}$$

Velocity of observer, $v_0 = 10\times\frac{5}{18} = \frac{25}{9}\text{m/s}$

$\therefore$ frequency detected by observer,

$$f' = \left[\frac{v + v_0}{v}\right]f$$

$$\therefore\ f' = \left[\frac{\frac{25}{9} + 330}{330}\right]660$$

$= 335.56 \times 2 = 671.12$

$\therefore$ closest answer is (4)

2. Ans. (2)

$$\frac{I_{max}}{I_{min}} = 16$$

$$\Rightarrow \frac{A_{max}}{A_{min}} = 4$$

$$\Rightarrow \frac{A_1 + A_2}{A_1 - A_2} = \frac{4}{1}$$

Using componendo & dividendo.

$$\frac{A_1}{A_2} = \frac{5}{3} \Rightarrow \frac{I_1}{I_2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

3. Ans. (1)

$$60 = \sqrt{\frac{Mg}{\mu}}$$

$$60.5 = \sqrt{\frac{M\left(g^2 + a^2\right)^{1/2}}{\mu}} \Rightarrow \frac{60.5}{60} = \sqrt{\sqrt{\frac{g^2 + a^2}{g^2}}}$$

$$\left(1 + \frac{0.5}{60}\right)^4 = \frac{g^2 + a^2}{g^2} = 1 + \frac{2}{60}$$

$$\Rightarrow g^2 + a^2 = g^2 + g^2 \times \frac{2}{60}$$

$$a = g\sqrt{\frac{2}{60}} = \frac{g}{\sqrt{30}} = \frac{g}{5.47}$$

$$\simeq \frac{g}{5}$$

4. Ans. (1)

For closed organ pipe, resonate frequency is odd multiple of fundamental frequency.

$\therefore$ $(2n + 1)\ f_0 \le 20{,}000$

(f_0 is fundamental frequency = 1.5 KHz)

$\therefore$ n = 6

$\therefore$ Total number of overtone that can be heared is 6. (1 to 6).

NTA answer is option (1) that is 7 overtones but correct answer is option (3) that is 6 overtones.

5. Ans. (2)

Velocity of wave on string

$$V=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{8}{5}\times 1000}=40\,m/s$$

Now, wavelength of wave $\lambda=\frac{v}{n}=\frac{40}{100}m$

Separation b/w successive nodes, $\frac{\lambda}{2}=\frac{20}{100}$ m

= 20 cm

6. Ans. (2)

$$f_{app}=f_0\left[\frac{v_2\pm v_0}{v_2\mp v_s}\right]$$

$$f_1=f_0\left[\frac{340}{340-34}\right]$$

$$f_2=f_0\left[\frac{340}{340-17}\right]$$

$$\frac{f_1}{f_2}=\frac{340-17}{340-34}=\frac{323}{306}\Rightarrow\frac{f_1}{f_2}=\frac{19}{18}$$

7. Ans. (2)

y = 0.03 sin(450 t – 9x)

$$v=\frac{\omega}{k}=\frac{450}{9}=50\,m/s$$

$$v=\sqrt{\frac{T}{\mu}}\Rightarrow\frac{T}{\mu}=2500$$

$\Rightarrow$ T = 2500 × 5 × 10^{-3}

= 12.5 N

8. Ans. (1)

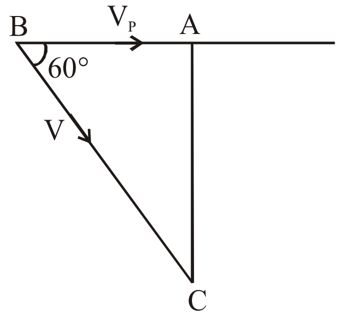
9. Ans. (1)

y= a sin(ωt + kx)

$\Rightarrow$ wave is moving along –ve x-axis with speed

$$\nu=\frac{\omega}{K}\Rightarrow\nu=\frac{50}{2}=25\,m/sec.$$

10. Ans. (3)



AB = $V_p\times t$

BC = Vt

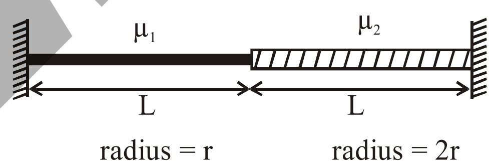
$$\cos 60°=\frac{AB}{BC}$$

$$\frac{1}{2}=\frac{V_p\times t}{Vt}$$

$$V_P=\frac{V}{2}$$

11. Ans. (4)

Sol.



Let mass per unit length of wires are μ_1 and μ_2 respectively.

$\because$ Materials are same, so density ρ is same.

$$\therefore\ \mu_1=\frac{\rho\pi r^2L}{L}=\mu\ \text{ and }\ \mu_2=\frac{\rho 4\pi r^2L}{L}=4\mu$$

Tension in both are same = T, let speed of wave in wires are V_1 and V_2

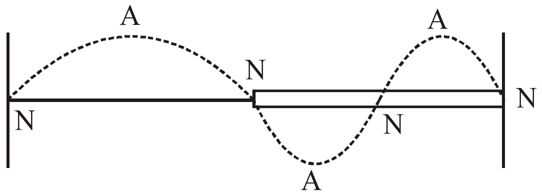
$$V_1=\sqrt{\frac{T}{\mu}}=V;\ V_2=\sqrt{\frac{T}{4\mu}}=\frac{V}{2}$$

So fundamental frequencies in both wires are

$$f_{01}=\frac{V_1}{2L}=\frac{V}{2L}\ \&\ f_{02}=\frac{V_2}{2L}=\frac{V}{4L}$$

Frequency at which both resonate is L.C.M of both frequencies i.e. $\frac{V}{2L}$.

Hence no. of loops in wires are 1 and 2 respectively.



So, ratio of no. of antinodes is 1 : 2.

12. Ans. (4)

Sol. $3\left(\frac{v}{2\ell}\right) = 240$

$3\frac{v}{2\times 2} = 240$

v = 320 m/s

fundamental frequency $= \frac{v}{2\ell} = \frac{320}{2\times 2} = 80$ Hz.

13. Ans. (3)

Sol. Speed of wave from wave equation

$v = -\frac{(\text{coeffecient of } t)}{(\text{coeffecient of } x)}$

$v = -\frac{1000}{(-3)} = \frac{1000}{3}$

since speed of wave $\propto \sqrt{T}$

$\text{so} = \frac{\frac{1000}{3}}{336} = \sqrt{\frac{273}{T}}$

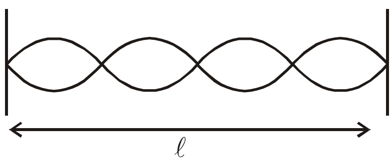
$\Rightarrow$ T = 277.41 K

T = 4.41°C

Option (3)

14. Ans. (2)

Sol. 4^{th} harmonic



$4\frac{\lambda}{2} = \ell$

$2\lambda = \ell$

From equation $\frac{2\pi}{\lambda} = 0.157$

$\lambda = 40$

$\ell = 2\lambda$

= 80 m

Option (2)

15. Ans. (3)

Sol. 50 m/s

O $\longleftarrow$ S

$f_{app} = \left(\frac{V-0}{V-50}\right)f_{source}$

$1000 = \left(\frac{350}{300}\right)f_{source}$

$f_{source} = \frac{1000\times 300}{350}$

50 m/s

$\longleftarrow$ S O

$f_{app} = \left(\frac{V}{V+50}\right)\cdot f_{source}$

$= \frac{350}{400}\times 1000\times\frac{300}{350}$

= 750 Hz

16. Ans. (2)

Sol. $f = 480 = \frac{300-v_1}{300}\times 500$

$\frac{1440}{5} = 300 - v_1$

$v_1 = \frac{60}{5} = 12 m/s$

$530 = \frac{300+v_2}{300}\times 500$

$1590 = 1500 + 5v_2$

$5v_2 = 90$

$v_2 = 18$ m/s

17. Ans. (3)

Sol. $\nu = 2f(l_2 - l_1)$

$\nu = 2 \times 480 \times (70 - 30) \times 10^{-2}$

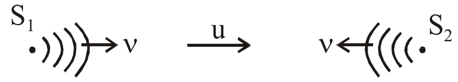
$\nu = 960 \times 40 \times 10^{-2}$

$\nu = 38400 \times 10^{-2}$ m/s

$\nu = 384$ m/s

18. Ans. (1)

Sol. f = 660 Hz, $\nu = 330$ m/s



$$f_1 = f\left(\frac{\nu - u}{\nu}\right)$$

$$f_2 = f\left(\frac{\nu + u}{\nu}\right)$$

$$f_2 - f_1 = \frac{f}{\nu}[\nu + u - (\nu - u)]$$

$$10 = f_2 - f_1 = \frac{f}{\nu}[2u]$$

u = 2.5 m/s

19. Ans. (3)

Sol. Loudness of sound is given by

$$dB = 10 \log \frac{I}{I_0} \left(\begin{array}{c} \text{I is intensity of sound} \\ I_0 \text{ is reference intensity of sound} \end{array}\right)$$

$$\therefore\ 120 = 10 \log\left(\frac{I}{I_0}\right)$$

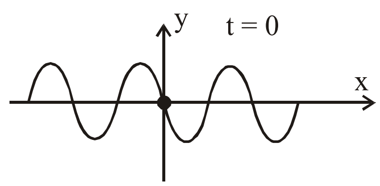
$\Rightarrow$ I = 1 W/m^2

$$\text{Also}\quad I = \frac{P}{4\pi r^2} = \frac{2}{4\pi r^2}$$

$$\therefore\ r = \sqrt{\frac{2}{4\pi}} = \sqrt{\frac{1}{2\pi}}\ \text{m} = 0.399\ \text{m} \approx 40\ \text{cm}$$

20. Ans. (3)

Sol.



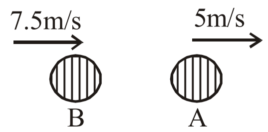
$y = A \sin(kx - wt + \phi)$

at x = 0, t = 0, y = 0 and slope is negative

$\Rightarrow \phi = \pi$

21. Ans. (2)

Sol.



$f_0 = 500$ Hz

frequency recieved by A

$$\Rightarrow \left(\frac{1500 - 5}{1500 - 7.5}\right) f_0 = f_1$$

and frequency recieved By B again =

$\longleftarrow$ 1500

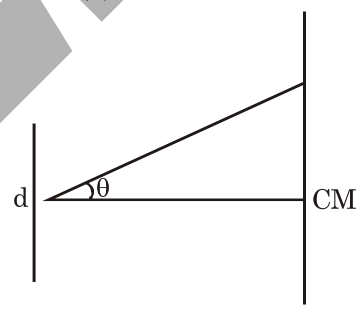
(B) (A) & $\Rightarrow$

7.5 m/s $\longrightarrow$ $\longrightarrow$ 5 m/sec

$$f_2 = \left(\frac{1500 + 7.5}{1500 + 5}\right) \times \left(\frac{1500 - 5}{1500 - 7.5}\right) f_0 = 502\ \text{Hz}$$

WAVE OPTICS

1. Ans. (2)



Pam difference

$d\sin\theta = n\lambda$

where d = seperation of slits

λ = wave length

n = no. of maximas

$0.32 \times 10^{-3} \sin 30 = n \times 500 \times 10^{-9}$

n = 320

Hence total no. of maximas observed in angular range $-30° \le \theta \le 30°$ is

maximas = 320 + 1 + 320 = 641

2. Ans. (4)

$$\sqrt{5}d - 2d = \frac{\lambda}{2}$$

3. Ans. (2)

Path difference = d $\sin\theta \approx d\theta$

$= 0.1 \times \frac{1}{40}$ mm = 2500nm

or bright fringe, path difference must be integral multiple of λ.

$\therefore$ 2500 = $n\lambda_1 = m\lambda_2$

$\therefore$ $\lambda_1 = 625$, $\lambda_2 = 500$ (from m=5)
(for n = 4)

4. Ans. (4)

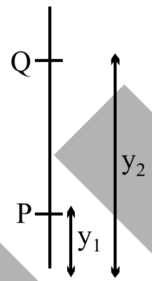
According to JEE-Mains Ans. key (3)

For diffraction

location of 1st minime

$$y_1 = \frac{D\lambda}{a} = 0.2469\,D\lambda$$

location of 2nd minima



$$y_2 = \frac{2D\lambda}{a} = 0.4938\,D\lambda$$

Now for interference

Path difference at P.

$$\frac{dy}{D} = 4.8\lambda$$

path difference at Q

$$\frac{dy}{D} = 9.6\lambda$$

So orders of maxima in between P & Q is
5, 6, 7, 8, 9

So 5 bright fringes all present between P & Q.

5. Ans. (3)

$$\Delta x = \frac{\lambda}{8}$$

$$\Delta\phi = \frac{(2\pi)}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$$

$$I = I_0\cos^2\left(\frac{\pi}{8}\right)$$

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{8}\right)$$

6. Ans. (2)

$$P_{refracted} = \frac{96}{100}P_I$$

$$\Rightarrow K_2A_t^2 = \frac{96}{100}K_1A_i^2$$

$$\Rightarrow r_2A_t^2 = \frac{96}{100}r_1A_i^2$$

$$\Rightarrow A_t^2 = \frac{96}{100}\times\frac{1}{\frac{3}{2}}\times(30)^2$$

$$A_t\sqrt{\frac{64}{100}\times(30)^2} = 24$$

7. Ans. (1)

C < i_b

here i_b is "brewester angle"
and c is critical angle

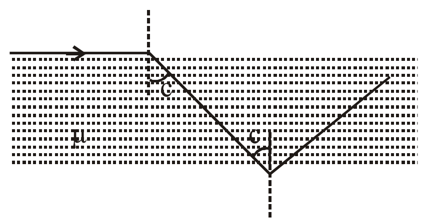
$\sin_c < \sin i_b$ since $\tan i_b = \mu_{0_{rel}} = \frac{1.5}{\mu}$

$$\frac{1}{\mu} < \frac{1.5}{\sqrt{\mu^2 + (1.5)^2}} \qquad \therefore \sin i_b = \frac{1.5}{\sqrt{\mu^2 + (1.5)^2}}$$

$$\sqrt{\mu^2 \times (1.5)^2} < 1.5 \times \mu$$

$\mu^2 + (1.5)^2 < (\mu \times 1.5)^2$

$$\mu < \frac{3}{\sqrt{5}}$$



slab $\mu = 1.5$

8. Ans. (2)

Sol. Given

$\frac{Y_A}{Y_B} = \frac{7}{4}$ $L_A = 2m$ $A_A = \pi R^2$

$L_B = 1.5m$ $A_B = \pi(2mm)^2$

$$\frac{F}{A} = Y\left(\frac{\ell}{L}\right)$$

given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

$$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B}$$

$$\Rightarrow \frac{(\pi R^2)\left(\frac{7}{4}Y_B\right)}{2} = \frac{\pi(2mm)^2 . Y_B}{1.5}$$

R = 1.74 mm

9. Ans. (1)

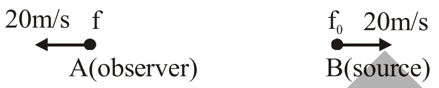
Sol. Given $\frac{a_1}{a_2} = \frac{1}{3}$

Ratio of intensities, $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{1}{9}$

Now, $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{1+3}{1-3}\right)^2 = 4$

10. Ans. (1)

Sol.



Applying Doppler effect for sound

$f = \frac{v+v_0}{v-v_s} f_0$ (v_0 & v_s is taken $\oplus$ when approaching each other)

$$2000 = \frac{340+(-20)}{340-(-20)} f_0$$

f_0 = 2250 Hz.

11. Allen Answer (Bonus)

Final Ans. by NTA (4)

Sol. Path difference at central maxima $\Delta x = (\mu - 1)t$, whole pattern will shift by same amount which will be given by

$(\mu - 1)t\frac{D}{d} = n\frac{\lambda D}{d}$, according to the question

$$t = \frac{n\lambda}{(\mu - 1)}$$

no option is matching, therefore question should be awarded bonus.

$\therefore$ Correct Option should be (Bonus)

12. Ans. (2)

Sol. $I_1 = 4I_0$

$I_2 = I_0$

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$= (2\sqrt{I_0} + \sqrt{I_0})^2 = 9I_0$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$= (2\sqrt{I_0} - \sqrt{I_0})^2 = I_0$

$\therefore \frac{I_{max}}{I_{min}} = \frac{9}{1}$

13. Ans. (4)

Sol. $f_{beat} = 11 - 9 = 2$ Hz

$\therefore$ Time period of oscillation of amplitude

$= \frac{1}{f_{beat}} = \frac{1}{2}$Hz

Although the graph of oscillation is not given, the equation of envelope is given by option (4)

14. Ans. (4)

Sol. Since unpolarised light falls on $P_1 \Rightarrow$

intensity of light transmitted from $P_1 = \frac{I_0}{2}$

Pass axis of P_2 will be at an angle of 30º with P_1

$\therefore$ Intensity of light transmitted from

$P_2 = \frac{I_0}{2}\cos^2 30^\circ = \frac{3I_0}{8}$

Pass axis of P_3 is at an angle of 60º with P_2

$\therefore$ Intensity of light transmitted from

$P_3 = \frac{3I_0}{8}\cos^2 60^\circ = \frac{3I_0}{32}$

$\therefore \left(\frac{I_0}{I}\right) = \frac{32}{3} = 10.67$

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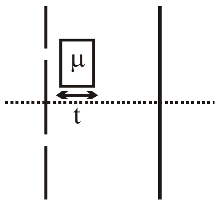
15. Ans. (4)

Sol.



$\Delta X = (\mu - 1)t = 1\lambda$

for one maximum shift

$$\boxed{t = \frac{\lambda}{\mu - 1}}$$

16. Ans. (1)

Sol. Limit of resolution of telescope = $\dfrac{1.22\lambda}{D}$

$\theta = \dfrac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}}$ $= 305 \times 10^{-9}$ radian

17. Ans. (3)

Sol. Limit of resolution = $\dfrac{1.22\ \lambda}{d}$

$= \dfrac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}}$

$= 2.9 \times 10^{-7}$ rad.

18. Ans. (1)

Sol. Numerical aperature of the microscope is given as

$NA = \dfrac{0.61\lambda}{d}$

Where d = minimum sparation between two points to be seen as distinct

$d = \dfrac{0.61\lambda}{NA} = \dfrac{(0.61) \times (5000 \times 10 m^{-10})}{1.25}$

$= 2.4 \times 10^{-7}$ m

$= 0.24\ \mu$m

WORK, POWER & ENERGY

1. Ans. (2)

$x = 3t^2 + 5$

$v = \dfrac{dx}{dt}$

$v = 6t + 0$

at $t = 0$ $\quad v = 0$

$t = 5$ sec $\quad v = 30$ m/s

W.D. = ΔKE

W.D. = $\dfrac{1}{2}mv^2 - 0 = \dfrac{1}{2}(2)(30)^2 = 900J$

2. Ans. (1)

Work done = $\vec{F} \cdot \vec{d}$

$= 12J$

work energy theorem

$w_{net} = \Delta K.E.$

$12 = K_f - 3$

$K_f = 15J$

3. Ans. (2)

$N - mg = \dfrac{mg}{2} \Rightarrow N = \dfrac{3mg}{2}$

Now, work done $W = \vec{N}\vec{S} = \left(\dfrac{3mg}{2}\right)\left(\dfrac{1}{2}gt^2\right)$

$\Rightarrow W = \dfrac{3mg^2t^2}{4}$

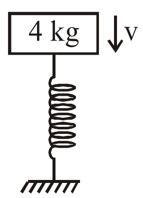
4. Ans. (Bonus)

According to JEE-Mains Ans. key (Bonus)

Velocity of 1 kg block just before it collides with 3kg block = $\sqrt{2gh} = \sqrt{2000}$ m/s

Applying momentum conversation just before and just after collision.

$1 \times \sqrt{2000} = 4v \Rightarrow v = \dfrac{\sqrt{2000}}{4}$ m/s



initial compression of spring

$1.25 \times 10^6\ x_0 = 30 \Rightarrow x_0 \approx 0$

applying work energy theorem,

$W_g + W_{sp} = \Delta KE$

$\Rightarrow 40 \times x + \frac{1}{2}\times 1.25\times 10^6(0^2 - x^2)$

$= 0 - \frac{1}{2}\times 4\times v^2$

solving $x \approx 2$ cm

5. Ans. (1)

Sol. According to work energy theorem

Work done by force on the particle = Change in KE

Work done = Area under F-x graph

$= \int F.dx$

$= 2\times 2 + \frac{(2+3)\times 1}{2}$

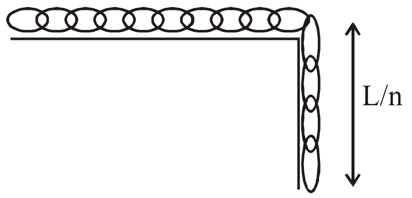
$W = KE_{final} - KE_{initial} = 6.5$

$KE_{initial} = 0$

$KE_{final} = 6.5$ J

6. Ans. (2)

Sol. Mass of the hanging part $= \frac{M}{n}$



$h_{COM} = \frac{L}{2n}$

work done

$W = mgh_{COM} = \left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$

Option (2)

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Chapter 02 Contents

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & APRIL 2019

CHEMISTRY

PHYSICAL CHEMISTRY

ORGANIC CHEMISTRY

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

Chapter 02 Contents

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & APRIL 2019

ORGANIC CHEMISTRY

INORGANIC CHEMISTRY

JANUARY & APRIL 2019 ATTEMPT (PC)

ATOMIC STRUCTURE

1. Ans.(3)

$h\nu = \phi + h\nu^\circ$

$$\frac{1}{2}mv^2 = hc\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)$$

$$h\nu=\phi+\frac{1}{2}mv^2$$

$$\phi=\frac{6.626\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}}-\frac{1}{2}\times9\times10^{-31}\times(6\times10^{5})^2$$

$\phi = 3.35 \times 10^{-19}$ J $\Rightarrow$ $\phi \simeq 2.1$ eV

2. Ans. (2)

According to de-broglie's hypothesis

$$2\pi r_n = n\lambda \Rightarrow 2\pi \,.\, a_0\frac{n^2}{z} = n\times1.5\pi a_0$$

$$\frac{n}{z}=0.75$$

3. Ans.(4)

Ozone protects most of the medium freequnecies ultravoilet light from 200 - 315 nm wave length.

4. Ans. (2)

5. Ans. (2)

For electron

$\lambda_{DB} = \dfrac{\lambda}{\sqrt{2mK.E.}}$ (de broglie wavelength)

By photoelectric effect

$h\nu = h\nu_0 + KE$

$KE = h\nu - h\nu_0$

$$\lambda_{DB}=\frac{h}{\sqrt{2m\times(h\nu-h\nu_0)}}$$

$$\lambda_{DB}\propto\frac{1}{(\nu-\nu_0)^{1/2}}$$

6. Ans. (1)

$$(E)_{n^{th}} = (E_1)_H\cdot\frac{Z^2}{n^2}$$

Second excited state, n = 3

$$E_{3^{rd}}(He^+)=(-13.6eV)\cdot\frac{2^2}{3^2}=-6.04\text{ eV}$$

7. Ans.(1)

Sol. In 'K', 2s orbital feel maximum attraction from nucleus (So having less energy) due to more Z_{eff}.

8. Ans. (3)

Number of ejected electrons are independent of frequency of light , & kinetic energy of electrons is independent of intensity of light.

K.E. = $h\nu + (-h\nu_0)$

y = mx + C

9. Ans. (4)

Refer Theory

10. Ans.(4)

$$\frac{1}{\lambda}=\bar{v}=R_H z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\bar{v}=R_H\times\left(\frac{1}{n_1^2}-\frac{1}{8^2}\right)$$

$$\bar{v}=R_H\times\frac{1}{n^2}-\frac{R_H}{8^2}$$

$$\bar{v}=R_H\times\frac{1}{n^2}-\frac{R_H}{64}$$

$m = R_H$

Linear with slope R_H

11. Ans.(4)

Sol. $h\nu-\phi=KE$

$$\Rightarrow\left(\frac{hc}{\lambda}\right)_{incident}=KE+\phi$$

$$\left(\frac{hc}{\lambda}\right)_{incident}\simeq KE$$

$$KE = \frac{p^2}{2m}=\frac{hc}{\lambda_{incident}}=\frac{hc}{\lambda} \quad ...(1)$$

$$\Rightarrow \frac{p^2\times(1.5)^2}{2m}=\frac{hc}{\lambda'} \quad ...(2)$$

divide (1) and (2)

$$(1.5)^2 = \frac{\lambda}{\lambda'}$$

$$\Rightarrow\lambda'=\frac{4\lambda}{9}$$

12. Ans.(4)

Sol. For Lyman

$$\bar{\nu}_{max} = R_H\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right) = R_H$$

$$\bar{\nu}_{min} = R_H\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3}{4}R_H$$

$$\Delta\bar{\nu}_{Lyman} = \frac{R_H}{4}$$

For Balmer

$$\bar{\nu}_{max} = R_H\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right) = \frac{R_H}{4}$$

$$\bar{\nu}_{min} = R_H\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = \frac{5}{36}R_H$$

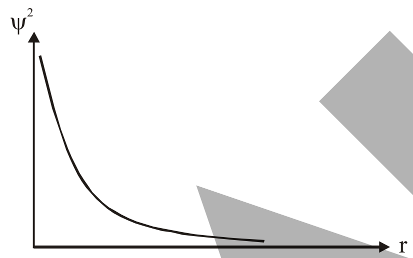
$$\Delta\bar{\nu}_{Balmer} = \frac{R_H}{4}-\frac{5R_H}{36} = \frac{4R_H}{36} = \frac{R_H}{9}$$

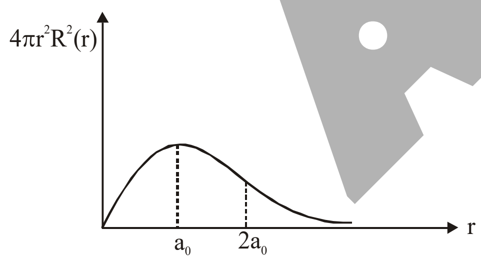
$$\frac{\Delta\bar{\nu}_{Lyman}}{\Delta\bar{\nu}_{Balmer}} = \frac{R_H/4}{R_H/9} = \frac{9}{4}$$

$\therefore$ Ans. is (4)

13. Ans.(4)

Sol.





14. Ans.(4)

Sol. Graph of $|\psi^2|$ v/s r, touches r axis at 1 point so it has one radial node and since at r = 0, it has some value so it should be for 's' orbital.

$\therefore$ $n - \ell - 1 = 1$ where $\ell = 0 \Rightarrow n - 1 = 1$

$\therefore$ $n = 2 \Rightarrow$ '2s' orbital

15. Ans. (2)

Sol.
$$\frac{\frac{1}{\lambda_2} = R_H\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)Z^2}{\frac{1}{\lambda_1} = R_H\left(\frac{1}{m_1^2}-\frac{1}{m_2^2}\right)Z^2}$$

as for shortest wavelengths both n_2 and m_2 are ∞

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{9}{1} = \frac{m_1^2}{n_1^2}$$

Now if $m_1 = 3$ & $n_1 = 1$ it will justify the statement hence Lyman and Paschen (2) is correct.

16. Ans.(2)

Sol. $P(x) = 4\pi x^2 \times [\Psi(x)]^2$

Probability will be maximum at a and c

CHEMICAL KINETICS

1. Ans.(1)

Rate constant (K) = 0.05 μg/year

means zero order reaction

$$t_{1/2} = \frac{a_0}{2K} = \frac{5\mu g}{2\times 0.05\mu g/year} = 50 \text{ year}$$

2. Ans. (1)

3. Ans. (1)

For zero order

$[A_0]-[A_t] = kt$

$0.2 - 0.1 = k\times 6$

$k = \frac{1}{60}$ M/hr

and $0.5-0.2 = \frac{1}{60}\times t$

t = 18 hrs.

4. Ans. (4)

On increasing E_a, k decreases.

In plot II initially k is shown to be almost constant with temperature while as in moderate temperature range increase of k is very sharp, therefore plot II is incorrect.

5. Ans. (3)

Ans.(3)

$$A_2 \underset{K_{-1}}{\overset{K_1}{\rightleftharpoons}} 2A$$

$$\frac{d[A]}{dt} = 2k_1[A_2] - 2k_{-1}[A]^2$$

6. Ans. (1)

$r = K[A]^x[B]^y$

$\Rightarrow 8 = 2^3 = 2^{x+y}$

$\Rightarrow x + y = 3$ ...(1)

$\Rightarrow 2 = 2^x$

$\Rightarrow x = 1, y = 2$

Order w.r.t. A = 1

Order w.r.t. B = 2

7. Ans.(2)

$$\ell n\frac{K_2}{K_1} = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$2.303\log\frac{K_2}{10^{-5}} = 4606\left[\frac{1}{400} - \frac{1}{500}\right]$$

$\Rightarrow K_2 = 10^{-4}\ s^{-1}$

8. Ans. (2)

$6.93 \times 10^{-3} = K \times (0.1)^x\ (0.2)^y$

$6.93 \times 10^{-3} = K \times (0.1)^x\ (0.25)^y$

So y = 0

and $1.386 \times 10^{-2} = K \times (0.2)^x\ (0.30)^y$

$$\frac{1}{2} = \left(\frac{1}{2}\right)^x \quad \boxed{x = 1}$$

So $r = K \times (0.1) \times (0.2)^0$

$6.93 \times 10^{-3} = K \times 0.1 \times (0.2)^0$

$$\boxed{K = 6.93 \times 10^{-2}}$$

$$t_{1/2} = \frac{0.693}{2K} = \frac{0.693}{0.693 \times 10^{-1} \times 2} = \frac{10}{2} = 5$$

9. Ans.(3)

Sol. $r = K\ [A]^x\ [B]^y$

$0.045 = K\ (0.05)^x\ (0.05)^y$ ....(1)

$0.090 = K\ (0.10)^x\ (0.05)^y$ ....(2)

$0.72 = K\ (0.20)^x\ (0.10)^y$ ....(3)

From (1) ÷ (2), $\frac{0.045}{0.090} = \left(\frac{0.05}{0.10}\right)^x \Rightarrow x = 1$

From (2) ÷ (3), $\frac{0.090}{0.720} = \left(\frac{0.10}{0.20}\right)^x \cdot \left(\frac{0.05}{0.10}\right)^y \Rightarrow y = 2$

Hence, $r = K\ [A]\ [B]^2$

Correct option : (3)

10. Ans.(1)

Sol. $A \xrightarrow{K_1} B \xrightarrow{K_2} C$

$$\frac{d[B]}{dt} = 0 = K_1[A] - K_2[B]$$

$$\Rightarrow [B] = \frac{K_1}{K_2}[A]$$

11. Ans.(1)

Sol. (i) $\ln[R] = \ln[R]_0 - Kt$ (Ist order)

$[R] = [R]_0 - Kt$ (zero order)

$\therefore$ Ans.(1)

12. Ans.(2)

Sol. $H_2(g) + I_2(g) \rightarrow 2HI(g)$

Apply Arrhenius equation

$$\log\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{600} - \frac{1}{800}\right)$$

$$\log\frac{1}{2.5 \times 10^{-4}} = \frac{E_a}{2.303 \times 8.31}\left(\frac{200}{600 \times 800}\right)$$

$\therefore\ E_a \approx 166$ kJ/mol

13. Ans.(2)

Sol. $\log\frac{-d[A]}{dt} = \log\frac{d[B]}{dt} + 0.3010$

$$\frac{-d[A]}{dt} = 2 \times \frac{d[B]}{dt}$$

$$\frac{1}{2} \times \frac{-d[A]}{dt} = \frac{d[B]}{dt}$$

$2A \longrightarrow B$

$2C_2H_4 \longrightarrow C_4H_8$

14. Ans.(4)

Sol. $2N_2O_5(g) \longrightarrow 4NO_2(g) + O_2(g)$

t=0 3.0M

t=30 2.75 M

$$\frac{-\Delta[N_2O_5]}{\Delta t} = \frac{0.25}{30}$$

$$\frac{1}{2} \times \frac{-\Delta[N_2O_5]}{\Delta t} = \frac{1}{4} \times \frac{\Delta[NO_2]}{\Delta t}$$

$$\frac{\Delta[NO_2]}{\Delta t} = \frac{0.25}{30} \times 2 = 1.66 \times 10^{-2}\ M/min$$

THERMODYNAMICS-01

1. Ans.(2)

$w = -nRT \ln \frac{V_2}{V_1}$

$w = -nRT \ln \frac{V_b}{V_i}$

$|w| = nRT \ln \frac{V_b}{V_i}$

$|w| = nRT (\ln V_b - \ln V_i)$

$|w| = nRT \ln V_b - nRT \ln V_i$

Y = m x – C

So, slope of curve 2 is more than curve 1 and intercept of curve 2 is more negative then curve 1.

2. Ans. (2)

Work done on isothermal irreversible for ideal gas

$= -P_{ext} (V_2 - V_1)$

$= -4 \text{ N/m}^2 (1\text{m}^3 - 5\text{m}^3)$

= 16 Nm

Isothermal process for ideal gas

$\Delta U = 0$

$q = -w$

= –16 Nm

= – 16 J

Heat used to increase temperature of $A\ell$

$q = n\, C_m\, \Delta T$

$16 \text{ J} = 1 \times 24 \frac{\text{J}}{\text{mol.K}} \times \Delta T$

$\Delta T = \frac{2}{3} \text{K}$

3. Ans.(3)

For cyclic process : $\Delta U = 0 \Rightarrow q = -w$

For isothermal process : $\Delta U = 0 \Rightarrow q = -w$

For adiabatic process : $q = 0 \Rightarrow \boldsymbol{\Delta U = W}$

For isochoric process : $w = 0 \Rightarrow \Delta U = q$

Correct option : (3)

4. Ans.(4)

Sol. $\Delta H = n\int_{T_1}^{T_2} C_{p,m} dT = 3 \times \int_{300}^{1000} (23 + 0.01T)dT$

$= 3 \left[23(1000 - 300) + \frac{0.01}{2}(1000^2 - 300^2)\right]$

= 61950 J $\approx$ 62 kJ

Correct option : (4)

5. Ans.(1)

Sol. n = 5; T_i = 100 K; T_f = 200 K;

C_V = 28 J/mol K; Ideal gas

$\Delta U = nC_V\Delta T$

= 5 mol × 28 J/mol K × (200 – 100) K

= 14,000 J = 14 kJ

$\Rightarrow C_p = C_v + R = (28 + 8)$ J/mol K

= 36 J/mol K

$\Rightarrow \Delta H = nC_p\Delta T$ = 5 mol × 36 J/mol K × 100 K

= 18000 J = 18 kJ

$\Delta H = \Delta U + \Delta(PV)$

$\Rightarrow \Delta(PV) = \Delta H - \Delta U$ = (18 – 14) kJ = 4 kJ

6. Ans.(3)

Sol. (A) q + w = $\Delta U \leftarrow$ definite quantity

(B) q $\rightarrow$ Path function

(C) w $\rightarrow$ Path function

(D) H – TS = G $\rightarrow$ state function

$\therefore$ Ans.(3)

7. Ans.(1)

$\Delta U = q + w$

q = – 2kJ, W = 10kJ

ΔU = 8kJ

8. Ans.(3)

$W = -P_{ext} (V_2 - V_1)$

= –1bar × (10–1)lit

= –9 bar–lit

= –900 J

= –0.9 kJ

THERMODYNAMICS-02

1. Ans.(3)

$\underset{T_1}{\boxed{1}}\ \underset{T_2}{\boxed{2}} \rightleftharpoons \underset{T_F}{\boxed{1}}\underset{T_F}{\boxed{1}}$

Heat lost by block - I = Heat gained by block - II $Cm(T_f - T_1) = C_m (T_2 - T_f)$

$T_f = \frac{T_1 + T_2}{2}$

$\Delta S_1 = Cp \ln \frac{T_f}{T_1}$

$$\Delta S_T = Cp \ln\left(\frac{T_f}{T_1}\right) + Cp \ln\left(\frac{T_f}{T_2}\right)$$

$$\Delta S_T = Cp \ln\left(\frac{T_f^{\,2}}{T_1 . T_2}\right)$$

2. Ans. (1)

At equilibrium

$$120 - \frac{3}{8}T = 0$$

$\Rightarrow T = 320$ K

If $T < 320$ K $\Rightarrow \Delta G = +ve \Rightarrow$ X is major product

If $T > 320$ K $\Rightarrow \Delta G = -ve \Rightarrow$ Y is major product.

3. Ans.(1)

$\Delta G^o = -RT \ln k$

if $K < 1 \Rightarrow \Delta G^o > 0$

4. Ans.(2)

$\Delta G = \Delta H - T\Delta S$

for spontaneous process at all temp. $\Delta G < 0$ and it is possible when $\Delta H < 0$ and $\Delta S > 0$.

5. Ans. (4)

$2H_2O = H_3O^+ + OH^-$ $K = 10^{-14}$

$\Delta G^\circ = -RT\ \ell n\ K$

$$= \frac{-8.314}{1000} \times 298 \times \ell n 10^{-14}$$

$= 80$ KJ/Mole

6. Ans. (4)

Compare with $\Delta G = \Delta H - T\Delta S$

7. Ans. (2)

$$T_{eq} = \frac{\Delta H}{\Delta S}$$

$$= \frac{491.1 \times 1000}{198}$$

$= 2480.3$ K

8. Ans. (1)

$\Delta G = \Delta H - T\Delta S$

$$T = \frac{\Delta H}{\Delta S} = \frac{200}{40} = 5K$$

9. Ans. (4)

$$\underset{273K}{H_2O(s)} \xrightarrow{\Delta S_1} \underset{273K}{H_2O(\ell)} \xrightarrow{\Delta S_2} \underset{373K}{H_2O(\ell)} \xrightarrow{\Delta S_3} \underset{373K}{H_2O(g)} \xrightarrow{\Delta S_4} \underset{383K}{H_2O(g)}$$

$$\Delta S_1 = \frac{\Delta H_{fusion}}{273} = \frac{334}{273} = 1.22$$

$$\Delta S_2 = 4.2 \ell N\left(\frac{373}{273}\right) = 1.31$$

$$\Delta S_3 = \frac{\Delta H_{vap}}{373} = \frac{2491}{373} = 6.67$$

$$\Delta S_4 = 2.0 \ell n\left(\frac{383}{373}\right) = 0.05$$

$\Delta S_{total} = 9.26$ kJ kg^{-1} K^{-1}

IONIC EQUILIBRIUM

1. Ans. (2)

$$Ag_2CO_3\ (s) \rightleftharpoons 2Ag^+(aq.) + CO_3^{-2}(aq)$$

$(0.1 + 2S)$ M $\quad$ S M

$$Ksp = [Ag^+]^2[CO_3^{-2}]$$

$8 \times 10^{-12} = (0.1 + 2S)^2\ (S)$

$S = 8 \times 10^{-10}$ M

2. Ans. (1)

HCl with Na_2CO_3

Eq. of HCl = Eq. of Na_2CO_3

$$\frac{25}{1000} \times M \times 1 = \frac{30}{1000} \times 0.1 \times 2$$

$$M = \frac{6}{25} M$$

Eq of HCl = Eq. of NaOH

$$\frac{6}{25} \times 1 \times \frac{V}{1000} = \frac{30}{1000} \times 0.2 \times 1$$

$V = 25$ ml

3. Ans. (3)

$Ca(OH)_2 + Na_2SO_4 \longrightarrow CaSO_4 + 2NaOH$

100 m mol 14 m mol — —

— — 14 m mol 28 m mol

$w_{CaSO_4} = 14 \times 10^{-3} \times 136 = 1.9 gm$

$[OH^-] = \frac{28}{100} = 0.28 M$

4. Ans. (3)

pH of rain water is approximate 5.6

5. Ans. (3)

20 ml 0.1 M $H_2SO_4 \Rightarrow \eta_{H^+} = 4$

30 ml 0.2 M $NH_4OH \Rightarrow \eta_{NH_4OH} = 6$

$NH_4OH + H^+ \rightleftharpoons NH_4^{\oplus} + H_2O$

$\Rightarrow$ 6 4 0 0

$\Rightarrow$ 2 0 4 4

Solution is basic buffer

$$pOH = pK_b + \log \frac{NH_4^+}{NH_4OH}$$

$= 4.7 + \log 2$

$= 4.7 + 0.3 = 5$

pH = 14 – 5 = 9

6. Ans.(4)

Sol. $Zr_3(PO_4)_4(s) \rightleftharpoons 3Zr^{4+}(aq.) + 4PO_4^{3-}(aq.)$

3S M 4S M

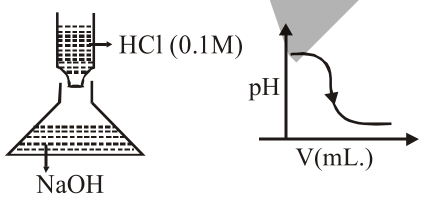
$K_{sp} = [Zr^{4+}]^3 [PO_4^{3-}]^4 = (3S)^3.(4S)^4 = 6912\ S^7$

$\therefore S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

Correct option : (4)

7. Ans. (1)

Sol.



8. Ans. (2)

Sol. (a) $H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$

400×.1=40 400×.1=40

20 0

$\therefore [H^+] = \frac{20 \times 2}{800} = \frac{1}{20} \Rightarrow pH = -\log\left(\frac{1}{20}\right)$

$\therefore$ pH = 1.3 so (a) is correct

(b) $\log\left(\frac{Kw_2}{Kw_1}\right) = \frac{\Delta H}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

so ionic product of water is temp. dependent hence (b) is correct.

(c) $K_a = 10^{-5}$, pH = 5 $\Rightarrow [H^+] = 10^{-5}$

$K_a = \frac{c\alpha^2}{(1-\alpha)} \Rightarrow K_a = \frac{[H^+].\alpha}{(1-\alpha)}$

$\therefore 10^{-5} = \frac{10^{-5}.\alpha}{(1-\alpha)} \Rightarrow 1 - \alpha = \alpha \Rightarrow \alpha = \frac{1}{2} =$ 50%

so (c) is correct.

(d) Le-chatelier's principle is applicable to common –Ion effect so option (d) is wrong

$\therefore$ correct answer (2)

9. Ans. (2)

Sol. For the salt of strong acid and weak base

$H^+ = \sqrt{\frac{K_w \times C}{K_b}}$

$[H^+] = \sqrt{\frac{10^{-14} \times 2 \times 10^{-2}}{10^{-5}}}$

$-\log[H^+] = 6 - \frac{1}{2}\log 20$

$\therefore$ pH = 5.35

10. Ans. (4)

Sol. $Al(OH)_3 \rightleftharpoons Al^{+3} + 3OH^-$

S' 0.2 + 3(S') $\simeq$ 0.2

$S' \times (0.2)^3 = k_{sp} = 2.4 \times 10^{-24}$

$(S') = 3 \times 10^{-22}$ M

11. Ans. (4)

Sol. $K_{sp} = 4\ (s)^3$

$= 4 \times (1.84 \times 10^{-5})^3$

$Cd(OH)_2 \rightleftharpoons Cd^{+2} + 2OH^-$

S' S' $(10^{-2} + S') \simeq 10^{-2}$

$S' \times (10^{-2})^2 = 4 \times (1.84 \times 10^{-5})^3$

$S' = 4 \times (1.84)^3 \times 10^{-11}$

$(S') = 2.491 \times 10^{-10}$ M

REAL GAS

1. Ans. (1)

$V_A = 2V_B$

$Z_A = 3Z_B$

$$\frac{P_A V_A}{n_A RT_A} = \frac{3 \cdot P_B \cdot V_B}{n_B.RT_B}$$

$2P_A = 3P_B$

2. Ans. (1)

Sol. $T_c = \dfrac{8a}{27Rb}$

Greater value of $\frac{a}{b} \Rightarrow$ higher is 'T_c'

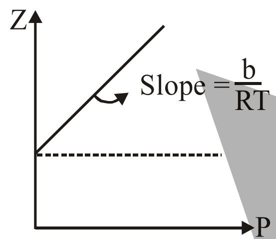
Gas	a/b
Ar	$\frac{1.3}{3.2} = 0.406$
Ne	$\frac{0.2}{1.7} = 0.118$
Kr	$\frac{5.1}{1} = 5.1$
Xe	$\frac{4.1}{5} = 0.82$

$\therefore$ T_c has order : Kr > Xe > Ar > Ne

$\therefore$ Ans. is (1)

3. Ans. (3)

Sol.



As b $\uparrow \Rightarrow$ slope $\uparrow$

Hence, Xe, will have highest slope

4. Ans. (3)

Sol.
- Gas A and C have same value of 'b' but different value of 'a' so gas having higher value of 'a' have more force of attraction so molecules will be more closer hence occupy less volume.
- Gas B and D have same value of 'a' but different value of 'b' so gas having lesser value of 'b' will be more compressible.

so option (3) is correct.

LIQUID SOLUTION

1. Ans. (2)

For same freezing point, molality of both solution should be same.

$m_x = m_y$

$$\frac{4\times 1000}{96\times M_x} = \frac{12\times 1000}{88\times M_y}$$

or, $M_y = \dfrac{96\times 12}{4\times 88} M_x = 3.27$ A

Closest option is 3A.

2. Ans. (2)

$$2(\underset{Wg}{C_6H_5COOH}) \xrightarrow[(30g)]{C_6H_6} \underset{diner}{(C_6H_5COOH)_2}$$

$\Delta_f T = i\ k_f\ m$

$2 = 0.6 \times 5 \times \dfrac{w\times 1000}{122\times 30}$

$(i = 1 - 0.8 + 0.4 = 0.6)$

$w = 2.44$ g

3. Ans. (3)

4. Ans. (1)

For $K_2[HgI_4]$

$i = 1 + 0.4\ (3-1)$

$= 1.8$

5. Ans. (2)

$$y_A = \frac{P_A}{P_{Total}} = \frac{P_A^o x_A}{P_A^o x_A + p_B^o x_B}$$

$$= \frac{7\times 10^3 \times 0.4}{7\times 10^3\times 0.4 + 12\times 10^3 \times 0.6} = \frac{2.8}{10} = 0.28$$

$y_B = 0.72$

6. Ans. (4)

$\Delta T_f = K_f\ .\ m$

$10 = 1.86 \times \dfrac{62/62}{W_{kg}}$

$W = 0.186$ kg

$\Delta W = (250 - 186) = 64$ gm

7. Ans. (2)

Liquid solution

$P_{gas} = K_H \times X_{gas}$

More is K_H less is solubility, lesser solubility is at higher temperature. So more is temperature more is K_H.

8. Ans. (2)

$$\frac{\Delta T_b}{\Delta T_f} = \frac{i.m \times k_b}{i \times m \times k_f}$$

$$\frac{2}{2} = \frac{1 \times 1 \times k_b}{1 \times 2 \times k_f}$$

$k_b = 2k_f$

9. Ans. (4)

$P_{total} = X_A . P_A^0 + X_B . P_B^0 = 0.5 \times 400 + 0.5 \times 600$
$= 500$ mmHg

Now, mole fraction of A in vapour,

$Y_A = \frac{P_A}{P_{total}} = \frac{0.5 \times 400}{500} = 0.4$

and mole fraction of B in vapour,

$Y_B = 1 - 0.4 = 0.6$

Correct option : (4)

10. Ans. (3)

$$p = k_H \times \left(\frac{n_{gas}}{n_{H_2O} + n_{gas}} \right)$$

$$= k_H \left(1 - \frac{n_{H_2O}}{n_{H_2O} + n_{gas}} \right)$$

$\Rightarrow p = k_H - k_H \times \chi_{H_2O}$

$p = (-k_H) \times \chi_{H_2O} + k_H$

11. Ans. (1)

$\pi_{XY} = 4\pi_{BaCl_2}$

$2 \times [XY] = 4 \times 3 \times 0.01$

(Assuming same temperature)

$\Rightarrow [XY] = 0.06$ M

$\therefore$ Ans. is (1)

12. Ans. (3)

$\because P_N^\circ > P_M^\circ$

$\therefore y_N > X_N$

& $X_M > y_M$

Multiply we get

$y_N X_M > X_N y_M$

$\therefore$ Ans. is (3)

13. Ans. (2)

Sol. $K_f = 4$ K-kg/mol

$m = 0.03$ mol/kg

$i = 3$

$\Delta T_f = iK_f \times m$

$\Delta T_f = 3 \times 4 \times 0.03 = 0.36K$

14. Ans. (3)

Sol. Lowering of vapour pressure $= p^0 - p = p^0.x_{solute}$

$$\therefore \Delta p = 35 \times \frac{0.6/60}{\frac{0.6}{60} + \frac{360}{18}}$$

$$= 35 \times \frac{.01}{.01 + 20} = 35 \times \frac{.01}{20.01}$$

$= .017$ mm Hg

15. Ans. (1)

Sol. $$\Pi = \frac{\left(\frac{0.6}{60} + \frac{1.8}{180} \right)}{0.1} \times 0.08206 \times 300$$

$\Pi = 4.9236$ atm

16. Ans. (3)

Sol. $\Delta T_b = K_b \times m$

$$\therefore \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{K_{b(A)}}{K_{b(B)}} \text{ as } m_A = m_B$$

$$\therefore \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{1}{5}$$

CHEMICAL EQUILIBRIUM

1. Ans.(2)

$$A + 2B \rightleftharpoons 2C + D$$

	A	2B	2C	D
t=0	a_0	$1.5a_0$	0	0
$t = t_{eq}$	$a_0 - x$	$1.5a_0 - 2x$	2x	x

At equilibrium [A] = [B]

$a_0 - x = 1.5a_0 - 2x \Rightarrow x = 0.5a_0$

$t = t_{eq}$ $0.5a_0$ $0.5a_0$ a_0 $0.5a_0$

$$K_C = \frac{[C]^2[D]}{[A][B]^2} = \frac{(a_0)^2\ (0.5a_0)}{(0.5a_0)\ (0.5a_0)^2} = 4$$

2. Ans. (3)

$A(s) \rightleftharpoons B(g) + C(g) \quad K_{P_1} = x = P_B \cdot P_C$

$P_1 \quad P_1 \quad x = P_1(P_1 + P_2) \quad ...(1)$

$D(s) \rightleftharpoons C(g) + E(g) \quad K_{P_2} = y = P_C \cdot P_E$

$P_2 \quad P_2 \quad y = (P_1 + P_2)(P_2) \quad ...(2)$

Adding (1) and (2)

$x + y = (P_1 + P_2)^2$

Now total pressure

$P_T = P_C + P_B + P_E$

$= (P_1 + P_2) + P_1 + P_2 = 2(P_1 + P_2)$

$P_T = 2\left(\sqrt{x+y}\right)$

3. Ans. (2)

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) \qquad \frac{1}{K_P}$

t = 0 P_1 – –

t = t $P_1 - 2P_2 \quad P_2 \quad 3P_2$

$(P1 - 2P^2) + P_2 + 3P_2 = P$

As $(P_1 - 2P_2) <<< P$

$P_2 = \frac{P}{4}$

$\frac{1}{K_P} = \frac{(P/4)(3P/4)^3}{P_{NH_3}^2}$

4. Ans. (2)

$A_2(g) + B_2(g) \underset{}{\overset{k_1}{\rightleftharpoons}} 2AB \quad ...(1)$

$\Rightarrow$ eq. (1) × 3

$6\ AB(g) \rightleftharpoons 3A_2(g) + 3B_2(g)$

$\Rightarrow \left(\frac{1}{k_1}\right)^3 = k_2 \Rightarrow k_2 = (k_1)^{-3}$

5. Ans. (3)

$NH_4SH(s) \rightleftharpoons NH_3(g) + H_2S(g)$

$n = \frac{5.1}{51} = 0.1\text{mole} \quad 0 \quad 0$

$0.1(1-\alpha) \quad 0.1\alpha \quad 0.1\alpha$

$\alpha = 30\% = .3$

so number of moles at equilibrium

$.1\ (1 - .3) \quad .1 \times .3 \quad .1 \times .3$

$= \quad .07 \quad = .03 \quad = .03$

Now use PV = nRT at equilibrium

$P_{total} \times 3 \text{ lit} = (.03 + .03) \times .082 \times 600$

$P_{total} = .984$ atm

At equilibrium

$P_{NH_3} = P_{H_2S} = \frac{P_{total}}{2} = .492$

So $k_p = P_{NH_3} \cdot P_{H_2S} = (.492)(.492)$

$k_p = 0.242 \text{ atm}^2$

6. Ans. (4)

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$\frac{k_p}{k_c} = (RT)^{\Delta n_g} = (RT)^0 = 1$

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$\frac{k_p}{k_c} = (RT)^1 = 24.62$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

$\frac{k_p}{k_c} = (RT)^{-2} = \frac{1}{(RT)^2} = 1.65 \times 10^{-3}$

7. Ans. (3)

Sol. $S(s) + O_2(g) \rightleftharpoons SO_2(g) \quad K_1 = 10^{52} \quad ...(1)$

$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g)\ K_2 = 10^{129} \quad ...(2)$

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)\ K_3 = x$

multiplying equation (1) by 2;

$2SO(s) + 2O_2(g) \rightleftharpoons 2SO_2(g) \quad K_1' = 10^{104} \quad ...(3)$

$\Rightarrow$ Substracting (3) from (2); we get

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g);$

8. Ans. (1)

Sol. In option (2)- Δn_g is –ve therfore increase in pressure will bring reaction in forward direction.

In option (3)- as the reaction is exothermic therefore increase in temperature will decrease the equilibrium constant.

In option (4)- Equillibrium constant changes only with temperature.

Hence, option (2), (3) and (4) are correct therefore option (1) is incorrect choice.

9. Ans. (4)

Sol. if $\Delta n_g \neq 0$

$Kp \neq Kc$

SURFACE CHEMISTRY

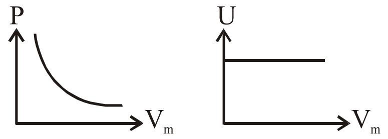
1. Ans. (1)

Colloidal solution fo rubber are negatively charged.

2. Ans. (3)

Isothermal expansion $PV_m = K$(Graph-C)

$$P = \frac{K}{V_m} \text{ (Graph-A)}$$



3. Ans. (2)

4. Ans. (4)

	Dispersed Phase	Dispersion Medium
Cheese	Liquid	Solid
Milk	Liquid	Liquid
Smoke	Solid	Gas

5. Ans. (4)

$\frac{x}{m} = K \times P^{1/n}$

$\log\frac{x}{m} = \log K + \frac{1}{n}\log P$

$m = \frac{1}{n} = \frac{2}{4} = \frac{1}{2} \Rightarrow n = 2$

So, $\frac{x}{m} = K \times P^{1/2}$

6. Ans.(4)

Haemoglobin $\longrightarrow$ positive sol

Gold sol $\longrightarrow$ negative sol

7. Ans. (3)

Sol. $\frac{x}{m} = K.p^{1/n}$

$\therefore \log\frac{x}{m} = \log K + \frac{1}{n}.\log P$

slope $= \frac{1}{n} = \frac{2}{3}$

$\therefore \frac{x}{m} = K.p^{2/3}$

Correct option : (3)

8. Ans. (2)

Aerosol is suspension of fine solid or liquid particles in air or other gas.

Ex. Fog, dust, smoke etc

$\therefore$ Ans.(2)

9. Ans. (3)

Freundlich adsorption isotherm $\frac{x}{m} = Kp^{0.5}$

so on increasing pressure, $\frac{x}{m}$ increases

physical adsorption decreases with increase in temperature so option (3) is correct.

10. Ans. (1)

Sol. In electrophoresis precipitation occurs at the electrode which is oppositely charged therefore (1) is correct.

11. Ans. (2)

12. Ans. (4)

Colligative properties of colloidal solution are smaller than true solution

13. Ans. (1)

Millimoles $= 10 \times 10^{-3} = 10^{-2}$

Moles $= 10^{-5}$

No. of molecules $= 6 \times 10^{23} \times 10^{-5} = 6 \times 10^{+18}$

surface area occupied by one molecule

$$= \frac{0.24}{6 \times 10^{18}} = 0.04 \times 10^{-18} cm^2$$

$4 \times 10^{-20} = a^2$

$a = 2 \times 10^{-10} cm = 2pm$

14. Ans. (1)

Sulphide is –ve charged colloid so cation with maximum charge will be most effective for coagulation.

$Al^{3+} > Ba^{2+} > Na^{+}$ coagulating power.

MOLE CONCEPT

1. Ans. (2)

$2NaHCO_3 + H_2C_2O_4 \rightarrow Na_2C_2O_4 + 2H_2O + 2CO_2$ moles of $CO_2 = \frac{0.25\times10^{-3}}{25} = 10^{-5}$

moles of $NaHCO_3 = 10^{-5}$ mol

mass of $NaHCO_3 = 10^{-5} \times 84$ gm

% of $naHCO_3 = \frac{84\times10^{-5}}{10\times10^{-3}} = 100$

$= 8.4\%$

2. Ans. (1)

moles of $C_{57}H_{110}O_6(s) = \frac{445}{890} = 0.5$ moles

$2C_{57}H_{110}O_6(s) + 163\,O_2(g) \rightarrow 114\,CO_2(g) + 110\,H_2O(l)$

$n_{H_2O} = \frac{110}{4} = \frac{55}{2}$

$m_{H_2O} = \frac{55}{2}\times18$

$= 495$gm

3. Ans. (4)

$[C_xH_yN_z] \xrightarrow[\text{Method}]{\text{Duma}} 6CO_2 + 4H_2O + N_2$

Hence, $C_6H_8N_2$

4. Ans. (4)

CH_4

% by mole of carbon $= \frac{1\,\text{mol atom}}{5\,\text{mol atom}}\times100$

$= 20\%$

5. Ans. (3)

$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$

(1) $\underset{(LR)}{0.5\text{ mol}}$ 2 mol

(2) 1 mol 3 mol (completion)

(3) 2 mol $\underset{(LR)}{5\text{ mol}}$

(4) $\underset{(LR)}{1.25\text{ mol}}$ 4 mol

$\therefore$ Ans.(3)

6. Ans. (3)

$\frac{w}{w}\% = 20$

100 gm solution has 20 gm KI

80 gm solvent has 20 gm KI

$$m = \frac{\frac{20}{166}}{\frac{80}{1000}} = \frac{20\times1000}{166\times80} = 1.506 \simeq 1.51\,\text{mol/kg}$$

7. Ans. (4)

$$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

$$10 \qquad 10\left(x+\frac{y}{4}\right) \qquad 10x$$

By given data, $10\left(x+\frac{y}{4}\right) = 55$ .... (1)

$10x = 40$ .... (2)

$\therefore$ x = 4, y = 6 $\Rightarrow C_4H_6$

8. Ans. (3)

Sol. $C_3H_8(g) + 5O_2(g) \longrightarrow 3CO_2(g) + 4H_2O(\ell)$

Each 1g of C_3H_8 requires 3.63 g of O_2

$P_4(s) + 5O_2(g) \longrightarrow P_4O_{10}(s)$

Each 1g of P_4 requires 1.29 g of O_2

$4Fe(s) + 3O_2(g) \longrightarrow 2Fe_2O_3(s)$

Each 1g of Fe requires 0.428 g of O_2

$2Mg(s) + O_2(g) \longrightarrow 2MgO(s)$

Each 1g of Mg requires 0.66 g of O_2

therefore least amount of O_2 is required in option (3).

9. Ans. (3)

$5[M_A + 2M_B] = 125$

$M_A + 2M_B = 25$ ....(1)

$2M_A + 2M_B = 30$ ....(2)

from eq. (1) & (2)

$M_A = 5$

$M_B = 10$

10. Ans. (2)

$$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

$$\left(\frac{25}{M}\right) \qquad x\times\frac{25}{M} = 2 \qquad \frac{y}{2}\times\frac{25}{M} = 0.5$$

C $\quad x\times\frac{25}{M}=2$

H $\quad y\times\frac{25}{M}=1$

$C_{2y}H_y \equiv$ 24y gm C + y gm H

or

24 : 1 ratio by mass

IDEAL GAS

1. Ans. (3)

$n_T = (0.5+x)$

PV = n × R × T

200 × 10 = (0.5 + x) × R × 1000

2 = (0.5 + x) R

$$\frac{2}{R}=\frac{1}{2}+x$$

$$\frac{4}{R}-1=2x$$

$$\boxed{\frac{4-R}{2R}=x}$$

2. Ans. (4)

$\frac{2}{5}$ air escaped from vessel,

$\therefore \frac{3}{5}$ air remain is vessel. P, V constant

$n_1T_1 = n_2T_2$

$n_1(300) = \left(\frac{3}{5}n_1\right)T_2 \Rightarrow T_2 = 500$ K

3. Ans. (4)

$$V_{mp}=\sqrt{\frac{2RT}{M}} \Rightarrow V_{mp}\propto\sqrt{\frac{T}{M}}$$

For N_2, O_2, H_2

$$\sqrt{\frac{300}{28}}<\sqrt{\frac{400}{32}}<\sqrt{\frac{300}{2}}$$

V_{mp} of N_2(300K) < V_{mp} of O_2(400K) < V_{mp} of H_2(300K)

CONCENTRATION TERMS

1. Ans. (2)

1L – 1M H_2O_2 solution will produce 11.35 L O_2 gas at STP.

2. Ans. (1)

8g NaOH, mol of NaOH = $\frac{8}{40}=0.2$mol

18g H_2O, mol of H_2O = $\frac{18}{18}=1$mol

$\therefore X_{NaOH} = \frac{0.2}{1.2}=0.167$

Molality = $\frac{0.2\times1000}{18}=11.11$ m

3. Ans. (3)

$n_{Na^+}=\frac{92}{23}=4$

So molality = 4

4. Ans. (1)

Molarity = $\frac{(n)_{solute}}{V_{solution}\text{ (in lit)}}$

$0.1=\frac{wt./342}{2}$

wt ($C_{12}H_{22}O_{11}$) = 68.4 gram

5. Ans. (2)

Volume strength = 11.2 × molarity = 11.2

$\Rightarrow$ molarity = 1 M

$\Rightarrow$ strength = 34 g/L

$\Rightarrow$ % w/w = $\frac{34}{1000}\times100 = 3.4\%$

6. **Ans. (3)**

$X_{solvent} = 0.8$

If $n_T = 1$

$n_{Solvent} = 0.8$

$n_{Solute} = 0.2$

$$\text{molality} = \frac{0.2}{\frac{0.8\times 18}{1000}} = 13.88$$

ELECTROCHEMISTRY

1. **Ans. (1)**

$\Delta G = \Delta H - \Delta S$

$$-nFE_{cell} = \Delta H - nFT\frac{dE_{cell}}{dT}$$

(n = 2)

2. **Ans. (2)**

$$\Lambda_m^0(HA) = \Lambda_m^0(HCl) + \Lambda_m^0(NaA) - \Lambda_m^0(NaCl)$$

= 425.9 + 100.5 - 126.4

= 400 S cm^2 mol^{-1}

$$\Lambda_m = \frac{1000K}{M} = \frac{1000\times 5\times 10^{-5}}{10^{-3}} = 50\text{ S cm}^2\text{ mol}^{-1}$$

$$\alpha = \frac{\Lambda_m}{\Lambda_m^0} = \frac{50}{400} = 0.125$$

3. **Ans. (2)**

Higher the oxidation potential better will be reducing power.

4. **Ans. (1)**

$$Pt(s)|H_2(g, 1\,bar)|HCl(aq)|AgCl(s)|Ag(s)|Pt(s)$$

$$10^{-6}\text{ m}$$

Anode: $H_2 \longrightarrow 2H^+ + 2e \times 1$

Cathode : $e^- + AgCl(s) \longrightarrow Ag(s) + Cl^-(aq) \times 2$

$$H_2(g) + 2AgCl(s) \longrightarrow 2H^+ + 2Ag(s) + 2Cl^-(aq)$$

$$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log_{10}\left((H^+)^2\cdot(Cl^-)^2\right)$$

$$0.92 = \left(E^0_{H_2/H^+} + E^0_{AgCl/Ag,\,Cl^-}\right) - \frac{0.06}{2}\log_{10}\left((10^{-6})^2(10^{-6})^2\right)$$

$$0.92 = 0 + E^0_{AgCl/Ag,Cl^-} - 0.03\log_{10}(10^{-6})^4$$

$E^0_{AgCl}/Ag, Cl^-$ = .92 + .03 × (–24) = 0.2 V

5. **Ans. (3)**

(A) $PbSO_4(s) + 2OH^- \rightarrow PbO_2 + H_2SO_4 + 2e^-$

0.05/2 mole 0.05F

(C) $PbSO_4 + 2e^- + 2H^+ \longrightarrow Pb(s) + H_2SO_4$

0.05/2 mole 0.05 F

$n_T(PbSO_4) = 0.05$ mole

$m_{PbSO_4} = 0.05\times 303 = 15.2$ gm

6. **Ans. (2)**

(1) $3Zn_{(s)} + 2Au^{+3}_{(aq)} \xrightarrow{n=6} 3Zn^{+2}_{(aq)} + 2Au_{(s)}$

$$\frac{E^0_{cell}}{n} = 0.36$$

(2) $Zn_{(s)} + 2Ag^+_{(aq)} \xrightarrow{n=2} Zn^{+2}_{(aq)} + 2Ag_{(s)}$

$$\frac{E^0_{cell}}{n} = 0.78$$

(3) $Zn_{(s)} + 2Fe^{+3}_{(aq)} \xrightarrow{n=2} Zn^{+2}_{(aq)} + 2Fe^{+2}_{(aq)}$

$$\frac{E^0_{cell}}{n} = 0.765$$

(4) $Zn_{(s)} + Fe^{+2}_{(aq)} \xrightarrow{n=2} Zn^{+2}_{(aq)} + Fe_{(s)}$

$$\frac{E^0_{cell}}{n} = 0.16$$

We have maximum value of $\left(\frac{E^0_{cell}}{n}\right)$ for reaction (2)

7. **Ans. (1)**

$\Delta G^\circ = -RT\ln k = -nFE^\circ_{cell}$

$$\ln k = \frac{n\times F\times E^\circ}{R\times T} = \frac{2\times 96000\times 2}{8\times 300}$$

$\ln k = 160$

$k = e^{160}$

8. Ans. (2)

$$E_{cell} = E^{o}_{cell} - \frac{0.059}{n}\log Q$$

At equilibrium

$$E^{\circ}_{Cell} = \frac{0.059}{2}\log 10^{16}$$

$= 0.059\times 8$

$= 0.472$ V

$\simeq 0.4736$ V

9. Ans. (3)

For strongest oxidising agent, standard reduction potential should be highest.

Correct option : (3)

10. Ans. (1)

Sol. $Fe^{+2}(aq) + Ag^{+}(aq) \rightarrow Fe^{+3}(aq) + Ag(s)$

Cell reaction

anode : $Fe^{+2}(aq) \rightarrow Fe^{+3}(aq) + e^{\ominus}$;

$$E^{o}_{Fe^{+2}/Fe^{+3}} = mV$$

cathode : $Ag^{+}(aq) + e^{\ominus} \rightarrow Ag(s)$;

$$E^{o}_{Ag^{+}/Ag} = xV$$

$\Rightarrow$ cell standard potential = (m + x)V

$\therefore$ to find 'm';

$Fe^{+2} + 2e^{\ominus} \rightarrow Fe$;

$$E^{o}_{1} = yV \Rightarrow \Delta^{o}_{1}G = -(2Fy)$$

$Fe^{+3} + 3e^{\ominus} \rightarrow Fe$;

$$E^{o}_{2} = zV \Rightarrow \Delta^{o}_{2}G = -(3Fz)$$

$Fe^{+2}(aq) \rightarrow Fe^{+3}(aq) + e^{\ominus}$;

$$E^{o}_{3} = mV \Rightarrow \Delta^{o}_{3}G = -(1Fm)$$

$\Delta^{o}_{3}G = \Delta G^{o}_{1} - \Delta G^{o}_{2} = (-2Fy + 3Fz) = -Fm$

$\Rightarrow m = (2y - 3z)$

$\Rightarrow E^{o}_{cell} = (x + 2y - 3z)V$

11. Ans. (1)

$\Delta G^{\circ} = -nFE^{\circ}_{cell}$

$= -2 \times 96000 \times 2$

$= -384000$ J

$= -384$ kJ

$\therefore$ Ans. is (1)

12. Ans. (2)

0.1 eq. of Ni^{+2} will be discharged.

No. of eq = (No of moles) × (n-factor)

0.1 = (No. of moles) × 2

No. of moles of Ni = $\frac{0.1}{2} = 0.05$

13. Ans. (2)

On dilution, no. of ions per ml decreases so conductivity decreases hence S1 is wrong.

$$\wedge_M = \frac{1000\times\kappa}{C}$$

On dilution C and κ both decreases but effect of C is more dominating so $\wedge_M$ increases hence S2 is right.

14. Ans. (2)

Both NaCl and KCl are strong electrolytes and as $Na^{+}(aq.)$ has less conductance than $K^{+}(aq.)$ due to more hydration therefore the graph of option (2) is correct.

15. Ans. (4)

$E^{\circ}_{Red}\uparrow \Rightarrow$ oxidizing power $\uparrow$

16. Ans. (3)

Order of acidic strength

A > C > B

Acidic strength $\uparrow \Rightarrow$ degree of ionization $\uparrow$

REDOX

1. Ans. (1)

ppm of $CaCO_3$

$(10^{-3} \times 10^{3}) \times 100 = 100$ ppm

2. BONUS

$$H_2C_2O_4 + 2NaOH \longrightarrow Na_2C_2O_4 + 2H_2O$$

m_{eq} of $H_2C_2O_4 = m_{eq}$ NaOH

$50 \times 0.5 \times 2 = 25 \times M_{NaOH} \times 1$

$\therefore \quad M_{NaOH} = 2$ M

Now 1000 ml solution = 2 × 40 gram NaOH

$\therefore \quad$ 50 ml solution = 4 gram NaOH

3. Ans. (3)

$$2\overset{+7}{M}nO_4 + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2\overset{+2}{M}n^{2+} + 10CO_2 + 8H_2O$$

10 e^- transfer for 10 molecules of CO_2 so per molecule of CO_2 transfer of e^- is '1'

4. Ans. (2)

H_2O_2 act as oxidising agent and reducing agent in acidic medium as well as basic medium.

H_2O_2 Act as oxidant :-

$H_2O_2 + 2H^{\oplus} + 2e^{\ominus} \rightarrow 2H_2O$ (In acidic medium)

$H_2O + 2e^{\ominus} \rightarrow 2OH^{\ominus}$ (In basic medium)

H_2O_2 Act as reductant :-

$H_2O_2 \rightarrow 2H^+ + O_2 + 2e^{\ominus}$ (In acidic medium)

$H_2O_2 + 2OH^{\ominus} \rightarrow 2H_2O + O_2 + 2e^{\ominus}$ (In basic medium)

5. Ans. (2)

Sol. $n_{eq.}KMnO_4 = n_{eq.}[FeC_2O_4 + Fe_2(C_2O_4)_3 + FeSO_4]$

or $n \times 5 = 1 \times 3 + 1 \times 6 + 1 \times 1$

$\therefore\ n = 2$

Correct option : (2)

6. Ans. (2)

Sol. $n_{eq.}CaCO_3 = n_{eq}Ca(HCO_3)_2 + n_{eq}Mg(HCO_3)_2$

or , $\frac{W}{100} \times 2 = \frac{0.81}{162} \times 2 + \frac{0.73}{146} \times 2$

$\therefore\ w = 1.0$

$\therefore$ Hardness $= \frac{1.0}{100} \times 10^6 = 10000\text{ppm}$

Correct option : (2)

7. Ans. (3)

Sol.

$$2\overset{+1}{Cu}Br \longrightarrow \overset{+2}{Cu}Br_2 + \overset{0}{Cu}$$

Oxidation (+1 → +2); Reduction (+1 → 0)

SOLID STATE

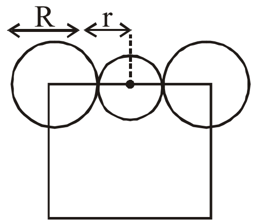
1. Ans. (4)

In Triclinic unit cell

$a \neq b \neq c$ & $\alpha \neq \beta \neq \gamma \neq 90°$

2. Ans. (2)

3. Ans. (3)



$a = 2(R + r)$

$\frac{a}{2} = (R + r)$ ...(1)

$a\sqrt{3} = 4R$ ...(2)

Using (1) & (2)

$\frac{a}{2} = \frac{a\sqrt{3}}{4} = r$

$a\left(\frac{2-\sqrt{3}}{4}\right) = r$

$r = 0.067\ a$

4. Ans. (4)

FCC unit cell Z = 4

$d = \frac{63.5 \times 4}{6 \times 10^{23} \times x^3 \times 10^{-24}}$ g/cm^3

$d = \frac{63.5 \times 4 \times 10}{6}$ g/cm^3

$d = \frac{423.33}{x^3} \simeq \left(\frac{422}{x^3}\right)$

5. Ans. (2)

Sol. Generally interstitial compounds are chemicaly inert.

6. Ans. (3)

Sol. p.f. $= \frac{\left(z_{eff} \times \frac{4}{3}\pi r_A^3\right)_A + \left(z_{eff} \times \frac{4}{3}\pi r_B^3\right)_B}{a^3}$

$2(r_A + r_B) = \sqrt{3}a$

$\Rightarrow 2(r_A + 2r_A) = \sqrt{3}a$

$\Rightarrow 2\sqrt{3}\, r_A = a$

$\Rightarrow$ p.f.$= \frac{1 \times \frac{4}{3}\pi r_A^3 + \frac{4}{3}\pi\left(8r_A^3\right)}{8 \times 3\sqrt{3}\, r_A^3} = \frac{9 \times \frac{4}{3}\pi}{8 \times 3\sqrt{3}} = \frac{\pi}{2\sqrt{3}}$

p. efficiency $= \frac{\pi}{2\sqrt{3}} \times 100 \simeq 90\%$

7. Ans. (1)

Sol. Distance between two nearest tetrahedral void = $\left(\frac{a}{2}\right)$

8. Ans. (1)

Sol. SC : BCC : FCC

1 : 2 : 4

9. Ans. (2)

A_2B_3 has HCP lattice

If A form HCP, then $\frac{3}{4}^{th}$ of THV must occupied by B to form A_2B_3

If B form HCP, then $\frac{1}{3}^{th}$ of THV must occupied by A to form A_2B_3

THERMOCHEMISTRY

1. Ans. (3)

$$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)\Delta_r H° = xkJ/mol \quad ...(1)$$

$$C_{(graphite)} + \frac{1}{2}O_2(g) \rightarrow CO(g)\Delta_r H° = ykJ/mol \quad ...(2)$$

$$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)\Delta_r H° = zkJ/mol \quad ...(3)$$

(1) = (2) + (3)

x = y + z

2. Ans. (2)

At higher temperature, rotational degree of freedom becomes active.

$C_P = \frac{7}{2}R$ (Independent of P)

$C_V = \frac{5}{2}R$ (Independent of V)

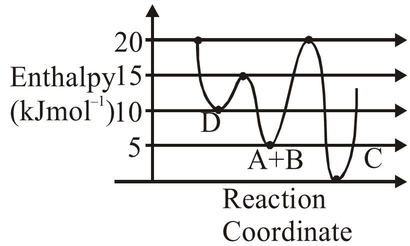
Variation of U vs T is similar as C_V vs T

3. Ans. (2)

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$; $\Delta n_g < 0$

4. Ans. (4)

Sol. $A + B \rightarrow C + D$



Activation enthalpy for C = 20 – 5 = 15kJ/mol

Activation enthalpy for D = 15 – 5 = 10kJ/mol.

5. Ans. (4)

Sol. $I_{2(s)} \rightarrow I_{2(g)}$: ΔH_1 = 24 cal/g at 200°C

$\Delta H_2 = \Delta H_1 + \Delta C_{P_{rxn}}(T_2 - T_1)$

= 24 + (0.031 –0.055) × 50

= 24–1.2

= 22.8 Cal/g

6. Ans. (3)

Sol. $C_7H_{16}(\ell) + 11O_2(g) \longrightarrow 7CO_2(g) + 8H_2O(\ell)$

$\Delta n_g = n_p - n_r = 7 - 11 = -4$

$\because$ $\Delta H = \Delta U + \Delta n_g RT$

$\therefore$ $\Delta H - \Delta U = -4$ RT

RADIOACTIVITY

1. Ans. (1)

Sol. From 0 to 1 hour, $N' = N_0 e^t$

From 1 hour onwards $\frac{dN}{dt} = -5N^2$

So at t = 1 hour, $N' = eN_0$

$$\frac{dN}{dt} = -5N^2$$

$$\int_{eN_0}^{N} N^{-2}\, dN = -5\int_1^t dt$$

$$\frac{1}{N} - \frac{1}{eN_0} = 5(t-1)$$

$$\frac{N_0}{N} - \frac{1}{e} = 5N_0(t-1)$$

$$\frac{N_0}{N} = 5N_0(t-1) + \frac{1}{e}$$

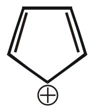
$$\frac{N_0}{N} = 5N_0 t + \left(\frac{1}{e} - 5N_0\right)$$

which is following y = mx + C

JANUARY 2019 ATTEMPT (OC)

GOC

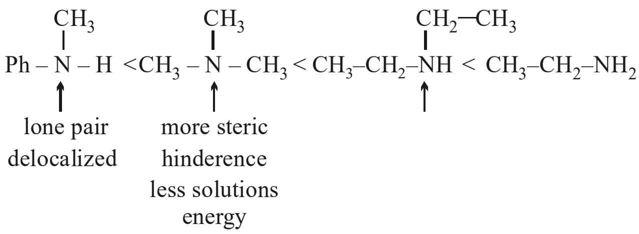
1. Ans. (3)



Do not have (4n + 2) π electron It has 4n π electrons

So it is Anti aromatic.

2. Ans. (1)

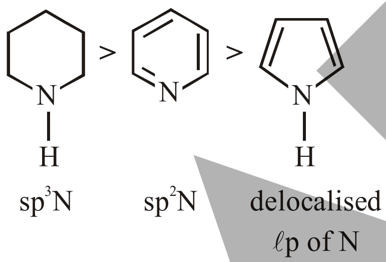


3. Ans. (4)

CN makes anino most stable so answer is $CH(CN)_3$

4. Ans. (4)

Order of basic strength :



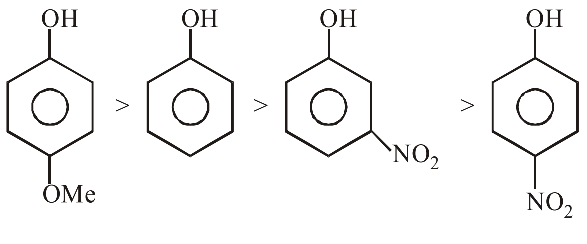
5. Ans. (1)

EWG increasea acidic strength

$NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

6. Ans. (4)

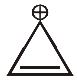
Acidic strength is inversely proportional to pka.



7. Ans. (1)

Localised lone pair e⁻.

8. Ans. (2)

out of the given options only  is aromatic.

Hence (B), (C) and (D) are not aromatic

9. Ans. (2)

$CH\equiv CH > CH_3–C\equiv CH > CH_2=CH_2$

(Acidic strength order)

10. Ans. (1)

M.P. of Napthalene ≃ 80°C

11. Ans. (1)

Sol. Basic strength order

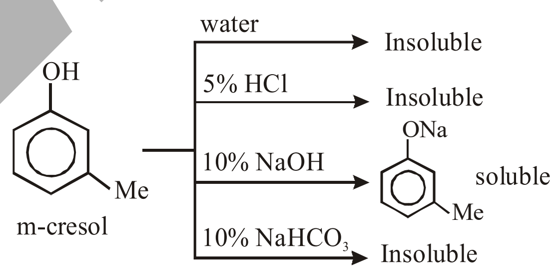
$(CH_3CH_2)_2NH > CH_3CH_2NH_2 > NH_3$

2° amine 1°amine

Correct option : (1)

12. Ans. (1)

Sol.



* Oleic acid is also soluble in $NaHCO_3$

* o-toluidine is not soluble in NaOH as well as $NaHCO_3$

* Benzamide is also not soluble in NaOH & $NaHCO_3$.

Correct option : (1)

13. Ans. (2)

Sol. B < D < A < C

Basicity $\propto +R \propto \frac{1}{-R}$

$\propto +H \propto \frac{1}{-H}$

CARBONYL COMPOUND

1. **Ans. (1)**

Aldehyde reacts at a faster rate than keton during aldol and stericall less hindered anion will be a better nucleophile so sefl aldol at

$CH_3-\overset{\overset{\large O}{\|}}{C}-H$ will be the major product.

2. **Ans. (1)**

$R{-}C{\equiv}N \xrightarrow{AlH(i-Bu_2)} R{-}CH{=}N{-}$

$\xrightarrow{H_2O} R{-}CH{=}O$

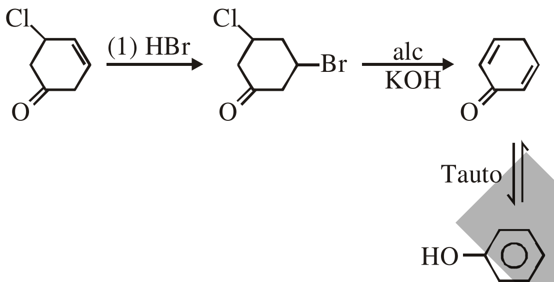
3. **Ans.(3)**

4. **Ans.(4)**

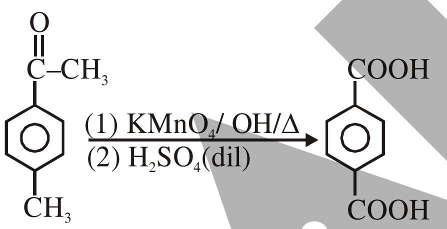
5. **Ans.(2)**

6. **Ans.(4)**

7. **Ans. (1)**



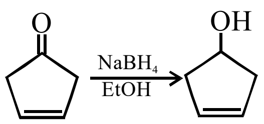
8. **Ans. (2)**



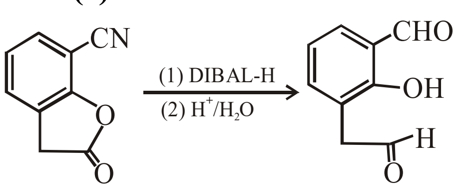
9. **Ans. (4)**

$NaBH_4$ can not reduce C=C

but can reduce $-\underset{\underset{\large O}{\|}}{C}-$ into OH.

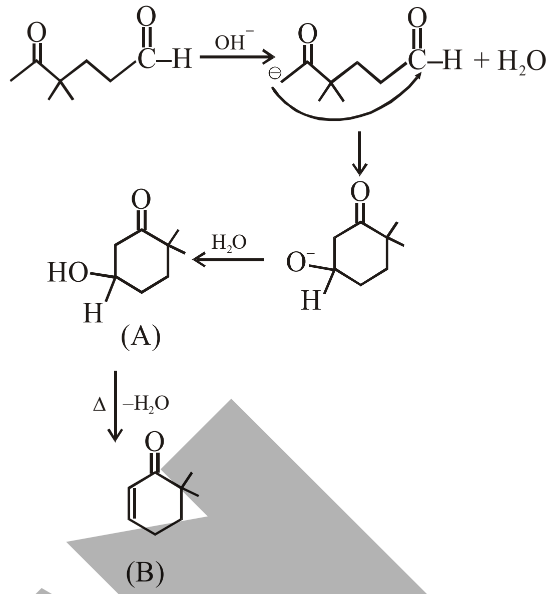


10. **Ans. (3)**

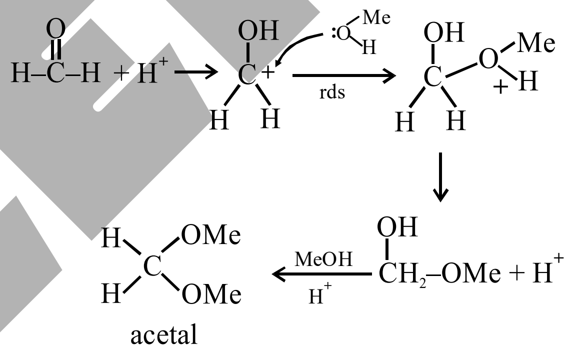


DIBAL-H will reduce cyanides & esters to aldehydes.

11. **Ans. (4)**



12. **Ans. (1)**

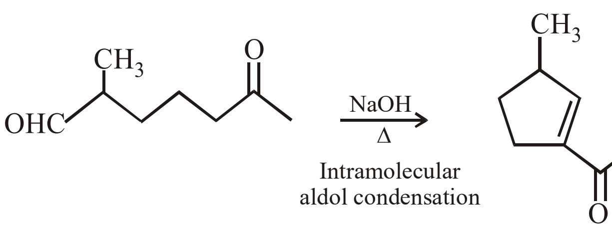


$$\text{rate} \propto \frac{1}{\text{steric crowding of aldehyde}}$$

t-butanol can show formation of carbocation in acidic medium.

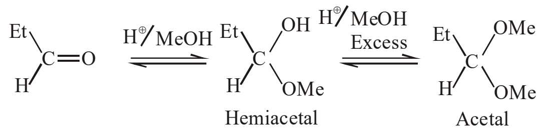
13. **Ans. (4)**

Sol.



14. **Ans. (4)**

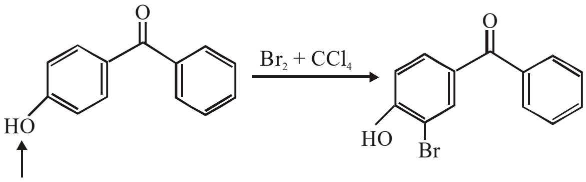
Sol.



Acetone as substrate is less rective than propanal towards neucleophilic addition.

15. Ans. (4)

Sol.



OH is activating and ortho para directing group towards ESR

16. Ans. (1)

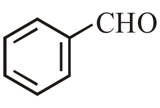
Sol.

$$CH_3-\underset{}{\overset{OH}{CH}}-CH_2-CH_2-NH_2 \xrightarrow[\text{triethylamine}]{\text{ethyl formate (1equiv.)}}$$

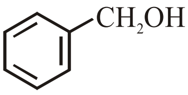
$$CH_3-\overset{OH}{CH}-CH_2-CH_2-NH-\overset{O}{\overset{\|}{C}}-H$$

as NH_2 is a better nucleophile than OH.

17. Ans. (4)

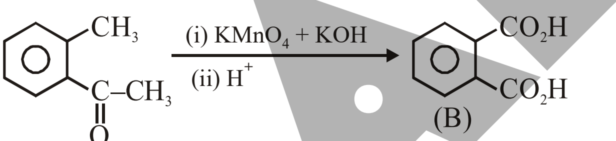
Sol.  $+ \ HCHO \xrightarrow[\text{(ii) } H_3O^{\oplus}]{\text{(i) 50\% NaOH}}$

This is cross cannizaro reaction so more reactive carbonyl compound is oxidized and less reactive is reduced so answer is

 $+ \ HCO_2H$

18. Ans. (1)

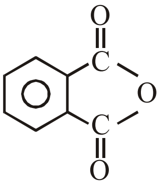
Sol.



(B)

+ve iodoform test

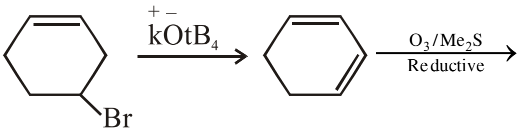
($C_8H_6O_4$) Pthleic acid

 is used for prepareation of phenolphthalein indicator

Pthleic anhydride

19. Ans. (2)

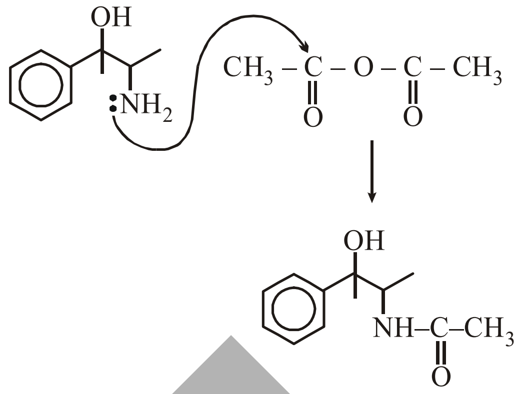
Sol.



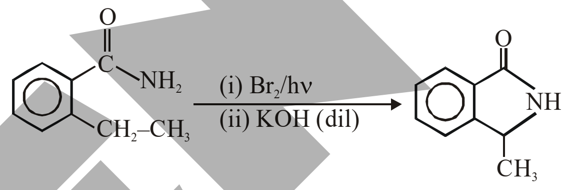
$CHO–CH_2–CH_2–CHO + OHC–CHO$

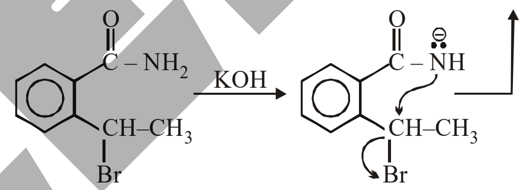
CAD

1. Ans. (3)



2. Ans.(3)





3. Ans.(4)

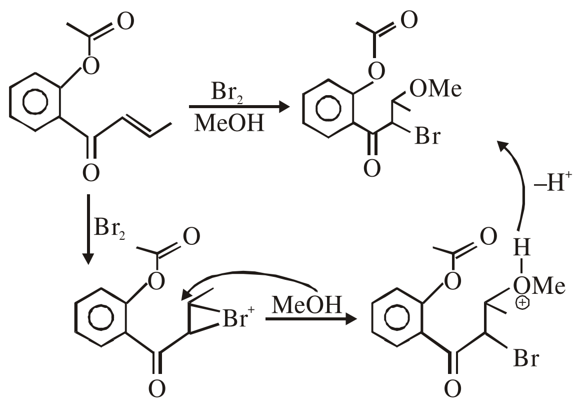
Adipic acid $CO_2H–(CH_2)_4–CO_2H$

$\xrightarrow[\text{agent}]{\text{dehydrating}}$ 7 membered cyclic anhydride (Very unstable)

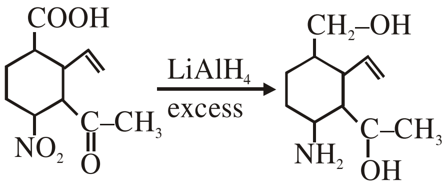
4. Ans. (2)

More is the electrophilic character of carbonyl group of ester faster is the alkaline hydrolysis.

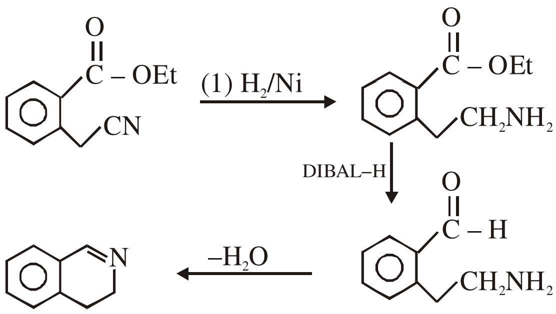
5. Ans. (2)



6. Ans. (2)



7. Ans. (2)

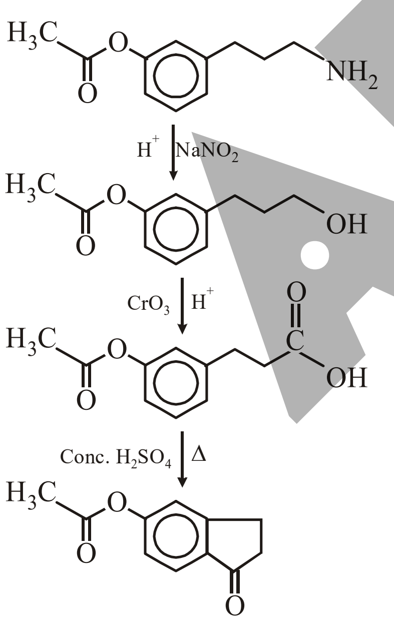


8. Ans. (1)

Rate of nucleophilic attack on carbonyl $\propto$ Electrophilicity of carbonyl group

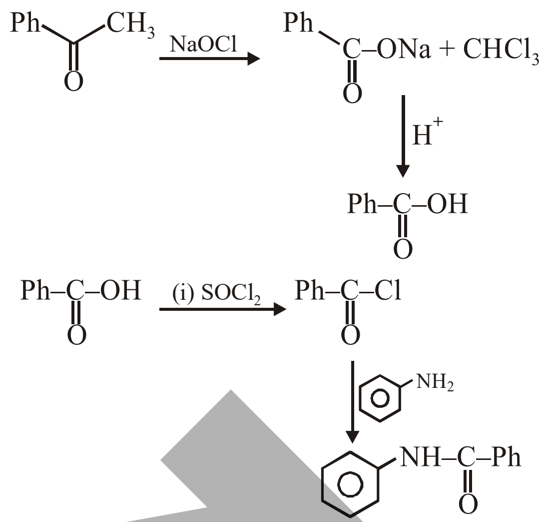
$$C_2H_5-\overset{O}{\overset{\|}{C}}-\underbrace{\ddot{N}H_2}_{+m} < C_2H_5-\overset{O}{\overset{\|}{C}}-\underbrace{OCH_3}_{+m} < C_2H_5-\overset{O}{\overset{\|}{C}}-\underbrace{O-\overset{O}{\overset{\|}{C}}}_{+m}-C_2H_5 < C_2H_5-\overset{O}{\overset{\|}{C}}-\underbrace{Cl}_{-I}$$

9. Ans. (4)



10. Ans. (1)

Sol.



BIOMOLECULE

1. Ans. (4)

Leusine

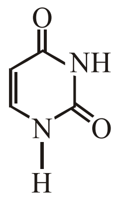
Serine $NO-CH_2-\underset{NH_2}{\underset{|}{CH}}-COOH$

Thrconine $H_3C-\underset{OH}{\underset{|}{CH}}-\underset{NH_2}{\underset{|}{CH}}-COOH$

2. Ans.(3)

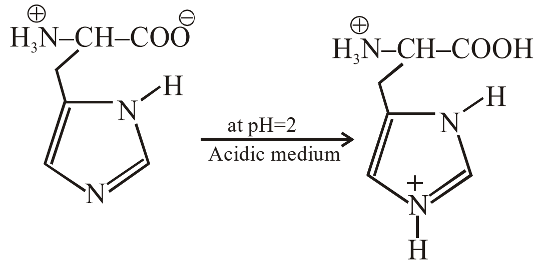
3. Ans.(3)

For the given structure 'uracil' is found in RNA



4. Ans. (1)

Histidine is

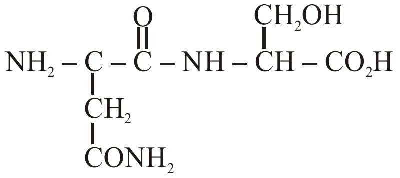


Zwitter ionic form

pIn = 7.59

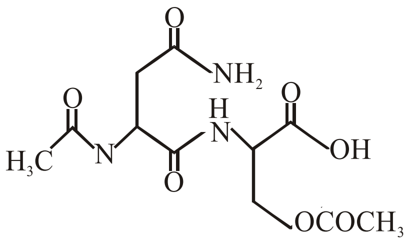
5. Ans. (1)

Asn–Ser is dipeptide having following structure



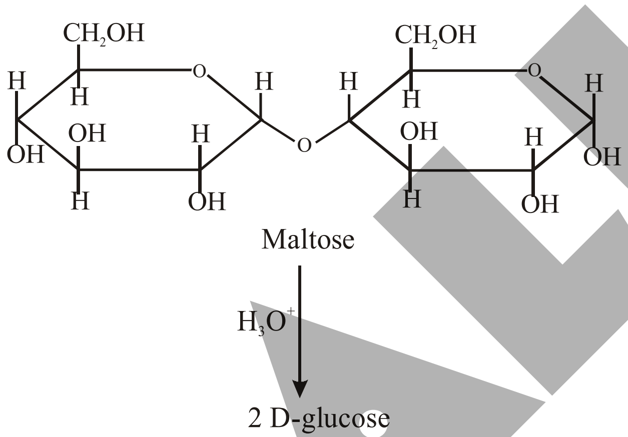
$$\text{Asn}-\text{Ser} + \underset{\text{excess}}{(CH_3CO)_2O} \xrightarrow{NEt_3} P$$

P is



6. Ans. (2)

Sol.



7. Ans. (4)

Sol. Seliwanoff's test is used to distinguished aldose and ketose group.

8. Ans. (2)

Sol. Sucrose $\xrightarrow{H_2O}$ α-D-glucose + β-D-fructose also named as invert sugar & it is a example of non-reducing sugar.

The glycosidic linkage is present between C_1 of α-glucose & C_2 of β-fructose.

9. Ans. (2)

Sol. Serine ⇒ $HO-\underset{\underset{O}{\|}}{C}-\underset{\underset{NH_2}{|}}{CH}-CH_2-OH$

Lysine ⇒ $H_2N-CH_2-CH_2-CH_2-CH_2-\underset{\underset{NH_2}{|}}{CH}-\overset{\overset{O}{\|}}{C}-OH$

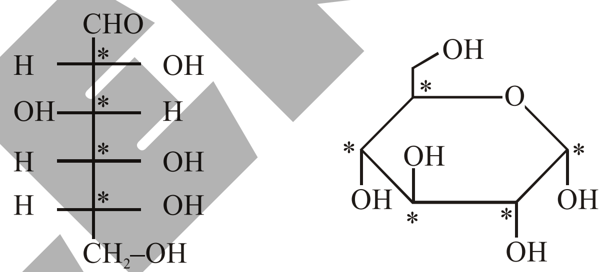
Lysine has $-NH_2$ group hence gives ⊕ve carbyl amine test and serine has –OH group hence gives ⊕ve serric ammonium nitrate test

10. Ans. (1)

Sol. Amylopetcin is a homopolymer of α-D-glucose where C_1–C_4 linkage and C_1–C_6 linkage are present.

11. Ans. (1)

Sol.



D–Glucose (Linear structure) α-D–Glucose (cyclic structure)

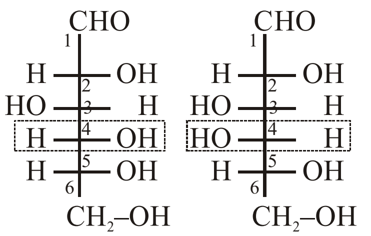
* :- Stereocenter

12. Ans. (1)

Sol. RNA is a single stranded structure.

13. Ans. (3)

Sol. Glucose and galactose are C-4 Epimer's



glucose galactose

14. Ans. (1)

Sol. Glycogen is an animal starch.

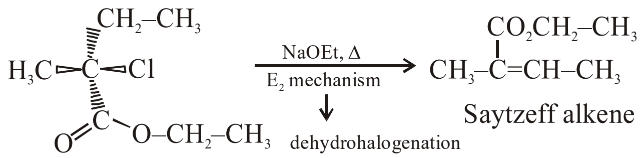
It consists of α-amylose and amylopectin.

Amylopectin is branched chain polysaccharide

Hence statement (1) is incorrect.

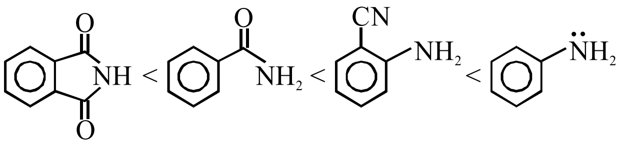
HALOGEN DERIVATIVE

1. Ans. (3)



2. Ans. (2)

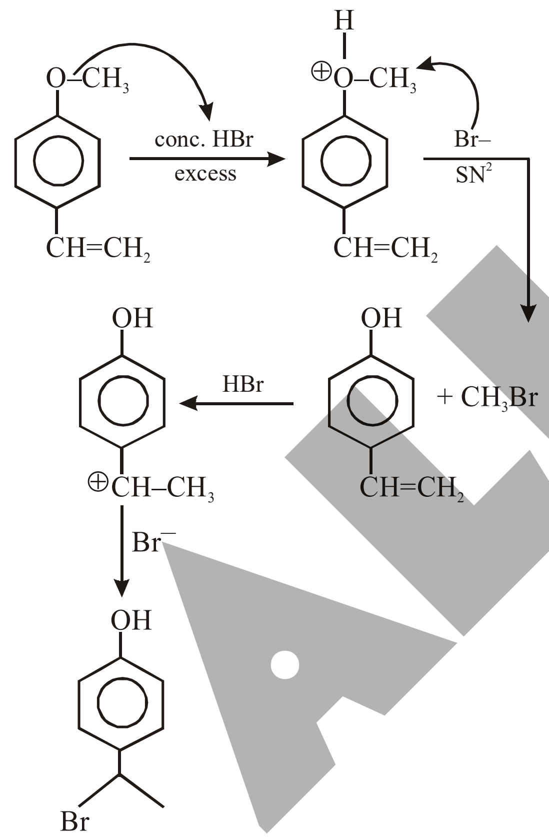
Nucleophilicity order



B A C D

3. Ans. (4)

Sol.



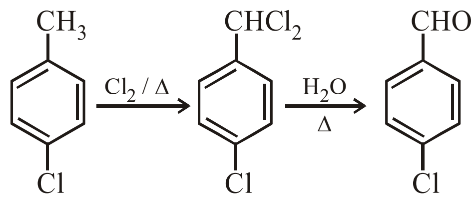
Correct option : (4)

4. Ans. (Bonus)

Sol. because one double bond is missing in all given option. So aromaticity is lost in both the ring.

5. Ans. (4)

Sol.



6. Ans. (1)

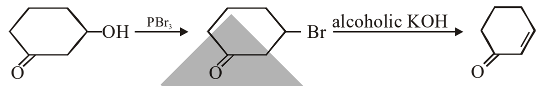
Sol. $CF_3-CH=CH_2 \xrightarrow{HCl} CF_3-\overset{H}{\overset{|}{C}H}-\overset{\oplus}{C}H_2 \longrightarrow CF_3-\overset{H}{\overset{|}{C}H}-\overset{Cl}{\overset{|}{C}H_2}$

Due to higher e^- withdrawing nature of CF_3 group.

It follow anti markovnikoff product

7. Ans. (4)

Sol.

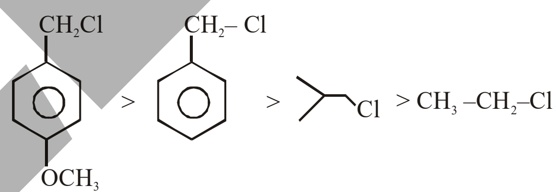


8. Ans. (4)

Sol. $CH_3-C\equiv CH \xrightarrow{DCl} H_3C-\underset{Cl}{\underset{|}{C}}=CHD \xrightarrow{DI} H_3C-\overset{I}{\overset{|}{\underset{Cl}{\underset{|}{C}}}}-CHD_2$

9. Ans. (3)

Sol. S_N1 Reactivity order

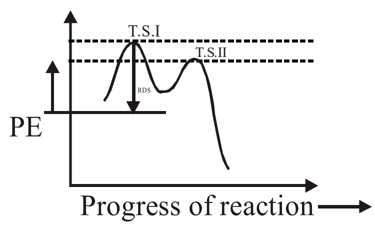


(C) (D) (A) (B)

Order C> D> A> B

10. Ans. (4)

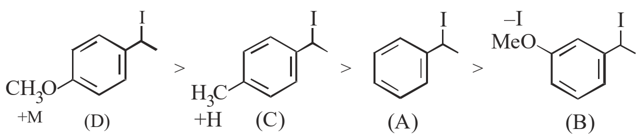
Sol. PE diagram for S_N1



$S_N1$is two step reaction where in step (1) formation of carbocation is RDS

11. Ans. (3)

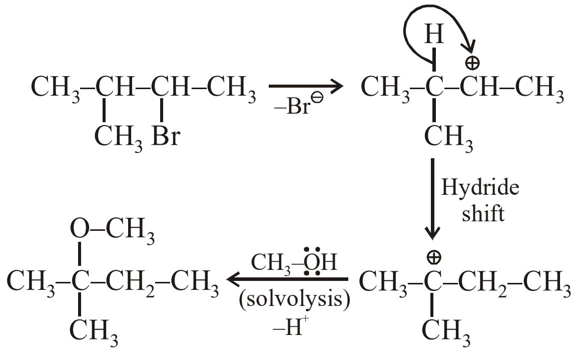
Sol. Rate of $S_{N'}$ is directly proposional to stability of first formed carbocation so answer is



12. Ans. (3)

Sol. $CH_3-\underset{CH_3}{\underset{|}{CH}}-\underset{Br}{\underset{|}{CH}}-CH_3 \xrightarrow{CH_3OH} ?$

In polar protic solvent S_{N^1} mechanism is favourable hence reaction complete via S_{N^1} mechanism



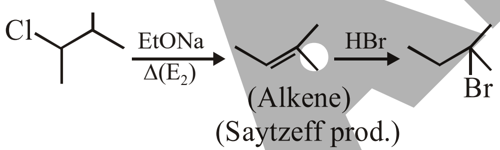
13. Ans. (1)

Sol. $\underbrace{\bar{O}H > CH_3-\underset{\underset{O}{\|}}{C}-O^- > CH_3-\underset{\underset{O}{\|}}{\overset{\overset{O}{\|}}{S}}-O^-}_{\text{Charged ion}} > \underbrace{H_2O}_{\text{neutral system}}$

ione pair donating tendency on oxygen is reduced, nucleophilicity reduced b < c < a < d

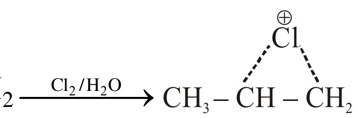
14. Ans. (3)

Sol.



15. Ans. (2)

Sol.



$\xrightarrow{H_2\ddot{O}} CH_3-\underset{OH}{\underset{|}{CH}}-\overset{Cl}{\overset{|}{CH_2}}$

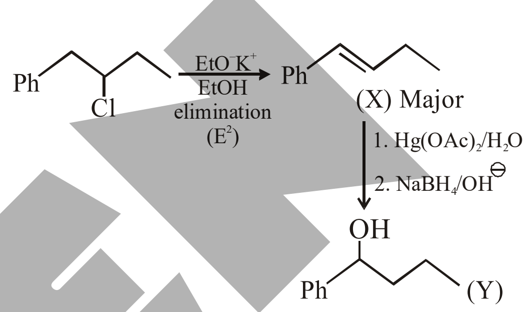
16. Ans. (4)

Sol. Vinyl halide $CH_2=CH-Cl$ do not undergo SN reaction

This is due to formation of highly unstable carbocation $(CH_2=\overset{\oplus}{C}H)$; which cannot be delocalised by the π-electron, also C–Cl has double bond character because of resonance

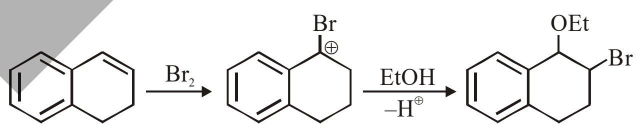
17. Ans. (4)

Sol.



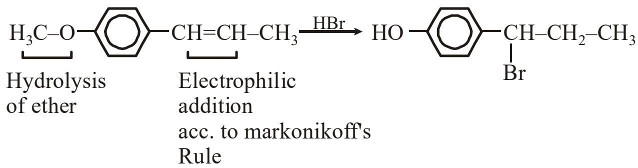
HYDROCARBON

1. Ans. (4)

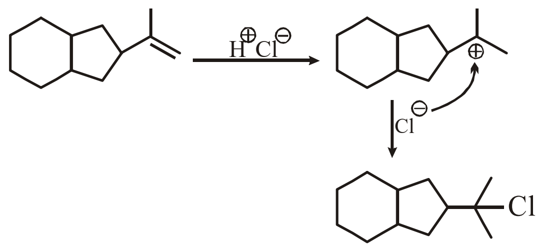


2. Ans. (2)

3. Ans. (2)



4. Ans. (1)



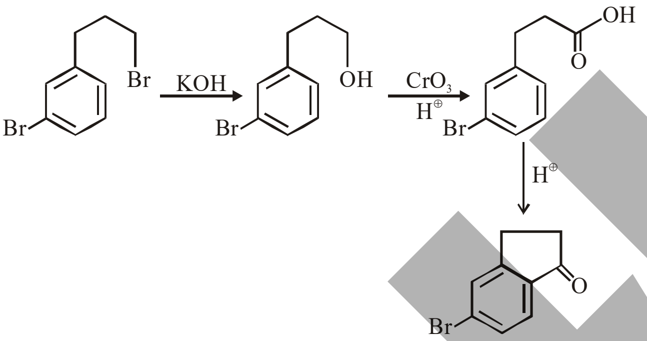
5. Ans. (2)

Sol. $CH_3-C\equiv CH \xrightarrow[\Delta]{Ag_2O} CH_3-C\equiv C-Ag\downarrow$
(terminal alkyne) (ppt)

$CH_3-C\equiv CH \xrightarrow[\text{dil.}H_2SO_4]{Hg^{2+}}$ (terminal alkyne) (Kucherov reaction) $\rightarrow$ $CH_3-\overset{OH}{\overset{|}{C}}=CH_2 \rightleftharpoons CH_3-\overset{O}{\overset{||}{C}}-CH_3 \xrightarrow{NaBH_4} CH_3-\underset{OH}{\underset{|}{CH}}-CH_3$ (2°alcohol) $\xrightarrow{\text{Conc. HCl+ZnCl}_2}$ Turbidity within 5 min. (Lucas Reagent Test)

AROMATIC

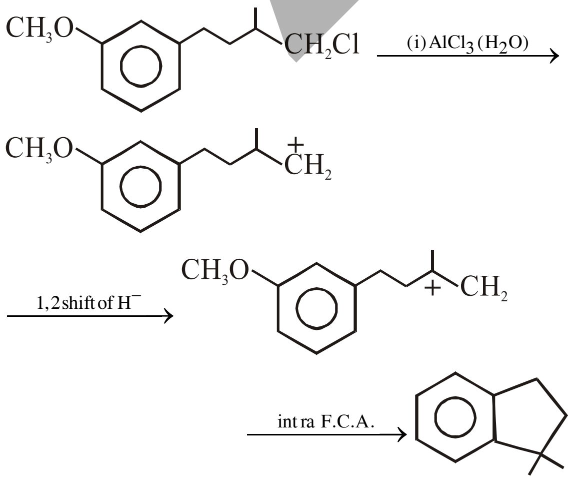
1. Ans. (2)



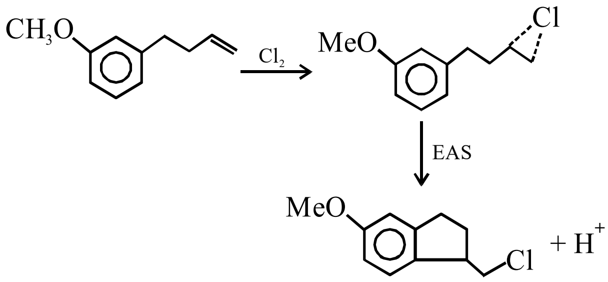
During AES Br is o/p directing and major product will be formed on less hindrance p position :

2. Ans. (3)

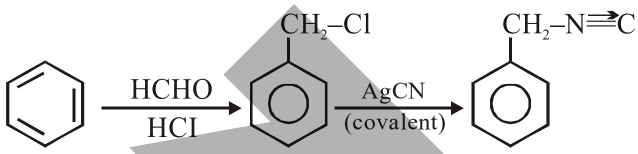
3. Ans. (2)



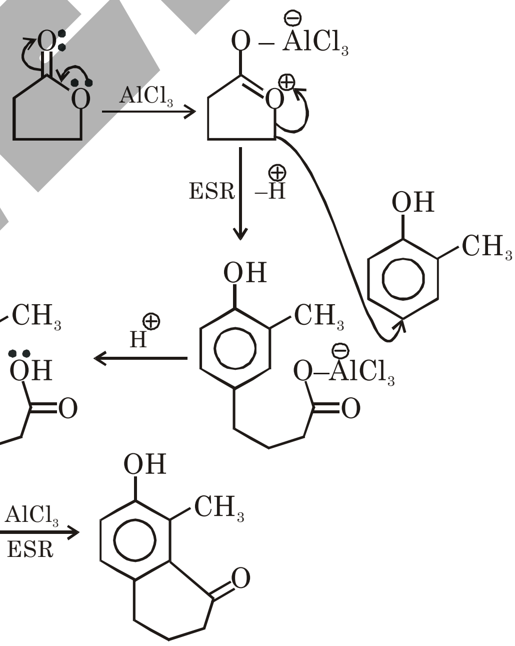
4. Ans.(4)



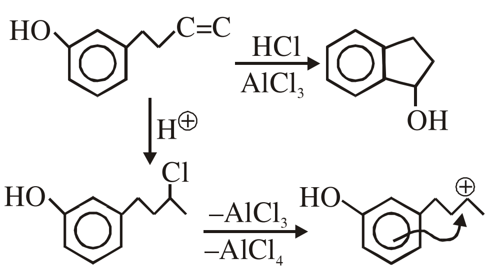
5. Ans. (4)



6. Ans. (1)

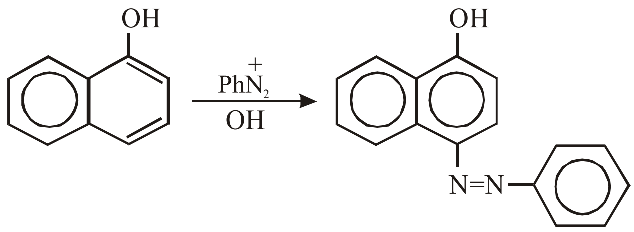


7. Ans. (2)



8. Ans. (3)

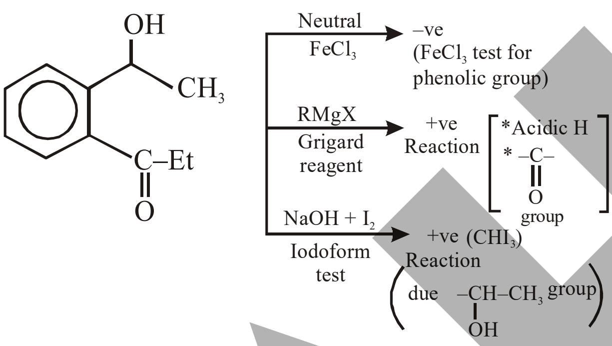
Sol.



α-naphthol (1-naphthol) → orange red dye

9. Ans. (1)

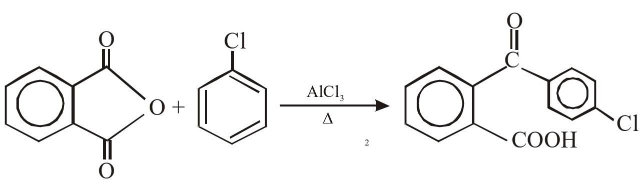
Sol.



Correct option : (1)

10. Ans. (3)

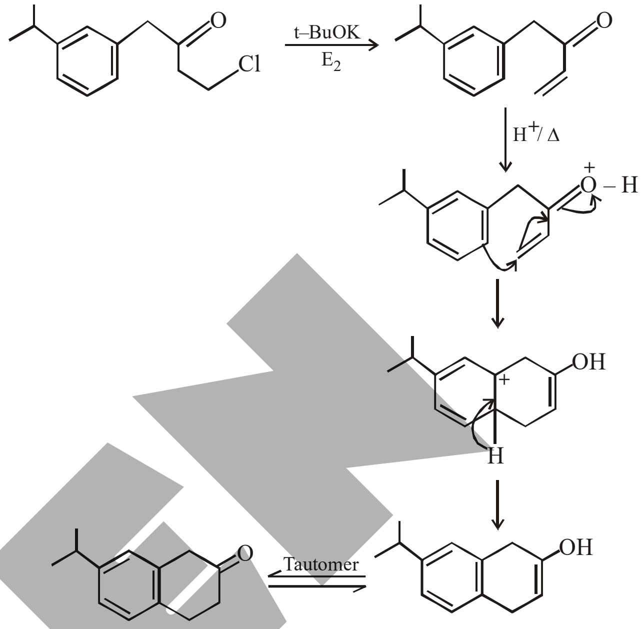
Sol.



Fridel-craft acylation. –Cl group is an ortho & para directing

11. Ans. (4)

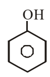
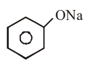
Sol.

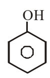
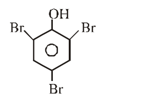


12. Ans. (3)

Sol. In Friedal crafts alkylation product obtained is more activated and hence polysubtitution will take place.

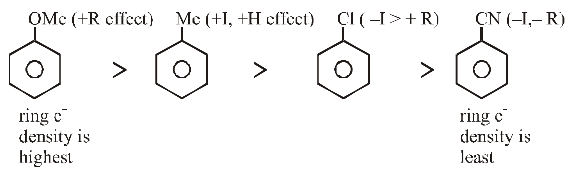
13. Ans. (1)

Sol.  is insoluble in dil. HCl but soluble in NaOH to form 

 decolorise Br_2 water to give 

(2,4,6- tribromophenol)

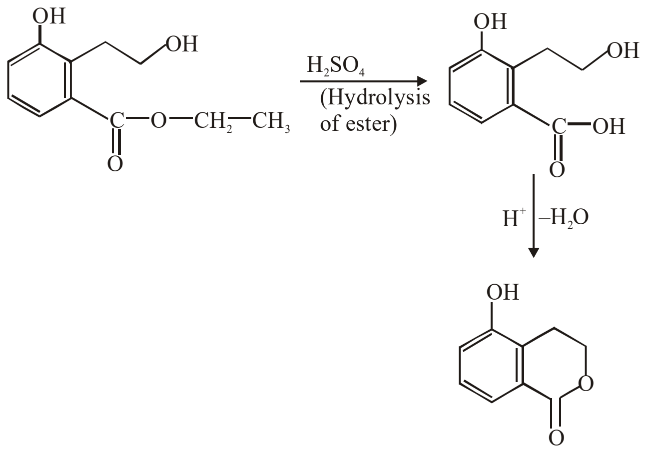
14. Ans. (3)

Sol. 

(More is the e density at ring faster is the reaction towards EAS)

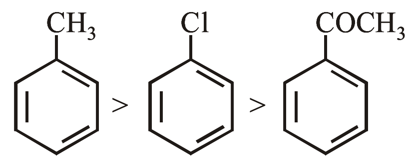
15. Ans. (3)

Sol.



16. Ans. (3)

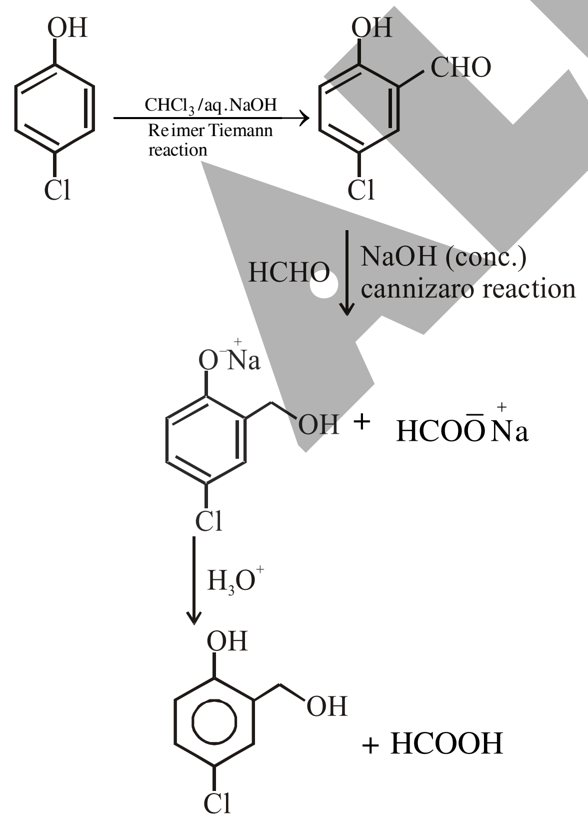
Sol. Rate of aromatic electophilic substitution is



(II) > (I) > (III)

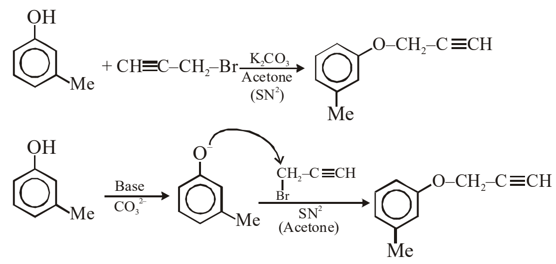
17. Ans. (3)

Sol.



18. Ans. (4)

Sol.



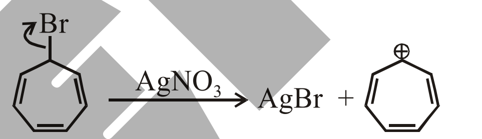
ALKAYLE HALIDE

1. Ans. (4)

2. Ans.(3)

Example of E_2 elimination and conjugated diene is formed with phenyl ring in conjugation which makes it very stable.

3. Ans. (4)



aromatic cation

as it can produce aromatic cation so will produce precipitate with $AgNO_3$.

4. Ans. (1)

$$CH_3-CH_2-\underset{\displaystyle Br}{\underset{|}{CH}}-\underset{\displaystyle Br}{\underset{|}{CH_2}}$$

$$\downarrow \text{Alc. KOH}$$

$$CH_3-CH_2-\underset{\displaystyle Br}{\underset{|}{C}}=CH_2$$

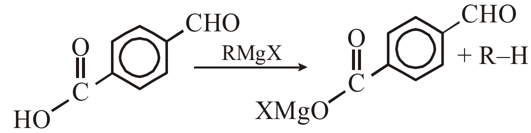
$$\text{NaNH}_2 \downarrow \text{in liq. } NH_3$$

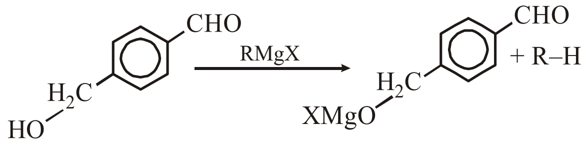
$$CH_3-CH_2-C\equiv CH$$

GRIGNARD REAGENT

1. Ans. (2)

Acid-base reaction of G.R are fast.





2. Ans. (1)

(1) $H-\overset{\overset{O}{\|}}{C}-H + Ph-\underset{\underset{CH_3}{|}}{CH}-CH_2MgX \longrightarrow Ph-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-OH$

POC

1. Ans. (2)

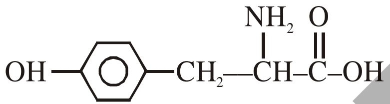
→ 2,4 – DNP test is given by aldehyde on ketone

→ Iodoform test is given by compound having $CH_3-\underset{\underset{O}{\|}}{C}-$ group.

2. Ans.(1)

3. Ans. (1)

(P) Tyrosine

Tyr 

(Q) Aspartic

ASP $H-O-\overset{\overset{O}{\|}}{C}-CH_2-\overset{\overset{NH_2}{|}}{CH}-\overset{\overset{O}{\|}}{C}-OH$ Acid

(R) Serine Ser $HO-CH_2-CH(NH_2)(COOH)$

(S) Lysine

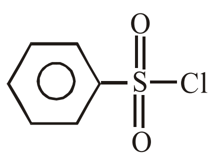
$NH_2-CH_2-CH_2-CH_2-CH_2-CH(NH_2)(COOH)$

(A) Ester test (Q) Aspartic acid (Acidic amino acid)

(B) Carbylamine (S) Lysine [NH_2 group present]

(C) Phthalein dye (P) Tyrosine {Phenolic group present}

4. Ans. (1)

Sol. Hinsberg Reagenl ⇒ 

[Benzene Sulphonyl chloride]

NOMENCLATURE

1. Ans.(2)

2. Ans. (3)

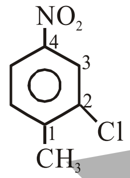
Sol.

$\underset{5}{CH_3}-\underset{4}{\overset{\overset{CH_3}{|}}{CH}}-\overset{3}{\underset{\underset{OH}{|}}{CH}}-\overset{2}{CH_2}-\overset{1}{COOH}$

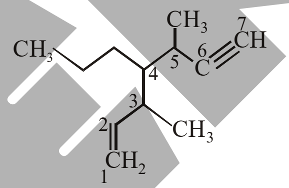
3-Hydroxy-4-methylpentanoic acid

–COOH principal functional group

3. Ans. (4)

Sol.  2-chloro–1-methyl–4–nitrobenzene

4. Ans. (4)

Sol. 

3,5-Dimethyl-4-propylhept-1-en-6-yne

Longest carbon chain, including multiple bonds, and numbering starts from double bond.

POLYMER

1. Ans. (4)

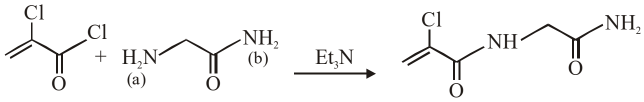
Nylon-6,6 is polymer of

Hexamethylene diamine & Adipic acid

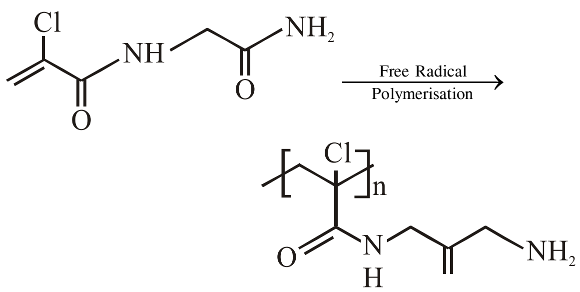
↓ ↓

$H_2N-(CH_2)_6-NH_2$ $HOOC-(CH_2)_4-COOH$

2. Ans. (4)

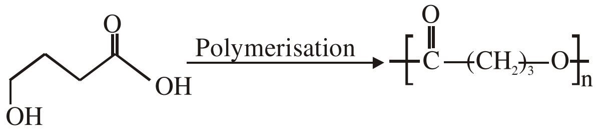


NH_2(a) will wact as nucleophile as (b) is having delocalised lonepair.

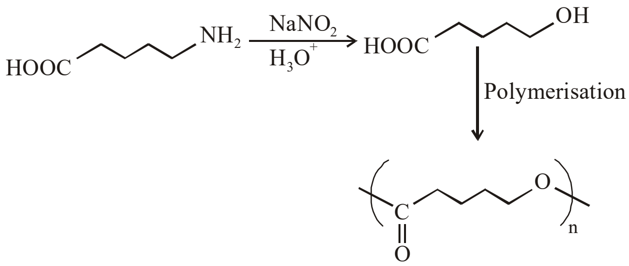


3. **Ans.(2)**

4. **Ans. (1)**



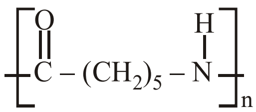
5. **Ans. (2)**



6. **Ans. (4)**

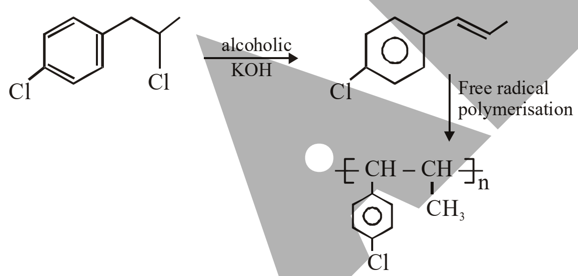
PHBV is a polymer of 3-hydroxybutanoic acid and 3-Hydroxy pentanoic acid.

7. **Ans. (3)**

Sol. 

Nylon-6

8. **Ans. (1)**

Sol. 

9. **Ans. (1)**

Sol. $H_2N-\underset{\underset{O}{\|}}{C}-NH_2+H-\underset{\underset{O}{\|}}{C}-H \longrightarrow \left[-HN-\underset{\underset{O}{\|}}{C}-NH-CH_2\right]_n$

Urea formaldehyde

10. **Ans. (2)**

Sol. Nylon-6,6 is a condensation polymer of hexamethylene diamine and adipic acid.

Buna-S, Teflon and Neoprene are addition polymer.

11. **Ans. (4)**

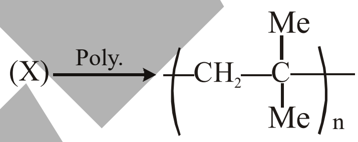
Sol.

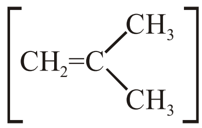
(a)	High density polythene	(III)	Ziegler-Natta Catalyst
(b)	Polyacrylonitrile	(I)	Peroxide catalyst
(c)	Novolac	(IV)	Acid or base catalyst
(d)	Nylon 6	(II)	Condensation at high temperature & pressure

12. **Ans. (3)**

Sol. Bakelite is thermoselting polymer

13. **Ans. (4)**

Sol.  Polyisobutylene

As per the given structure of the polymer the monomer is –  Isobutylene

CHEMISTRY IN EVERYDAY LIFE

1. **Ans. (2)**

(A) Norethindrone – Antifertility

(B) Ofloaxacin – Anti-Biotic

(C) Equanil – Hypertension (traiquilizer)

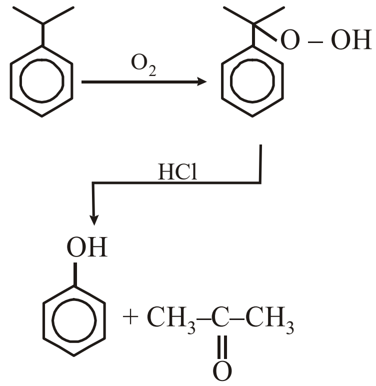
2. **Ans. (1)**

Sol. Nor adrenaline is a neutro transmitter and it belongs to catecholamine family that fanctions in brain & body as a hermone & neutro transmitter.

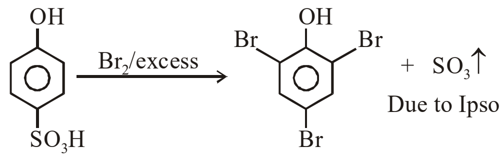
PHENOL

1. **Ans. (3)**

Cummene hydroperoxide reaction



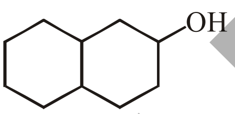
2. **Ans. (1)**



Due to Ipso attack

AMINE

1. **(Bonus)**

Answer should be 

2. **Ans. (3)**

$$[X]\xrightarrow[NaOH]{Br_2}C_3H_9N\xrightarrow[KOH]{CHCl_3}CH_3CH_2CH_2\text{–}NC$$

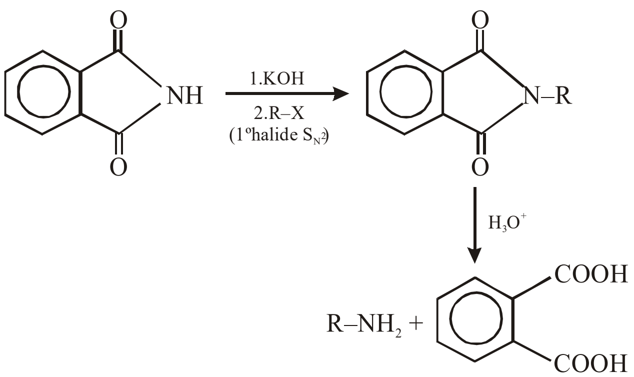
Hoff mann's Bromaide degradation — Carbylamine Reaction

Thus [X] must be aride with oen carbon more than is amine.

Thus [X] is $CH_3CH_2CH_2CONH_2$

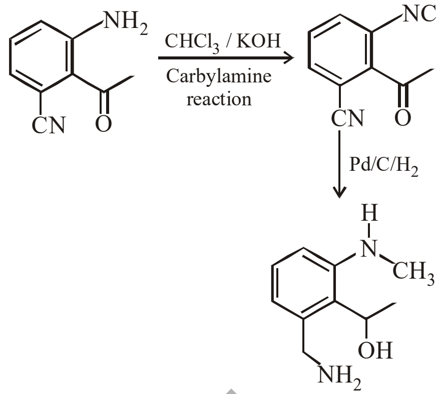
3. **Ans. (2)**

Sol. Gabriel phthalimide synthesis :



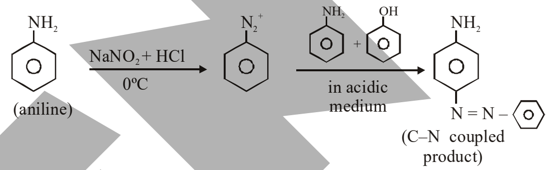
4. **Ans. (1)**

Sol.



5. **Ans. (1)**

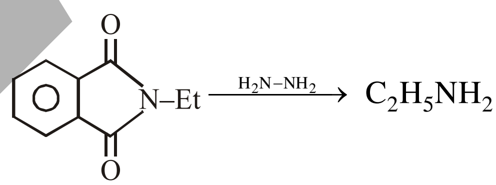
Sol.



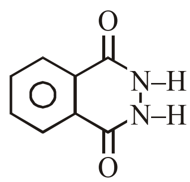
Aniline undergoes diazo coupling in acidic medium with $Ph\overset{+}{N}_2$

6. **Ans. (4)**

Sol.

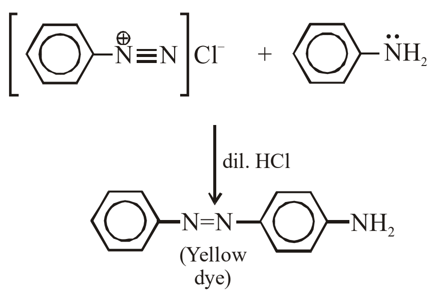


reagent is $NH_2\text{–}NH_2$ byproduct will be



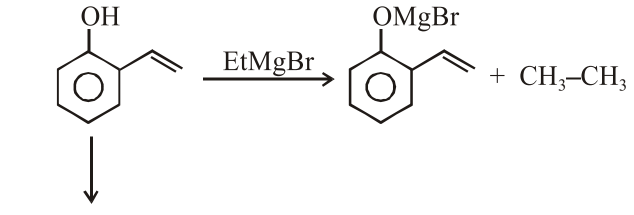
7. **Ans. (3)**

Sol.



ORGANO METALIC

1. Ans.(4)

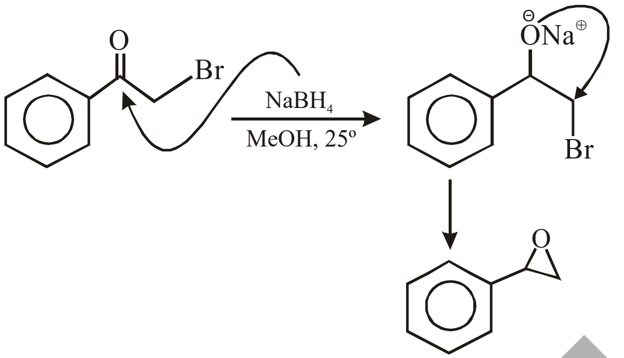


declolourizes Bromin water

REDUCTION

1. Ans. (4)

Sol.



2. Ans. (2)

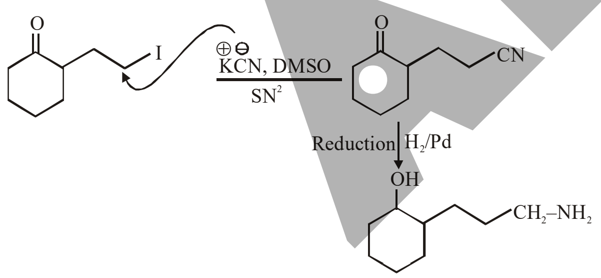
Sol. $CH_3 - CH = CH - \underset{\underset{O}{\|}}{C} - OCH_3$

$\downarrow LiAlH_4$

$CH_3 - CH = CH - CH_2OH$

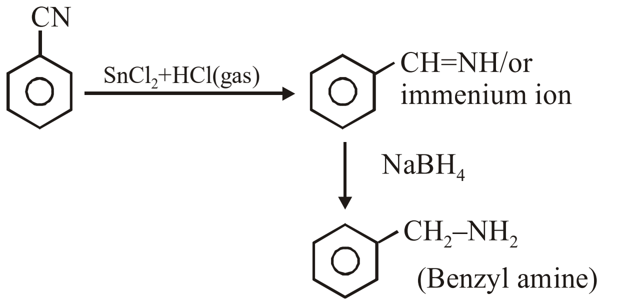
3. Ans. (2)

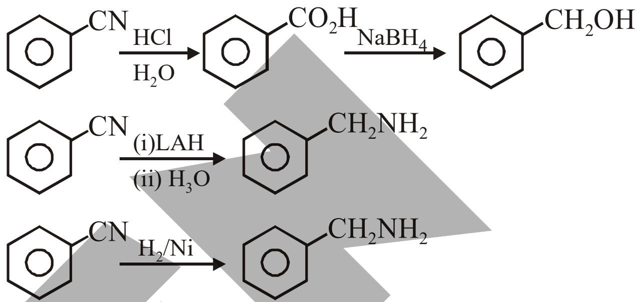
Sol.



4. Ans. (1)

Sol.

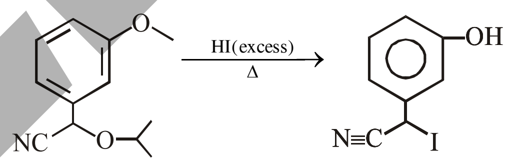




ALCOHOL & ETHER

1. Ans. (1)

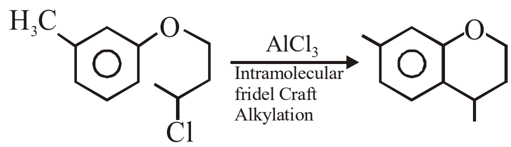
Sol.



Phenolic –OH does not react with HI and benzylic –O– having –CN attached will react with HI by S_N2 mechanism.

2. Ans. (4)

Sol.



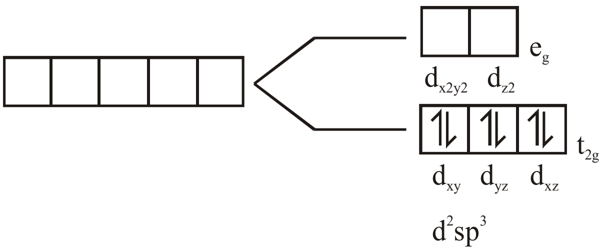
JANUARY+APRIL 2019 ATTEMPT (IOC)

COORDINATION COMPOUND

1. Ans. (4)

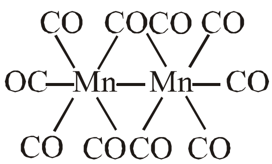
$K_3[Co(CN)_6]$

$Co^{+3} \rightarrow [Ar]_{18}\ 3d^6$

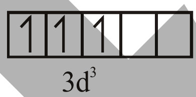


2. Ans. (2)

Compounds having at least one bond between carbon and metal are known as organometallic compounds.



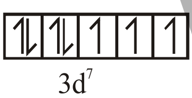
3. Ans. (2) V^{2+} and Co^{2+}

$V^{2+} \rightarrow [V(H_2O)_6]Cl_2$; $[Ar]_{18}$ 

3 unpaired e^- , spin only magnetic moment

= 3.89 B.M.

$Co^{2+} \rightarrow [Co(H_2O)_6]Cl_2$;

$[Ar]_{18}$ 

3 unpaired e^-, spin only magnetic moment

= 3.89 B.M.

4. Ans. (2)

$\mu = 5.9$ BM $\therefore$ n (no of unpaired.e^-) = 5

Cation Mn^{II} – $3d^5$ confn only possible for relatively weak ligand.

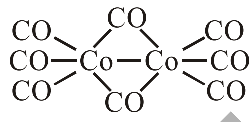
$\therefore$ NCS^-

5. Ans. (2)

$C_2O_4^{2-}$ (oxalato) : bidentate

H_2O (aqua) : Monodentate

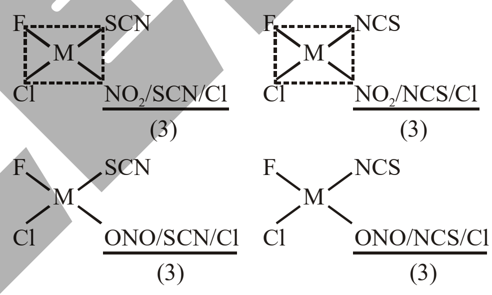
6. Ans. (4)



Bridging CO are 2 and Co – Co bond is 1.

7. Ans. (1)

The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is 12.



8. Ans. (1)

Wilkinsion catalyst is $[(ph_3P)_3RhCl]$

9. Ans. (2)

Δ_0 order will be compared by spectro chemical series not by energies of violet & yellow light

so Δ_0 order is $[Cr(H_2O)_6]Cl_3 < [Cr(NH_3)_6]Cl_3$

10. Ans. (1)

$\mu = \sqrt{n(n+2)}$ B.M.

n = Number of unpaired electrons

n = Maximum number of unpaired electron = 5

Ex : Mn^{2+} complex.

11. Ans. (1)

As complex $K_3[Co(CN)_6]$ have CN^- ligand which is strongfield ligand amongst the given ligands in other complexes.

12. Ans. (2)

13. Ans. (1)

14. Ans.(1)

(1) EDTA (ethylene diamine tetra acetate) is used for lead poisoning

(2) Cis platin is used as a anti cancer drug

(3) D-penicillamine is used for copper poisoning

(4) desferrioxime B is used for iron poisoning

15. Ans.(3)

en and $C_2O_4^{2-}$ are bidentate ligand. So coordination number of $[Co(Cl)(en)_2]Cl$ is 5 and $K_3[Al(C_2O_4)_3]$ is 6.

16. Ans.(1)

$[Fe(H_2O)_6]Cl_2$, $Fe^{2+} \rightarrow 3d^6 \rightarrow (t_{2g})^4(e_g)^2$

C.F.S.E. $= 4 \times(-0.4\Delta_o) + 2 \times 0.6\Delta_o = -0.4\Delta_o$

$K_2[NiCl_4]$, $Ni^{2+} \rightarrow 3d^8 \rightarrow (e)^4(t_2)^4$

C.F.S.E. $= 4\times(-0.6\Delta_t) + 4 \times (0.4\Delta_t) = -0.8\Delta_t$

17. Ans.(3)

(1) $[Fe(H_2O)_6]^{2+}$, $Fe^{2+} \rightarrow 3d^6 \rightarrow$ 4 unpaired electron

$[Cr(H_2O)_6]^{2+}$, $Cr^{2+} \rightarrow 3d^4 \rightarrow$ 4 unpaired electron

(2) $[Ni(NH_3)_4(H_2O)_2]^{2+} = Ni^{2+} \rightarrow 3d^8$

$\rightarrow$ 2 unpaired electron

$\mu_m = 2.83$ B.M

(3) In gemstone, ruby has Cr^{3+} ion occupying the octahedral sites of aluminium oxide (Al_2O_3) normally occupied by Al^{3+} ion.

(4) Complimenry color of violet is yellow

18. Ans.(1)

Towards common transition element and inner transition metal ion given ligand can have maximum denticities of 6 and 8 respectively.

19. Ans.(2)

cis–$[PtCl_2(NH_3)_2]$ is used in chemotherapy to inhibits the growth of tumors.

20. Ans.(3)

Donating atoms are both nitrogen & oxygen.

Correct option : (3)

21. Ans.(1)

According to question all the complexes are **low spin.**

Complex	Configuration	No. of unpaired electrons
$[V(CN)_6]^{4-}$	$t_{2g}^3e_g^0$	3
$[Cr(NH_3)_6]^{2+}$	$t_{2g}^4e_g^0$	2
$[Ru(NH_3)_6]^{3+}$	$t_{2g}^5e_g^0$	1
$[Fe(CN)_6]^{4-}$	$t_{2g}^6e_g^0$	0

Correct option : (1)

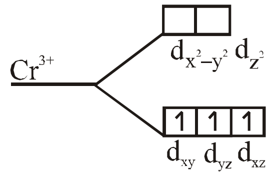
22. Ans.(Bonus)

Complex is $[Fe(H_2O)_6]_2[Fe(CN)_6]$

Complex ion	Configuration	No. of unpaired electrons	Magnetic moment
$[Fe(H_2O)_6]^{2+}$	$t_{2g}^4 e_g^2$	4	4.9 BM
$[Fe(CN)_6]^{4-}$	$t_{2g}^6e_g^0$	0	0

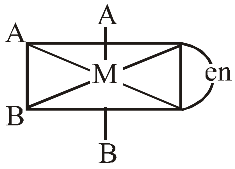
23. Ans.(3)

Degenerate orbitals of $[Cr(H_2O)_6]^{3+}$



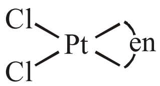
Hence according to the options given, degenerate orbitals are d_{xz} & d_{yz}

24. Ans.(3)

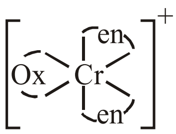


This structure does not contain plane of symmetry hence it is optically active, rest of all options has plane of symmetry and they are optically inactive.

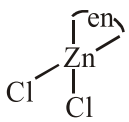
25. Ans.(4)

(1) 

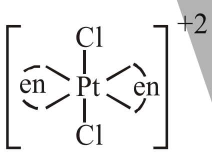
no trans isomer possible because bidentate ligand will be co-ordinating only at 90° angle in square planar complex

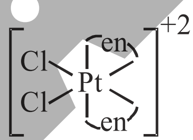
(2) 

no trans isomer possible

(3) 

sp^3 hybridized so no trans possible

(4) 



trans and cis both are possible

26. Ans.(2)

A complex having strong field ligand has tendency to absorb light of highest energy.

Among the three complexes.

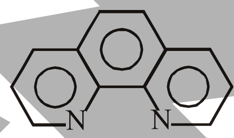
$[Co(NH_3)_6]^{+3}$ will absorb radiation of highest energy and least wavelength.

$[Co(NH_3)_5H_2O]^{+3}$ has field weaker than the above compound and therefore absorb radiation of lesser energy and more wavelength.

$[CoCl(NH_3)_5]^{+2}$ has the weakest field and therefore will absorb light of least energy and highest wavelength.

Strength of ligand $NH_3 > H_2O > Cl$.

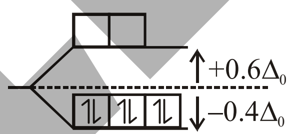
27. Ans.(1)

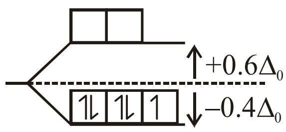
Phen = 

is a strong field symmetrical bidentate ligand.

(1) $[Fe(phen)_3]^{2+} \xrightarrow{-e^-} [Fe(phen)_3]^{3+}$

$Fe^{2+} : 3d^6$ $\quad$ $Fe^{3+} : 3d^5$





$\boxed{C.F.S.E = -2.4\,\Delta_0}$ $\quad$ $\boxed{C.F.S.E = -2.0\Delta_0}$

By oxidation of Fe^{2+} into Fe^{3+}, the CFSE value decrease.

(2) $[Zn(phen)_3]^{2+} \xrightarrow{-e^-}$ $[Zn(phen)_3]^{3+}$

$Zn^{2+} : 3d^{10}$ $\quad$ $Zn^{3+} : 3d^9$

$\boxed{C.F.S.E = 0}$ $\quad$ $\boxed{C.F.S.E = -0.6\Delta_0}$

By oxidation of Zn^{2+} into Zn^{3+}, the CFSE value increase.

(3) $[Ni(phen)_3]^{2+} \xrightarrow{-e^-} [Ni(phen)_3]^{3+}$

$Ni^{2+} : 3d^8$ $\quad$ $Ni^{3+} : 3d^7$

$\boxed{C.F.S.E = -1.2\,\Delta_0}$ $\quad$ $\boxed{C.F.S.E = -1.8\Delta_0}$

by oxidation of Ni^{2+} into Ni^{3+}, the CFSE value increase.

(4) $[Co(phen)_3]^{2+} \xrightarrow{-e^-} [Co(phen)_3]^{3+}$

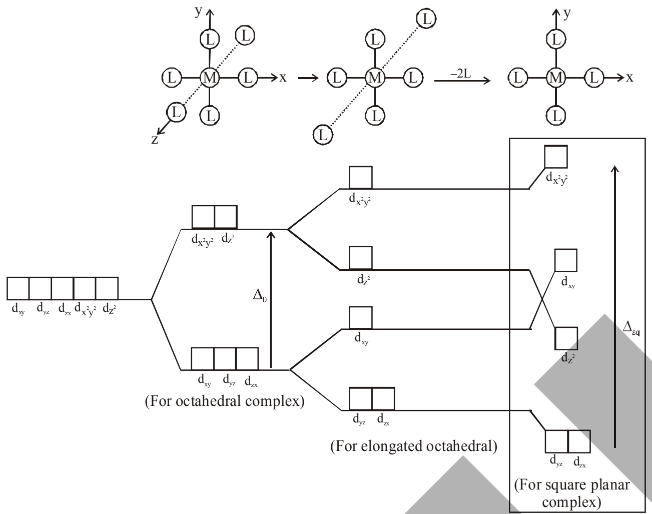
Co^{2+} : $3d^7$ $\qquad$ Co^{3+} : $3d^6$

$\boxed{C.F.S.E = -1.8\,\Delta_0}$ $\qquad$ $\boxed{C.F.S.E = -2.4\,\Delta_0}$

by oxidation of Co^{2+} into Co^{3+}, the CFSE value increase.

28. **Ans.(1)**

If both ligands present along z-axis removed from octachedral field and converted into square planar field, then



CHEMICAL BONDING

1. **Ans. (4)**

Carbon atom have 2p orbitals able to form strongest $p\pi$ - $p\pi$ bonds

2. **Ans. (4)**

Catenation is not shown by lead.

3. **Ans. (4)**

CCl_4 cannot get hydrolyzed due to the absence of vacant orbital at carbon atom.

4. **Ans. (3)**

Due to inert pair effect as we move down the group in 13th group lower oxidation state becomes more stable.

Al < Ga < In < Tℓ

5. **Ans. (3)**

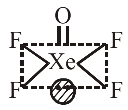
(1) B_2H_6 : Electron deficient

(2) AlH_3 : Electron deficient

(3) SiH_4 : Electron precise

(4) GaH_3 : Electron deficient

6. **Ans. (3)**

 $sp^3d^2 \Rightarrow$ [5σ-bond +1 l.p.]

7. **Ans. (1)**

$N_2^{\oplus} \Rightarrow BO = 2.5 \Rightarrow \left[\pi - \text{Bond} = 2 \;\&\; \sigma - \text{Bond} = \frac{1}{2}\right]$

$N_2 \Rightarrow$ B.O. = 3.0 $\Rightarrow$ [π-Bond = 2 & σ-Bond = 1]

$O_2^{\oplus}$ = B.O. $\Rightarrow$ 2.5 $\Rightarrow$ [π-Bond = 1.5 & σ-Bond = 1]

$O_2 \Rightarrow$ B.O. $\Rightarrow$ 2 $\Rightarrow$ [π-Bond $\Rightarrow$ 1 & σ-Bond = 1]

8. **Ans. (1)**

9. **Ans. (4)**

Both Li_2^+ and Li_2^- has 0.5 bond order and hence both are stable.

10. **Ans.(1)**

In C_{60} molecule there are 20 hexagons and 12 pentagons

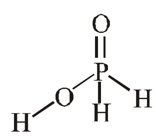
$\therefore$ Ans.(1)

11. **Ans. (4)**

Inert pair effect is promenent character of p-block element.

12. **Ans. (3)**

H_3PO_2 is good reducing agent due to presence of two P–H bonds.



13. Ans. (2)

Process	Change in magnetic nature	Bond Order Change
$N_2 \rightarrow N_2^+$	Dia $\rightarrow$ para	$3 \rightarrow 2.5$
$NO \rightarrow NO^+$	Para $\rightarrow$ Dia	$2.5 \rightarrow 3$
$O_2 \rightarrow O_2^{-2}$	Para $\rightarrow$ Dia	$2 \rightarrow 1$
$O_2 \rightarrow O_2^+$	Para $\rightarrow$ Para	$2 \rightarrow 2.5$

14. Ans. (4)

15. Ans.(3)

In diamond C–C bond have only σ bond character while in case of graphite and fullerene (C_{60} and C_{70}) C–C bond contain double bond character. That's why diamond having maximum C–C bond length.

16. Ans.(2)

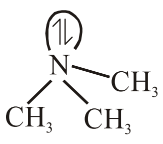
Thermal stability of Alkaline earth metals carbonates increases down the group.

because down the group polarizing power of cation decreases. So thermal stability increases.

Hence, Thermal stability order :

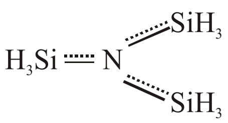
$MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

17. Ans.(4)

(a) 

nitrogen is sp^3 hybrid and pyramidal

no back-bonding i.e.more basic

(b) 

Nitrogen sp^2 hybrid and planar due to back bonding and less basic because lone pair is not available for donation.

18. Ans.(3)

Fledespar - $KAlSi_3O_8 - NaAlSi_3O_8$ - $CaAl_2Si_2O_8$

Zeolites - $NaAlSi_2O_6.H_2O$

mica - $KAl_3Si_3O_{10}(OH)_2$

asbestos - $Mg_3Si_2O_5(OH)_4$

These all are silicates having basic unit $(SiO_4)^{4-}$

19. Ans.(3)

Total No. of pentagons in C_{60} = 12

Total no. of trigons (triangles) in white phosphorus (P_4) = 4

20. Ans.(3)

Chemical species	Hybridisation of central atom
ICl_2^-	sp^3d
IF_6^-	sp^3d^3
ICl_4^-	sp^3d^2
BrF_2^-	sp^3d

21. Ans.(3)

All halides of Be are predominantly covalent in nature.

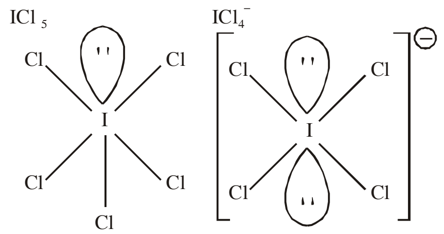
22. Ans.(1)

Chemical Species	Bond Order	Magnetic behaviour
C_2^{2-}	3	diamagnetic
N_2^{2-}	2	paramagnetic
O_2	2	paramagnetic
O_2^{2-}	1	diamagnetic

$$\text{B.O.} \propto \frac{1}{\text{bond length}}$$

23. Ans.(3)

Chemical species	Hybridisation	Shape
ICl_5	sp^3d^2	Square pyramidal
ICl_4^-	sp^3d^2	Square planar



24. Ans.(4)

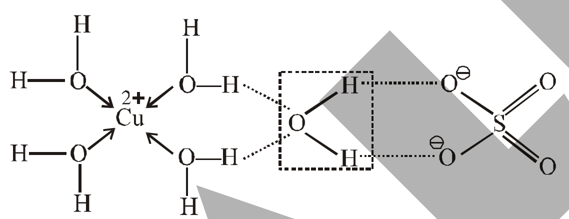
Correct order of oxidation state of nitrogen in oxides of nitrogen is following

$$\overset{+1}{N}_2O < \overset{+2}{N}O < \overset{+3}{N}_2O_3 < \overset{+4}{N}O_2$$

25. Ans.(2)

In case of only C_2, incoming electron will enter in the bonding molecular orbital which increases the bond order and stability too. Whereas rest of all takes electron in their antibonding molecular orbital which decreases bond order and stability.

26. Ans.(3)



One water molecule as shown in the diagram, is not coordinated to copper ion directly.

27. Ans.(4)

O_2,NO,B_2 are paramagnetic according to M.O.T. where as CO is diamagnetic.

28. Ans.(1)

$BeCl_2$ exist as $(BeCl_2)_n$ polymeric chain in solid form, while $BeCl_2$ exist as dimer $(BeCl_2)_2$ in vapour phase.

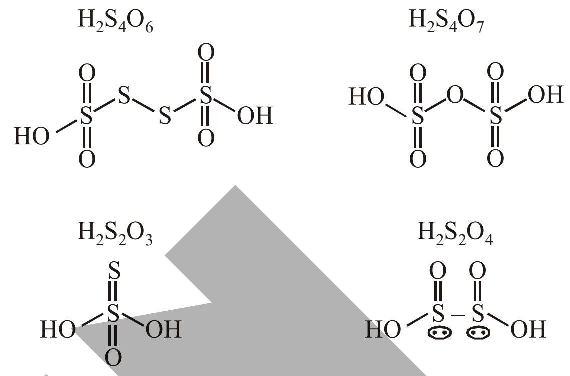
29. Ans.(3)

HF has highest boiling point among hydrogen halides because it has strongest hydrogen bonding

30. Ans.(3)

Based on NCERT, statement of limitations of VBT, I & III are correct

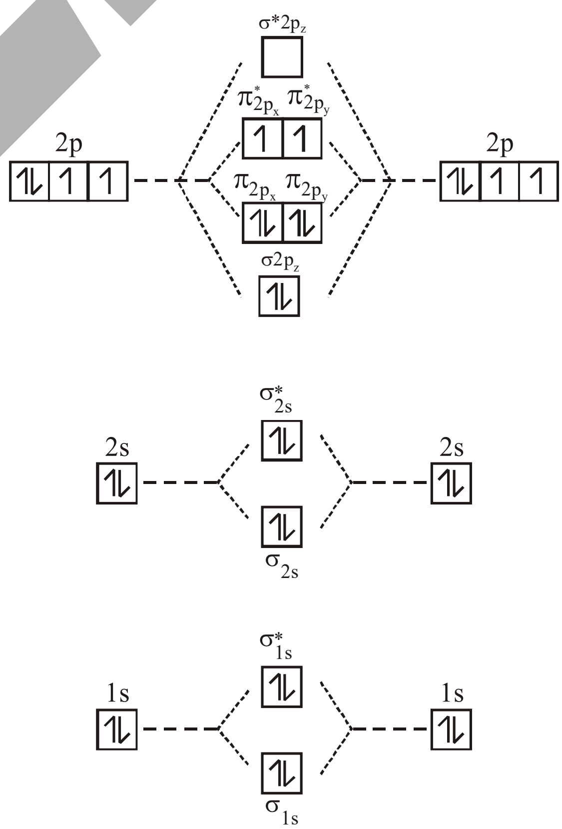
31. Ans.(2)



$H_2S_2O_7$ does not contain bond between sulphur atoms.

32. Ans.(3)

Molecular orbital diagram of O_2 is



An incoming electron will go in $\pi^*_{2p_x}$ orbital.

S-BLOCK

1. **Ans. (1)**

$Rb + O_{2(excess)} \longrightarrow RbO_2$

$2RbO_2 + 2H_2O \longrightarrow 2RbOH + H_2O_2 + O_2$

2. **Ans. (2)**

$Na_2CO_3.10H_2O \rightarrow$ Solvay process

$Mg(HCO_3)_2 \rightarrow$ Temporary hardness

$NaOH \rightarrow$ Castner-kellner cell

$Ca_3Al_2O_6 \rightarrow$ Portland cement

3. **Ans. (4)**

"Be" Metal is used in x-ray window is due to transparent to x-rays.

4. **Ans. (4)**

Smaller in size of center atoms more water molecules will crystallize hence $Ba(NO_3)_2$ is answer due to its largest size of '+ve' ion.

5. **Ans. (3)**

Only Li react directly with N_2 out of alkali metals

$6Li + N_2 \rightarrow 2Li_3N$

6. **Ans. (4)**

7. **Ans.(3)**

$$Mg + \underset{O_2/N_2}{air} \xrightarrow{\Delta} MgO + Mg_3N_2$$

8. **Ans.(1)**

$$\underset{\substack{\text{washing soda}\\(X)}}{Na_2CO_3.10H_2O(s)} \xrightarrow{\Delta} \underset{(Y)}{Na_2CO_3.H_2O} \xrightarrow[T > 373k]{\Delta} \underset{\substack{\text{(soda ash)}\\(Z)}}{Na_2CO_3}$$

9. **Ans.(3)**

Temporary hardness is due to soluble $Mg(HCO_3)_2$ and $Ca(HCO_3)_2$

$Mg(HCO_3)_2 \xrightarrow{Boil} Mg(OH)_2 \downarrow + 2CO_2 \uparrow$

$Ca(HCO_3)_2 \xrightarrow{Boil} CaCO_3 \downarrow + H_2O + CO_2 \uparrow$

10. **Ans.(4)**

$$(4)\ 2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2(g) + \frac{1}{2}O_2(g)$$

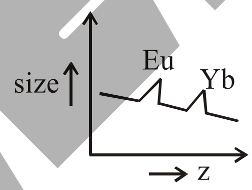
PERIODIC TABLE

1. **Ans. (3)**

Z = 120

Its general electronic configuration may be represented as [Nobal gas] ns^2 , like other alkaline earth metals.

2. **Ans. (3)**

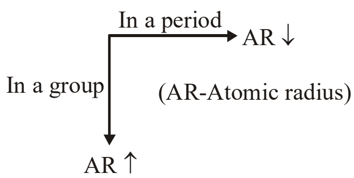


Eu > Ce > Ho > N.

3. **Ans. (2)**

$Be(OH)_2$ is amphoteric in nature while rest all alkaline earth metal hydroxide are basic in nature.

4. **Ans. (4)**



Atomic radii order : C < S < Al < Cs

Atomic radius of C : 170 pm

Atomic radius of S : 180 pm

Atomic radius of Al : 184 pm

Atomic radius of Cs : 300 pm

5. Ans. (1)

B C

Al Si

$\boxed{Ga < Ge}$

Along the period electronegativity increases

6. Ans. (1)

Due to Lanthanoid contraction both atomic radii and ionic radii decreases gradually in the lanthanoid series.

7. Ans. (4)

E.N. of Al = (1.5) $\approxeq$ Be (1.5)

8. Ans. (2)

Electronegativity decreases as we go down the group and atomic radius increases as we go down the group.

9. Ans. (3)

Second electron gain enthalpy is always positive for every element.

$O^{-}_{(g)} + e^{-} \rightarrow O^{-2}_{(g)}$; ΔH = positive

10. Ans.(3)

Mo and W has nearly similar atomic radius due to lanthanoid contraction.

11. Ans.(1)

In case of 'Be' electron remove from '2s' orbital while in case of 'B' electron remove from '2p' orbital. '2s' orbital have greater penetration effect then '2p' orbitals. So 'Be' having more I.E. then 'B'

12. Ans.(3)

Atomic number (Z) = 15 $\Rightarrow$ P $\rightarrow$ [Ne] $3s^2\ 3p^3$

Phosphorus belongs to 15^{th} group

number of valence electrons = 5

and valency = 3 in ground state.

13. Ans.(4)

The highest oxidation state of U and Pu is 6+ and 7+ respectively

14. Ans.(Bonus)

In question noble gas asked, which does not exist in the atmosphere and answer is given Ra.

Ra is a alkaline earth metal not noble gas it should be Rn. It is printing error in JEE Main paper

15. Ans.(2)

Ti $\rightarrow$ |Ar| $3d^2\ 4s^2$

Mn $\rightarrow$ |Ar| $3d^5\ 4s^2$

Ni $\rightarrow$ |Ar| $3d^8\ 4s^2$

Zn $\rightarrow$ |Ar| $3d^{10}\ 4s^2$

Correct order of I.P. is

[Ti < Mn < Ni < Zn]

16. Ans.(1)

Hydration enthalpy depends upon ionic potential (charge / size). As ionic potential increases hydration enthalpy increases.

Correct option : (1)

17. Ans.(4)

Symbol	Atomic number
unh	106
uun	110
une	109
uue	119

18. Ans.(2)

For isoelectronic species the size is compared by nuclear charge.

Correct option: (2)

19. Ans.(2)

K = 2, 8, 8 ,1

After removal of one electron, second electron we have to remove from another shell, hence there is large difference between first and second ionization energies.

METALLURGY

1. **Ans. (2) Carbon**

In the Hall-Heroult process the cathode is made of carbon.

2. **Ans. (1)**

ZnO & MgO both are in oxide form therefore no change on calcination.

3. **Ans. (3)**

Siderite : $FeCO_3$

Kaolinite : $Al_2(OH)_4Si_2O_5$

Malachite : $Cu(OH)_2.CuCO_3$

Calamine : $ZnCO_3$

4. **Ans. (4)**

Copper pyrites : $CuFeS_2$

Malachite : $Cu(OH)_2 . CuCO_3$

Azurite $Cu(OH)_2 . 2CuCO_3$

Dolomite $CaCO_3 . MgCO_3$

5. **Ans. (4)**

According to the given diagram Al can reduce ZnO.

$3ZnO+2Al\rightarrow3Zn+Al_2O_3$

6. **Ans. (4)**

Calcination in carried out for carbonates and oxide ores in absence of oxygen. Roasting is carried out mainly for sulphide ores in presence of excess of oxygen.

7. **Ans. (4)**

In Hall-Heroult's process is given by

$2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$

$2Al_2O_3(\ell) \rightleftharpoons 4Al^{3+}(\ell) + 6O^{2\ominus}(\ell)$

At cathode :- $4Al^{3+}_{(\ell)} + 12e^{\ominus} \rightarrow 4Al(\ell)$

At Anode : $6O^{2\ominus}_{(\ell)} \rightarrow 3O_2(g) + 12e^{\ominus}$

$3C + 3O_2 \rightarrow 3CO_2 (\uparrow)$

8. **Ans.(3)**

The idea of froth floatation method came from washerwoman and this process is related to concentration of sulphide ores.

9. **Ans.(1)**

(1) During leaching when bauxite is treated with concentrated NaOH, then sodium aluminate and sodium silicate is formed in the soluble form, whereas Fe_2O_3 is precipitated

(2) The blistered appearance of copper during the metallurgical process is due to the evolution of SO_2.

(3) Cast iron is obtained from pig iron.

(4) Hall-Heroult process is used for production of only aluminium.

10. **Ans.(2)**

(1) Zincite is ZnO

(2) Aniline is the forth stablizer.

(3) Zone refining process is not used for refining of 'Ti'

(4) Sodium cyanide is used in the metallurgy of silver

11. **Ans.(4)**

Ellingham diagram helps in predicting the feasibiltiy of thermal reduction of ores.

Correct option : (4)

12. **Ans.(2)**

Mond's process is used for the purification of Nickel.

13. **Ans.(4)**

$Na_3AlF_6 \rightarrow$ Cryolite is the fluoride ore.

Magnetite Fe_3O_4

Sphalerite ZnS

Malachite $Cu(OH)_2.CuCO_3$

14. Ans.(1)

1. Bauxite–$AlO_x(OH)_{3-2x}$ where $0 < x < 1$
2. Siderite – $FeCO_3$
3. Calamine – $ZnCO_3$
4. Malachite – $CuCO_3.Cu(OH)_2$

15. Ans.(3)

Assertion is correct as Haemetite ore is used for extraction of Fe.

Haemetite is an oxide ore so reason is incorrect

16. Ans.(4)

Liquation is used for Sn.

Zone refining is used for Ga.

Mond's process is used for Ni.

Van arkel process is used for Zr.

17. Ans.(3)

Mg – Al alloy is used for construction of aircrafts.

QUANTUM NUMBER

1. Ans. (3)

Total number of isotopes of hydrogen is 3

$\Rightarrow {}_1^1H,\ {}_1^2H$ or ${}_1^2D,\ {}_1^3H$ or ${}_1^3T$

and only ${}_1^3H$ or ${}_1^3T$ is an Radioactive element.

2. Ans. (4)

Isotopes of hydrogen is :

Proteium Deuterium Tritium

3. Ans. (1)

4. Ans.(2)

According to $(n+\ell)$ rule : $3p < 3d < 4p < 4d$

Correct option : (2)

5. Ans.(4)

$N^{-3} \rightarrow 1s^2\ 2s^2\ 2p^6$

$Li^+ \rightarrow 1s^2$

$Mg^{+2} \rightarrow 1s^2\ 2s^2\ 2p^6$

$O^{-2} \rightarrow 1s^2\ 2s^2\ 2p^6$

$F^- \rightarrow 1s^2\ 2s^2\ 2p^6$

$Na^+ \rightarrow 1s^2\ 2s^2\ 2p^6$

N^{-3}, O^{-2}, F^- and Na^+ are isoelectronic

P-BLOCK

1. Ans. (1)

2. Ans. (3)

Then is no catalyst is required for combustion of coal.

3. Ans.(3)

$B_2H_6 + 3H_2O \longrightarrow 2H_3BO_3 + 3H_2$

$B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

Correct option : (3)

4. Ans. (1)

Quartz (Information)

5. Ans.(2)

Kieselguhr is amorphous form of silica, it's a fact

6. Ans.(1)

All statements are correct

$B_2O_3 \rightarrow$ acidic

Al_2O_3 & Ga_2O_3 are amphoteric

oxides of In & Tl are basic

7. Ans.(1)

As we move down the group, bond strength decreases, thereby decreasing the catenation tendency.

Hence the order is as expected

$C > Si > Ge \approx Sn$

8. Ans.(3)

water gas = $CO + H_2$

is also called syn gas because it is used for synthesis of methanol.

D-BLOCK

1. **Ans. (3)**
Usally Sc(Scandium) does not show variable oxidation states.
Most common oxidation states of :
(i) Sc : +3
(ii) V : +2, +3, +4, +5
(iii) Ti : +2, +3, +4
(iv) Cu : +1, +2

2. **Ans. (3)**
$MnO_2(A) \xrightarrow{4KOH, O_2} 2K_2MnO_4(B) + 2H_2O$
(Green)
$3K_2MnO_4(B) \xrightarrow{4HCl} 2KMnO_4(C) + 2H_2O$
(Purple)
$2KMnO_4(C) \xrightarrow{H_2O, KI} 2MnO_2(A) + 2KOH + KIO_3(D)$
$A \rightarrow MnO_2$
$D \rightarrow KIO_3$

3. **Ans. (2)**
Since Zn is not a transition element so transition element having lowest atomisation energy out of Cu, V, Fe is Cu.

4. **Ans.(2)**
V_2O_5 is catalyst → contact process for H_2SO_4
$TiCl_4/Al(Me)_3$ → Ziegler Natta salt used as catalyst for polymerisation of ethene.
$PdCl_2$ → used as catalyst for ethanal (Wacker process).
Iron oxide → is used as catalyst in Haber's synthesis.

5. **Ans.(1)**
$Ti^{+2} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^2$
unpaired electrons = 2.
spin only magnetic moment $(\mu) = \sqrt{2(2+2)}$
$= \sqrt{8}$ B.M
$Ti^{+3} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^1$
unpaired electrons = 1
$\mu = \sqrt{1(1+2)} = \sqrt{3}$ B.M
$V^{+2} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^3$
$\mu = \sqrt{3(3+2)} = \sqrt{15}$ B.M
$Sc^{+3} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$
$\mu = 0$

HYDROGEN & IT'S COMPOUND

1. **Ans. (3)**
NaH is an example of ionic hydride which is also known as saline hydride.

2. **Ans. (2)**
Option (a), (b) & (c) are correct answer (NCERT THEORY BASED)

3. **Ans. (1)**
$Ca(HCO_3)_2$ is reponsible for temporary hardness of water

4. **Ans. (2)**
H_2O_2 act as oxidising agent and reducing agent in acidic medium as well as basic medium.
H_2O_2 Act as oxidant :-
$H_2O_2 + 2H^{\oplus} + 2e^{\ominus} \rightarrow 2H_2O$ (In acidic medium)
$H_2O + 2e^{\ominus} \rightarrow 2OH^{\ominus}$ (In basic medium)
H_2O_2 Act as reductant :-
$H_2O_2 \rightarrow 2H^+ + O_2 + 2e^{\ominus}$ (In acidic medium)
$H_2O_2 + 2OH^{\ominus} \rightarrow 2H_2O + O_2 + 2e^{\ominus}$ (In basic medium)

5. **Ans.(1)**
$Zn + 2HCl \rightarrow ZnCl_2 + H_2 \uparrow$
$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2 \uparrow$

ENVIRONMENTAL CHEMISTRY

1. **Ans. (3)**
Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

2. **Ans. (4)**
Ozone protects most of the medium freequnecies ultravoilet light from 200 - 315 nm wave length.

3. **Ans. (3)**
Freons (CFC's) are not common components of photo chemical smog.

4. **Ans. (4)**
Taj mahal is slowely disfigured and

discoloured due to acid rain.

5. **Ans. (1)**

Due to acid rain in plants high concentration of SO_2 makes the flower buds stiff and makes them fall.

6. **Ans. (2)**

Photochemical smog produce chemicals such as formaldehyde, acrolein and peroxyacetyl nitrate (PAN).

7. **Ans.(1)**

The common component of photochemical smog are ozone,nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate (PAN).

8. **Ans.(1)**

Photochemical smog occurs in warm (sunlight) and has high concentration of oxidising agent therefore it is called photochemical smog/oxidising smog.

9. **Ans.(1)**

The upper stratosphere consists of ozone (O_3), which protect us from harmful ultraviolet (UV) radiations coming from sun.

Correct option : (1)

10. **Ans.(3)**

Correct option : (3)

11. **Ans.(4)**

Excessive release of CO_2 into the atmosphere results in **global warming**.

12. **Ans.(3)**

It's a fact, the layer of atmosphere between 10km to 50km above sea level is called as stratosphere.

13. **Ans.(3)**

Troposphere is the lowest region of atmosphere bounded by Earth beneath and the stratosphere above where most of the clouds form and where life form exists.

14. **Ans.(4)**

Nitrogen oxides and hydrocarbons (unburnt fuel) are primary pollutant that leads to photochemical smog.

SALT ANALYSIS

1. **Ans. (2)**

$3Cl_2 + 6\ OH^- \rightarrow 5Cl^- + ClO_3^- + 3H_2O$

2. **Ans. (1)**

$I_2 + 10HNO_3 \longrightarrow 2HIO_3 + 10NO_2 + 4H_2O$

In HIO_3 oxidation state of iodine is +5.

3. **Ans.(4)**

The phosphorus containing organic compound are detected by 'Lassaigne's test' by heated with an oxidizing agent (sodium peroxide)

The phosphorus present in the compound in oxidised to phosphate.

The solution is boiled with nitric acid and then treated with ammonium molybdate to produced canary yellow precipitate.

$Na_3PO_4 + 3HNO_3 \rightarrow H_3PO_4 + 3NaNO_3$

$H_3PO_4 + 12\ (NH_4)_2MoO_4 + 21HNO_3 \rightarrow$

(Ammonium molybdate)

$(NH_4)_3PO_4.12MoO_3\downarrow + 21\ NH_4NO_3 + 12\ H_2O$

(Ammonium phosphomolybdate)

(canary yellow precipitate)

4. **Ans.(1)**

$$\begin{array}{c} CH_3 \\ | \\ CH_3\text{–}C\text{–}Cl \\ | \\ CH_3 \end{array} + AgNO_3 \longrightarrow (CH_3)_3C^{\oplus} + \underset{\text{(white ppt)}}{AgCl_{(s)}\downarrow}$$

Reason :- Due to most stable carbocation formation tert-butyl chloride given the ppt immediately

F-BLOCK

1. **Ans.(1)**

Sm^{3+} ($4f^5$) = yellow colour

Correct option : (1)

2. **Ans.(4)**

Np and Pu show maximum no. of oxidations states starting from +3 to +7 all oxidation states.

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Chapter 03 Contents

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & APRIL 2019

MATHEMATICS

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path to success

Chapter 03 Contents

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & APRIL 2019

MATHEMATICS

JANUARY AND APRIL 2019 ATTEMPT (MATHEMATICS)

COMPOUND ANGLE

1. Ans. (1)

We have,

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6\theta$

$= 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta \cdot \cos^2\theta + 4(1 - \cos^2\theta)^3$

$= 13 - 4\cos^6\theta$

2. Ans. (3)

$$2\sin\frac{\pi}{2^{10}}\cos\frac{\pi}{2^{10}}\ldots\ldots\cos\frac{\pi}{2^2}$$

$$\frac{1}{2^9}\sin\frac{\pi}{2} = \frac{1}{512}$$

Option (3)

3. Ans. (4)

$f_4(x) - f_6(x)$

$$= \frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{2}\sin^2 2x\right) - \frac{1}{6}\left(1 - \frac{3}{4}\sin^2 2x\right) = \frac{1}{12}$$

4. Ans. (1)

$$y = 3\cos\theta + 5\left(\sin\theta\frac{\sqrt{3}}{2} - \cos\theta\frac{1}{2}\right)$$

$$\frac{5\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$$

$$y_{max} = \sqrt{\frac{75}{4} + \frac{1}{4}} = \sqrt{19}$$

5. Official Ans. by NTA (4)

Sol. $0 < \alpha + \beta = \frac{\pi}{2}$ and $\frac{-\pi}{4} < \alpha - \beta < \frac{\pi}{4}$

if $\cos(\alpha + \beta) = \frac{3}{5}$ then $\tan(\alpha + \beta) = \frac{4}{3}$

and if $\sin(\alpha - \beta) = \frac{5}{13}$ then $\tan(\alpha - \beta) = \frac{5}{12}$

(since $\alpha - \beta$ here lies in the first quadrant)

Now $\tan(2\alpha) = \tan\{(\alpha + \beta) + (\alpha - \beta)\}$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta).\tan(\alpha - \beta)} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}.\frac{5}{12}} = \frac{63}{16}$$

6. Official Ans. by NTA (2)

Sol. $\frac{1}{2}\left(2\cos^2 10° - 2\cos 10°\cos 50° + 2\cos^2 50°\right)$

$$\Rightarrow \frac{1}{2}\left(1 + \cos 20° - \left(\cos 60° + \cos 40°\right) + 1 + \cos 100°\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20° + 2\sin 70°\sin(-30°)\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20° - \sin 70°\right)$$

$$\Rightarrow \frac{3}{4} \text{ Ans. (2)}$$

7. Official Ans. by NTA (4)

Sol. $(\sin 10° \sin 30° \sin 70°) \sin 30°$

$$\frac{1}{4}(\sin 30°)^2 = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$$

QUADRATIC EQUATION

1. Ans. (3)

We have

$(x + 1)^2 + 1 = 0$

$\Rightarrow (x + 1)^2 - (i)^2 = 0$

$\Rightarrow (x + 1 + i)(x + 1 - i) = 0$

$\therefore \; x = \underbrace{-(1+i)}_{\alpha(\text{let})} \; \underbrace{-(1-i)}_{\beta(\text{let})}$

So, $\alpha^{15} + \beta^{15} = (\alpha^2)^7\alpha + (\beta^2)^7\beta$

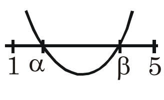
$= -128(-i + 1 + i + 1)$

$= -256$

2. Ans. (1)

$x^2 - mx + 4 = 0$

$\alpha, \beta \in [1, 5]$



(1) $D > 0 \Rightarrow m^2 - 16 > 0$

$\Rightarrow m \in (-\infty, -4) \cup (4, \infty)$

(2) $f(1) \geq 0 \Rightarrow 5 - m \geq 0 \Rightarrow m \in (-\infty, 5]$

(3) $f(5) \geq 0 \Rightarrow 29 - 5m \geq 0 \Rightarrow m \in \left(-\infty, \frac{29}{5}\right]$

(4) $1 < \frac{-b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2,10)$

$\Rightarrow m \in (4,5)$

No option correct : Bonus

* If we consider $\alpha, \beta \in (1,5)$ then option (1) is correct.

3. **Ans. (3)**

$6x^2 - 11x + \alpha = 0$

given roots are rational

$\Rightarrow$ D must be perfect square

$\Rightarrow 121 - 24\alpha = \lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha = 1 \Rightarrow \lambda \notin I$

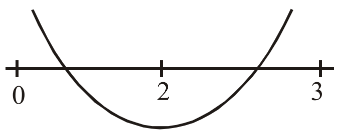
$\alpha = 2 \Rightarrow \lambda \notin I$

$\alpha = 3 \Rightarrow \lambda \in I$ $\Rightarrow$ 3 integral values

$\alpha = 4 \Rightarrow \lambda \in I$

$\alpha = 5 \Rightarrow \lambda \in I$

4. **Ans. (1)**



Let $f(x) = (c - 5)x^2 - 2cx + c - 4$

$\therefore f(0)f(2) < 0$ .....(1)

& $f(2)f(3) < 0$ .....(2)

from (1) & (2)

$(c - 4)(c - 24) < 0$

& $(c - 24)(4c - 49) < 0$

$\Rightarrow \frac{49}{4} < c < 24$

$\therefore$ s = {13, 14, 15, ... 23}

Number of elements in set S = 11

5. **Ans. (1)**

$\alpha + \beta = \lambda - 3$

$\alpha\beta = 2 - \lambda$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (\lambda - 3)^2 - 2(2 - \lambda)$

$= \lambda^2 + 9 - 6\lambda - 4 + 2\lambda$

$= \lambda^2 - 4\lambda + 5$

$= (\lambda - 2)^2 + 1$

$\therefore \lambda = 2$

Option (1)

6. **Ans. (3)**

$81x^2 + kx + 256 = 0$; $x = \alpha, \alpha^3$

$\Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm\frac{4}{3}$

Now $-\frac{k}{81} = \alpha + \alpha^3 = \pm\frac{100}{27} \Rightarrow k = \pm 300$

7. **Ans. (1)**

$D = (1 + \sin\theta\cos\theta)^2 - 4\sin\theta\cos\theta$

$= (1 - \sin\theta\cos\theta)^2$

$\Rightarrow$ roots are $\beta = \operatorname{cosec}\theta$ and $\alpha = \cos\theta$

$\Rightarrow \sum_{n=0}^{\infty}\left(\alpha^n + \left(-\frac{1}{\beta}\right)^n\right) = \sum_{n=0}^{\infty}(\cos\theta)^n + \sum_{n=0}^{n}(-\sin\theta)^n$

$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

8. **Ans. (2)**

$3m^2x^2 + m(m - 4)x + 2 = 0$

$\lambda + \frac{1}{\lambda} = 1$, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$, $\alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 3\alpha\beta$

$\left(-\frac{m(m-4)}{3m^2}\right)^2 = \frac{3(2)}{3m^2}$, $\frac{(m-4)^2}{9m^2} = \frac{6}{3m}$

$(m - 4)^2 = 18$, $m = 4 \pm \sqrt{18}, 4 \pm 3\sqrt{2}$

9. **Ans. (2)**

Exprsssion is always positve it

$2m + 1 > 0 \Rightarrow m > -\frac{1}{2}$ & $D < 0$

$\Rightarrow m^2 - 6m - 3 < 0$

$3 - \sqrt{12} < m < 3 + \sqrt{12}$ .... (iii)

$\therefore$ Common interval is

$3 - \sqrt{12} < m < 3 + \sqrt{12}$

$\therefore$ Intgral value of m {0,1,2,3,4,5,6}

10. **Official Ans. by NTA (3)**

Sol. $(x - 1)^2 + 1 = 0 \Rightarrow x = 1 + i, 1 - i$

$\therefore \left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm i)^n = 1$

$\therefore$ n (least natural number) = 4

11. Official Ans. by NTA (3)

Sol. $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0$

$\left|\sqrt{x}-2\right|+\left(\sqrt{x}\right)^2-4\sqrt{x}+2=0$

$\left|\sqrt{x}-2\right|^2+\left|\sqrt{x}-2\right|-2=0$

$\left|\sqrt{x}-2\right|=-2$ (not possible) or $\left|\sqrt{x}-2\right|=1$

$\sqrt{x}-2=1,-1$

$\sqrt{x}=3,1$

$x = 9, 1$

Sum = 10

12. Official Ans. by NTA (1)

Sol. $D<0$

$4(1+3m)^2-4(1+m^2)(1+8m)<0$

$\Rightarrow m(2m-1)^2>0 \Rightarrow m>0$

13. Official Ans. by NTA (2)

ALLEN Ans. (2) or (Bonus)

Sol. In given question p, q $\in$ R. If we take other root as any real number α,

then quadratic equation will be

$x^2-(\alpha+2-\sqrt{3})x+\alpha.\left(2-\sqrt{3}\right)=0$

Now, we can have none or any of the options can be correct depending upon 'α'

Instead of p, q $\in$ R it should be p, q $\in$ Q then other root will be $2+\sqrt{3}$

$\Rightarrow\ p=-\left(2+\sqrt{3}-2-\sqrt{3}\right)=-4$

and $q=\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)=1$

$\Rightarrow p^2-4q-12=(-4)^2-4-12$

$= 16-16=0$

Option (2) is correct

14. Official Ans. by NTA (4)

Sol. $\text{SOR}=\dfrac{3}{m^2+1}\Rightarrow(\text{S.O.R})_{\max}=3$

when $m = 0$

$x^2-3x+1=0 \rightarrow \alpha, \beta$

$\alpha+\beta=3$

$\alpha\beta=1$

$|\alpha^3-\beta^2|=||\alpha-\beta|(\alpha^2+\beta^2+\alpha\beta)|$

$=\left|\sqrt{(\alpha-\beta)^2-\alpha\beta}\ \ ((\alpha+\beta)^2-\alpha\beta)\right|$

$=\left|\sqrt{9-4}\,(9-1)\right|$

$=\sqrt{5}\times 8$

15. Official Ans. by NTA (4)

Sol. $\dfrac{\alpha^{12}+\beta^{12}}{\left(\dfrac{1}{\alpha^{12}}+\dfrac{1}{\beta^{12}}\right)(\alpha-\beta)^{24}}=\dfrac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$

$=\dfrac{(\alpha\beta)^{12}}{\left[(\alpha+\beta)^2-4\alpha\beta\right]^{12}}=\left[\dfrac{\alpha\beta}{(\alpha+\beta)^2-4\alpha\beta}\right]^{12}$

$=\left(\dfrac{-2\sin\theta}{\sin^2\theta+8\sin\theta}\right)^{12}=\dfrac{2^{12}}{(\sin\theta+8)^{12}}$

SEQUENCE & PROGRESSION

1. Ans. (4)

$\dfrac{b}{r}, b, br \rightarrow$ G.P. $\qquad (|r| \neq 1)$

given $a+b+c=xb$

$\Rightarrow b/r+b+br=xb$

$\Rightarrow b=0$ (not possible)

or $1+r+\dfrac{1}{r}=x\ \Rightarrow\ x-1=r+\dfrac{1}{r}$

$\Rightarrow x-1>2$ or $x-1<-2$

$\Rightarrow x>3$ or $x<-1$

So x can't be '2'

2. Ans. (4)

$S = a_1 + a_2 + ... + a_{30}$

$$S=\frac{30}{2}[a_1+a_{30}]$$

$S = 15(a_1 + a_{30}) = 15\ (a_1 + a_1 + 29d)$

$T = a_1 + a_3 + ... + a_{29}$

$= (a_1) + (a_1 + 2d)\ ..... + (a_1 + 28d)$

$= 15a_1 + 2d(1 + 2 + ... + 14)$

$T = 15a_1 + 210\ d$

Now use $S - 2T = 75$

$\Rightarrow 15\ (2a_1 + 29d) - 2\ (15a_1 + 210\ d) = 75$

$\Rightarrow d = 5$

Given $a_5 = 27 = a_1 + 4d \Rightarrow a_1 = 7$

Now $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

3. Ans. (1)

$$T_n=\frac{\left(3+(n-1)\times3\right)\left(1^2+2^2+....+n^2\right)}{(2n+1)}$$

$$T_n=\frac{3.\dfrac{n(n+1)(2n+1)}{6}}{2n+1}=\frac{n^2(n+1)}{2}$$

$$S_{15}=\frac{1}{2}\sum_{n=1}^{15}\left(n^3+n^2\right)=\frac{1}{2}\left[\left(\frac{15(15+1)}{2}\right)^2+\frac{15\times16\times31}{6}\right]$$

$= 7820$

4. Ans. (2)

$a = A + 6d$

$b = A + 10d$

$c = A + 12d$

a,b,c are in G.P.

$\Rightarrow (A + 10d)^2 = (A + 6d)\ (a + 12d)$

$$\Rightarrow\frac{A}{d}=-14$$

$$\frac{a}{c}=\frac{A+6d}{A+12d}=\frac{6+\dfrac{A}{d}}{12+\dfrac{A}{d}}=\frac{6-14}{12-14}=4$$

5. Ans. (3)

$$\frac{a}{1-r}=3 \qquad ...(1)$$

$$\frac{a^3}{1-r^3}=\frac{27}{19}\Rightarrow\frac{27(1-r)^3}{1-r^3}=\frac{27}{19}$$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$$\Rightarrow r=\frac{2}{3}\text{ as }|r|<1$$

6. Ans. (3)

$a_1, a_2, ..., a_{10}$ are in G.P.,

Let the common ratio be r

$$\frac{a_3}{a_1}=25\Rightarrow\frac{a_1r^2}{a_1}=25\Rightarrow r^2=25$$

$$\frac{a_9}{a_5}=\frac{a_1r^8}{a_1r^4}=r^4=5^4$$

7. Ans. (1)

$a + 18d = 0 \qquad ...(1)$

$$\frac{a+48d}{a+28d}=\frac{-18d+48d}{-18d+28d}=\frac{3}{1}$$

8. Ans. (2)

$$\frac{x^my^n}{\left(1+x^{2m}\right)\left(1+y^{2n}\right)}=\frac{1}{\left(x^m+\dfrac{1}{x^m}\right)\left(y^n+\dfrac{1}{y^n}\right)}\le\frac{1}{4}$$

using AM $\geq$ GM

9. Ans. (4)

Let terms are $\frac{a}{r},a,ar\rightarrow$ G.P

$\therefore\ a^3 = 512 \Rightarrow a = 8$

$$\frac{8}{r}+4,12,8r\rightarrow\text{A.P.}$$

$$24=\frac{8}{r}+4+8r$$

$$r=2,\ r=\frac{1}{2}$$

$r = 2\ (4, 8, 16)$

$$r=\frac{1}{2}(16,8,4)$$

Sum = 28

10. Ans. (1)

$$S_K=\frac{K+1}{2}$$

$$\Sigma S_k^2=\frac{5}{12}A$$

$$\sum_{K=1}^{10}\left(\frac{K+1}{2}\right)^2=\frac{2^2+3^2+--+11^2}{4}=\frac{5}{12}A$$

$$\frac{11\times12\times23}{6}-1=\frac{5}{3}A$$

$$505=\frac{5}{3}A,\quad A = 303$$

11. Ans (2)

A.M. $\geq$ G.M.

$$\frac{\sin^4\alpha+4\cos^4\beta+1+1}{4}\geq(\sin^4\alpha.4\cos^4\beta.1.1)^{\frac{1}{4}}$$

$\sin^4\alpha + 4\cos^2\beta + 2 \geq 4\sqrt{2}\sin\alpha\cos\beta$

given that $\sin^4\alpha + 4\cos^4\beta + 2$

$= 4\sqrt{2}\ \sin\alpha\cos\beta$

$\Rightarrow$ A.M. = G.M. $\Rightarrow \sin^4\alpha = 1 = 4\cos^4\beta$

$\sin\alpha = 1,\ \cos\beta = \pm\dfrac{1}{\sqrt{2}}$

$\Rightarrow \sin\beta = \dfrac{1}{\sqrt{2}}$ as $\beta \in [0, \pi]$

$\cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= -2\sin\alpha\sin\beta$

$= -\sqrt{2}$

12. Ans. (2)

$$S=\left(\frac{3}{4}\right)^3+\left(\frac{6}{4}\right)^3+\left(\frac{9}{4}\right)^3+\left(\frac{12}{4}\right)^3+............15\text{ term}$$

$$=\frac{27}{64}\sum_{r=1}^{15}r^3$$

$$=\frac{27}{64}.\left[\frac{15(15+1)}{2}\right]^2$$

$= 225$ K (Given in question)

K = 27

13. Official Ans. by NTA (2)

Sol. S_A = sum of numbers between 100 & 200 which are divisible by 7.

$\Rightarrow\ S_A = 105 + 112 + ... + 196$

$$S_A=\frac{14}{2}[105+196]=2107$$

S_B = Sum of numbers between 100 & 200 which are divisible by 13.

$S_B = 104 + 117 + ... + 195 = \dfrac{8}{2}[104+195]=1196$

S_C = Sum of numbers between 100 & 200 which are divisible by both 7 & 13.

$S_C = 182$

$\Rightarrow$ H.C.F. (91, n) > 1 $= S_A + S_B - S_C = 3121$

14. Official Ans. by NTA (2)

Sol. $S=\sum_{k=1}^{20}\dfrac{1}{2^k}$

$$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{3^2}+...+\frac{20}{2^{20}}$$

$$S\times\frac{1}{2}=\frac{1}{2^2}+\frac{2}{2^3}+...+\frac{19}{2^{20}}+\frac{20}{2^{21}}$$

$$\Rightarrow\left(1-\frac{1}{2}\right)S=\frac{1}{2}+\frac{1}{2^2}+...+\frac{1}{2^{20}}-\frac{20}{2^{21}}$$

$$\Rightarrow S=2-\frac{11}{2^{19}}$$

15. Official Ans. by NTA (3)

Sol. a, b, c in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

$x = -r$ is the common root, satisfies second equation $d(-r)^2 + 2e(-r) + f = 0$

$$\Rightarrow d.\frac{c}{a}-\frac{2ce}{b}+f=0$$

$$\Rightarrow \frac{d}{a}+\frac{f}{c}=\frac{2e}{b}$$

16. Official Ans. by NTA (1)

Sol. $S_n=50n+\dfrac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$$=50n+\frac{n(n-7)}{2}A-50(n-1)-\frac{(n-1)(n-8)}{2}A$$

$= 50 + \dfrac{A}{2}[n^2 - 7n - n^2 + 9n - 8]$

$= 50 + A(n - 4)$

$d = T_n - T_{n-1}$

$= 50 + A(n - 4) - 50 - A(n - 5)$

$= A$

$T_{50} = 50 + 46A$

$(d, A_{50}) = (A, 50+46A)$

17. Official Ans. by NTA (1)

Sol. $a - d + a + a + d = 33 \Rightarrow a = 11$

$a(a^2 - d^2) = 1155$

$121 - d^2 = 105$

$d^2 = 16 \Rightarrow d = \pm 4$

If $d = 4$ then I^{st} term $= 7$

If $d = -4$ then I^{st} term $= 15$

$T_{11} = 7 + 40 = 47$

OR $T_{11} = 15 - 40 = -25$

18. Official Ans. by NTA (2)

Sol. $T_r = r(2r - 1)$

$S = \Sigma 2r^2 - \Sigma r$

$$S = \frac{2.n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$S_{11} = \frac{2}{6}.(11)(12)(23) - \frac{11(12)}{2} = (44)(23) - 66 = 946$$

19. Official Ans. by NTA (1)

Sol. $$T_n = \frac{(3+(n-1)\times 2)(1^3+2^3+\ldots+n^3)}{(1^2+2^2+\ldots+n^2)}$$

$$= \frac{3}{2}n(n+1) = \frac{n(n+1)(n+2)-(n-1)n(n+1)}{2}$$

$$\Rightarrow S_n = \frac{n(n+1)(n+2)}{2}$$

$\Rightarrow S_{10} = 660$

20. Official Ans. by NTA (3)

Sol. $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$$\Rightarrow \frac{6}{2}(a_1 + a_{16}) = 114$$

$\Rightarrow a_1 + a_{16} = 38$

$$\text{So}, a_1 + a_6 + a_{11} + a_{16} = \frac{4}{2}(a_1 + a_{16})$$

$= 2 \times 38 = 76$

21. Official Ans. by NTA (4)

Sol. $$\text{Sum} = \sum_{n=1}^{15} \frac{1^3+2^3+\ldots.n^3}{1+2+\ldots.+n} - \frac{1}{2}.\frac{15.16}{2}$$

$$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - 60$$

$$= \sum_{n=1}^{15} \frac{n(n+1)(n+2-(n-1))}{6} - 60$$

$$= \frac{15.16.17}{6} - 60 = 620$$

22. Official Ans. by NTA (2)

Sol. $b = ar$

$c = ar^2$

3a, 7b and 15 c are in A.P.

$\Rightarrow 14b = 3a + 15c$

$\Rightarrow 14(ar) = 3a + 15\,ar^2$

$\Rightarrow 14r = 3 + 15r^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow (3r-1)(5r-3) = 0$

$$r = \frac{1}{3}, \frac{3}{5}.$$

Only acceptable value is $r = \frac{1}{3}$, because

$$r \in \left(0, \frac{1}{2}\right]$$

$$\therefore \text{c. d} = 7b - 3a = 7ar - 3a = \frac{7}{3}a - 3a = -\frac{2}{3}a$$

$$\therefore 4^{th} \text{ term} = 15\,c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$$

23. Official Ans. by NTA (3)

Sol. $375x^2 - 25x - 2 = 0$

$$\alpha + \beta = \frac{25}{375}, \quad \alpha\beta = \frac{-2}{375}$$

$\Rightarrow (\alpha + \alpha^2 + \ldots$ upto infinite terms$)$

$+ (\beta + \beta^2 + \ldots$ upto infinite terms$)$

$$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} = \frac{1}{12}$$

24. Official Ans. by NTA (1)

Sol. $2\{2a+3d\} = 16$

$3(2a + 5d) = -48$

$2a + 3d = 8$

$2a + 5d = -16$

$\boxed{d = -12}$

$S_{10} = 5\,\{44 - 9 \times 12\}$

$= -320$

25. Official Ans. by NTA (1)

Sol. $a_1 + a_7 + a_{16} = 40$

$a + a + 6d + a + 15d = 40$

$$\Rightarrow 3a + 21d = 40 \quad \Rightarrow \boxed{a + 7d = \frac{40}{3}}$$

$$S_{15} = \frac{15}{2}(2a + 14d) = 15(a + 7d)$$

$$S_{15} = 15 \times \frac{40}{3} \Rightarrow 200 \quad \boxed{S_{15} = 200}$$

26. Official Ans. by NTA (1)

Sol. $\alpha x^2 + 2\beta x + \gamma = 0$

Let $\beta = \alpha t$, $\gamma = \alpha t^2$

$\therefore \alpha x^2 + 2\alpha tx + \alpha t^2 = 0$

$\Rightarrow x^2 + 2tx + t^2 = 0$

$\Rightarrow (x + t)^2 = 0$

$\Rightarrow x = -t$

it must be root of equation $x^2 + x - 1 = 0$

$\therefore t^2 - t - 1 = 0$ (1)

Now

$\alpha(\beta + \gamma) = \alpha^2(t + t^2)$

Option 1 $\beta\gamma = \alpha t \cdot \alpha t^2 = \alpha^2 t^3 = a^2 (t^2 + t)$

(from equation 1)

TRIGONOMETRIC EQUATION

1. Ans. (1)

$\sin x - \sin 2x + \sin 3x = 0$

$\Rightarrow (\sin x + \sin 3x) - \sin 2x = 0$

$\Rightarrow 2\sin x . \cos x - \sin 2x = 0$

$\Rightarrow \sin 2x(2\cos x - 1) = 0$

$\Rightarrow \sin 2x = 0$ or $\cos x = \frac{1}{2}$

$\Rightarrow x = 0, \frac{\pi}{3}$

2. Ans. (1)

$\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$, $\theta \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$

$\Rightarrow (2\cos^2 2\theta - 1)^2 = 0$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} = \cos^2 \frac{\pi}{4}$

$\Rightarrow 2\theta = n\pi \pm \frac{\pi}{4}$, $n \in I$

$\Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{\pi}{2} - \frac{\pi}{8}$

Sum of solutions $\frac{\pi}{2}$

3. Official Ans. by NTA (3)

Sol. $2(1 - \sin^2\theta) + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}; \sin\theta = 2(\text{reject})$

roots : $\pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, -\frac{\pi}{6}, -\pi + \frac{\pi}{6}$

$\Rightarrow$ sum of values $= 2\pi$

4. Official Ans. by NTA (1)

Sol. $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot 4^{-\sin^2 y} \le 1$

$\Rightarrow 2^{\sqrt{(\sin x - 1)^2 + 4}} \le 2^{2\sin^2 y}$

$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \le 2\sin^2 y$

$\Rightarrow \sin x = 1$ and $|\sin y| = 1$

5. Official Ans. by NTA (1)

Sol. $1 + \sin^4 x = \cos^2 3x$

$\sin x = 0$ & $\cos 3x = 1$

$0, 2\pi, -2\pi, -\pi, \pi$

6. Official Ans. by NTA (1)

Sol. $\cos 2x + \alpha \sin x = 2\alpha - 7$

$\Rightarrow 2\sin^2 x - \alpha \sin x + 2\alpha - 8 = 0$

$\sin^2 x - \frac{\alpha}{2}\sin x + \alpha - 4 = 0$

$\Rightarrow \sin x = 2$ (rejected) or $\sin x = \frac{\alpha - 4}{2}$

$\Rightarrow \left|\frac{\alpha - 4}{2}\right| \le 1$

$\Rightarrow \alpha \in [2, 6]$

SOLUTION OF TRIANGLE

1. Ans. (4)

r = 1 is obviously true.

Let 0 < r < 1

$\Rightarrow$ r + r² > 1

$\Rightarrow r^2 + r - 1 > 0$

$$\left(r-\frac{-1-\sqrt{5}}{2}\right)\left(r-\left(\frac{-1+\sqrt{5}}{2}\right)\right)$$

$$\Rightarrow r - \frac{-1-\sqrt{5}}{2} \text{ or } r > \frac{-1+\sqrt{5}}{2}$$

$$r \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$$

$$\frac{\sqrt{5}-1}{2} < r < 1$$

When r > 1

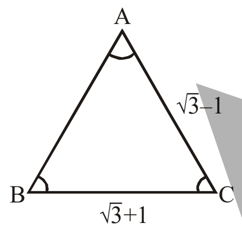
$$\Rightarrow \frac{\sqrt{5}+1}{2} > \frac{1}{r} > 1$$

$$\Rightarrow r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$$

Now check options

2. Ans. (1)

A + B = 120°



$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})}\cot(30°) = \frac{1}{\sqrt{3}}.\sqrt{3} = 1$$

$$\frac{A-B}{2} = 45° \quad \Rightarrow \quad A - B = 90°$$

$$A + B = 120°$$

$$2A = 210°$$

$$A = 105°$$

$$B = 15°$$

$\therefore$ Option (1)

3. Ans. (2)

Given a + b = x and ab = y

If $x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = -\frac{1}{2}$$

$$\Rightarrow \cos C = -\frac{1}{2}$$

$$\Rightarrow \angle C = \frac{2\pi}{3}$$

$$R = \frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$$

4. Ans. (3)

b + c = 11λ, c + a = 12λ, a + b = 13λ

$\Rightarrow$ a = 7λ, b = 6λ, c = 5λ

(using cosine formula)

$$\cos A = \frac{1}{5}, \cos B = \frac{19}{35}, \cos C = \frac{5}{7}$$

$\alpha : \beta : \gamma \Rightarrow 7 : 19 : 25$

5. Official Ans. by NTA (3)

Sol. a < b < c are in A.P.

$\angle C = 2\angle A$ (Given)

$\Rightarrow \sin C = \sin 2A$

$\Rightarrow \sin C = 2\sin A.\cos A$

$$\Rightarrow \frac{\sin C}{\sin A} = 2\cos A$$

$$\Rightarrow \frac{c}{a} = 2\frac{b^2+c^2-a^2}{2bc}$$

put a = b – λ, c = b + λ, λ > 0

$$\Rightarrow \lambda = \frac{b}{5}$$

$$\Rightarrow a = b - \frac{b}{5} = \frac{4}{5}b, c = b + \frac{b}{5} = \frac{6b}{5}$$

$\Rightarrow$ required ratio = 4 : 5 : 6

6. Official Ans. by NTA (3)

Sol. $\angle B = \frac{\pi}{3}$, by sine Rule

$$\sin A = \frac{1}{2}$$

$\Rightarrow$ A = 30°, a = 2 , b = $2\sqrt{3}$, c = 4

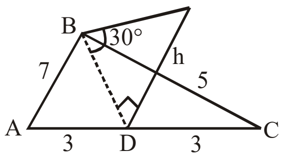
$$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3} \text{ sq. cm}$$

HEIGHT & DISTANCE

1. Ans. (2)

BD = hcot30° = $h\sqrt{3}$

So, $7^2 + 5^2 = 2((h\sqrt{3})^2 + 3^2)$

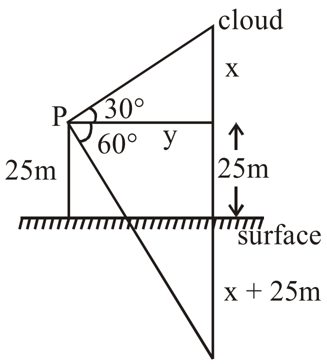


$\Rightarrow 37 = 3h^2 + 9.$

$\Rightarrow 3h^2 = 28$

$\Rightarrow h = \sqrt{\frac{28}{3}} = \frac{2}{3}\sqrt{21}$

2. Ans (2)



$\tan 30° = \frac{x}{y} \Rightarrow y = \sqrt{3}\,x$ .....(i)

$\tan 60° = \frac{25 + x + 25}{y}$

$\Rightarrow \sqrt{3}\,y = 50 + x$

$\Rightarrow 3x = 50 + x$

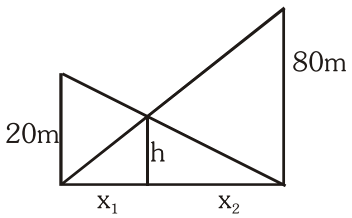
$\Rightarrow x = 25$ m

$\therefore$ Height of cloud from surface

$= 25 + 25 = 50$m

3. Official Ans. by NTA (3)

Sol.



by similar triangle

$\frac{h}{x_1} = \frac{80}{x_1 + x_2}$ ...(1)

by $\frac{h}{x_2} = \frac{20}{x_1 + x_2}$ ..(2)

by (1) and (2)

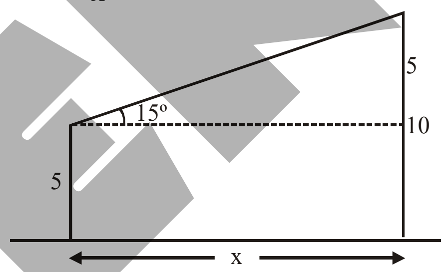
$\frac{x_2}{x_1} = 4$ or $x_2 = 4x_1$

$\Rightarrow \frac{h}{x_1} = \frac{80}{5x_1}$

or h = 16m

4. Official Ans. by NTA (3)

Sol. $\tan 15° = \frac{5}{x}$



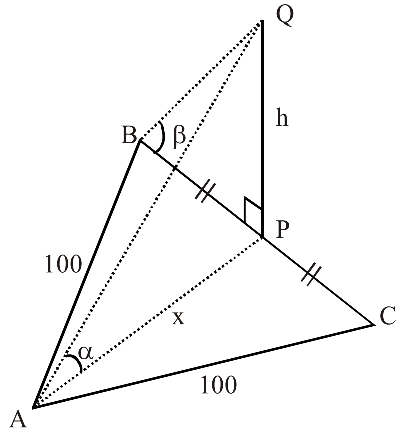
$2 - \sqrt{13} = \frac{5}{x}$

$x = 5(2 + \sqrt{3})$

5. Official Ans. by NTA (3)

Sol. cot α = $3\sqrt{2}$

& cosec β = $2\sqrt{2}$



So, $\frac{x}{h} = 3\sqrt{2}$ ...(i)

And $\dfrac{h}{\sqrt{10^4 - x^2}} = \dfrac{1}{\sqrt{7}}$ ...(ii)

So, from (i) & (ii)

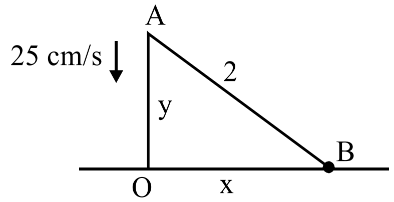
$\Rightarrow \dfrac{h}{\sqrt{10^4 - 18h^2}} = \dfrac{1}{\sqrt{7}}$

$\Rightarrow 25h^2 = 100 \times 100$

$\Rightarrow h = 20.$

6. Official Ans. by NTA (3)

Sol.



$x^2 + y^2 = 4 \left(\dfrac{dy}{dt} = -25\right)$

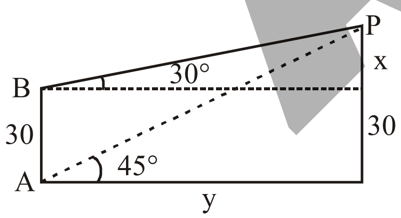
$x\dfrac{dx}{dt} + y\dfrac{dy}{dt} = 0$

$\sqrt{3}\dfrac{dx}{dt} - 1(25) = 0$

$\dfrac{dx}{dt} = \dfrac{25}{\sqrt{3}}$ cm / sec

7. Official Ans. by NTA (2)

Sol.



$\tan 45° = 1 = \dfrac{x+30}{y} \Rightarrow x + 30 = y$ (i)

$\tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{x}{y} \Rightarrow x = \dfrac{y}{\sqrt{3}}$ (ii)

from (i) and (ii) $y = 15\left(3+\sqrt{3}\right)$

DETERMINANT

1. Ans. (2)

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix} = a^2 - 3$$

$$D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 3 & 2 \\ a+1 & 3 & a^2-1 \end{vmatrix} = a^2 - a + 1$$

$$D_2 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 2 & a+1 & a^2-1 \end{vmatrix} = a^2 - 3$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 2 & 3 & a+1 \end{vmatrix} = a - 4$$

D = 0 at $|a| = \sqrt{3}$ but $D_3 = \pm\sqrt{3} - 4 \neq 0$

So the system is Inconsistant for $|a| = \sqrt{3}$

2. Ans. (2)

$P_1 \equiv x - 4y + 7z - g = 0$

$P_2 \equiv 3x - 5y - h = 0$

$P_3 \equiv -2x + 5y - 9z - k = 0$

Here $\Delta = 0$

$2P_1 + P_2 + P_3 = 0$ when $2g + h + k = 0$

3. Ans. (4)

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \alpha-1 \end{vmatrix} = (\alpha-1) - 4 = (\alpha-5)$$

for infinite solutions $D = 0 \Rightarrow \alpha = 5$

$$D_x = 0 \Rightarrow \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 3 \\ \beta-15 & -2 & 5 \end{vmatrix} = 0$$

$\Rightarrow 2 + \beta - 15 = 0 \Rightarrow \beta - 13 = 0$

on $\beta = 13$ we get $D_y = D_z = 0$

$\alpha = 5, \beta = 13$

4. Ans. (3)

$$\det A = \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & -\sin\theta+2+2d \end{vmatrix}$$

$(R_1 \to R_1 + R_3 - 2R_2)$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & 2+2d-\sin\theta \end{vmatrix}$$

$= (2+\sin\theta)(2+2d-\sin\theta) - d(2\sin\theta - d)$

$= 4 + 4d - 2\sin\theta + 2\sin\theta + 2d\sin\theta - \sin^2\theta - 2d\sin\theta + d^2$

$= d^2 + 4d + 4 - \sin^2\theta$

$= (d+2)^2 - \sin^2\theta$

For a given d, minimum value of

$\det(A) = (d+2)^2 - 1 = 8$

$\Rightarrow d = 1$ or -5

5. Ans. (1)

Apply

$C_3 \to C_3 - C_2$

$C_2 \to C_2 - C_1$

We get $D = 0$

Option (1)

6. Ans. (4)

$$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$(8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta) + 7(-\cos 2\theta - 4\sin 3\theta) = 0$

$14 - 7\cos 2\theta + 21\sin 3\theta - 7\cos 2\theta - 28\sin 3\theta = 0$

$14 - 7\sin 3\theta - 14\cos 2\theta = 0$

$14 - 7(3\sin\theta - 4\sin^3\theta) - 14(1 - 2\sin^2\theta) = 0$

$-21\sin\theta + 28\sin^3\theta + 28\sin^2\theta = 0$

$7\sin\theta[-3 + 4\sin^2\theta + 4\sin\theta] = 0$

$\sin\theta, 4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$

$2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

$\sin\theta = \frac{-3}{2}$; $\sin\theta = \frac{1}{2}$

Hence, 2 solutions in $(0, \pi)$

Option (4)

7. Ans. (1)

$P_1 : 2x + 2y + 3z = a$

$P_2 : 3x - y + 5z = b$

$P_3 : x - 3y + 2z = c$

We find

$P_1 + P_3 = P_2 \Rightarrow a + c = b$

8. Ans. (4)

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 2c & c-a-b \end{vmatrix}$$

$= (a+b+c)(a+b+c)^2$

$\Rightarrow x = -2(a+b+c)$

9. Ans. (3)

For unique solution

$$\Delta \neq 0 \Rightarrow \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0 \Rightarrow \alpha + \beta \neq -2$$

10. Ans. (2)

$$\begin{vmatrix} \lambda-1 & 2 & 2 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow (\lambda-1)^3 = 0 \Rightarrow \lambda = 1$$

11. Official Ans. by NTA (1)

Sol. For non-trivial solution

$D = 0$

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0 \Rightarrow 2c^3 - 3c^2 - 1 = 0$$

$\Rightarrow (c+1)^2(2c-1) = 0$

$\therefore$ Greatest value of c is $\frac{1}{2}$

12. Official Ans. by NTA (3)

Sol. $x - 2y + kz = 1$ ...(1)

$2x + y + z = 2$ ...(2)

$3x - y - kz = 3$ ...(3)

(1) +(3)

$\Rightarrow 4x - 3y = 4$

13. Official Ans. by NTA (3)

Sol. $\begin{vmatrix} 2 & 3 & -1 \\ 1 & K & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0$

By solving $K = \frac{9}{2}$

$2x + 3y - z = 0$ …(1)

$x + \frac{9}{2}y - 2z = 0$ …(2)

$2x - y + z = 0$ …(3)

(1)-(3) $\Rightarrow 4y - 2z = 0$

$2y = z$ …(4)

$\boxed{\frac{y}{z} = \frac{1}{2}}$ …(5)

put z from eqn. (4) into (1)

$2x + 3y - 2y = 0$

$2x + y = 0$

$\boxed{\frac{x}{y} = -\frac{1}{2}}$ …(6)

$\frac{(6)}{(5)}$ $\boxed{\frac{z}{x} = -4}$

$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + K = \frac{1}{2}$

14. Official Ans. by NTA (2)

Sol. $\Delta_1 = f(\theta) = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3$

and $\Delta_2 = f(2\theta) = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix} = -x^3$

So $\Delta_1 + \Delta_2 = -2x^3$

15. Official Ans. by NTA (2)

Sol. $D = 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$

16. Official Ans. by NTA (3)

Sol. By expansion, we get

$-5x^3 + 30x - 30 + 5x = 0$

$\Rightarrow -5x^3 + 35x - 30 = 0$

$\Rightarrow x^3 - 7x + 6 = 0$, All roots are real

So, sum of roots = 0

17. Official Ans. by NTA (3)

Sol. $R_1 \to R_1 - R_2$

$\begin{vmatrix} 1 & -1 & 0 \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$

$R_2 \to R_2 - R_3$

$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$

$\Rightarrow (1 + 4\cos 6\theta) + \sin^2\theta + 1(\cos^2\theta) = 0$

$1 + 2\cos 6\theta = 0 \Rightarrow \cos 6\theta = -1/2$

$6\theta = \frac{2\pi}{3} \Rightarrow \boxed{\theta = \frac{\pi}{9}}$

18. Official Ans. by NTA (2)

Sol. $[\sin\theta]x + [-\cos\theta]y = 0$ and $[\cos\theta]x + y = 0$

for infinite many solution

$\begin{vmatrix} [\sin\theta] & [-\cos\theta] \\ [\cos\theta] & 1 \end{vmatrix} = 0$

ie $[\sin\theta] = -[\cos\theta][\cot\theta]$ (1)

when $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \Rightarrow \sin\theta \in \left(0, \frac{1}{2}\right)$

$-\cos\theta \in \left(0, \frac{1}{2}\right)$

$\cot\theta \in \left(-\frac{1}{\sqrt{3}}, 0\right)$

when $\theta \in \left(\pi, \frac{7\pi}{6}\right) \Rightarrow \sin\theta \in \left(-\frac{1}{2}, 0\right)$

$-\cos\theta \in \left(\frac{\sqrt{3}}{2}, 1\right)$

$\cot\theta \in \left(\sqrt{3}, \infty\right)$

when $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ then equation (i) satisfied there fore infinite many solution.

when $\theta \in \left(\pi, \frac{7\pi}{6}\right)$ then equation (i) not satisfied there fore infinite unique solution.

STRAIGHT LINE

1. Ans. (4)

Given set of lines px + qy + r = 0

given condition 3p + 2q + 4r = 0

$\Rightarrow \frac{3}{4}p + \frac{1}{2}q + r = 0$

$\Rightarrow$ All lines pass through a fixed point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

2. Ans. (4)

Equation of AB is

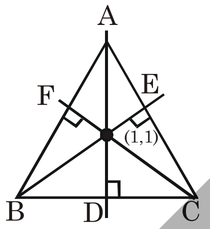
3x – 2y + 6 = 0

equation of AC is

4x + 5y – 20 = 0

Equation of BE is

2x + 3y – 5 = 0



Equation of CF is 5x – 4y – 1 = 0

$\Rightarrow$ Equation of BC is 26x – 122y = 1675

3. Ans. (4)

Let A(α,0) and B(0,β)

be the vertices of the given triangle AOB

$\Rightarrow |\alpha\beta| = 100$

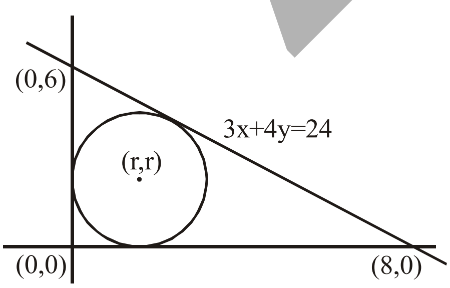
$\Rightarrow$ Number of triangles

= 4 × (number of divisors of 100)

= 4 × 9 = 36

4. Ans. (2)

$\left|\frac{3r + 4r - 24}{5}\right| = r$



7r – 24 = ±5r

2r = 24 or 12r + 24

r = 14, r = 2

then incentre is (2, 2)

5. Ans. (2)

Let the centroid of ΔPQR is (h, k) & P is (α, β), then

$\frac{\alpha + 1 + 3}{3} = h$ and $\frac{\beta + 4 - 2}{3} = k$

$\alpha = (3h - 4)$ $\qquad \beta = (3k - 4)$

Point P(α, β) lies on line 2x – 3y + 4 = 0

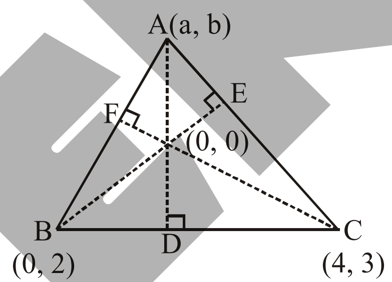
$\therefore$ 2(3h – 4) – 3(3k – 2) + 4 = 0

$\Rightarrow$ locus is 6x – 9y + 2 = 0

6. Ans. (2)

$m_{BD} \times m_{AD} = -1 \Rightarrow \left(\frac{3-2}{4-0}\right) \times \left(\frac{b-0}{a-0}\right) = -1$

$\Rightarrow$ b + 4a = 0 ……(i)



$m_{AB} \times m_{CF} = -1 \Rightarrow \left(\frac{(b-2)}{a-0}\right) \times \left(\frac{3}{4}\right) = -1$

$\Rightarrow$ 3b – 6 = –4a $\Rightarrow$ 4a + 3b = 6 ……(ii)

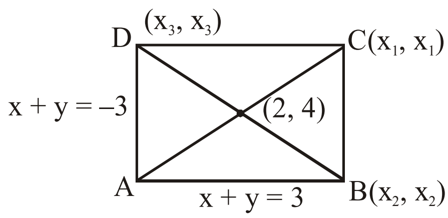
From (i) and (ii)

$a = \frac{-3}{4}$, b = 3

$\therefore$ IInd quadrant.

Option (2)

7. Ans. (4)



Solving x + y = 3

and x – y = –3 $\Big\rangle$ A(0, 3)

$\frac{x_1 + 0}{2} = 2$; $x_i = 4$ similarly $y_1 = 5$

C $\Rightarrow$ (4, 5)

Now equation of BC is $x - y = -1$

and equation of CD is $x + y = 9$

Solving $x + y = 9$ and $x - y = -3$

Point D is (3, 6)

Option (4)

8. **Ans. (4)**

co-ordinates of point D are (4,7)

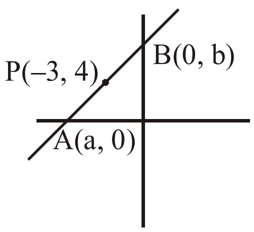
$\Rightarrow$ line AD is $5x - 3y + 1 = 0$

9. **Ans. (4)**

$$\frac{17-\beta}{-8} \times \frac{2}{3} = -1$$

$\beta = 5$

10. **Ans (4)**



Let the line be $\frac{x}{a} + \frac{y}{b} = 1$

$$(-3, 4) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

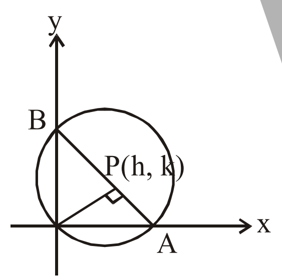
$a = -6, b = 8$

equation of line is $4x - 3y + 24 = 0$

11. **Ans. (3)**

Slope of AB $= \frac{-h}{k}$

Equation of AB is $hx + ky = h^2 + k^2$



$$A\left(\frac{h^2+k^2}{h}, 0\right), \quad B\left(0, \frac{h^2+k^2}{k}\right)$$

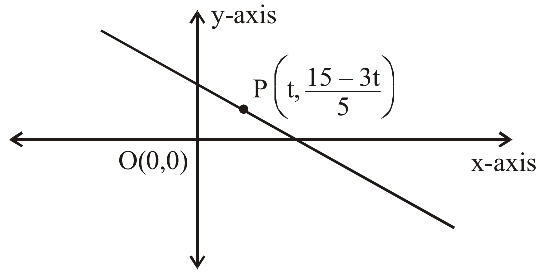
$AB = 2R$

$\Rightarrow (h^2 + k^2)^3 = 4R^2h^2k^2$

$\Rightarrow (x^2 + y^2)^3 = 4R^2x^2y^2$

12. **Official Ans. by NTA (1)**

Sol.



Now, $\left|\frac{15-3t}{5}\right| = |t|$

$$\Rightarrow \frac{15-3t}{5} = t \text{ or } \frac{15-3t}{5} = -t$$

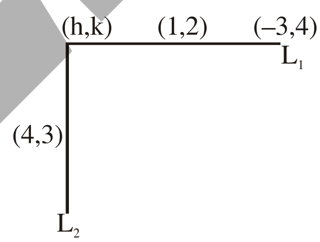
$$\therefore \quad t = \frac{15}{8} \text{ or } t = \frac{-15}{2}$$

So, $P\left(\frac{15}{8}, \frac{15}{8}\right) \in I^{st}$ quadrant

or $P\left(\frac{-15}{2}, \frac{15}{2}\right) \in II^{nd}$ quadrant

13. **Official Ans. by NTA (3)**

Sol.



equation of L_1 is

$$y = -\frac{1}{2}x + \frac{5}{2} \qquad ...(1)$$

equation of L_2 is

$y = 2x - 5$ ...(2)

by (1) and (2)

$x = 3$

$y = 1 \Rightarrow h = 3, k = 1$

$$\frac{k}{h} = \frac{1}{3}$$

14. **Official Ans. by NTA (3)**

Sol. $x = 2 + r\cos\theta$

$y = 3 + r\sin\theta$

$\Rightarrow 2 + r\cos\theta + 3 + r\sin\theta = 7$

$\Rightarrow r(\cos\theta + \sin\theta) = 2$

$\Rightarrow \sin\theta + \cos\theta = \frac{2}{r} = \frac{2}{\pm 4} = \pm\frac{1}{2}$

$\Rightarrow 1 + \sin 2\theta = \frac{1}{4}$

$\Rightarrow \sin 2\theta = -\frac{3}{4}$

$\Rightarrow \frac{2m}{1+m^2} = -\frac{3}{4}$

$\Rightarrow 3m^2 + 8m + 3 = 0$

$\Rightarrow m = \frac{-4 \pm \sqrt{7}}{1-7}$

$\frac{1-\sqrt{7}}{1+\sqrt{7}} = \frac{\left(1-\sqrt{7}\right)^2}{1-7} = \frac{8-2\sqrt{7}}{-6} = \frac{-4+\sqrt{7}}{3}$

15. Official Ans. by NTA (4)

Sol. $\left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$

$2 = -(a^2)(a-1)$

$a^3 - a^2 + 2 = 0$

$(a+1)(a^2 - 2a + 2) = 0$

$\therefore\ a = -1$

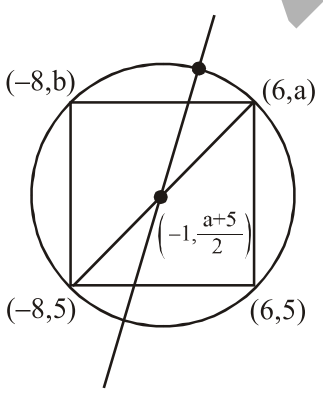
$\left.\begin{array}{l} L_1 : x - 2y + 1 = 0 \\ L_2 : 2x + y - 1 = 0 \end{array}\right\}$

$0(0, 0) \quad P\left(\frac{1}{5}, \frac{3}{5}\right)$

$OP = \sqrt{\frac{1}{25} + \frac{9}{25}} = \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$

16. Official Ans. by NTA (2)

Sol.



$\frac{3(a+5)}{2} = -1 + 7$

$a + 5 = \frac{2(6)}{3}$

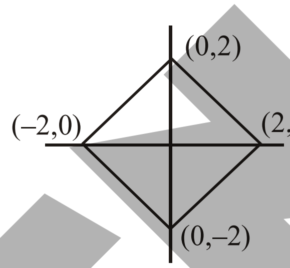
$a = -1$

sides = 6 and 14

$\Rightarrow A = 84$

17. Official Ans. by NTA (1)

Sol. $|x-y| \le 2$ and $|x+y| \le 2$

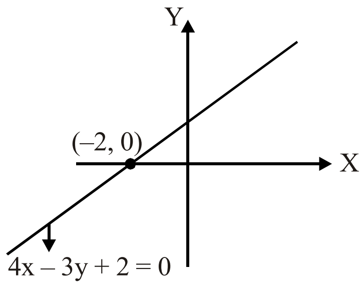


Square whose side is $2\sqrt{2}$

18. Official Ans. by NTA (1)

Sol. Required line is $4x - 3y + \lambda = 0$

$\left|\frac{\lambda}{5}\right| = \frac{3}{5}$



$\Rightarrow \lambda = \pm 3.$

So, required equation of line is

$4x - 3y + 3 = 0$ and $4x - 3y - 3 = 0$

(1) $4\left(-\frac{1}{4}\right) - 3\left(\frac{2}{3}\right) + 3 = 0$

19. Official Ans. by NTA (2)

Sol. $2y = 2\sin x \sin(x+2) - 2\sin^2(x+1)$

$2y = \cos 2 - \cos(2x+2) - (1 - \cos(2x+2))$

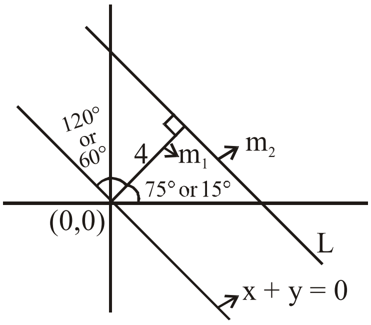
$= \cos 2 - 1$

$2y = -2\sin^2\frac{1}{2}$

$y = -\sin^2\frac{1}{2} \le 0$

20. Official Ans. by NTA (1) or (2)

Sol.



$m_1 = \tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

or $m = \tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$m_2 = \frac{-1}{m_1} = \frac{-(\sqrt{3}-1)}{\sqrt{3}+1}$

or $m_2 = \frac{-1}{m_1} = \frac{-(\sqrt{3}+1)}{\sqrt{3}-1}$

$\Rightarrow y = m_2x + C$

$\Rightarrow y = \frac{-(\sqrt{3}-1)x}{\sqrt{3}+1} + C \Rightarrow L$

or $y = \frac{-(\sqrt{3}+1)x}{\sqrt{3}-1} + C \Rightarrow L$

Distance from origin = 4

$\therefore \left|\frac{C}{\sqrt{1+\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)^2}}}\right| = 4$ or $\left|\frac{C}{\sqrt{1+\frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)^2}}}\right| = 4$

$\Rightarrow C = \frac{8\sqrt{2}}{(\sqrt{3}+1)}$ or $C = \frac{8\sqrt{2}}{(\sqrt{3}-1)}$

$\Rightarrow (\sqrt{3}-1)y + (\sqrt{3}+1)x = 8\sqrt{2}$

or $(\sqrt{3}-1)x + (\sqrt{3}+1)y = 8\sqrt{2}$

21. Official Ans. by NTA (2)

Sol. Let $B(\alpha,\beta)$ and $C(\gamma,\delta)$

$\frac{\alpha+1}{2} = -1 \Rightarrow \alpha = -3$

$\frac{\beta+2}{2} = 1 \Rightarrow \beta = 0$

$\Rightarrow B(-3,0)$

Now $\frac{\gamma+1}{2} = 2 \Rightarrow \gamma = 3$

$\frac{\delta+2}{2} = 3 \Rightarrow \delta = 4$

$\Rightarrow C(3,4)$

$\Rightarrow$ centroid of triangle is $G\left(\frac{1}{3}, 2\right)$

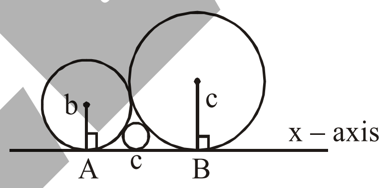
CIRCLE

1. Ans. (1)

AB = AC + CB

$\sqrt{(b+c)^2-(b-c)^2}$

$= \sqrt{(b+a)^2-(b-a)^2} + \sqrt{(a+c)^2-(a-c)^2}$



$\sqrt{bc} = \sqrt{ab} + \sqrt{ac}$

$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}}$

2. Ans. (2)

$x^2 + y^2 - 16x - 20y + 164 = r^2$

$A(8,10)$, $R_1 = r$

$(x-4)^2 + (y-7)^2 = 36$

$B(4,7)$, $R_2 = 6$

$|R_1 - R_2| < AB < R_1 + R_2$

$\Rightarrow 1 < r < 11$

3. Ans. (4)

$x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at (1, –1)

$x - y + 2(x+1) - 3(y-1) - 12 = 0$

$3x - 4y - 7 = 0$

$\therefore$ Equation of circle is

$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$

It passes through (4, 0) :

$(16 + 16 - 12) + \lambda(12 - 7) = 0$

$\Rightarrow 20 + \lambda(5) = 0$

$\Rightarrow \lambda = -4$

$\therefore (x^2+y^2+4x-6y-12)-4(3x-4y-7)=0$

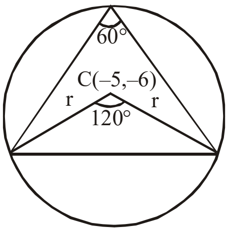
or $x^2 + y^2 - 8x + 10y + 16 = 0$

Radius = $\sqrt{16+25-16}=5$

4. Ans. (2)

$$3\left(\frac{1}{2}r^2.\sin 120°\right)=27\sqrt{3}$$

$$\frac{r^2}{2}\frac{\sqrt{3}}{2}=\frac{27\sqrt{3}}{3}$$



$r^2 = \frac{108}{3}=36$

Radius = $\sqrt{25+36-C}=\sqrt{36}$

$\boxed{C=25}$

$\therefore$ Option (2)

5. Ans. (4)

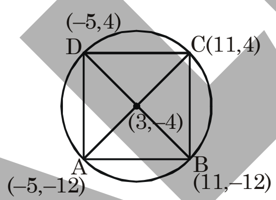
$R=\sqrt{9+16+103}=8\sqrt{2}$

OA = 13

$OB=\sqrt{265}$

$OC=\sqrt{137}$

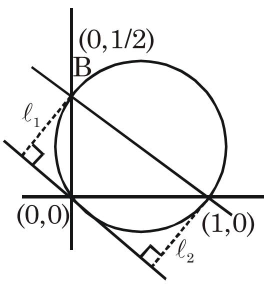
$OD=\sqrt{41}$



6. Ans. (2)

Equation of circle

$$(x-1)(x-0)+(y-0)\left(y-\frac{1}{2}\right)=0$$



$\Rightarrow x^2 + y^2 - x - \frac{y}{2} = 0$

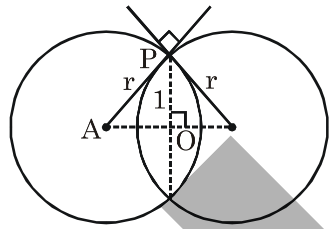
Equation of tangent of origin is $2x + y = 0$

$$\ell_1+\ell_2=\frac{2}{\sqrt{5}}+\frac{1}{2\sqrt{5}}$$

$$=\frac{4+1}{2\sqrt{5}}=\frac{\sqrt{5}}{2}$$

7. Ans. (4)

In ΔAPO



$$\left(\frac{\sqrt{2}\,r}{2}\right)^2+1^2=r^2$$

$\Rightarrow \boxed{r=\sqrt{2}}$

So distance between centres = $\sqrt{2}\,r = 2$

8. Ans. (2)

Let equation of circle is

$x^2 + y^2 + 2fx + 2fy + e = 0$, it passes through (0, 2b)

$\Rightarrow 0 + 4b^2 + 2g \times 0 + 4f + c = 0$

$\Rightarrow 4b^2 + 4f + c = 0$ ...(i)

$2\sqrt{g^2-c}=4a$ ...(ii)

$g^2 - c = 4a^2 \Rightarrow c=\left(g^2-4a^2\right)$

Putting in equation (1)

$\Rightarrow 4b^2 + 4f + g^2 - 4a^2 = 0$

$\Rightarrow x^2 + 4y + 4(b^2 - a^2) = 0$, it represent a parabola.

9. Ans. (1)

Centre of circles are opposite side of line

$(3 + 4 - \lambda)(27 + 4 - \lambda) < 0$

$(\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7, 31)$

distance from S_1

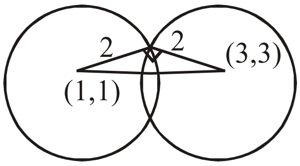
$$\left|\frac{3+4-\lambda}{5}\right|\geq 1 \Rightarrow \lambda\in(-\infty, 2]\cup[(12,\infty]$$

distance from S_2

$$\left|\frac{27+4-\lambda}{5}\right|\geq 2 \Rightarrow \lambda\in(-\infty, 21]\cup[41, \infty)$$

so $\lambda\in[12, 21]$

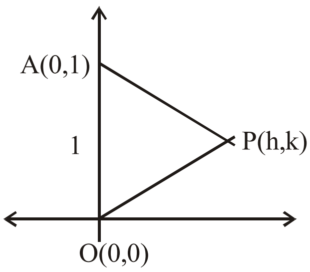
10. Ans. (4)



Area = $2 \times \frac{1}{2}.4 = 2$

11. Official Ans. by NTA (2)

Sol.



AP + OP + AO = 4

$\sqrt{h^2+(k-1)^2}+\sqrt{h^2+k^2}+1=4$

$\sqrt{h^2+(k-1)^2}+\sqrt{h^2+k^2}=3$

$h^2+(k-1)^2=9+h^2+k^2-6\sqrt{h^2+k^2}$

$-2k-8=-6\sqrt{h^2+k^2}$

$k+4=3\sqrt{h^2+k^2}$

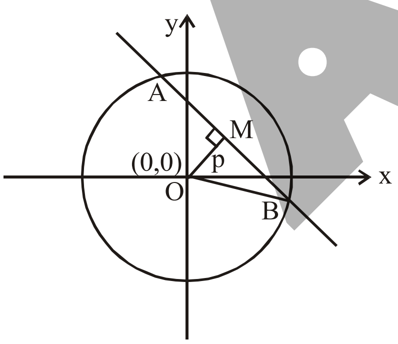
$k^2 + 16 + 8k = 9(h^2 + k^2)$

$9h^2 + 8k^2 - 8k - 16 = 0$

Locus of P is $9x^2 + 8y^2 - 8y - 16 = 0$

12. Official Ans. by NTA (4)

Sol.



$p=\frac{n}{\sqrt{2}}$, but $\frac{n}{\sqrt{2}}<4 \Rightarrow n = 1, 2, 3, 4, 5.$

Length of chord AB $=2\sqrt{16-\frac{n^2}{2}}$

$=\sqrt{64-2n^2}=\ell(\text{say})$

For $n = 1, \ell^2 = 62$

$n = 2, \ell^2 = 56$

$n = 3, \ell^2 = 46$

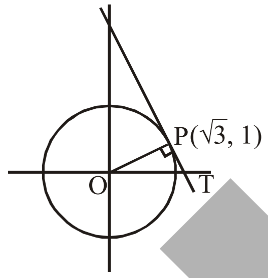
$n = 4, \ell^2 = 32$

$n = 5, \ell^2 = 14$

$\therefore$ Required sum $= 62 + 56 + 46 + 32 + 14 = 210$

13. Official Ans. by NTA (4)

Sol.



Given $x^2 + y^2 = 4$

equation of tangent

$\Rightarrow \sqrt{3}x+y=4$ ...(1)

Equation of normal

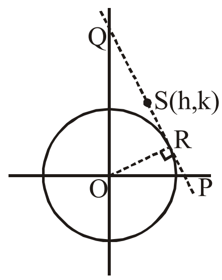
$x-\sqrt{3}y=0$ ...(2)

Coordinate of $T\left(\frac{4}{\sqrt{3}},0\right)$

$\therefore$ Area of triangle = $=\frac{2}{\sqrt{3}}$

14. Official Ans. by NTA (3)

Sol.



Let the mid point be S(h,k)

$\therefore$ P(2h,0) and Q(0,2k)

equation of PQ : $\frac{x}{2h}+\frac{y}{2k}=1$

$\because$ PQ is tangent to circle at R(say)

$\therefore$ OR = 1 $\Rightarrow \left|\frac{-1}{\sqrt{\left(\frac{1}{2h}\right)^2+\left(\frac{1}{2k}\right)^2}}\right|=1$

$\Rightarrow \dfrac{1}{4h^2}+\dfrac{1}{4k^2}=1$

$\Rightarrow x^2 + y^2 - 4x^2y^2 = 0$

Aliter :

tangent to circle

$x\cos\theta + y\sin\theta = 1$

P : $(\sec\theta, 0)$

Q : $(0, \operatorname{cosec}\theta)$

$2h = \sec\theta \Rightarrow \cos\theta = \dfrac{1}{2h}$ & $\sin\theta = \dfrac{1}{2k}$

$\dfrac{1}{(2x)^2}+\dfrac{1}{(2y)^2}=1$

15. Official Ans. by NTA (2)

Sol. Circle touches internally

$C_1(0, 0); r_1 = 2$

$C_2 : (-3, -4); r_2 = 7$

$C_1C_2 = |r_1 - r_2|$

$S_1 - S_2 = 0 \Rightarrow$ eqn. of common tangent

$6x + 8y - 20 = 0$

$3x + 4y = 10$

$(6, -2)$ satisfy it

16. Official Ans. by NTA (3)

Sol. Equation of common chord

$4kx + \dfrac{1}{2}y + k + \dfrac{1}{2} = 0$ ....(1)

and given line is $4x + 5y - k = 0$ .....(2)

On comparing (1) & (2), we get

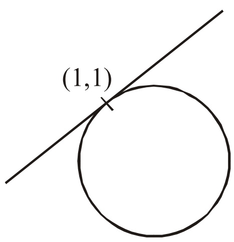
$k = \dfrac{1}{10} = \dfrac{k+\frac{1}{2}}{-k}$

$\Rightarrow$ No real value of k exist

17. Official Ans. by NTA (1)

ALLEN Ans. (3)

Sol.



Equation of circle can be written as $(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

It passes through $(1, -3)$

$16 + \lambda(4) = 0 \Rightarrow \lambda = -4$

So $(x-1)^2 + (y-1)^2 - 4(x-y) = 0$

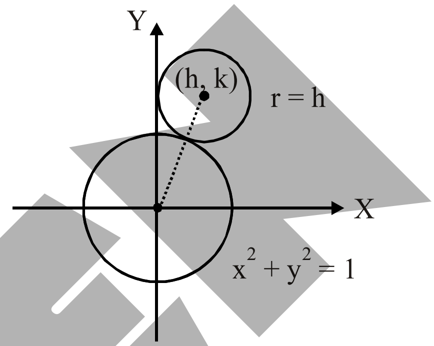
$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

$\Rightarrow r = 2\sqrt{2}$

(correct key is 3)

18. Official Ans. by NTA (4)

Sol.



$\sqrt{h^2+k^2} = |h| + 1$

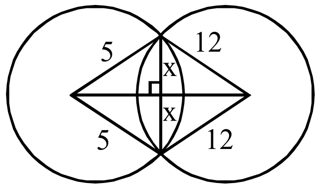
$\Rightarrow x^2 + y^2 = x^2 + 1 + 2x$

$\Rightarrow y^2 = 1 + 2x$

$\Rightarrow y = \sqrt{1+2x}$; $x \ge 0$.

19. Official Ans. by NTA (2)

Sol.



Let length of common chord = 2x

$\sqrt{25-x^2}+\sqrt{144-x^2}=13$

after solving

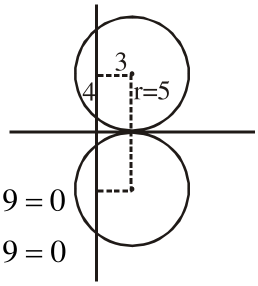
$x = \dfrac{12\times5}{13}$

$2x = \dfrac{120}{13}$

20. Official Ans. by NTA (1)

Sol. Equaiton of circles are

$$\begin{cases}(x-3)^2+(y-5)^2=25\\(x-3)^2+(y+5)^2=25\end{cases}$$



$$\Rightarrow\begin{cases}x^2+y^2-6x-10y+9=0\\x^2+y^2-6x+10y+9=0\end{cases}$$

PERMUTATION & COMBINATION

1. Ans. (2)

Required number of ways

= Total number of ways – When A and B are always included.

$= {}^5C_2 . {}^7C_3 - {}^5C_1 {}^5C_2 = 300$

2. Ans. (2)

a_1	a_2	a_3

Number of numbers = $5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Number of numbers = 2×5^3

Required number = $5^3 + 2 \times 5^3 - 1$

= 374

3. Ans. (4)

$$\sum_{r=2}^{13}(7r+2) = 7.\frac{2+13}{2}\times 6+2\times 12$$

= 7 × 90 + 24 = 654

$$\sum_{r=1}^{13}(7r+5)=7\left(\frac{1+13}{2}\right)\times 13+5\times 13=702$$

Total = 654 + 702 = 1356

4. Ans. (1)

S = {1,2,3------100}

= Total non empty subsets-subsets with product of element is odd

$= 2^{100}-1-1[(2^{50}-1)]$

$= 2^{100} - 2^{50}$

$= 2^{50}(2^{50}-1)$

5. Ans (3)

Let m-men, 2-women

${}^mC_2 \times 2 = {}^mC_1\, {}^2C_1 . 2 + 84$

$m^2 - 5m - 84 = 0 \Rightarrow (m-12)(m+7) = 0$

m = 12

6. Ans. (1)

$2.{}^nC_5 = {}^nC_4 + {}^nC_6$

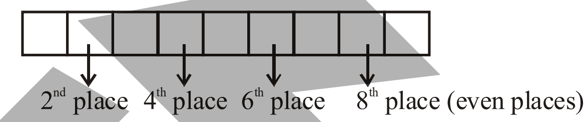
$$2.\frac{\lfloor n}{\lfloor 5\lfloor n-5}=\frac{\lfloor n}{\lfloor 4\lfloor n-4}+\frac{\lfloor n}{\lfloor 6\lfloor n-6}$$

$$\frac{2}{5}.\frac{1}{n-5}=\frac{1}{(n-4)(n-5)}+\frac{1}{30}$$

n = 14 satisfying equation.

7. Official Ans. by NTA (4)

Sol.



Number of such numbers

$$={}^4C_3\times\frac{3!}{2!}\times\frac{6!}{2!4!}=180$$

8. Official Ans. by NTA (4)

Sol. (1) The number of four-digit numbers Starting with 5 is equal to $6^3 = 216$

(2) Starting with 44 and 55 is equal to $36 \times 2 = 72$

(3) Starting with 433,434 and 435 is equal to $6 \times 3 = 18$

(3) Remaining numbers are 4322,4323,4324,4325is equal to 4

so total numbers are

216 + 72 + 18 + 4 = 310

9. Official Ans. by NTA (1)

Sol. Since there are 8 males and 5 females. Out of these 13, if we select 11 persons, then there will be at least 6 males and atleast 3 females in the selection.

$$m=n=\binom{13}{11}=\binom{13}{2}=\frac{13\times 12}{2}=78$$

10. Official Ans. by NTA (1)

Sol. $\frac{n(n+1)}{2}+99=(n-2)^2$

$n^2 + n + 198 = 2(n^2 + 4 - 4n)$

$n^2 - 9n - 190 = 0$

$n^2 - 19n + 10 - 190 = 0$

$n(n - 19) + 10(n - 19) = 0$

$n = 19$

11. Official Ans. by NTA (2)

Sol. Sum of given digits 0, 1, 2, 5, 7, 9 is 24.

Let the six digit number be abcdef and to be divisible by 11

so $|(a + c + e) - (b + d + f)|$ is multiple of 11.

Hence only possibility is $a + c + e = 12 = b + d + f$

Case-I $\{a, c, e\} = \{9, 2, 1\}$ & $\{b, d, f\} = \{7, 5, 0\}$

So, Number of numbers $= 3! \times 3! = 36$

Case-II $\{a,c,e\} = \{7,5,0\}$ and $\{b,d,f\} = \{9,2,1\}$

So, Number of numbers $2 \times 2! \times 3! = 24$

Total = 60

12. Official Ans. by NTA (3)

Sol. Total cases = number of diagonals

$= {}^{20}C_2 - 20 = 170$

13. Official Ans. by NTA (1)

Sol. ${}^5C_1 \cdot {}^nC_2 + {}^5C_2 \cdot {}^nC_1 = 1750$

$n^2 + 3n = 700$

$\therefore\ n = 25$

14. Official Ans. by NTA (1)

Sol.

10 Identical	21Distinct	10 Object
0	10	${}^{21}C_{10} \times 1$
1	9	${}^{21}C_9 \times 1$
⋮	⋮	⋮
10	0	${}^{21}C_0 \times 1$

${}^{21}C_0 + \dots + {}^{21}C_{10} + {}^{21}C_1 + \dots + {}^{21}C_0 = 2^{21}$

$({}^{21}C_0 + \dots + {}^{21}C_{10}) = 2^{20}$

BINOMIAL THEOREM

1. Ans. (4)

$$\frac{2^{403}}{15} = \frac{2^3.(2^4)^{100}}{15} = \frac{8}{15}(15+1)^{100}$$

$$= \frac{8}{15}(15\lambda+1) = 8\lambda + \frac{8}{15}$$

$\because$ 8λ is integer

$\Rightarrow$ fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15} \Rightarrow k = 8$

2. Ans. (2)

$(1 - t^6)^3 (1 - t)^{-3}$

$(1 - t^{18} - 3t^6 + 3t^{12}) (1 - t)^{-3}$

$\Rightarrow$ cofficient of t^4 in $(1 - t)^{-3}$ is

${}^{3+4-1}C_4 = {}^6C_2 = 15$

3. Ans. (3)

$$\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{21}C_i}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \frac{1}{(21)^3}\left[\frac{20(21)}{2}\right]^2 = \frac{k}{21}$$

$\Rightarrow 100 = k$

4. Ans. (4)

$(1 + x^{\log_2 x})^5$

$T_3 = {}^5C_2.(x^{\log_2 x})^2 = 2560$

$\Rightarrow 10.x^{2\log_2 x} = 2560$

$\Rightarrow x^{2\log_2 x} = 256$

$\Rightarrow 2(\log_2 x)^2 = \log_2 256$

$\Rightarrow 2(\log_2 x)^2 = 8$

$\Rightarrow (\log_2 x)^2 = 4 \quad \Rightarrow \log_2 x = 2$ or -2

$x = 4$ or $\frac{1}{4}$

5. Ans. (2)

$$x^2\left({}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r\right)$$

$$x^2\left[{}^{10}C_r (x)^{\frac{10-r}{2}} (\lambda)^r (x)^{-2r}\right]$$

$$x^2\left[{}^{10}C_r\ \lambda^r\ x^{\frac{10-5r}{2}}\right]$$

$\therefore\ r = 2$

Hence, ${}^{10}C_2\ \lambda^2 = 720$

$\lambda^2 = 16$

$\lambda = \pm 4$

Option (2)

6. Ans. (3)

$$\sum_{r=0}^{25} {}^{50}C_r \cdot {}^{50-r}C_{25-r}$$

$$= \sum_{r=0}^{25} \frac{50!}{r!\,(50-r)!} \times \frac{(50-r)!}{(25)!\,(25-r)!}$$

$$= \sum_{r=0}^{25} \frac{50!}{25!\,25!} \times \frac{25!}{(25-r)!\,(r!)}$$

$$= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = \left(2^{25}\right) {}^{50}C_{25}$$

$\therefore$ $K = 2^{25}$

Option (3)

7. Ans. (3)

$$T_5 = {}^{8}C_4 \frac{x^{12}}{81} \times \frac{81}{x^4} = 5670$$

$\Rightarrow 70x^8 = 5670$

$\Rightarrow x = \pm\sqrt{3}$

8. Ans. (1)

Given sum = coefficient of x^r in the expansion of $(1 + x)^{20}(1 + x)^{20}$,

which is equal to ${}^{40}C_r$

It is maximum when r = 20

9. Ans. (4)

$(10 + x)^{50} + (10 - x)^{50}$

$\Rightarrow a_2 = 2.{}^{50}C_2 10^{48},\ a_0 = 2.10^{50}$

$$\frac{a_2}{a_0} = \frac{{}^{50}C_2}{10^2} = 12.25$$

10. Ans. (1)

${}^{101}C_1 + {}^{101}C_2S_1 + ... + {}^{101}C_{101}S_{100} = \alpha T_{100}$

${}^{101}C_1 + {}^{101}C_2(1 + q) + {}^{101}C_3(1 + q + q^2) + ...$

$+ {}^{101}C_{101}(1 + q + ... + q^{100})$

$$= 2\alpha \frac{\left(1 - \left(\frac{1+q}{2}\right)^{101}\right)}{(1-q)}$$

$\Rightarrow {}^{101}C_1(1 - q) + {}^{101}C_2(1 - q^2) + ... + {}^{101}C_{101}(1 - q^{101})$

$$= 2\alpha\left(1 - \left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow (2^{101} - 1) - ((1 + q)^{101} - 1)$

$$= 2\alpha\left(1 - \left(\frac{1+q}{2}\right)^{101}\right)$$

$$\Rightarrow 2^{101}\left(1 - \left(\frac{1+q}{2}\right)^{101}\right) = 2\alpha\left(1 - \left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow \alpha = 2^{100}$

11. Ans. (4)

$$\frac{T_5}{T_5^1} = \frac{{}^{10}C_4 (2^{1/3})^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2(3^{1/3})}\right)^{10-4}(2^{1/3})^4} = 4.(36)^{1/3}$$

12. Ans. (4)

General term $T_{r+1} = {}^{60}C_r\ 7^{\frac{60-r}{5}}\ 3^{\frac{r}{10}}$

$\therefore$ for rational term, r = 0, 10, 20, 30, 40, 50, 60

$\Rightarrow$ no of rational terms = 7

$\therefore$ number of irrational terms = 54

Also, $-27\times {}^{15}C_3 + 9a \times {}^{15}C_2 - 3b\times {}^{15}C_1 = 0$

$\Rightarrow 9 \times {}^{15}C_2\ a - 45\ b - 27 \times {}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ...(ii)

(i) + (ii)

$-24\ a + 672 = 0$

$\Rightarrow a = 28$

So, b = 315

13. Official Ans. by NTA (2)

Sol. $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + \ldots + 62.{}^{20}C_{20}$

$$= \sum_{r=0}^{20}(3r+2)\,{}^{20}C_r$$

$$= 3\sum_{r=0}^{20} r.\,{}^{20}C_r + 2\sum_{r=0}^{20} {}^{20}C_r$$

$$= 3\sum_{r=0}^{20} r\left(\frac{20}{r}\right){}^{19}C_{r-1} + 2.2^{20}$$

$= 60.2^{19} + 2.2^{20} = 2^{25}$

14. Official Ans. by NTA (4)

Sol. $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$

$= 2[{}^6C_0x^6 + {}^6C_2x^4(x^3 - 1) + {}^6C_4x^2(x^3 - 1)^2 + {}^6C_6(x^3 - 1)^3]$

$= 2[{}^6C_0x^6 + {}^6C_2x^7 - {}^6C_2x^4 + {}^6C_4x^8 + {}^6C_4x^2 - 2{}^6C_4x^5 + (x^9 - 1 - 3x^6 + 3x^3)]$

$\Rightarrow$ Sum of coefficient of even powers of x $= 2[1 - 15 + 15 + 15 - 1 - 3] = 24$

15. Official Ans. by NTA (4)

Sol. $200 = {}^6C_3\left(x^{\frac{1}{x+\log_{10}x}}\right)^{\frac{3}{2}} \times x^{\frac{1}{4}}$

$\Rightarrow 10 = x^{\frac{3}{2(1+\log_{10}x)}+\frac{1}{4}}$

$\Rightarrow 1 = \left(\frac{3}{2(1+t)}+\frac{1}{4}\right)t$

where $t = \log_{10}x$

$\Rightarrow t^2 + 3t - 4 = 0$

$\Rightarrow t = 1, -4$

$\Rightarrow x = 10, 10^{-4}$

$\Rightarrow x = 10$ (As $x > 1$)

16. Official Ans. by NTA (2)

Sol. $T_4 = T_{3+1} = \binom{6}{3}\left(\frac{2}{x}\right)^3 .\left(x^{\log_8 x}\right)^3$

$20 \times 8^7 = \frac{160}{x^3}.x^{3\log_8 x}$

$8^6 = x^{\log_2 x} - 3$

$2^{18} = x^{\log_2 x - 3}$

$\Rightarrow 18 = (\log_2 x - 3)(\log_2 x)$

Let $\log_2 x = t$

$\Rightarrow t^2 - 3t - 18 = 0$

$\Rightarrow (t-6)(t+3) = 0$

$\Rightarrow t = 6, -3$

$\log_2 x = 6 \Rightarrow x = 2^6 = 8^2$

$\log_2 x = -3 \Rightarrow x = 2^{-3} = 8^{-1}$

17. Official Ans. by NTA (4)

Sol. $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15}$

$$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$$

$\frac{r}{n-r+1} = \frac{2}{15}$

$15r = 2n - 2r + 2$

$\boxed{17r = 2n + 2}$

$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$$

$\frac{r+1}{n-r} = \frac{3}{14}$

$14r + 14 = 3n - 3r$

$3n - 17r = 14$

$\underline{2n - 17r = -2}$

$n = 16$

$17r = 34$, $r = 2$

${}^{16}C_1$, ${}^{16}C_2$, ${}^{16}C_3$

$\frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$

$\frac{680+16}{3} = \frac{696}{3} = 232$

18. Official Ans. by NTA (1)

Sol. Coefiicient of $x^2 = {}^{15}C_2 \times 9 - 3a\left({}^{15}C_1\right) + b = 0$

$\Rightarrow -45a + b + {}^{15}C_2 \times 9 = 0$ ....(i)

Also, $-27\times{}^{15}C_3 + 9a \times {}^{15}C_2 - 3b\times{}^{15}C_1 = 0$

$\Rightarrow 9 \times {}^{15}C_2\, a - 45\, b - 27 \times {}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ...(ii)

(i) + (ii)

$-24a + 672 = 0$

$\Rightarrow a = 28$

So, $b = 315$

19. Official Ans. by NTA (2)

Sol. $T_r = \sum_{r=0}^{n} {}^nC_r x^{2n-2r}.x^{-3r}$

$2n - 5r = 1 \Rightarrow 2n = 5r + 1$

for r = 15. n = 38

smallest value of n is 38.

20. Official Ans. by NTA (2)

Sol. $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$

$(1 - x^2)(1 - x^3)^9$

${}^9C_6 = 84$

21. Official Ans. by NTA (1)

Sol. $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$

Diff. w.r.t. x

$\Rightarrow n(1+x)^{n-1} = {}^nC_1 + {}^nC_2(2x) + ...+ {}^nC_n\, n(x)^{n-1}$

Multiply by x both side

$\Rightarrow nx(1+x)^{n-1} = {}^nC_1\, x + {}^nC_2(2x^2) + ...+ {}^nC_n(n\, x^n)$

Diff w.r.t. x

$\Rightarrow n\left[(1+x)^{n-1} + (n-1)x(1+x)^{n-2}\right]$

$= {}^nC_1 + {}^nC_2\, 2^2x + ... {}^nC_n\,(n^2)x^{n-1}$

Put x = 1 and n = 20

$\Rightarrow {}^{20}C_1 + 2^2\, {}^{20}C_2 + 3^2\, {}^{20}C_3 + ... + (20)^2\, {}^{20}C_{20}$

$= 20 \times 2^{18}[2 + 19] = 420(2^{18}) = A(2^{\beta})$

22. Official Ans. by NTA (4)

Sol. $\frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{81}\cdot x^8\left(2x^2 - \frac{3}{x^2}\right)^6$

its general term

$\frac{1}{60}\,{}^6C_r 2^{6-r}(-3)^r x^{12-r} - \frac{1}{81}\,{}^6C_r 2^{6-r}(-3)^r 12^{20-4r}$

for term independent of x, r for Ist expression is 3 and r for second expression is 5

$\therefore$ term independent of x = – 36

SET

1. Ans. (4)

Let n(A) = number of students opted Mathematics = 70,

n(B) = number of students opted Physics = 46,

n(C)=number of students opted Chemistry= 28,

$n(A \cap B) = 23$,

$n(B \cap C) = 9$,

$n(A \cap C) = 14$,

$n(A \cap B \cap C) = 4$,

Now $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$

$- n(A \cap C) + n(A \cap B \cap C)$

$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$

So number of students not opted for any course

$= \text{Total} - n(A \cup B \cup C)$

$= 140 - 102 = 38$

2. Official Ans. by NTA (3)

Sol. Let population = 100

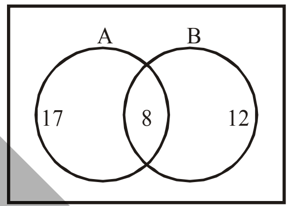
n(A) = 25

n(B) = 20

$n(A \cap B) = 8$

$n(A \cap \bar{B}) = 17$

$n(\bar{A} \cap B) = 12$

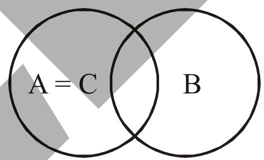


$\frac{30}{100}\times 17 + \frac{40}{100}\times 12 + \frac{50}{100}\times 8$

$5.1 + 4.8 + 4 = 13.9$

3. Official Ans. by NTA (1)

Sol.



for A = C, $A - C = \phi$

$\Rightarrow \phi \subseteq B$

But $A \not\subseteq B$

$\Rightarrow$ option 1 is **NOT** true

Let $x \in (C\, x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow (x \in C \text{ or } x \in A)$ and $(x \in C \text{ or } x \in B)$

$\Rightarrow x \in C \text{ or } x \in (A \cap B)$

$\Rightarrow x \in C \text{ or } x \in C \quad (\text{as } A \cup B \subseteq C)$

$\Rightarrow x \in C$

$\Rightarrow (C \cup A) \cap (C \cup B) \subseteq C \qquad (1)$

Now $x \in C \Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow C \subseteq (C \cup A) \cap (C \cup B) \qquad (2)$

$\Rightarrow$ from (1) and (2)

$C = (C \cup A) \cap (C \cup B)$

$\Rightarrow$ option 2 is true

Let $x \in A$ and $x \notin B$

$\Rightarrow x \in (A - B)$

$\Rightarrow x \in C$ (as $A - B \subseteq C$)

Let $x \in A$ and $x \in B$

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in C$ (as $A \cap B \subseteq C$)

Hence $x \in A \Rightarrow x \in C$

$\Rightarrow A \subseteq C$

$\Rightarrow$ Option 3 is true

as $C \supseteq (A \cap B)$

$\Rightarrow B \cap C \supseteq (A \cap B)$

as $A \cap B \neq \phi$

$\Rightarrow B \cap C \neq \phi$

$\Rightarrow$ Option 4 is true.

RELATION

1. Ans (3)

$A = \{x \in z : 2^{(x+2)(x^2-5x+6)} = 1\}$

$2^{(x+2)(x^2-5x+6)} = 2^0 \Rightarrow x = -2, 2, 3$

$A = \{-2, 2, 3\}$

$B = \{x \in Z : -3 < 2x - 1 < 9\}$

$B = \{0, 1, 2, 3, 4\}$

$A \times B$ has is 15 elements so number of subsets of $A \times B$ is 2^{15}.

7. Official Ans. by NTA (1)

Sol. $f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x| < 1$

$$f\left(\frac{2x}{1+x^2}\right) = \ell n\left(\frac{1-\frac{2x}{1+2x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$= \ell n\left(\frac{(x-1)^2}{(x+1)^2}\right) = 2\ell n\left|\frac{1-x}{1+x}\right| = 2f(x)$$

8. Official Ans. by NTA (1)

Sol. $f(x) = a^x, a > 0$

$$f(x) = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x) = \frac{a^x + a^{-x}}{2}$$

$$f_2(x) = \frac{a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x+y) + f_1(x-y)$$

$$= \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{-x+y}}{2}$$

$$= \frac{(a^x + a^{-x})}{2}(a^y + a^{-y})$$

$$= f_1(x) \times 2f_1(y)$$

$= 2f_1(x)\, f_1(y)$

9. Official Ans. by NTA (2)

Sol. From the given functional equation :

$f(x) = 2^x \quad \forall\, x \in N$

$2^{a+1} + 2^{a+2} + ... + 2^{a+10} = 16(2^{10} - 1)$

$2^a (2 + 2^2 + ... + 2^{10}) = 16(2^{10} - 1)$

$$2^a . \frac{2.(2^{10} - 1)}{1} = 16(2^{10} - 1)$$

$2^{a+1} = 16 = 2^4$

$a = 3$

10. Official Ans. by NTA (1)

Sol. $y = \frac{x^2}{1-x^2}$

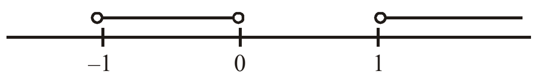
Range of y : R –[–1,0)

for surjective funciton, A must be same as above range.

11. Official Ans. by NTA (3)

Sol. $4 - x^2 \neq 0$; $x^3 - x > 0$

$x = \pm 2 \qquad x(x - 1)(x + 1) > 0$



$\therefore D_f \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

12. Official Ans. by NTA (3)

Sol. $g(S) = [-2, 2]$

So, $f(g(S)) = [0, 4] = S$

And $f(S) = [0, 16] \Rightarrow f(g(S) \neq f(S)$

Also, $g(f(S)) = [-4, 4] \neq g(S)$

So, $g(f(S) \neq S$

13. Official Ans. by NTA (4)

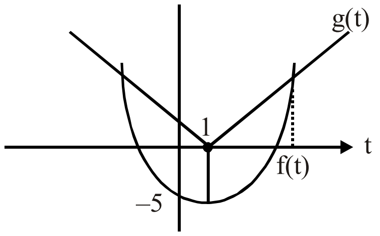
Sol. Let $2^x = t$

$5 + |t-1| = t^2 - 2t$

$\Rightarrow |t-1| = (t^2 - 2t - 5)$

$\quad g(t) \qquad f(t)$

From the graph



$\because t > 0$

So, number of real root is 1.

14. Official Ans. by NTA (3)

Sol. $f(x) = \sqrt{x}$, $g(x) = \tan x$, $h(x) = \dfrac{1-x^2}{1+x^2}$

$fog\,(x) = \sqrt{\tan x}$

$hofog\,(x) = h(\sqrt{\tan x}) = \dfrac{1-\tan x}{1+\tan x}$

$$= -\tan\left(\frac{\pi}{4} - x\right)$$

$\phi(x) = \tan\left(\dfrac{\pi}{4} - x\right)$

$$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$$

$$= \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$$

15. Official Ans. by NTA (2)

Sol. $$\underbrace{\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \ldots + \left[-\frac{1}{3} - \frac{66}{100}\right]}_{(-1)67}$$

$$+\underbrace{\left[-\frac{1}{3} - \frac{67}{100}\right] + \ldots + \left[-\frac{1}{3} - \frac{99}{100}\right]}_{-2(33)} = -133$$

FUNCTION

1. Ans. (1)

Given $f_1(x) = \dfrac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \dfrac{1}{1-x}$

$(f_2 \circ J \circ f_1)(x) = f_3(x)$

$f_2 \circ \left(J\left(f_1(x)\right)\right) = f_3\,(x)$

$f_2 \circ \left(J\left(\dfrac{1}{x}\right)\right) = \dfrac{1}{1-x}$

$1 - J\left(\dfrac{1}{x}\right) = \dfrac{1}{1-x}$

$J\left(\dfrac{1}{x}\right) = 1 - \dfrac{1}{1-x} = \dfrac{-x}{1-x} = \dfrac{x}{x-1}$

Now $x \to \dfrac{1}{x}$, $J(x) = \dfrac{\frac{1}{x}}{\frac{1}{x} - 1} = \dfrac{1}{1-x} = f_3(x)$

2. Ans. (1)

$f(x) = 2\left(1 + \dfrac{1}{x-1}\right)$

$f'(x) = -\dfrac{2}{(x-1)^2}$

$\Rightarrow f$ is one-one but not onto

3. Ans. (4)

$f(x) = \begin{cases} \dfrac{n+1}{2} & n \text{ is odd} \\ n/2 & n \text{ is even} \end{cases}$

$g(x) = n - (-1)^n \begin{cases} n+1\ ; & n \text{ is odd} \\ n-1\ ; & n \text{ is even} \end{cases}$

$f(g(n)) = \begin{cases} \dfrac{n}{2}; & n \text{ is even} \\ \dfrac{n+1}{2}; & n \text{ is odd} \end{cases}$

$\therefore$ many one but onto

Option (4)

4. Ans. (2)

$f(0) = 0$ & $f(x)$ is odd.

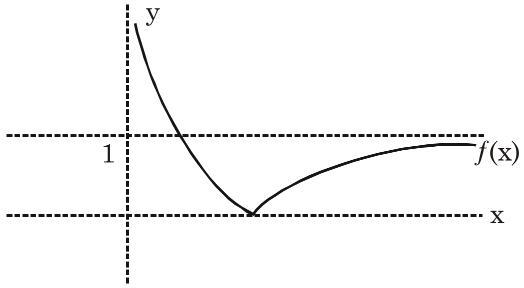
Further, if $x > 0$ then

$f(x) = \dfrac{1}{x + \frac{1}{x}} \in \left(0, \dfrac{1}{2}\right]$

Hence, $f(x) \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

5. Ans. (Bonus)

$$f(x) = \left|1 - \frac{1}{x}\right| = \frac{|x-1|}{x} = \begin{cases} \dfrac{1-x}{x} & 0 < x \le 1 \\ \dfrac{x-1}{x} & x \ge 1 \end{cases}$$



$\Rightarrow f(x)$ is not injective

but range of function is $[0,\infty)$

Remark : If co-domain is $[0,\infty)$, then $f(x)$ will be surjective

6. Ans. (1)

$f(k) = 3m$ (3,6,9,12,15,18)

for k = 4,8,12,16,20   6.5.4.3.2 ways

For rest numbers 15! ways

Total ways = 6!(15!)

7. Official Ans. by NTA (1)

Sol. $f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x|<1$

$$f\left(\frac{2x}{1+x^2}\right) = \ell n\left(\frac{1-\frac{2x}{1+2x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$= \ell n\left(\frac{(x-1)^2}{(x+1)^2}\right) = 2\ell n\left|\frac{1-x}{1+x}\right| = 2f(x)$$

8. Official Ans. by NTA (1)

Sol. $f(x) = a^x, a > 0$

$$f(x) = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x) = \frac{a^x + a^{-x}}{2}$$

$$f_2(x) = \frac{a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x+y) + f_1(x-y)$$

$$= \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{-x+y}}{2}$$

$$= \frac{(a^x + a^{-x})}{2}(a^y + a^{-y})$$

$$= f_1(x) \times 2f_1(y)$$

$$= 2f_1(x)\, f_1(y)$$

9. Official Ans. by NTA (2)

Sol. From the given functional equation :

$f(x) = 2^x \quad \forall x \in N$

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$2^a (2 + 2^2 + ... + 2^{10}) = 16(2^{10} - 1)$

$$2^a.\frac{2.(2^{10}-1)}{1} = 16(2^{10}-1)$$

$2^{a+1} = 16 = 2^4$

a = 3

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Sol. $y = \frac{x^2}{1-x^2}$

Range of y : R –[–1,0)

for surjective funciton, A must be same as above range.

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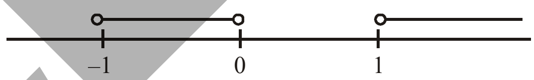
Sol. $4 - x^2 \neq 0$; $x^3 - x > 0$

$x = \pm 2$ $x(x-1)(x+1) > 0$



$\therefore D_f \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

12. Official Ans. by NTA (3)

Sol. g(S) = [–2, 2]

So, f(g(S)) = [0, 4] = S

And f(S) = [0, 16] $\Rightarrow$ f(g(S) $\neq$ f(S)

Also, g(f(S)) = [–4, 4] $\neq$ g (S)

So, g(f(S) $\neq$ S

13. Official Ans. by NTA (4)

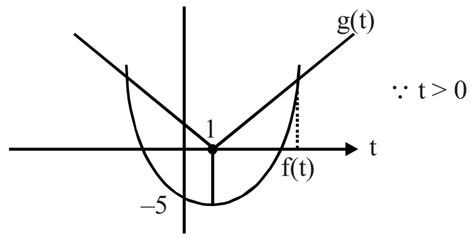
Sol. Let $2^x = t$

$5 + |t-1| = t^2 - 2t$

$\Rightarrow |t-1| = (t^2 - 2t - 5)$

g(t)   f(t)

From the graph



So, number of real root is 1.

14. Official Ans. by NTA (3)

Sol. $f(x) = \sqrt{x}$, $g(x) = \tan x$, $h(x) = \dfrac{1-x^2}{1+x^2}$

$\text{fog}(x) = \sqrt{\tan x}$

$\text{hofog}(x) = h(\sqrt{\tan x}) = \dfrac{1-\tan x}{1+\tan x}$

$= -\tan\left(\dfrac{\pi}{4}-x\right)$

$\phi(x) = \tan\left(\dfrac{\pi}{4}-x\right)$

$\phi\left(\dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{4}-\dfrac{\pi}{3}\right) = \tan\left(-\dfrac{\pi}{12}\right) = -\tan\dfrac{\pi}{12}$

$= \tan\left(\pi-\dfrac{\pi}{12}\right) = \tan\dfrac{11\pi}{12}$

15. Official Ans. by NTA (2)

Sol. $\underbrace{\left[-\dfrac{1}{3}\right]+\left[-\dfrac{1}{3}-\dfrac{1}{100}\right]+\ldots+\left[-\dfrac{1}{3}-\dfrac{66}{100}\right]}_{(-1)67}$

$+\underbrace{\left[-\dfrac{1}{3}-\dfrac{67}{100}\right]+\ldots+\left[-\dfrac{1}{3}-\dfrac{99}{100}\right]}_{-2(33)} = -133$

INVERSE TRIGONOMETRIC FUNCTION

1. Ans. (1)

$\cos^{-1}\left(\dfrac{2}{3x}\right) + \cos^{-1}\left(\dfrac{3}{4x}\right) = \dfrac{\pi}{2} \quad \left(x > \dfrac{3}{4}\right)$

$\cos^{-1}\left(\dfrac{3}{4x}\right) = \dfrac{\pi}{2} - \cos^{-1}\left(\dfrac{2}{3x}\right)$

$\cos^{-1}\left(\dfrac{3}{4x}\right) = \sin^{-1}\left(\dfrac{2}{3x}\right)$

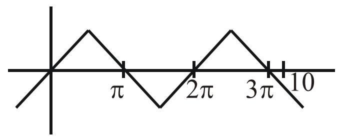
$\cos\left(\cos^{-1}\left(\dfrac{3}{4x}\right)\right) = \cos\left(\sin^{-1}\dfrac{2}{3x}\right)$

$\dfrac{3}{4x} = \dfrac{\sqrt{9x^2-4}}{3x}$

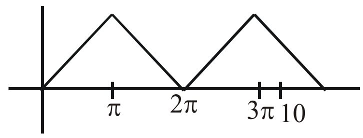
$\dfrac{81}{16}+4 = 9x^2$

$x^2 = \dfrac{145}{16\times 9} \quad \Rightarrow \quad x = \dfrac{\sqrt{145}}{12}$

2. Ans. (1)



$x = \sin^{-1}(\sin 10) = 3\pi - 10$



$y = \cos^{-1}(\cos 10) = 4\pi - 10$

$y - x = \pi$

3. Ans. (3)

$\cot\left(\sum_{n=1}^{19}\cot^{-1}(1+n(n+1))\right)$

$\cot\left(\sum_{n=1}^{19}\cot^{-1}(n^2+n+1)\right) = \cot\left(\sum_{n=1}^{19}\tan^{-1}\dfrac{1}{1+n(n+1)}\right)$

$\sum_{n=1}^{19}(\tan^{-1}(n+1)-\tan^{-1}n)$

$\cot(\tan^{-1}20 - \tan^{-1}1) = \dfrac{\cot A\cot\beta+1}{\cot\beta-\cot A}$

(Where tanA=20, tanB=1) $\quad \dfrac{1\left(\dfrac{1}{20}\right)+1}{1-\dfrac{1}{20}} = \dfrac{21}{19}$

$\therefore$ Option (3)

4. Ans. (3)

$\cot^{-1}x > 5$ (reject), $\cot^{-1}x < 2$

$\therefore\ x > \cot 2$

$\therefore\ x \in (\cot 2, \infty)$

5. Ans. (4)

$\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$\Rightarrow \dfrac{5x}{1-6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$x = -1$ or $x = \dfrac{1}{6}$

$x = \dfrac{1}{6} \quad \because x > 0$

6. **Official Ans. by NTA (1)**

Sol. $\cos\alpha = \frac{3}{5}, \tan\beta = \frac{1}{3}$

$\Rightarrow \quad \tan\alpha = \frac{4}{3}$

$\Rightarrow \quad \tan(\alpha-\beta) = \frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{3}.\frac{1}{3}} = \frac{9}{13}$

$\Rightarrow \quad \sin(\alpha-\beta) = \frac{9}{5\sqrt{10}}$

$\Rightarrow \quad \alpha-\beta = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

7. **Official Ans. by NTA (3)**

Sol. $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\cos(\cos^{-1}x - \cos^{-1}\frac{y}{2}) = \cos\alpha$

$\Rightarrow x \times \frac{y}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

$\Rightarrow \left(\cos\alpha - \frac{xy}{2}\right)^2 = \left(1-x^2\right)\left(1-\frac{y^2}{4}\right)$

$x^2 + \frac{y^2}{4} - xy\cos\alpha = 1 - \cos^2\alpha = \sin^2\alpha$

8. **Official Ans. by NTA (3)**

Sol. $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

$= \sin^{1}\left(\frac{33}{65}\right) = \cos^{-1}\left(\frac{56}{65}\right) = \frac{\pi}{2} - \sin^{1}\left(\frac{56}{65}\right)$

LIMIT

1. **Ans. (1)**

$\lim\limits_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$= \lim\limits_{y\to 0} \frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$

$= \lim\limits_{y\to 0} \frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$

$= \lim\limits_{y\to 0} \frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$

$= \lim\limits_{y\to 0} \frac{1}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)} = \frac{1}{4\sqrt{2}}$

2. **Ans. (1)**

$\lim\limits_{x\to 0^-} \frac{x\left([x]+|x|\right)\sin[x]}{|x|}$

$x \to 0^-$

$[x] = -1 \Rightarrow \lim\limits_{x\to 0^-} \frac{x(-x-1)\sin(-1)}{-x} = -\sin 1$

$|x| = -x$

3. **Ans. (4)**

$\lim\limits_{x\to 1^+} \frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$

$= \lim\limits_{x\to 1^+} \frac{(1-x)+\sin(x-1)}{(x-1)(-1)}\sin\left(\frac{\pi}{2}(-1)\right)$

$= \lim\limits_{x\to 1^+} \left(1-\frac{\sin(x-1)}{(x-1)}\right)(-1) = (1-1)(-1) = 0$

4. **Ans. (4)**

R.H.L. $= \lim\limits_{x\to 0^+} \frac{\tan(\pi\sin^2 x)+\left(|x|-\sin(x[x])\right)^2}{x^2}$

(as $x \to 0^+ \Rightarrow [x] = 0$)

$= \lim\limits_{x\to 0^+} \frac{\tan(\pi\sin^2 x)+x^2}{x^2}$

$= \lim\limits_{x\to 0^+} \frac{\tan(\pi\sin^2 x)}{(\pi\sin^2 x)} + 1 = \pi + 1$

L.H.L. $= \lim\limits_{x\to 0^-} \frac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$

(as $x \to 0^- \Rightarrow [x] = -1$)

$$\lim_{x\to 0+} \frac{\tan(\pi \sin^2 x)}{\pi \sin^2 x} \cdot \frac{\pi \sin^2 x}{x^2} + \left(-1 + \frac{\sin x}{x}\right)^2 \Rightarrow \pi$$

R.H.L. $\neq$ L.H.L.

5. Ans. (4)

$$\lim_{x\to 0} \frac{x \tan^2 2x}{\tan 4x \sin^2 x} = \lim_{x\to 0} \frac{x\left(\frac{\tan^2 2x}{4x^2}\right)4x^2}{\left(\frac{\tan 4x}{4x}\right)4x\left(\frac{\sin^2 x}{x^2}\right)x^2} = 1$$

6. Ans. (3)

$$\lim_{x\to \pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$$

$$\lim_{x\to \pi/4} \frac{(1-\tan^4 x)}{\cos(x+\pi/4)}$$

$$2 \lim_{x\to \pi/4} \frac{(1-\tan^2 x)}{\cos(x+\pi/4)}$$

$$R \lim_{x\to \pi/4} \frac{\cos^2 x - \sin^2 x}{\frac{\cos x - \sin x}{\sqrt{2}}} \frac{1}{\cos^2 x}$$

$$4\sqrt{2} \lim_{x\to \pi/4} (\cos x + \sin x) = 8$$

7. Ans. (3)

$$\lim_{x\to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}} \times \frac{\sqrt{\pi} + \sqrt{2\sin^{-1} x}}{\sqrt{\pi} + \sqrt{2\sin^{-1} x}}$$

$$\lim_{x\to 1^-} \frac{2\left(\frac{\pi}{2} - \sin^{-1} x\right)}{\sqrt{1-x}\cdot\left(\sqrt{\pi} + \sqrt{2\sin^{-1} x}\right)}$$

$$\lim_{x\to 1^-} \frac{2\cos^{-1} x}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{\pi}}$$

Put $x = \cos\theta$

$$\lim_{\theta\to 0^+} \frac{2\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)} \cdot \frac{1}{2\sqrt{\pi}} = \sqrt{\frac{2}{\pi}}$$

8. Official Ans. by NTA (2)

Sol. $$\lim_{x\to 0} \frac{\left(\frac{\sin^2 x}{x^2}\right)\left(\sqrt{2} + \sqrt{1+\cos x}\right)}{\left(\frac{1-\cos x}{x^2}\right)}$$

$$= \frac{(1)^2 \cdot \left(2\sqrt{2}\right)}{\frac{1}{2}} = 4\sqrt{2}$$

9. Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0} \left(\frac{1 + f(3+x) - f(3)}{1 + f(2-x) - f(2)}\right)^{\frac{1}{x}} \quad (1^\infty \text{ form})$$

$$\Rightarrow e^{\lim_{x\to 0} \frac{f(3+x) - f(2-x) - f(3) + f(2)}{x(1 + f(2-x) - f(2))}}$$

using L'Hopital

$$\Rightarrow e^{\lim_{x\to 0} \frac{f'(3+x) + f'(2-x)}{-xf'(2-x) + (1 + f(2-x) - f(2))}}$$

$$\Rightarrow e^{\frac{f'(3)+f'(2)}{1}} = 1$$

10. Official Ans. by NTA (4)

Sol. $f(x) = [x] - \left[\frac{x}{4}\right]$

$$\lim_{x\to 4+} f(x) = \lim_{x\to 4+} \left(\left([x] - \left[\frac{x}{4}\right]\right)\right) = 4 - 1 = 3$$

$$\lim_{x\to 4-} f(x) = \lim_{x\to 4-} \left([x] - \frac{x}{4}\right) = 3 - 0 = 3$$

f(x) = 3

$\therefore$ continuous at x = 4

11. Official Ans. by NTA (4)

Sol. $$\lim_{x\to 1} \frac{x^4 - 1}{x - 1} = \lim_{x\to k} \frac{x^3 - k^3}{x^2 - k^2}$$

$$\Rightarrow \lim_{x\to 1} (x+1)(x^2+1) = \frac{k^2 + k^2 + k^2}{2k}$$

$\Rightarrow$ k = 8/3

12. Official Ans. by NTA (1)

Sol. $\lim\limits_{x\to 1}\dfrac{x^2-ax+b}{x-1}=5$

$1 - a + b = 0$ ...(i)

$2 - a = 5$ ...(ii)

$\Rightarrow a + b = -7.$

13. Official Ans. by NTA (2)

Sol. Rationalize

$$\lim_{x\to 0}\frac{(x+2\sin x)\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right)}{x^2+2\sin x+1-\sin^2 x+x-1}$$

$$\lim_{x\to 0}\frac{(x+2\sin x)(2)}{x^2+2\sin x-\sin^2 x+x}$$

$\dfrac{0}{0}$ form using L' hospital

$$\Rightarrow \lim_{x\to 0}\frac{(1+2\cos x)\times 2}{2x+2\cos x-2\sin x\cos x+1}\ \Rightarrow \frac{2\times 3}{(2+1)}=2$$

14. Official Ans. by NTA (1)

Sol. Maxima of f(x) occured at x = 2 i.e. $\alpha = 2$

Minima of g(x) occured at x = –1 i.e. $\beta = -1$

$$\therefore \lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}=\frac{1}{2}$$

CONTINUITY

1. Ans. (4)

$$f(x)=\begin{cases} 5 & \text{if} \quad x\le 1\\ a+bx & \text{if} \quad 1<x<3\\ b+5x & \text{if} \quad 3\le x<5\\ 30 & \text{if} \quad x\ge 5\end{cases}$$

$f(1) = 5$, $f(1^-) = 5$, $f(1^+) = a + b$

$f(3^-) = a + 3b$, $f(3) = b + 15$, $f(3^+) = b + 15$

$f(5^-) = b + 25$; $f(5) = 30$ $f(5^+) = 30$

from above we concluded that f is not continuous for any values of a and b.

2. Official Ans. by NTA (4)

Sol. $$f(x)=\begin{cases}-(x+1) & , \quad -1\le x<0\\ x & , \quad 0\le x<1\\ 2x & , \quad 1\le x<2\\ x+2 & , \quad 2\le x<3\\ x+3 & , \quad x=3\end{cases}$$

function discontinuous at $x = 0,1,3$

3. Official Ans. by NTA (1)

Sol. $\therefore$ function should be continuous at $x=\dfrac{\pi}{4}$

$$\therefore \lim_{x\to \frac{\pi}{4}} f(x)=f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1}=k$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x}=k \quad \text{(Using L'H}\hat{o}\text{pital rule)}$$

$$\lim_{x\to\frac{\pi}{4}}\sqrt{2}\sin^3 x=k$$

$$\Rightarrow k=\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3=\frac{1}{2}$$

4. Official Ans. by NTA (1)

Sol. $$f(x)\begin{cases} a|\pi-x|+1; x\ge 5\\ b|\pi-x|+3; x>5\end{cases}$$

$a|\pi-5|+1=b|5-\pi|+3$

$a(5-\pi)+1=b(5-\pi)+3$

$(a-b)\,(5-\pi)=2$

$$a-b=\frac{2}{5-\pi}$$

5. Official Ans. by NTA (4)

Sol. $$\text{RHL}=\lim_{x\to 0^+}\frac{\sqrt{x+x^2}-\sqrt{x}}{x^{\frac{3}{2}}}=\lim_{x\to 0^+}\frac{\sqrt{1+x}-1}{x}=\frac{1}{2}$$

$$\text{LHL}=\lim_{x\to 0}\frac{\sin(p+1)x+\sin x}{x}=(p+1)+1=p+2$$

for continuity LHL = RHL = f(0)

$$\Rightarrow (p,q)=\left(\frac{-3}{2},\frac{1}{2}\right)$$

DIFFERENTIABILITY

1. Ans. (4)

$|f(x) - f(y)| \le 2|x - y|^{3/2}$

divide both sides by $|x - y|$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le 2.|x-y|^{1/2}$$

apply limit $x \to y$

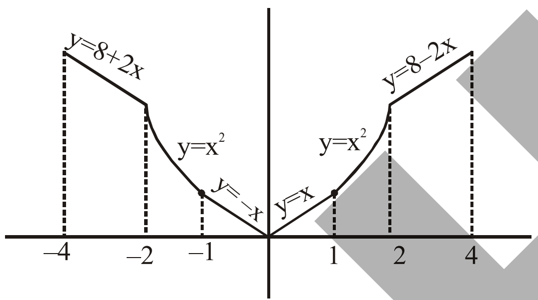
$|f'(y)| \le 0 \Rightarrow f'(y) = 0$

$\Rightarrow f(y) = c \Rightarrow f(x) = 1$

$$\int_0^1 1.dx = 1$$

2. Ans. (3)

$$f(x)=\begin{cases}8+2x, & -4\le x<-2\\ x^2, & -2\le x\le -1\\ |x|, & -1<x<1\\ x^2, & 1\le x\le 2\\ 8-2x, & 2<x\le 4\end{cases}$$



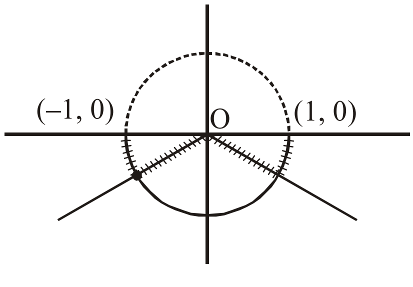
$f(x)$ is not differentiable at $x = \{-2,-1,0,1,2\}$

$\Rightarrow S = \{-2, -1, 0, 1, 2\}$

3. Ans. (1)

$f : (-1, 1) \to R$

$$f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$$



Non-derivable at 3 points in $(-1, 1)$

Option (1)

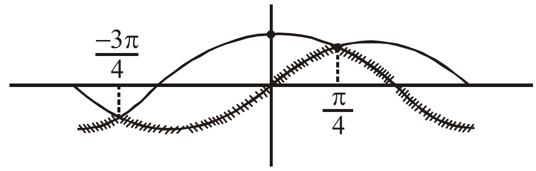
4. Ans. (3)

$f(x) = \sin|x| - |x| + 2(x - \pi) \cos x$

$\because \sin|x| - |x|$ is differentiable function at $x=0$

$\therefore k = \phi$

5. Ans. (1)



6. Ans. (4)

$$|f(x)| = \begin{cases}1 & , & -2\le x<0\\ 1-x^2 & , & 0\le x<1\\ x^2-1 & , & 1\le x\le 2\end{cases}$$

and $f(|x|) = x^2 - 1$, $x \in [-2, 2]$

$$\text{Hence } g(x) = \begin{cases}x^2 & , & x\in[-2,0)\\ 0 & , & x\in[0,1)\\ 2(x^2-1) & , & x\in[1,2]\end{cases}$$

It is not differentiable at $x = 1$

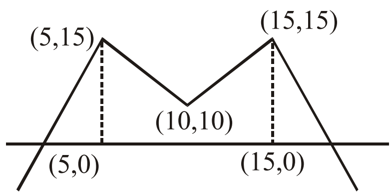
7. Official Ans. by NTA (3)

Sol. $f(x) = 15 - |x - 10|$, $x \in R$

$f(f(x)) = 15 - |f(x) - 10|$

$= 15 - |15 - |x - 10| - 10|$

$= 15 - |5 - |x - 10||$



$x = 5, 10, 15$ are points of non differentiability

Aliter :

At $x = 10$ $f(x)$ is non differentiable

also, when $15 - |x - 10| = 10$

$\Rightarrow x = 5, 15$

$\therefore$ non differentiability points are $\{5, 10, 15\}$

8. Official Ans. by NTA (1)

Sol. $$g'(c) = \lim_{h\to 0}\frac{|f(c+h)|-|f(c)|}{h}$$

$$= \lim_{h\to 0}\frac{|f(c+h)|}{h} = \lim_{h\to 0}\frac{|f(c+h)-f(c)|}{h}$$

$$= \lim_{h\to 0}\left|\frac{f(c+h)-f(c)}{h}\right|\frac{|h|}{h}$$

$$= \lim_{h\to 0}|f'(c)|\frac{|h|}{h}=0, \text{ if } f'(c) = 0$$

i.e., g(x) is differentiable at x = c, if f '(c) = 0

METHOD OF DIFFERENTIATION

1. **Ans. (4)**

$$\frac{dx}{dt}=3\sec^2 t$$

$$\frac{dy}{dt}=3\sec t\tan t$$

$$\frac{dy}{dx}=\frac{\tan t}{\sec t}=\sin t$$

$$\frac{d^2y}{dx^2}=\cos t\frac{dt}{dx}$$

$$=\frac{\cos t}{3\sec^2 t}=\frac{\cos^3 t}{3} \quad =\frac{1}{3.2\sqrt{2}}=\frac{1}{6\sqrt{2}}$$

2. **Ans. (2)**

$f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$

$\Rightarrow f'(x) = 3x^2 + 2xf'(1) + f''(x)$ .....(1)

$\Rightarrow f''(x) = 6x + 2f'(1)$ .....(2)

$\Rightarrow f'''(x) = 6$ .....(3)

put x = 1 in equation (1) :

$f'(1) = 3 + 2f'(1) + f''(2)$ .....(4)

put x = 2 in equation (2) :

$f''(2) = 12 + 2f'(1)$ .....(5)

from equation (4) & (5) :

$-3 - f'(1) = 12 + 2f'(1)$

$\Rightarrow 3f'(1) = -15$

$\Rightarrow f'(1) = -5 \;\Rightarrow\; f''(2) = 2$ ....(2)

put x = 3 in equation (3) :

$f'''(3) = 6$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

$f(2) = 8 - 20 + 4 + 6 = -2$

3. **Ans. (3)**

Differentiating with respect to x,

$$x.\frac{1}{\ell nx}.\frac{1}{x}+\ell n(\ell nx)-2x+2y.\frac{dy}{dx}=0$$

at x = e we get

$$1-2e+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{2e-1}{2y}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2e-1}{2\sqrt{4+e^2}} \text{ as } y(e)=\sqrt{4+e^2}$$

4. **Ans. (4)**

$(2x)^{2y} = 4e^{2x-2y}$

$2y\ell n2x = \ell n4 + 2x - 2y$

$$y=\frac{x+\ell n2}{1+\ell n2x}$$

$$y'=\frac{(1+\ell n2x)-(x+\ell n2)\frac{1}{x}}{(1+\ell n2x)^2}$$

$$y'(1+\ell n2x)^2=\left[\frac{x\ell n2x-\ell n2}{x}\right]$$

5. **Ans (1)**

$$\frac{f'(x)}{f(x)} = 1 \;\forall\; x \in R$$

Intergrate & use f(1) = 2

$f(x) = 2e^{x-1} \Rightarrow f'(x) = 2e^{x-1}$

$h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x))\, f'(x)$

$h'(1) = f'(f(1))\, f'(1)$

$= f'(2)\, f'(1)$

$= 2e \cdot 2 = 4e$

6. **Official Ans. by NTA (4)**

Sol. Consider $\cot^{-1}\left(\dfrac{\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x}{\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\sin x}\right)$

$$=\cot^{-1}\left(\frac{\sin\left(x+\frac{\pi}{3}\right)}{\cos\left(x+\frac{\pi}{3}\right)}\right)$$

$$=\cot^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right)=\frac{\pi}{2}-\tan^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right)$$

$$\begin{cases}\frac{\pi}{2}-\left(x+\frac{\pi}{3}\right)=\left(\frac{\pi}{6}-x\right); & 0<x<\frac{\pi}{6}\\ \frac{\pi}{2}-\left(\left(x-\frac{\pi}{3}\right)-\pi\right)=\left(\frac{7\pi}{6}-x\right); & \frac{\pi}{6}<x<\frac{\pi}{2}\end{cases}$$

$$\therefore\quad 2y=\begin{cases}\left(\frac{\pi}{6}-x\right)^2; & 0<x<\frac{\pi}{6}\\ \left(\frac{7\pi}{6}-x\right)^2; & \frac{\pi}{6}<x<\frac{\pi}{2}\end{cases}$$

$$\therefore\quad 2\frac{dy}{dx}=\begin{cases}2\left(\frac{\pi}{6}-x\right).(-1); & 0<x<\frac{\pi}{6}\\ 2\left(\frac{7\pi}{6}-x\right).(-1); & \frac{\pi}{6}<x<\frac{\pi}{2}\end{cases}$$

7. Official Ans. by NTA (2)

Sol. $y=f(f(f(x)))+(f(x))^2$

$$\frac{dy}{dx}=f'(f(f(x)))f'(f(x))f'(x)+2f(x)f'(x)$$

$=f'(1)f'(1)f'(1)+2f(1)f'(1)$

$=3\times5\times3+2\times1\times3$

$=27+6$

$=33$

8. Official Ans. by NTA (4)

Sol. $fog(x)=(-x)\Rightarrow(fg(\alpha))=-\alpha=b$

$(fg(x))'=-1\Rightarrow(fg(\alpha))'=-1=a$

9. Official Ans. by NTA (1)

Sol. $e^y=xy=e$

differentiate w.r.t. x

$$e^y\frac{dy}{dx}+x\frac{dy}{dx}+y=0$$

$$\frac{dy}{dx}(x+e^y)=-y,\ \left.\frac{dy}{dx}\right|_{(0,1)}=-\frac{1}{e}$$

again differentiate w.r.t. x

$$e^y\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot e^y\cdot\frac{dy}{dx}+x\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=0$$

$$(x+e^y)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\cdot e^y+2\frac{dy}{dx}=0$$

$$e\frac{d^2y}{dx^2}+\frac{1}{e^2}e+2\left(-\frac{1}{e}\right)=0$$

$$\therefore\quad\frac{d^2y}{dx^2}=\frac{1}{e^2}$$

10. Official Ans. by NTA (4)

Sol. $f(x)=\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$

$$=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)=\tan^{-1}\left(\tan\left(x-\frac{\pi}{4}\right)\right)$$

$$\because\ x-\frac{\pi}{4}\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$$

$$\therefore\ f(x)=x-\frac{\pi}{4}$$

$\Rightarrow$ its derivative w.r.t. $\frac{x}{2}$ is $\frac{1}{1/2}=2$

INDEFINITE INTEGRATION

1. (1 or 3)

Put $(x^2-1)=1$

$\Rightarrow 2xdx=dt$

$$\therefore\ I=\frac{1}{2}\int\sqrt{\frac{1-\cos t}{1+\cos t}}\,dt$$

$$=\frac{1}{2}\int\tan\left(\frac{t}{2}\right)dt$$

$$=\ln\left|\sec\frac{t}{2}\right|+c$$

$$I=\ln\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$$

2. Ans. (4)

$$\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx$$

$$=\int\frac{5x^{-6}+7x^{-8}}{\left(\frac{1}{x^7}+\frac{1}{x^5}+2\right)^2}dx=\frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}}+C$$

As $f(0) = 0,\ f(x) = \frac{x^7}{2x^7+x^2+1}$

$f(1) = \frac{1}{4}$

3. Ans. (3)

$$\int \frac{(\sin^n\theta - \sin\theta)^{1/n}\cos\theta}{\sin^{n+1}\theta}d\theta$$

$$= \int \frac{\sin\theta\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{1/n}}{\sin^{n+1}\theta}d\theta$$

Put $1-\frac{1}{\sin^{n-1}\theta} = t$

So $\frac{(n-1)}{\sin^n\theta}\cos\theta d\theta = dt$

Now $\frac{1}{n-1}\int (t)^{1/n}dt$

$$= \frac{1}{(n-1)}\frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1}+C$$

$$= \frac{1}{(n-1)}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{1}{n}+1}+C$$

4. Ans. (1)

$$\int x^5\cdot e^{-4x^3}dx = \frac{1}{48}e^{-4x^3}f(x)+c$$

Put $x^3 = t$

$3x^2\,dx = dt$

$\int x^3\cdot e^{-4x^3}\cdot x^2dx$

$\frac{1}{3}\int t\cdot e^{-4t}\,dt$

$\frac{1}{3}\left[t\cdot\frac{e^{-4t}}{-4}-\int\frac{e^{-4t}}{-4}dt\right]$

$-\frac{e^{-4t}}{48}[4t+1]+c$

$\frac{-e^{-4x^3}}{48}[4x^3+1]+c$

$\therefore\ f(x) = -1-4x^3$

Option (1)

(From the given options (1) is most suitable)

5. Ans. (2)

$$\int\frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$$

$$\int\frac{|x|\sqrt{\frac{1}{x^2}-1}}{x^4}dx,$$

Put $\frac{1}{x^2}-1 = t \Rightarrow \frac{dt}{dx} = \frac{-2}{x^3}$

Case-1 $x \ge 0$

$-\frac{1}{2}\int\sqrt{t}\ dt \Rightarrow -\frac{t^{3/2}}{3}+C$

$\Rightarrow -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2} \Rightarrow \frac{\left(\sqrt{1-x^2}\right)^3}{-3x^2}+C$

$A(x) = -\frac{1}{3x^3}$ and m = 3

$\left(A(x)\right)^m = \left(-\frac{1}{3x^3}\right)^3 = -\frac{1}{27x^9}$

Case-II $x \le 0$

We get $\frac{\left(\sqrt{1-x^2}\right)^3}{-3x^3}+C$

$A(x) = \frac{1}{-3x^3}$, m = 3

$\left(A(x)\right)^m = \frac{-1}{27x^9}$

6. Ans. (1)

$\sqrt{2x-1} = t \Rightarrow 2x-1 = t^2 \Rightarrow 2dx = 2t.dt$

$$\int\frac{x+1}{\sqrt{2x-1}}dx = \int\frac{\frac{t^2+1}{2}+1}{t}t\,dt = \int\frac{t^2+3}{2}dt$$

$$= \frac{1}{2}\left(\frac{t^3}{3}+3t\right) = \frac{t}{6}\left(t^2+9\right)+c$$

$$= \sqrt{2x-1}\left(\frac{2x-1+9}{6}\right)+c = \sqrt{2x-1}\left(\frac{x+4}{3}\right)+c$$

$\Rightarrow f(x) = \frac{x+4}{3}$

7. Ans. (2)

$I = \int\cos(\ell n\,x)dx$

$I = \cos(\ln x).x + \int\sin(\ell n\,x)\,dx$

$\cos(\ell n\,x)x + [\sin(\ell n\,x).x - \int\cos(\ell n\,x)dx]$

$I = \frac{x}{2}[\sin(\ell n\,x)+\cos(\ell n\,x)]+C$

8. Ans (2)

$$\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$$

$$\int \frac{\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}$$

Let $\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right) = t$

$$-\frac{1}{2}\int\frac{dt}{t^4}=\frac{1}{6t^3}+C \Rightarrow \frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$$

9. Official Ans. by NTA (3)

Sol. $$\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx=\int\frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$$

$$=\int\frac{\sin 3x+\sin 2x}{\sin x}dx$$

$$=\int\frac{3\sin x-4\sin^3 x-2\sin x\cos x}{\sin x}dx$$

$$=\int(3-4\sin^2 x+2\cos x)dx$$

$$=\int\left(3-2\left(1-\cos 2x\right)+2\cos x\right)dx$$

$$=\int\left(1+2\cos 2x+2\cos x\right)dx$$

$= x + \sin 2x + 2\sin x + c$

10. Official Ans. by NTA (4)

Sol. $$\int\frac{dx}{x^3\left(1+x^6\right)^{2/3}}=xf(x)(1+x^6)^{1/3}+c$$

$$\int\frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{2/3}}=xf(x)(1+x^6)^{1/3}+c$$

Let $t=\frac{1}{x^6}+1$

$$dt=\frac{-6}{x^7}dx$$

$$-\frac{1}{6}\int\frac{dt}{t^{2/3}}=-\frac{1}{2}t^{1/3}$$

$$=-\frac{1}{2}\left(\frac{1}{x^6}+1\right)^{1/3}=-\frac{1}{2}\frac{\left(1+x^6\right)^{1/3}}{x^2}$$

$$\therefore\ f(x)=-\frac{1}{2x^3}$$

11. Official Ans. by NTA (4)

Sol. $$I=\int\frac{dx}{\left(\sin x\right)^{4/3}.\left(\cos x\right)^{2/3}}$$

$$I=\int\frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3}.\cos^2 x}$$

$$\Rightarrow I=\int\frac{\sec^2 x}{\left(\tan x\right)^{4/3}}dx$$

put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\therefore\ I=\int\frac{dt}{t^{4/3}} \Rightarrow I=\frac{-3}{t^{1/3}}+c$$

$$\Rightarrow I=\frac{-3}{\left(\tan x\right)^{1/3}}+c$$

12. Official Ans. by NTA (4)

Sol. $$\int\frac{dx}{\left(\left(x-1\right)^2+9\right)^2}=\frac{1}{27}\int\cos^2\theta d\theta$$

(Put $x-1= 3\tan\theta$)

$$=\frac{1}{54}\int\left(1+\cos 2\theta\right)d\theta=\frac{1}{54}\left(\theta+\frac{\sin 2\theta}{2}\right)+C$$

$$=\frac{1}{54}\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{3\left(x-1\right)}{x^2-2x+10}\right)+C$$

13. Official Ans. by NTA (1)

Sol. Let $x^2 = t$ $\qquad 2x dx = dt$

$$\Rightarrow \frac{1}{2}\int t^2.e^{-t}dt =\frac{1}{2}\left[-t^2.e^{-t}+\int 2t.e^{-t}.dt\right]$$

$$=\frac{1}{2}\left(-t^2.e^{-t}\right)+\left(-t.e^{-t}+\int 1.e^{-t}.dt\right)$$

$$= -\frac{t^2e^{-t}}{2} - te^{-t} - e^{-t} = \left(-\frac{t^2}{2} - t - 1\right)e^{-t}$$

$$= \left(-\frac{x^4}{2} - x^2 - 1\right)e^{-x^2} + C$$

$$g(x) = -1 - x^2 - \frac{x^4}{2} + ke^{x^2}$$

for k = 0

$$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$$

14. Official Ans. by NTA (1)

Sol. $\int \frac{2x^3-1}{x^4+x}dx$

$$\int \frac{2x - \frac{1}{x^2}}{x^2 + \frac{1}{x}}dx$$

$$x^2 + \frac{1}{x} = t$$

$$\left(2x - \frac{1}{x^2}\right)dx = dt$$

$$\int \frac{dt}{t} = \ell n(t) + C$$

$$= \ell n\left(x^2 + \frac{1}{x}\right) + C$$

15. Official Ans. by NTA (3)

Sol. $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha}dx = \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$

Let $x - \alpha = t$

$$\Rightarrow \int \frac{\sin(t+2\alpha)}{\sin t}dt = \int \cos 2\alpha dt + \int \cot(t)\sin 2\alpha dt$$

$$= t.\cos 2\alpha + \ell n|\sin t|.\sin 2\alpha + C$$

$$= (x - \alpha)\cos 2\alpha + \ln|\sin(x-\alpha)|.\sin 2\alpha + C$$

16. Official Ans. by NTA (4)

Sol. $\int e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x)\, dx$

$$= e^{\sec x} f(x) + C$$

Diff. both sides w.r.t. 'x'

$$e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x))$$

$$= e^{\sec x} \cdot \sec x \tan x\, f(x) + e^{\sec x} f'(x)$$

$f'(x) = \sec^2 x + \tan x \sec x$

$\Rightarrow f(x) = \tan x + \sec x + c$

DEFINITE INTEGRATION

1. Ans. (4)

$$\int_0^{\pi} |\cos x|^3 dx = \int_0^{\pi/2} \cos^3 x\, dx - \int_{\pi/2}^{\pi} \cos^3 x\, dx$$

$$= \int_0^{\pi/2}\left(\frac{\cos 3x + 3\cos x}{4}\right)dx - \int_{\pi/2}^{\pi}\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\left[\left(\frac{\sin 3x}{3} + 3\sin x\right)_0^{\pi/2} - \left(\frac{\sin 3x}{3} + 3\sin x\right)_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{4}\left[\left(\frac{-1}{3} + 3\right) - (0+0) - \left\{(0+0) - \left(\frac{-1}{3} + 3\right)\right\}\right]$$

$$= \frac{4}{3}$$

2. Ans. (1)

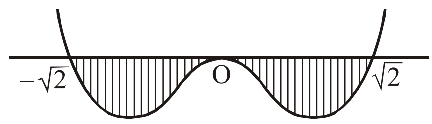
$$\frac{1}{\sqrt{2k}}\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{\sec\theta}}d\theta = \frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}d\theta$$

$$= -\frac{1}{\sqrt{2k}} 2\sqrt{\cos\theta}\Big|_0^{\pi/3} = -\frac{\sqrt{2}}{\sqrt{k}}\left(\frac{1}{\sqrt{2}} - 1\right)$$

given it is $1 - \frac{1}{\sqrt{2}} \Rightarrow k = 2$

3. Ans. (2)

Let $f(x) = x^2(x^2 - 2)$



As long as $f(x)$ lie below the x-axis, definite integral will remain negative,

so correct value of (a, b) is $(-\sqrt{2}, \sqrt{2})$ for minimum of I

4. Ans. (4)

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x] + [\sin x] + 4}$$

$$= \int_{-\frac{\pi}{2}}^{-1}\frac{dx}{-2-1+4} + \int_{-1}^{0}\frac{dx}{-1-1+4} + \int_0^1 \frac{dx}{0+0+4} + \int_1^{\frac{\pi}{2}}\frac{dx}{1+0+4}$$

$$\int_{-\pi/2}^{-1}\frac{dx}{1}+\int_{-1}^{0}\frac{dx}{2}+\int_{0}^{1}\frac{dx}{4}+\int_{1}^{\pi/2}\frac{dx}{5}$$

$$\left(-1+\frac{\pi}{2}\right)+\frac{1}{2}(0+1)+\frac{1}{4}+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$$

$$-1+\frac{1}{2}+\frac{1}{4}-\frac{1}{5}+\frac{\pi}{2}+\frac{\pi}{10}$$

$$\frac{-20+10+5-4}{20}+\frac{6\pi}{10}$$

$$\frac{-9}{20}+\frac{3\pi}{5}$$

Option (4)

5. Ans. (2)

$$\int_0^x f(t)\,dt = x^2+\int_x^1 t^2 f(t)\,dt \qquad f'\left(\frac{1}{2}\right)=?$$

Differentiate w.r.t. 'x'

$f(x) = 2x + 0 - x^2 f(x)$

$$f(x)=\frac{2x}{1+x^2}\ \Rightarrow\ f'(x)=\frac{(1+x^2)2-2x(2x)}{(1+x^2)^2}$$

$$f'(x)=\frac{2x^2-4x^2+2}{(1+x^2)^2}$$

$$f'\left(\frac{1}{2}\right)=\frac{2-2\left(\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2}=\frac{\left(\frac{3}{2}\right)}{\frac{25}{16}}=\frac{48}{50}=\frac{24}{25}$$

Option (2)

6. Ans. (4)

$$I=\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$$

$$I=\int_0^2\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}+\frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx$$

$$\left(\left[\frac{x}{\pi}\right]+\left[-\frac{x}{\pi}\right]=-1 \text{ as } \quad x\neq n\pi\right)$$

$$I=\int_0^2\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}+\frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx=0$$

7. Ans. (1)

$$I=\int_{\pi/6}^{\pi/4}\frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$$

$$I=\frac{1}{2}\int_{\pi/6}^{\pi/4}\frac{\tan^4 x\sec^2 x\,dx}{\left(1+\tan^{10}x\right)}$$ Put $\tan^5 x = t$

$$I=\frac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1}\frac{dt}{1+t^2}=\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\frac{1}{9\sqrt{3}}\right)$$

8. Ans. (2)

$$I=\int_0^a f(x)g(x)dx$$

$$I=\int_0^a f(a-x)g(a-x)dx$$

$$I=\int_0^a f(x)(4-g(x)dx$$

$$I=4\int_0^a f(x)dx-I$$

$$\Rightarrow I=2\int_0^a f(x)dx$$

9. Ans. (4)

$$\int_1^e\left(\frac{x}{e}\right)^{2x}\log_e x.dx-\int_1^e\left(\frac{e}{x}\right)\log_e x.dx$$

Let $\left(\frac{x}{e}\right)^{2x}=t$, $\left(\frac{e}{x}\right)^{x}=v$

$$=\frac{1}{2}\int_{\left(\frac{1}{e}\right)^2}^{1}dt+\int_e^1 dv$$

$$=\frac{1}{2}\left(1-\frac{1}{e^2}\right)+(1-e)=\frac{3}{2}-\frac{1}{2e^2}-e$$

10. Ans. (2)

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$$

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)}=\int_0^2\frac{dx}{1+x^2}=\tan^{-1}2$$

11. Official Ans. by NTA (4)

Sol. $g(f(x)) = \ell n(f(x)) = \ell n\left(\frac{2-x.\cos x}{2+x.\cos x}\right)$

$\therefore \quad I = \int_{-\pi/4}^{\pi/4} \ell n\left(\frac{2-x.\cos x}{2+x.\cos x}\right)dx$

$= \int_0^{\pi/4}\left(\ell n\left(\frac{2-x.\cos x}{2+x.\cos x}\right) + \ell n\left(\frac{2+x.\cos x}{2-x.\cos x}\right)\right)dx$

$= \int_0^{\pi/2}(0)dx = 0 = \log_e(1)$

12. Official Ans. by NTA (1)

Sol. $f(x) = \int_0^x g(t)dt$

$f(-x) = \int_0^{-x} g(t)dt$

put $t = -u$

$= -\int_0^x g(-u)du$

$= -\int_0^x g(u)d(u) = -f(x)$

$\Rightarrow f(-x) = -f(x)$

$\Rightarrow f(x)$ is an odd function

Also $f(5 + x) = g(x)$

$f(5 - x) = g(-x) = g(x) = f(5 + x)$

$\Rightarrow f(5 - x) = f(5 + x)$

Now

$I = \int_0^x f(t)dt$

$t = u + 5$

$I = \int_{-5}^{x-5} f(u+5)du$

$= \int_{-5}^{x-5} g(u)du$

$= \int_{-5}^{x-5} f'(u)du$

$= f(x - 5) - f(-5)$

$= -f(5 - x) + f(5)$

$= f(5) - f(5+x)$

$= \int_{5+x}^{5} f'(t)dt = \int_{5+x}^{5} g(t)dt$

13. Official Ans. by NTA (3)

Sol. $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}dx$

$\Rightarrow I = \int_0^{\pi/4} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}dx$

$= \int_0^{\pi/4}(1 - \sin x\cos x)\,dx$

$= \left(x - \frac{\sin^2 x}{2}\right)_0^{\pi/4}$

$= \frac{\pi}{4} - \frac{1}{4}$

$= \frac{\pi - 1}{4}$

14. Official Ans. by NTA (3)

Sol. $\lim_{x\to 2}\frac{\int_6^{f(x)} 2t\,dt}{x-2}$

L Hopital Rule

$\lim_{x\to 2}\frac{2f(x)f'(x)}{1} = 2f(2) = f'(2) = 12f'(2)$

15. Official Ans. by NTA (3)

Sol. $I = \int_0^{2\pi}\left[\sin 2x(1 + \cos 3x)\right]dx$

$I = \int_0^{\pi}\left(\left[\sin 2x + \sin 2x\cos 3x\right] + \left[-\sin 2x - \sin 2x\cos 3x\right]\right)dx$

$= \int_0^{\pi} -dx = -\pi$

16. Official Ans. by NTA (3)

Sol. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{n+r}{n}\right)^{1/3}$

$= \int_0^1 (1+x)^{1/3}dx = \frac{3}{4}\left(2^{4/3} - 1\right)$

17. Official Ans. by NTA (1)

Sol. $I = \int \frac{1}{\cos^{2/3} x \sin^{1/3} x . \sin x} dx$

$= \int \frac{\tan^{2/3} x}{\tan^2 x} . \sec^2 x . dx$

$= \int \frac{\sec^2 x}{\tan^{4/3} x} . dx \qquad \{\tan x = t,\ \sec^2 x dx = dt\}$

$= \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{-1/3} = -3\left(t^{-1/3}\right)$

$\Rightarrow I = -3\tan(x)^{-1/3}$

$\Rightarrow I = \frac{3}{(\tan x)^{1/3}}\Bigg|_{\pi/6}^{\pi/3} = -3\left(\frac{1}{\left(\sqrt{3}\right)^{1/3}} - \left(\sqrt{3}\right)^{1/3}\right)$

$= 3\left(3^{1/3} - \frac{1}{3^{1/6}}\right) = 3^{7/6} - 3^{5/6}$

18. Official Ans. by NTA (1)

Sol. $\int_0^{\pi/2} \frac{\cot x dx}{\cot x + \operatorname{cosec} x}$

$\int_0^{\pi/2} \frac{\cos x}{\cos x + 1} = \int \frac{2\cos^2 \frac{x}{2} - 1}{2\cos^2 \frac{x}{2}}$

$\int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2 \frac{x}{2}\right) dx$

$\left[x - \tan \frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$\frac{1}{2}[\pi - 2] \qquad m = \frac{1}{2},\ n = -2$

$mn = -1$

19. Official Ans. by NTA (1)

Sol. $I = \int_0^1 x \tan\left(\frac{1}{1 + x^2(x^2 - 1)}\right) dx$

$I = \int_0^1 x\left(\tan^{-1} x^2 - \tan^{-1}(x^2 - 1)\right) dx$

$x^2 = t \Rightarrow 2xdx = dt$

$I = \frac{1}{2}\int_0^1 \left(\tan^{-1} t - \tan^{-1}(t - 1)\right) dx$

$= \frac{1}{2}\int_0^1 \tan^{-1} t\, dt - \frac{1}{2}\int_0^1 \tan^{-1}(t - 1)\, dt$

$= \frac{1}{2}\int_0^1 \tan^{-1} t\, dt - \frac{1}{2}\int_0^1 \tan^{-1} dt = \int_0^1 \tan^{-1} dt$

$\tan^{-1} t = \theta \Rightarrow t = \tan\theta$

$dt = \sec^2\theta d\theta$

$\int_0^{\pi/4} \theta \cdot \sec^2\theta\, d\theta$

$I = (\theta . \tan\theta)\Big|_0^{\pi/4} - \int_0^{\pi/4} \tan\theta\, d\theta$

$= \left(\frac{\pi}{4} - 0\right) - \ln(\sec\theta)\Big|_0^{\pi/4}$

$= \frac{\pi}{4} - \left(\ell n\sqrt{2} - 0\right)$

$= \frac{\pi}{4} - \frac{1}{2}\ell n 2$

20. Official Ans. by NTA (4)

Sol. $\lim_{x \to 2} g(x) = \lim_{x \to 2} \frac{\int_6^{f(x)} 4t^3 dt}{x - 2}$

$= \lim_{x \to 2} \frac{4.f^3(x) \cdot f'(x)}{1}$

$= 4f^3(2)\, f'(2) = 18$

21. Official Ans. by NTA (4)

Sol. $\int_\alpha^{\alpha+1} \frac{(x + \alpha + 1) - (x + \alpha)}{(x + \alpha)(x + \alpha + 1)} dx = \left(\ell n|x + \alpha| - \ell n|x + \alpha + 1|\right)_\alpha^{\alpha+1}$

$= \ell n\left|\frac{2\alpha + 1}{2\alpha + 2} \times \frac{2\alpha + 1}{2\alpha}\right| = \ell n\frac{9}{8}$

$\Rightarrow \alpha = -2, 1$

TANGENT & NORMAL

1. **Ans. (4)**

Point of intersection is P(2,6).

Also, $m_1 = \left(\frac{dy}{dx}\right)_{P(2,6)} = -2x = -4$

$m_2 = \left(\frac{dy}{dx}\right)_{P(2,6)} = 2x = 4$

$\therefore |\tan\theta| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{8}{15}$

2. **Ans (4)**

$y = x^2 - 5x + 5$

$\frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$

at $x = \frac{7}{2}$, $y = \frac{-1}{4}$

Equation of tangent at $\left(\frac{7}{2}, \frac{-1}{4}\right)$ is $2x - y - \frac{29}{4} = 0$

Now check options

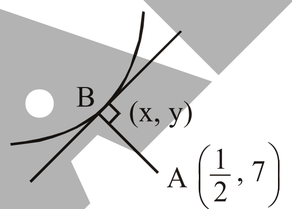
$x = \frac{1}{8}$, $y = -7$

3. **Ans. (3)**

$y - x^{3/2} = 7\ (x \geq 0)$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right) = -1$



$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{-x^{3/2}}{\frac{1}{2}-x}\right) = -1$

$\frac{3}{2}\cdot x^2 = \frac{1}{2} - x$

$3x^2 = 1 - 2x$

$3x^2 + 2x - 1 = 0$

$3x^2 + 3x - x - 1 = 0$

$(x + 1)(3x - 1) = 0$

$\therefore\ x = -1$ (rejected)

$x = \frac{1}{3}$

$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3}\right)^{3/2}$

$\ell_{AB} = \sqrt{\left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3} = \sqrt{\frac{1}{36} + \frac{1}{27}}$

$= \sqrt{\frac{3+4}{9 \times 12}}$

$= \sqrt{\frac{7}{108}} = \frac{1}{6}\sqrt{\frac{7}{3}}$

Option (3)

4. **Official Ans. by NTA (3)**

Sol. $f(1) = 1 - 1 - 2 = -2$

$f(-1) = -1 - 1 + 2 = 0$

$m = \frac{f(1) - f(-1)}{1+1} = \frac{-2-0}{2} = -1$

$\frac{dy}{dx} = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow (x - 1)(3x + 1) = 0$

$\Rightarrow x = 1, -\frac{1}{3}$

5. **Official Ans. by NTA (2)**

Sol. $y = x^3 + ax - b$

(1, –5) lies on the curve

$\Rightarrow -5 = 1 + a - b \Rightarrow a - b = -6$ ... (i)

Also, $y' = 3x^2 + a$

$y'_{(1,-5)} = 3 + a$ (slope of tangent)

$\because$ this tangent is $\perp$ to $-x + y + 4 = 0$

$\Rightarrow (3 + a)(1) = -1$

$\Rightarrow a = -4$ ....(ii)

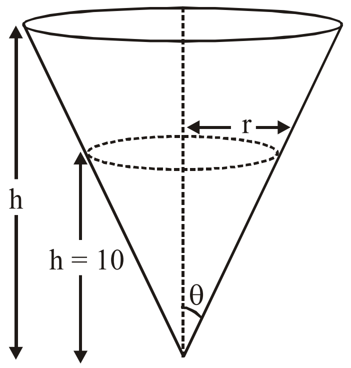
By (i) and (ii) : $a = -4$, $b = 2$

$\therefore\ y = x^3 - 4x - 2.$

(2,–2) lies on this curve.

6. Official Ans. by NTA (2)

Sol. $\tan\theta = \dfrac{1}{2} = \dfrac{r}{h}$



$r = \dfrac{h}{2}$

$V = \dfrac{1}{3}\pi r^2 h$

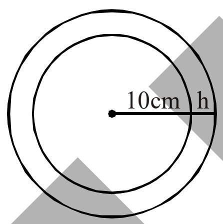
$V = \dfrac{1}{3}\pi . \dfrac{h^3}{4}$

$$\frac{dV}{dt} = \frac{\pi}{12}(3h)^2\left(\frac{dh}{dt}\right)$$

$$5 = \frac{\pi}{4}.(100)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{5\pi}$$

7. Official Ans. by NTA (3)

Sol. $V = \dfrac{4}{3}\pi\left((10+h)^3 - 10^3\right)$



$$\frac{dV}{dt} = 4\pi(10+h)^2\frac{dh}{dt}$$

$$-50 = 4\pi\ (10 + 5)^2\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = -\frac{1}{18}\frac{cm}{min}$$

8. Official Ans. by NTA (1)

Sol. $\left.\dfrac{dy}{dx}\right|_{(\alpha,\beta)} = \dfrac{-\alpha^2-3}{\left(\alpha^2-3\right)^2}$

Given that :

$$\frac{-\alpha^2-3}{\left(\alpha^2-3\right)^2} = -\frac{1}{3}$$

$\Rightarrow \alpha = 0, \pm 3$ $\quad$ $(\alpha \neq 0)$

$\Rightarrow \beta = \pm\ \dfrac{1}{2}.$ $\quad$ $(\beta \neq 0)$

$|6\alpha + 2\beta| = 19$

MONOTONICITY

1. Ans. (4)

$$f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$f'(x) = \frac{a^2}{\left(a^2+x^2\right)^{3/2}} + \frac{b^2}{\left(b^2+(d-x)^2\right)^{3/2}} > 0\ \forall x \in R$$

$f(x)$ is an increasing function.

2. Ans (3)

$f'(x) = 3x^2 - 6(a - 2)x + 3a$

$f'(x) \geq 0\ \forall\ x \in (0, 1]$

$f'(x) \leq 0\ \forall\ x \in [1, 5)$

$\Rightarrow f'(x) = 0$ at $x = 1 \Rightarrow a = 5$

$f(x) - 14 = (x - 1)^2\ (x - 7)$

$$\frac{f(x)-14}{(x-1)^2} = x - 7$$

3. Official Ans. by NTA (2)

Sol. $\phi(x) = f(x) + f(2 - x)$

$\phi'(x) = f'(x) - f'(2 - x)$ ......(1)

Since $f''(x) > 0$

$\Rightarrow\ f'(x)$ is increasing $\forall\ x \in (0, 2)$

Case-I : When $x > 2 - x \ \Rightarrow\ x > 1$

$\Rightarrow\ \phi'(x) > 0\ \forall\ x \in (1, 2)$

$\therefore\ \phi(x)$ is increasing on $(1, 2)$

Case-II : When $x < 2 - x \ \Rightarrow\ x < 1$

$\Rightarrow\ \phi'(x) < 0\ \forall\ x \in (0,1)$

$\therefore\ \phi(x)$ is decreasing on $(0, 1)$

4. Official Ans. by NTA (2)

Sol. $h(x) = f(g(x))$

$\Rightarrow h'(x) = f'(g(x)).\ g'(x)$ and $f'(x) = e^x - 1$

$\Rightarrow h'(x) = (e^{g(x)} - 1)\ g'(x)$

$\Rightarrow h'(x) = \left(e^{x^2-x} - 1\right)\ (2x - 1) \geq 0$

Case-I $e^{x^2-x} \geq 1$ and $2x - 1 \geq 0$

$\Rightarrow\ x \in [1, \infty)$ ......(1)

Case-II $e^{x^2-x} \leq 1$ and $2x - 1 \leq 0$

$\Rightarrow\ x \in \left[0, \dfrac{1}{2}\right]$.....(2)

Hence, $x \in \left[0, \dfrac{1}{2}\right] \cup [1, \infty)$

5. Official Ans. by NTA (2)

Sol. $f(x) = x\sqrt{kx - x^2}$

$$f'(x) = \frac{3kx - 4x^2}{2\sqrt{kx - x^2}}$$

For $\uparrow$ $f'(x) \geq 0$	$3kx - 4x^2 \geq 0$
$kx - x^2 \geq 0$	$4x^2 - 3kx \leq 0$
$x^2 - kx \leq 0$	$4x(x - \frac{3k}{4}) \leq 0$
$x(x-k) \leq 0$ so $x \in [0, 3]$	
$3 - \frac{3k}{4} \leq 0$	
+ve $\boxed{x \geq 3}$	$\boxed{k \geq 4}$

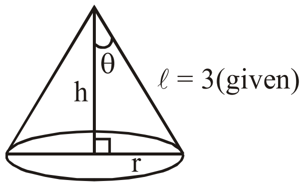
minimum value of k is $\boxed{m = 4}$

$f(x) = x\sqrt{kx - x^2}$

$$= 3\sqrt{4 \times 3 - 3^2} = 3\sqrt{3},\ M = 3\sqrt{3}$$

MAXIMA & MINIMA

1. Ans. (3)



$\therefore$ h = 3 cos θ

r = 3 sin θ

Now,

$$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}\left(9\sin^2\theta\right).\left(3\cos\theta\right)$$

$$\therefore \frac{dV}{d\theta} = 0 \Rightarrow \sin\theta = \sqrt{\frac{2}{3}}$$

Also, $\left.\frac{d^2V}{d\theta^2}\right]_{\sin\theta = \sqrt{\frac{2}{3}}}$ = negative

$\Rightarrow$ Volume is maximum,

when $\sin\theta = \sqrt{\frac{2}{3}}$

$\therefore$ $V_{max}\left(\sin\theta = \sqrt{\frac{2}{3}}\right) = 2\sqrt{3}\pi$ (in cu. m)

2. Ans. (1)

Let points $\left(\frac{3}{2}, 0\right)$, (t^2, t), $t > 0$

$$\text{Distance} = \sqrt{t^2 + \left(t^2 - \frac{3}{2}\right)^2}$$

$$= \sqrt{t^4 - 2t^2 + \frac{9}{4}} = \sqrt{(t^2 - 1)^2 + \frac{5}{4}}$$

So minimum distance is $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

3. Ans. (1)

$S = \{x \in R,\ x^2 + 30 - 11x \leq 0\}$

$= \{x \in R,\ 5 \leq x \leq 6\}$

Now $f(x) = 3x^3 - 18x^2 + 27x - 40$

$\Rightarrow f'(x) = 9(x - 1)(x - 3)$,

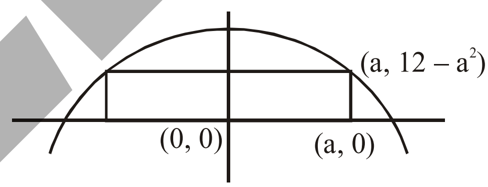
which is positive in [5, 6]

$\Rightarrow f(x)$ increasing in [5, 6]

Hence maximum value = $f(6) = 122$

4. Ans. (3)

$f(a) = 2a(12 - a)^2$



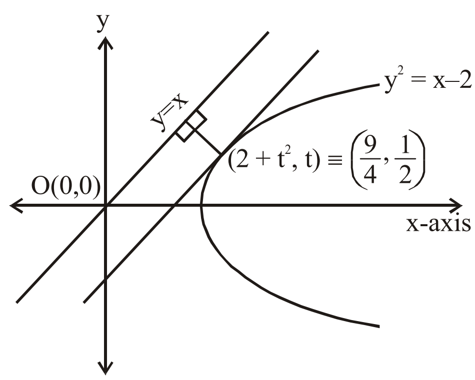
$f'(a) = 2(12 - 3a^2)$

maximum at a = 2

maximum area = f(2) = 32

5. Official Ans. by NTA (1)

Sol.



we have, $2y.\frac{dy}{dx} = 1 \Rightarrow \left.\frac{dy}{dx}\right]_{P(2+t^2, t)} = \frac{1}{2t} = 1$

$$\Rightarrow t = \frac{1}{2}$$

$\therefore \quad P\left(\frac{9}{4},\frac{1}{2}\right)$

So, shortest distance

$$=\frac{\left|\frac{9}{4}-\frac{2}{4}\right|}{\sqrt{2}}=\frac{7}{4\sqrt{2}}$$

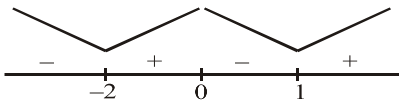
6. Official Ans. by NTA (1)

Sol. $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

$f'(x) = 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2)$

$= 36x(x - 1)(x + 2)$

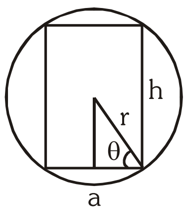


Points of minima = $\{-2, 1\} = S_1$

Point of maxima = $\{0\} = S_2$

7. Official Ans. by NTA (1)

Sol.



$h = 2r\sin\theta$

$a = 2r\cos\theta$

$v = \pi(r\cos\theta)^2(2r\sin\theta)$

$v = 2\pi r^3\cos^2\theta\sin\theta$

$$\frac{dv}{d\theta}=\pi r^3\left(-2\cos\theta\sin^2\theta+\cos^3\theta\right)=0$$

$$\text{or } \tan\theta=\frac{1}{\sqrt{2}}$$

$$\because\ h=2\times3\times\frac{1}{\sqrt{3}}=2\sqrt{3}$$

8. Official Ans. by NTA (2)

Sol. $f'(x) = \lambda(x + 1)(x - 0)(x - 1) = \lambda(x^3 - x)$

$$\Rightarrow\ f(x)=\lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+\mu$$

Now $f(x) = f(0)$

$$\Rightarrow\ \lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+\mu=\mu$$

$\Rightarrow x = 0, 0, \pm\sqrt{2}$

Two irrational and one rational number

9. Official Ans. by NTA (2)

Sol. Let a is first term and d is common difference

then, $a + 5d = 2$ (given) ...(1)

$f(d) = (2 - 5d)(2 - 2d)(2 - d)$

$f'(d) = 0 \quad \Rightarrow d = \frac{2}{3},\frac{8}{5}$

$f''(d) < 0$ at $d = 8/5$

$\Rightarrow d = \frac{8}{5}$

DIFFERENTIAL EQUATION

1. Ans. (3)

$$\frac{dy}{dx}+\left(\frac{2}{x}\right)y=x$$

$\Rightarrow$ I.F. $= x^2$

$\therefore\ yx^2 = \frac{x^4}{4}+\frac{3}{4}$ (As, $y(1) = 1$)

$$\therefore\ y\left(x=\frac{1}{2}\right)=\frac{49}{16}$$

2. Ans. (2)

$f(xy) = f(x).\,f(y)$

$f(0) = 1$ as $f(0) \neq 0$

$\Rightarrow f(x) = 1$

$$\frac{dy}{dx}=f(x)=1$$

$\Rightarrow y = x + c$

At, $x = 0$, $y = 1 \Rightarrow c = 1$

$y = x + 1$

$$\Rightarrow y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)=\frac{1}{4}+1+\frac{3}{4}+1=3$$

3. Ans. (1)

$$\frac{dy}{dx}+3\sec^2x.y=\sec^2x$$

I.F. $= e^{3\int\sec^2x\,dx} = e^{3\tan x}$

or $y.e^{3\tan x} = \int\sec^2x.e^{3\tan x}dx$

or $y.e^{3\tan x} = \frac{1}{3}e^{3\tan x}+C$ ....(1)

Given

$y\left(\frac{\pi}{4}\right)=\frac{4}{3}$

$\therefore\ \frac{4}{3}.e^{3}=\frac{1}{3}e^{3}+C$

$\therefore\ C = e^3$

Now put $x = -\frac{\pi}{4}$ in equation (1)

$\therefore\ y.e^{-3}=\frac{1}{3}e^{-3}+e^{3}$

$\therefore\ y=\frac{1}{3}+e^{6}$

$\therefore\ y\left(-\frac{\pi}{4}\right)=\frac{1}{3}+e^{6}$

4. Ans. (1)

$f'(x)=7-\frac{3}{4}\frac{f(x)}{x} \qquad (x>0)$

Given $f(1)\neq 4 \qquad \lim\limits_{x\to 0^{+}} xf\left(\frac{1}{x}\right) = ?$

$\frac{dy}{dx}+\frac{3}{4}\frac{y}{x}=7$ (This is LDE)

$\text{IF} = e^{\int\frac{3}{4x}dx}=e^{\frac{3}{4}\ln|x|}=x^{\frac{3}{4}}$

$y.x^{\frac{3}{4}}=\int 7.x^{\frac{3}{4}}dx$

$y.x^{\frac{3}{4}}=7.\frac{x^{\frac{7}{4}}}{\frac{7}{4}}+C$

$f(x)=4x+C.x^{-\frac{3}{4}}$

$f\left(\frac{1}{x}\right)=\frac{4}{x}+C.x^{\frac{3}{4}}$

$\lim\limits_{x\to 0^{+}} xf\left(\frac{1}{x}\right)=\lim\limits_{x\to 0^{+}}\left(4+C.x^{\frac{7}{4}}\right)=4$

$\therefore$ Option (1)

5. Ans. (2)

$(x^2 - y^2)\,dx + 2xy\,dy = 0$

$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$

Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

Solving we get,

$\int\frac{2v}{v^2+1}dv=\int-\frac{dx}{x}$

$\ln(v^2 + 1) = -\ln x + C$

$(y^2 + x^2) = Cx$

$1 + 1 = C \Rightarrow C = 2$

$\boxed{y^2+x^2=2x}$

$\therefore$ Option (2)

6. Ans. (2)

$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$

$\text{I.F.}=e^{\int\left(\frac{2x+1}{x}\right)dx}=e^{\int\left(2+\frac{1}{x}\right)dx}=e^{2x+\ell nx}=e^{2x}.x$

So, $y(xe^{2x})=\int e^{-2x}.xe^{2x}+C$

$\Rightarrow xye^{2x}=\int x\,dx+C$

$\Rightarrow 2xye^{2x} = x^2 + 2C$

It passess through $\left(1,\frac{1}{2}e^{-2}\right)$ we get C = 0

$y=\frac{xe^{-2x}}{2}$

$\Rightarrow \frac{dy}{dx}=\frac{1}{2}e^{-2x}(-2x+1)$

$\Rightarrow f(x)$ is decreasing in $\left(\frac{1}{2},1\right)$

$y(\log_e 2)=\frac{(\log_e 2)e^{-2(\log_e 2)}}{2}$

$=\frac{1}{8}\log_e 2$

7. Ans. (4)

$x - y = t \Rightarrow \frac{dy}{dx}=1-\frac{dt}{dx}$

$\Rightarrow 1-\frac{dt}{dx}=t^2 \Rightarrow \int\frac{dt}{1-t^2}=\int 1dx$

$\Rightarrow \frac{1}{2}\ell n\left(\frac{1+t}{1-t}\right)=x+\lambda$

$\Rightarrow \frac{1}{2}\ell n\left(\frac{1+x-y}{1-x+y}\right)=x+\lambda$ given $y(1) = 1$

$\Rightarrow \frac{1}{2}\ell n(1)=1+\lambda \Rightarrow \lambda=-1$

$\Rightarrow \ell n\left(\frac{1+x-y}{1-x+y}\right)=2(x-1)$

$\Rightarrow -\ell n\left(\frac{1-x+y}{1+x-y}\right)=2(x-1)$

8. Ans. (2)

$\frac{dy}{dx}=\frac{y}{x}=\ell n x$

$e^{\int\frac{1}{x}dx}=x$

$xy=\int x\ell nx+C$

$\ell n x\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}$

$xy=\frac{x}{2}\ell n x-\frac{x^2}{4}$+C, for 2y(2)= $2\ell n2 - 1$

$\Rightarrow C=0$

$y=\frac{x}{2}\ell n x-\frac{x}{4}$

$y(e)=\frac{e}{4}$

9. Ans. (2)

$\frac{dy}{dx}=\frac{x^2-2y}{x}$ (Given)

$\frac{dy}{dx}+2\frac{y}{x}=x$

I.F = $e^{\int\frac{2}{x}dx}=x^2$

$\therefore\ y.x^2=\int x.x^2dx+C=\frac{x^4}{y}+C$

hence bpasses through (1, –2) $\Rightarrow$ C = $-\frac{9}{4}$

$\therefore\ yx^2=\frac{x^4}{4}-\frac{9}{4}$

Now check option(s) , Which is satisly by option (ii)

10. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}+\left(\frac{2x}{x^2+1}\right)y=\frac{1}{(x^2+1)^2}$

(Linear differential equation)

$\therefore$ I.F. = $e^{\ell n(x^2+1)}=(x^2+1)$

So, general solution is $y.(x^2 + 1) = \tan^{-1}x + c$

As y(0) = 0 $\Rightarrow$ c = 0

$\therefore\ y(x)=\frac{\tan^{-1}x}{x^2+1}$

As, $\sqrt{a}.\ y(1)=\frac{\pi}{32}$

$\Rightarrow\ \sqrt{a}=\frac{1}{4}\Rightarrow a=\frac{1}{16}$

11. Official Ans. by NTA (1)

Sol. given $\frac{dy}{dx}=\frac{2y}{x^2}$

$\Rightarrow\int\frac{dy}{2y}=\int\frac{dx}{x^2}$

$\Rightarrow\frac{1}{2}\ell ny=-\frac{1}{x}+c$

passes through centre (1,1)

$\Rightarrow c=1$

$\Rightarrow x\ell ny=2(x-1)$

12. Official Ans. by NTA (4)

Sol. $x\frac{dy}{dx}+2y=x^2 : y(1)=1$

$\frac{dy}{dx}+\left(\frac{2}{x}\right)y=x$ (LDE in y)

IF $=e^{\int\frac{2}{x}dx}=e^{2\ell nx}=x^2$

$y.(x^2)=\int x.x^2dx=\frac{x^4}{4}+C$

y(1) =1

$1=\frac{1}{4}+C\Rightarrow C=1-\frac{1}{4}=\frac{3}{4}$

$yx^2=\frac{x^4}{4}+\frac{3}{4}$

$y=\frac{x^2}{4}+\frac{3}{4x^2}$

13. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} - y\tan x = 6x\sec x$

$y\left(\frac{\pi}{3}\right) = 0\,;\, y\left(\frac{\pi}{6}\right) = 7$

$e^{\int p dx} = e^{-\int \tan x dx} = e^{\ell n \cos x} = \cos x$

$y\,.\,\cos x = \int 6x\sec x\cos x\, dx$

$y.\cos x = \frac{6x^2}{2} + C$

$y = 3x^2\sec x + C\sec x$

$0 = 3\cdot\frac{\pi^2}{9}\cdot(2) + C(2)$

$2C = \frac{-2\pi^2}{3} \Rightarrow \boxed{C = -\frac{\pi^2}{3}}$

$y(\pi/6) = 3\cdot\frac{\pi^2}{36}\cdot\left(\frac{2}{\sqrt{3}}\right) + \left(\frac{2}{\sqrt{3}}\right)\cdot\left(-\frac{\pi^2}{3}\right)$

$\Rightarrow\ y = -\frac{\pi^2}{2\sqrt{3}}$

14. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx}$ = (tan x –y) $\sec^2 x$

Now, put tanx = t $\Rightarrow \frac{dt}{dx} = \sec^2 x$

So $\frac{dy}{dt} + y = t$

On solving, we get $ye^t = e^t(t-1) + c$

$\Rightarrow y = (\tan x - 1) + ce^{-\tan x}$

$\Rightarrow y(0) = 0 \Rightarrow c = 1$

$\Rightarrow y = \tan x - 1 + e^{-\tan x}$

So $y\left(-\frac{\pi}{4}\right) = e - 2$

15. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + y(\tan x) = 2x + x^2\tan x$

I.F $= e^{\int \tan x\, dx} = e^{\ln .\sec x} = \sec x$

$\therefore\ y\,.\,\sec x = \int\left(2x + x^2\tan x\right)\sec x.dx$

$= \int 2x\sec x\, dx + \int x^2(\sec x.\tan x)dx$

$y\sec x = x^2\sec x + \lambda$

$\Rightarrow y = x^2 + \lambda\cos x$

$y(0) = 0 + \lambda = 1 \qquad \Rightarrow \lambda = 1$

$y = x^2 + \cos x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y'(x) = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(\frac{-\pi}{4}\right) = \frac{-\pi}{2} + \frac{1}{\sqrt{2}}$

$y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

16. Official Ans. by NTA (4)

Sol. $y^2dx + xdy = \frac{dy}{y}$

$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

IF $= e^{\int\frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

$e^{-\frac{1}{y}}\cdot x = \int e^{-\frac{1}{y}}\cdot\frac{1}{y^3}dy + C$

$xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{e^{-\frac{1}{y}}}{y} + C$

$C = -\frac{1}{e}$

$x = \frac{3}{2} - \frac{1}{\sqrt{e}}$ when $y = 2$

17. Official Ans. by NTA (1)

Sol. $xy\dfrac{dy}{dx}-y^2+x^3=0$

put $y^2=k \Rightarrow y\dfrac{dy}{dx}=\dfrac{1}{2}\dfrac{dk}{dx}$

$\therefore$ given differential equation becomes

$$\frac{dk}{dx}+k\left(-\frac{2}{x}\right)=-2x^2$$

I.F. $= e^{\int -\frac{2}{x}dx}=\dfrac{1}{x^2}$

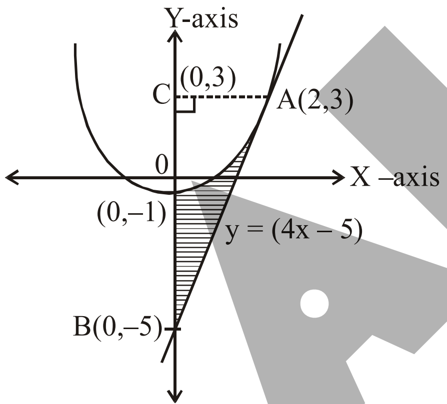
$\therefore$ solution is $k\cdot\dfrac{1}{x^2}=\int -2x^2\cdot\dfrac{1}{x^2}dx+\lambda$

$y^2+2x^3=\lambda x^2$

take $\lambda = -c$ (integration constant)

AREA UNDER THE CURVE

1. Ans. (3)



Equation of tangent at (2,3) on

$y = x^2 - 1$, is $y = (4x - 5)$ ....(i)

$\therefore$ Required shaded area

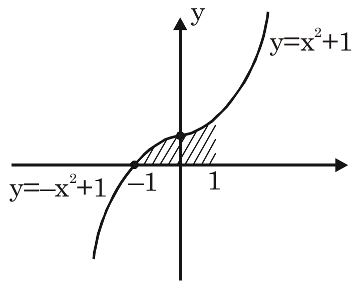
$= \text{ar}(\Delta ABC) - \int_{-1}^{3}\sqrt{y+1}\,dy$

$=\dfrac{1}{2}.(8).(2)-\dfrac{2}{3}\left((y+1)^{3/2}\right)_{-1}^{3}$

$=8-\dfrac{16}{3}=\dfrac{8}{3}$ (square units)

2. Ans. (3)

The graph is a follows



$$\int_{-1}^{0}\left(-x^2+1\right)dx+\int_{0}^{1}\left(x^2+1\right)dx=2$$

3. Ans. (1)

Area bounded by $y^2 = 4ax$ & $x^2 = 4by$,

a, b $\neq 0$ is $\left|\dfrac{16ab}{3}\right|$

by using formula : $4a=\dfrac{1}{k}=4b, k>0$

Area $=\left|\dfrac{16.\frac{1}{4k}.\frac{1}{4k}}{3}\right|=1$

$\Rightarrow k^2=\dfrac{1}{3}$

$\Rightarrow k=\dfrac{1}{\sqrt{3}}$

4. Ans. (1)

$y = x e^{x^2}$

$\left.\dfrac{dy}{dx}\right|_{(1,e)}=\left.\left(e\cdot e^{x^2}\cdot 2x+e^{x^2}\right)\right|_{(1,e)} = 2\cdot e + e = 3e$

T : $y - e = 3e(x - 1)$

$y = 3ex - 3e + e$

$y = (3e)x - 2e$

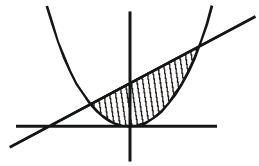
$\left(\dfrac{4}{3}, 2e\right)$ lies on it

Option (1)

5. Ans. (2)

$x = 4y - 2$ & $x^2 = 4y$

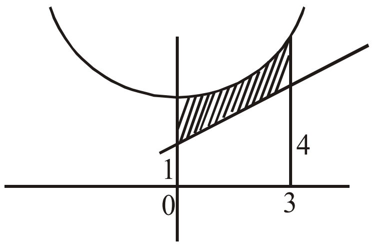
$\Rightarrow x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$



$x = 2, -1$

So, $\displaystyle\int_{-1}^{2}\left(\frac{x+2}{4}-\frac{x^2}{4}\right)dx=\frac{9}{8}$

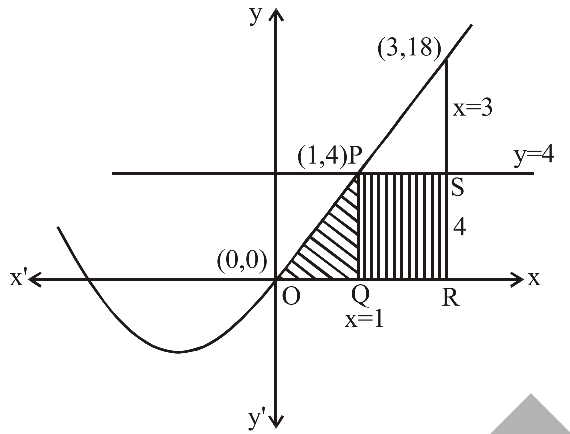
6. Ans. (2)



Req. area = $\int_0^3 (x^2+2)dx - \frac{1}{2}.5.3 = 9+6-\frac{15}{2} = \frac{15}{2}$

7. Official Ans. by NTA (2)

Sol.



Required Area

$= \int_0^1 (x^2+3x)dx + \text{Area of rectangle PQRS}$

$= \frac{11}{6} + 8 = \frac{59}{6}$

8. Official Ans. by NTA (2)

Sol. $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$

$A(\alpha) = 2\int_0^\alpha \sqrt{x}dx = 2\alpha^{\frac{3}{2}}$

$A(4) = 2\times 4^{3/2} = 16$

$A(\lambda) = 2\times\lambda^{3/2}$

$\frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \lambda = 4.\left(\frac{4}{25}\right)^{1/3}$

9. Official Ans. by NTA (2)

Sol. $x^2 \le y \le x+2$

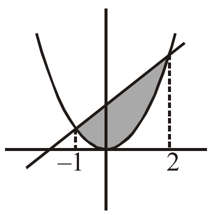
$x^2 = y$; $y = x+2$

$x^2 = x+2$

$x^2 - x - 2 = 0$

$(x-2)(x-1) = 0$

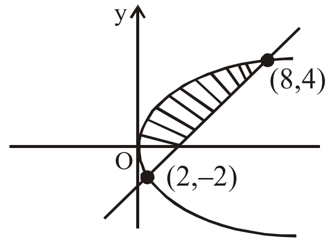
$x = 2, -1$



Area = $\int_{-1}^{2} (x+2) - x^2 \ dx = \frac{9}{2}$

10. Official Ans. by NTA (2)

Sol.



$y^2 = 2x$

$x - y - 4 = 0$

$(x-4)^2 = 2x$

$x^2 + 16 - 8x - 2x = 0$

$x^2 - 10x + 16 = 0$

$x = 8, 2$

$y = 4, -2$

$A = \int_{-2}^{4}\left(y+4-\frac{y^2}{2}\right)dy$

$= \left.\frac{y^2}{2}\right|_{-2}^{4} + 4y\big|_{-2}^{4} - \left.\frac{y^3}{6}\right|_{-2}^{4}$

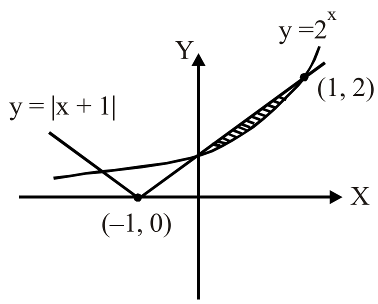
$= (8-2) + 4(6) - \frac{1}{6}(64+8)$

$= 6 + 24 - 12 = 18$

11. Official Ans. by NTA (1)

Sol. Required Area

$\int_0^1 \left((x+1) - 2^x\right)dx$



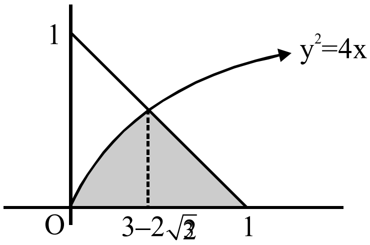
$= \left(\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right)_0^1$

$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(0 + 0 - \frac{1}{\ln 2}\right)$

$= \frac{3}{2} - \frac{1}{\ln 2}$

12. Official Ans. by NTA (3)

Sol. $\{(x, y) : y^2 \le 4x,\ x + y \le 1,\ x \ge 0,\ y \ge 0\}$



$$A \int_0^{3-2\sqrt{2}} 2\sqrt{x}dx + \frac{1}{2}\left(1-\left(3-2\sqrt{2}\right)\right)\left(1-(3-2\sqrt{2})\right)$$

$$= \frac{2[x^{3/2}]_0^{3-2\sqrt{2}}}{3/2} + \frac{1}{2}\left(2\sqrt{2}-2\right)\left(2\sqrt{2}-2\right)$$

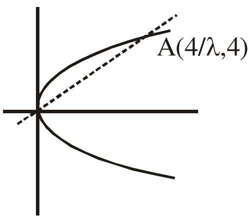
$$= \frac{8\sqrt{2}}{3} + \left(-\frac{10}{3}\right)$$

$a = \dfrac{8}{3}$, $b = -\dfrac{10}{3}$

$a - b = 6$

13. Official Ans. by NTA (1)

Sol.



$$\text{Area } = \frac{1}{9} = \int_0^{\frac{4}{\lambda}} \left(\sqrt{4\lambda x} - \lambda x\right)dx$$

$\Rightarrow \lambda = 24$

MATRIX

1. Ans. (1)

Here, $AA^T = I$

$$\Rightarrow A^{-1} = A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\text{Also, } A^{-n} = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$$

$$\therefore\ A^{-50} = \begin{bmatrix} \cos(50)\theta & \sin(50)\theta \\ -\sin(50)\theta & \cos(50)\theta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

2. Ans. (3)

$$|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$= e^{-t}[5\cos^2 t + 5\sin^2 t]\ \forall\ t \in R$

$= 5e^{-t} \ne 0\ \forall\ t \in R$

3. Ans. (4)

$$A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}\ (b > 0)$$

$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1\ (b^2 - b^2 - 1)$

$|A| = 2(b^2 + 2) - b^2 - 1$

$|A| = b^2 + 3$

$$\frac{|A|}{b} = b + \frac{3}{b} \Rightarrow \frac{b + \frac{3}{b}}{2} \ge \sqrt{3}$$

$$b + \frac{3}{b} \ge 2\sqrt{3}$$

Option (4)

4. Ans. (1)

A is orthogonal matrix

$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \dfrac{1}{\sqrt{2}}$

5. Ans. (2)

$|A|^2.|B| = 8$ and $\dfrac{|A|}{|B|} = 8 \Rightarrow |A| = 4$ and $|B| = \dfrac{1}{2}$

$\therefore\ \det(BA^{-1}.B^T) = \dfrac{1}{4} \times \dfrac{1}{4} = \dfrac{1}{16}$

6. Ans. (4)

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 9+9+9 & 3+3 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3+3 & 1 & 0 \\ 6.9 & 3+3+3 & 1 \end{bmatrix}$$

$$P^n = \begin{bmatrix} 1 & 0 & 0 \\ 3n & 1 & 0 \\ \frac{n(n+1)}{2}3^2 & 3n & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5.3 & 1 & 0 \\ 15.9 & 5.3 & 1 \end{bmatrix}$$

$Q = P^5 + I_3$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = 10$$

Aliter

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$P = I + X$

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}$$

$X^3 = 0$

$P^5 = I + 5X + 10X^2$

$Q = P^5 + I = 2I + 5X + 10X^2$

$$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 15 & 0 & 0 \\ 15 & 15 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 90 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow Q = \begin{pmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{pmatrix}$$

7. Ans (2)

$$|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$= 2(1 + \sin^2\theta)$

$$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \Rightarrow \frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow 0 \le \sin^2\theta < \frac{1}{2}$$

$\therefore |A| \in [2, 3)$

8. Official Ans. by NTA (4)

Sol. put $b = \frac{2+c}{2}$ in determinant of A

$$|A| = \frac{c^3 - 6c^2 + 12c - 8}{4} \in [2,16]$$

$\Rightarrow (c-2)^3 \in [8, 64]$

$\Rightarrow c \in [4, 6]$

9. Official Ans. by NTA (4)

Sol. $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3\alpha & -\sin 3\alpha \\ \sin 3\alpha & \cos 3\alpha \end{bmatrix}$$

Similarly

$$A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\Rightarrow \quad \cos 32\alpha = 0$ & $\sin 32\alpha = 1$

$$\Rightarrow \quad 32\alpha = (4n+1)\frac{\pi}{2}, n \in I$$

$$\alpha = (4n+1)\frac{\pi}{64}, n \in I$$

$$\alpha = \frac{\pi}{64} \text{ for } n = 0$$

10. Official Ans. by NTA (1)

Sol. $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}.....\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 1+2+3+.....+n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \dfrac{n(n-2)}{2}=78 \Rightarrow n = 13, -12\text{(reject)}$

$\therefore$ We have to find inverse of $\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}$

$\therefore \begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

11. Official Ans. by NTA (4)

Sol. $A^TA = 3I_3$

$$\begin{pmatrix}0 & 2x & 2x\\2y & y & -y\\1 & -1 & 1\end{pmatrix}\begin{pmatrix}0 & 2y & 1\\2x & y & -1\\2x & -y & 1\end{pmatrix}=\begin{pmatrix}3 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{pmatrix}$$

$$\begin{pmatrix}8x^2 & 0 & 0\\0 & 6y^2 & 0\\0 & 0 & 3\end{pmatrix}=\begin{bmatrix}3 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{bmatrix}$$

$8x^2 = 3$

$6y^2 = 3$

$x^2 = 3/8$

$y^2 = 1/2$

$x=\pm\sqrt{\dfrac{3}{8}};y=\pm\sqrt{\dfrac{1}{2}}$

12. Official Ans. by NTA (3)

Sol. $|B| = 5(-5) -2\alpha(-\alpha) -2\alpha$

$= 2\alpha^2 - 2\alpha -25$

$1 + |A| = 0$

$\alpha^2 - \alpha - 12 = 0$

Sum = 1

13. Official Ans. by NTA (3)

Sol. $A = A'$, $B = -B'$

$A + B = \begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$ ...(1)

$A' + B' = \begin{bmatrix}2 & 5\\3 & -1\end{bmatrix}$

$A - B = \begin{bmatrix}2 & 5\\3 & -1\end{bmatrix}$ ...(2)

After adding Eq. (1) & (2)

$A=\begin{bmatrix}2 & 4\\4 & -1\end{bmatrix},\ B=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

$AB = \begin{bmatrix}4 & -2\\-1 & -4\end{bmatrix}$

VECTORS

1. Ans. (1)

$\vec{a}\times\vec{c}=-\vec{b}$

$(\vec{a}\times\vec{c})\times\vec{a}=-\vec{b}\times\vec{a}$

$\Rightarrow (\vec{a}\times\vec{c})\times\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}.\vec{a})\vec{c}-(\vec{c}.\vec{a})\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow 2\vec{c}-4\vec{a}=\vec{a}\times\vec{b}$

Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\1 & -1 & 0\\1 & 1 & 1\end{vmatrix}=-\hat{i}-\hat{j}+2\hat{k}$

So, $2\vec{c}=4\hat{i}-4\hat{j}-\hat{i}-\hat{j}+2\hat{k}$

$=3\hat{i}-5\hat{j}+2\hat{k}$

$\Rightarrow \vec{c}=\dfrac{3}{2}\hat{i}-\dfrac{5}{2}\hat{j}+\hat{k}$

$|\vec{c}|=\sqrt{\dfrac{9}{4}+\dfrac{25}{4}+1}=\sqrt{\dfrac{38}{4}}=\sqrt{\dfrac{19}{2}}$

$|\vec{c}|^2=\dfrac{19}{2}$

2. Ans. (4)

Projection of $\vec{b}$ on $\vec{a}=\dfrac{\vec{a}.\vec{b}}{|\vec{a}|}=|\vec{a}|$

$\Rightarrow b_1 + b_2 = 2$ ...(1)

and $(\vec{a}+\vec{b})\perp\vec{c}\Rightarrow(\vec{a}+\vec{b}).\vec{c}=0$

$\Rightarrow 5b_1 + b_2 = -10$ ...(2)

from (1) and (2) $\Rightarrow b_1 = -3$ and $b_2 = 5$

then $|\vec{b}|=\sqrt{b_1^2+b_2^2+2}=6$

3. Ans. (2)

$4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}=4\hat{i}+2\lambda_1\hat{j}+6\hat{k}$

$\Rightarrow 3 - \lambda_2 = 2\lambda_1 \Rightarrow 2\lambda_1 + \lambda_2 = 3$ ....(1)

Given $\vec{a}.\vec{c}=0$

$\Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$

$\Rightarrow 2\lambda_1 + \lambda_3 = -1$ .....(2)

Now $(\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1)$

Now check the options, option (2) is correct

4. Ans. (4)

$\vec{\alpha}=(\lambda-2)\vec{\alpha}+\vec{b}$

$\vec{\beta}=(4\lambda-2)\vec{\alpha}+3\vec{b}$

$\frac{\lambda-2}{4\lambda-2}=\frac{1}{3}$

$3\lambda - 6 = 4\lambda - 2$

$\boxed{\lambda=-4}$

$\therefore$ Option (4)

5. Ans. (3)

$\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=0$

$$\Rightarrow \begin{vmatrix}1 & 2 & 4\\ 1 & \lambda & 4\\ 2 & 4 & \lambda^2-1\end{vmatrix}=0$$

$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$\Rightarrow \lambda^2(\lambda - 2) - 9(\lambda - 2) = 0$

$\Rightarrow (\lambda - 3)(\lambda + 3)(\lambda - 2) = 0$

$\Rightarrow \lambda = 2, 3, -3$

So, $\lambda = 2$ (as $\vec{a}$ is parallel to $\vec{c}$ for $\lambda = \pm 3$)

Hence $\vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 4\\ 2 & 4 & 3\end{vmatrix}=-10\hat{i}+5\hat{j}$

6. Ans. (2)

Angle bisector is $x - y = 0$

$\Rightarrow \frac{\left|\beta-(1-\beta)\right|}{\sqrt{2}}=\frac{3}{\sqrt{2}}$

$\Rightarrow \left|2\beta-1\right|=3$

$\Rightarrow \beta=2 \text{ or } -1$

7. Ans. (3)

$$\begin{vmatrix}\mu & 1 & 1\\ 1 & \mu & 1\\ 1 & 1 & \mu\end{vmatrix}=0$$

$\mu(\mu^2 - 1)-1(\mu-1) + 1(1-\mu) = 0$

$\mu^3 - \mu - \mu + 1 + 1\ \mu = 0$

$\mu^3 - 3\mu + 2 = 0$

$\mu^3 - 1 - 3(\mu-1) = 0$

$\mu = 1, \mu^2 + \mu - 2 = 0$

$\mu = 1, \mu = -2$

sum of distinct solutions $= -1$

8. Ans. (2)

$\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right).\vec{c}=\frac{1}{2}\vec{b}$

$\because$ $\vec{b}$ & $\vec{c}$ are linearly independent

$\therefore$ $\vec{a}.\vec{c}=\frac{1}{2}$ & $\vec{a}.\vec{b}=0$

(All given vectors are unit vectors)

$\therefore$ $\vec{a}\wedge\vec{c}=60^\circ$ & $\vec{a}\wedge\vec{b}=90^\circ$

$\therefore |\alpha - \beta| = 30^\circ$

9. Official Ans. by NTA (2)

Sol. Vector perpendicular to plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ & $\hat{i}+2\hat{j}+3\hat{k}$ is parallel to vector

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}=\hat{i}-2\hat{j}+\hat{k}$$

$\therefore$ Required magnitude of projection

$$=\frac{\left|(2\hat{i}+3\hat{j}+\hat{k}).(\hat{i}-2\hat{j}+\hat{k})\right|}{\left|\hat{i}-2\hat{j}+\hat{k}\right|}$$

$$=\frac{|2-6+1|}{\left|\sqrt{6}\right|}=\frac{3}{\sqrt{6}}=\sqrt{\frac{3}{2}}$$

10. Official Ans. by NTA (4)

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & x\\ 1 & -1 & 1\end{vmatrix}$$

$=(2+x)\hat{i}+(x-3)\hat{j}-5k$

$\left|\vec{a}\times\vec{b}\right|=\sqrt{4+x^2+4x+x^2+9-6x+25}$

$=\sqrt{2x^2-2x+38}$

$\Rightarrow \left|\vec{a}\times\vec{b}\right|\geq\sqrt{\frac{75}{2}}$

$\Rightarrow \left|\vec{a}\times\vec{b}\right|\geq 5\sqrt{\frac{3}{2}}$

11. Official Ans. by NTA (3)

Sol. $\vec{\alpha} = 3\hat{i} + \hat{j}$

$\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$

$\vec{\beta}_1 = \lambda\left(3\hat{i} + \hat{j}\right), \vec{\beta}_2 = \lambda\left(3\hat{i} + \hat{j}\right) - 2\hat{i} + j - 3\hat{k}$

$\vec{\beta}_2.\vec{\alpha} = 0$

$(3\lambda - 2).3 + (\lambda + 1) = 0$

$9\lambda - 6 + \lambda + 1 = 0$

$\lambda = \frac{1}{2}$

$\Rightarrow \vec{\beta}_1 = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\Rightarrow \vec{\beta}_2 = -\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

Now $\vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -3 \end{vmatrix}$

$= \hat{i}\left(-\frac{3}{2} - 0\right) - \hat{j}\left(-\frac{9}{2} - 0\right) + \hat{k}\left(\frac{9}{4} + \frac{1}{4}\right)$

$= -\frac{3}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{5}{2}\hat{k}$

$= \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

Aliter :

$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2 \Rightarrow \vec{\beta}.\hat{\alpha} = \vec{\beta}_1.\hat{\alpha} = \left|\vec{\beta}_1\right|$

$\Rightarrow \vec{\beta}_1 = \left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha}$

$\Rightarrow \vec{\beta}_2 = \left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha} - \vec{\beta}$

$\Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = -\left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha} \times \vec{\beta}$

$= \frac{-5}{10}\left(3\hat{i} + \hat{j}\right) \times \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$= \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

12. Official Ans. by NTA (3)

Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

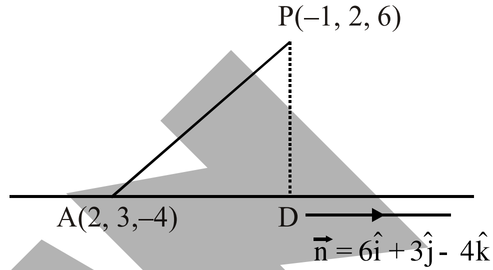
$\frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$

$\cos^2\gamma = 1 - \frac{3}{4} = \frac{1}{4}$

$\cos^2\gamma = \pm\frac{1}{2} \Rightarrow \gamma = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$

13. Official Ans. by NTA (1)

Sol.



$AD = \left|\frac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \sqrt{61}$

$\Rightarrow PD = \sqrt{AP^2 - AD^2} = \sqrt{110 - 61} = 7$

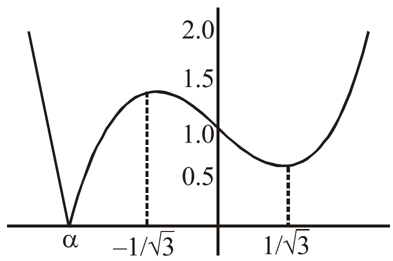
14. Official Ans. by NTA (3)

ALLEN Ans. Bonus

Sol. Volume of parallelopiped $= \left\|\begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix}\right\|$

$f(\lambda) = |\lambda^3 - \lambda + 1|$

Its graph as follows



where $\alpha \approx -1.32$

$\because$ Question is asking minimum value of volume of parallelopiped & corresponding value of λ; the minimum value is zero, $\because$ cubic always has atleast one real root. Hence answer to the question must be root of cubic $\lambda^3 - \lambda + 1 = 0$. None of the options satisfies the cubic.

Hence Question must be Bonus.

In JEE (Screening) 2003 same Question was asked and answer was given to be none of these, where the options were :

(A) –3 (B) 3

(C) $\frac{1}{\sqrt{3}}$ (D) none of these

15. Official Ans. by NTA (3)

Sol. $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = 2(\vec{b}\times\vec{a})$

$$= 2\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -2\\ 3 & 2 & 2\end{vmatrix} = 2(8\hat{i}-8\hat{j}+4\hat{k})$$

Required vector = $\pm 12\frac{(2\hat{i}-2\hat{j}-\hat{k})}{3}$

$= \pm 4(2\hat{i}-2\hat{j}-\hat{k})$

16. Official Ans. by NTA (4)

Sol. $\begin{vmatrix}\alpha & 1 & 3\\ 2 & 1 & -4\\ \alpha & -2 & 3\end{vmatrix} = 0$

$\Rightarrow 3\alpha^2 + 18 = 0$

$\Rightarrow \alpha \in \phi$

3D

1. Ans. (3)

Equation of plane

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow (1+2\lambda)x + (1+3\lambda)y + (1-\lambda)z - 1 + 4\lambda = 0$

dr's of normal of the plane are

$1 + 2\lambda,\ 1 + 3\lambda,\ 1 - \lambda$

Since plane is parallel to y - axis, $1 + 3\lambda = 0$

$\Rightarrow \lambda = -1/3$

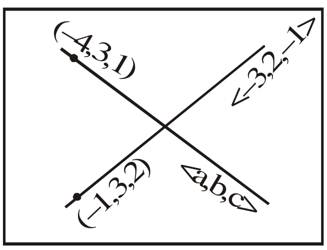
So the equation of plane is

$x + 4z - 7 = 0$

Point (3, 2, 1) satisfies this equation

Hence Answer is (3)

2. Ans. (2)



Normal vector of plane containing two intersecting lines is parallel to vector.

$$(\vec{V}_1) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 0 & 1\\ -3 & 2 & -1\end{vmatrix}$$

$= -2\hat{i} + 6\hat{k}$

$\therefore$ Required line is parallel to vector

$$(\vec{V}_2) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ -2 & 0 & 6\end{vmatrix} = 3\hat{i}-\hat{j}+\hat{k}$$

$\Rightarrow$ Required equation of line is

$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

3. Ans. (2)

Vector along the normal to the plane containing the lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$

is $(8\hat{i}-\hat{j}-10\hat{k})$

vector perpendicular to the vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $8\hat{i}-\hat{j}-10\hat{k}$ is $26\hat{i}-52\hat{j}+26\hat{k}$

so, required plane is

$26x - 52y + 26z = 0$

$x - 2y + z = 0$

4. Ans. (2)

Line $x = ay + b,\ z = cy + d \Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$

Line $x = a'z + b',\ y = c'z + d'$

$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1}$

Given both the lines are perpendicular

$\Rightarrow aa' + c' + c = 0$

5. Ans. (4)

Let $\vec{n}$ be the normal vector to the plane passing through (4, –1, 2) and parallel to the lines L_1 & L_2

then $\vec{n} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 2\\ 1 & 2 & 3\end{vmatrix}$

$\therefore\ \vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$\therefore$ Equation of plane is

$-1(x-4) - 1(y+1) + 1(z-2) = 0$

$\therefore\ x + y - z - 1 = 0$

Now check options

6. Ans. (3)

Let point A is

$(1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$

and point B is (3, 2, 6)

then $\overrightarrow{AB} = (2+3\mu)\hat{i} + (3-\mu)\hat{j} + (4-5\mu)\hat{k}$

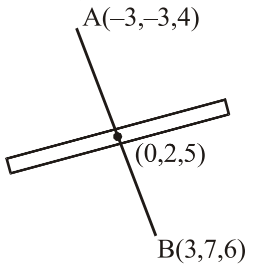
which is parallel to the plane $x - 4y + 3z = 1$

$\therefore\ 2 + 3\mu - 12 + 4\mu + 12 - 15\mu = 0$

$8\mu = 2$

$\mu = \frac{1}{4}$

7. Ans. (2)



$\vec{n} = 3\hat{i} + 5\hat{j} + \hat{k}$

$p : 3(x-0) + 5(y-2) + 1(z-5) = 0$

$3x + 5y + z = 15$

$\therefore$ Option (2)

8. Ans. (3)

General point on the given line is

$x = 2\lambda + 4$

$y = 2\lambda + 5$

$z = \lambda + 3$

Solving with plane,

$2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$

$5\lambda + 12 = 2$

$5\lambda = -10$

$\boxed{\lambda = -2}$

$\therefore$ Option (3)

9. Ans. (2, 4)

Let the equation of plane be

$a(x-0) + b(y+1) + c(z-0) = 0$

It passes through (0,0,1) then

$b + c = 0$ ...(1)

Now $\cos\frac{\pi}{4} = \frac{a(0)+b(1)+c(-1)}{\sqrt{2}\sqrt{a^2+b^2+c^2}}$

$\Rightarrow a^2 = -2bc$ and $b = -c$

we get $a^2 = 2c^2$

$\Rightarrow\ a = \pm\sqrt{2}\,c$

$\Rightarrow$ direction ratio (a, b, c) = $(\sqrt{2}, -1, 1)$ or $(\sqrt{2}, 1, -1)$

10. Ans. (1)

The normal vector of required plane

$= (2\hat{i} - \hat{j} + 3\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$= -8\hat{i} + 8\hat{j} + 8\hat{k}$

So, direction ratio of normal is (–1, 1, 1)

So required plane is

$-(x-3) + (y+2) + (z-1) = 0$

$\Rightarrow -x + y + z + 4 = 0$

Which is satisfied by (2, 0, –2)

11. Ans. (4)

Normal vector of plane

$= \begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 4 & -4 & 4 \end{vmatrix} = -4(5\hat{i} + 2\hat{j} - 3\hat{k})$

equation of plane is $5(x-7) + 2y - 3(z-6) = 0$

$5x + 2y - 3z = 17$

12. Ans. (3)

Point on L_1 $(\lambda + 3, 3\lambda - 1, -\lambda + 6)$

Point on L_2 $(7\mu - 5, -6\mu + 2, 4\mu + 3)$

$\Rightarrow \lambda + 3 = 7\mu - 5$ ...(i)

$3\lambda - 1 = -6\mu + 2$ ...(ii)

$\Rightarrow \lambda = -1,\ \mu = 1$

point R(2,–4,7)

Reflection is (2,–4,–7)

13. Ans. (1)

$\begin{vmatrix} i & j & k \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$

$\hat{i}(35-28) - \hat{j}(21.7) + \hat{k}(12-5)$

$7\hat{i} - 14\hat{j} + 7\hat{k}$

$\hat{i} - 2\hat{j} + \hat{k}$

$1(x + 2) - 2(y - 2) + 1\ (z+15) = 0$

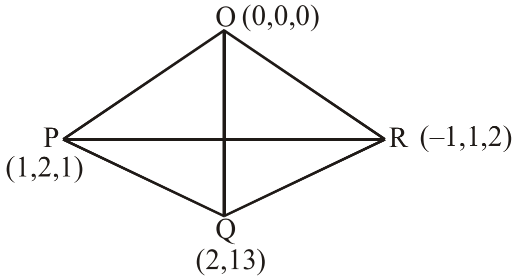
$x - 2y + z + 11 = 0$

$$\frac{11}{\sqrt{4+1+1}} = \frac{11}{\sqrt{6}}$$

14. Ans. (2)

$\overrightarrow{OP} \times \overrightarrow{OQ} = (\hat{i} + 2\hat{j} + \hat{k}) \times (2\hat{i} + \hat{j} + 3\hat{k})$

$5\hat{i} - \hat{j} - 3\hat{k}$



$\overrightarrow{PQ} \times \overrightarrow{PR} = (\hat{i} - \hat{j} + 2\hat{k}) \times (-2\hat{i} - \hat{j} + \hat{k})$

$\hat{i} - 5\hat{j} - 3\hat{k}$

$$\cos\theta = \frac{5+5+9}{\left(\sqrt{25+9+1}\right)^2} = \frac{19}{35}$$

15. Ans (3)

DR's of line are 2, 1, –2

normal vector of plane is $\hat{i} - 2\hat{j} - k\hat{k}$

$$\sin\alpha = \frac{(2\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} - 2\hat{j} - k\hat{k})}{3\sqrt{1+4+k^2}}$$

$$\sin\alpha = \frac{2k}{3\sqrt{k^2+5}} \quad ....(1)$$

$$\cos\alpha = \frac{2\sqrt{2}}{3} \quad ....(2)$$

$(1)^2 + (2)^2 = 1 \Rightarrow k^2 = \dfrac{5}{3}$

16. Ans (2)

All four points are coplaner so

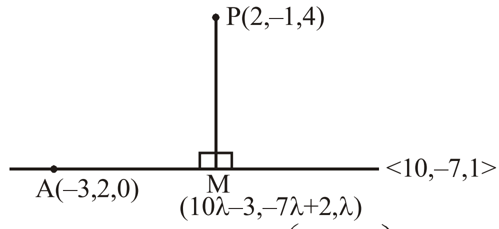
$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & -\lambda^2+1 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$(\lambda^2 + 1)^2\ (3 - \lambda^2) = 0$

$\lambda = \pm\sqrt{3}$

17. Official Ans. by NTA (2)

Sol.



$$\equiv \left(2, \frac{-3}{2}, \frac{1}{2}\right)$$

Now, $\overrightarrow{MP}.\left(10\hat{i} - 7\hat{j} + \hat{k}\right) = 0$

$\Rightarrow \quad \lambda = \dfrac{1}{2}$

$\therefore$ Length of perpendicular

$$(= PM) = \sqrt{0 + \frac{1}{4} + \frac{49}{4}}$$

$$= \sqrt{\frac{50}{4}} = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}},$$

which is greater than 3 but less than 4.

18. Official Ans. by NTA (2)

Sol. The required plane is

$(2x - y - 4) + \lambda(y + 2z - 4) = 0$

it passes through (1, 1, 0)

$\Rightarrow \quad (2 - 1 - 4) + \lambda(1 - 4) = 0$

$\Rightarrow \quad -3 - 3\lambda = 0 \quad \Rightarrow \quad \lambda = -1$

$\Rightarrow \quad x - y - z = 0$

19. Official Ans. by NTA (3)

Sol. Let the plane be

$(x + y + z - 1) + \lambda\ (2x + 3y + 4z - 5) = 0$

$\Rightarrow (2\lambda + 1)x + (3\lambda + 1)y + (4\lambda + 1)z - (5\lambda + 1) = 0$

$\perp$ to the plane $x - y + z = 0$

$\Rightarrow \lambda = -\dfrac{1}{3}$

$\Rightarrow$ the required plane is $x - z + 2 = 0$

20. Official Ans. by NTA (1)

Sol. $\dfrac{4}{2} = \dfrac{-y}{y+3} = \dfrac{10-z}{z-4}$

$\Rightarrow z = 6\ \&\ y = -2$

$\Rightarrow R(4, -2, 6)$

dist. from origin $= \sqrt{16+4+36} = 2\sqrt{14}$

21. Official Ans. by NTA (2)

Sol. Let ax + by + cz = 1 be the equation of the plane

$\Rightarrow 0 - b + 0 = 1$

$\Rightarrow b = -1$

0 + 0 + c = 1

$\Rightarrow c = 1$

$$\cos\theta = \left|\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right|$$

$$\frac{1}{\sqrt{2}} = \frac{|0-1-1|}{\sqrt{(a^2+1+1)}\sqrt{0+1+1}}$$

$\Rightarrow a^2 + 2 = 4$

$\Rightarrow a = \pm\sqrt{2}$

$\Rightarrow \pm\sqrt{2}x - y + z = 1$

Now for –sign

$-\sqrt{2}.\sqrt{2} - 1 + 4 = 1$

option (2)

22. Official Ans. by NTA (1)

Sol. Any point on the given line can be

$(1 + 2\lambda\ ,\ -1 + 3\lambda,\ 2 + 4\lambda)\ ;\ \lambda \in R$

Put in plane

$1 + 2\lambda + (-2 + 6\lambda) + (6 + 12\lambda) = 15$

$20\lambda + 5 = 15$

$20\lambda = 10$

$\lambda = \frac{1}{2}$

$\therefore$ Point $\left(2, \frac{1}{2}, 4\right)$

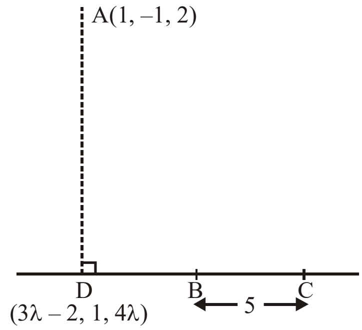
Distance from origin

$$= \sqrt{4 + \frac{1}{4} + 16} = \frac{\sqrt{16+1+64}}{2} = \frac{\sqrt{81}}{2}$$

$$= \frac{9}{2}$$

23. Official Ans. by NTA (2)

Sol.



$\overrightarrow{AD}\cdot(3\hat{i} + 4\hat{k}) = 0$

$3(3\lambda - 3) + 0 + 4(4\lambda - 2) = 0$

$(9\lambda - 9) + (16\lambda - 8) = 0$

$25\lambda = 17 \Rightarrow \lambda = \frac{17}{25}$

$$\therefore\ \overrightarrow{AD} = \left(\frac{51}{25} - 3\right)\hat{i} + 2\hat{j} + \left(\frac{68}{25} - 2\right)\hat{k}$$

$$= \frac{24}{25}\hat{i} + 2\hat{j} + \frac{18}{25}\hat{k}$$

$$|\overrightarrow{AD}| = \sqrt{\frac{576}{625} + 4 + \frac{324}{625}}$$

$$= \sqrt{\frac{900}{625} + 4} = \sqrt{\frac{3400}{625}}$$

$$= \sqrt{34}\cdot\frac{10}{25} = \frac{2}{5}\sqrt{34}$$

Area of $\Delta = \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34}$

24. Official Ans. by NTA (4)

Sol. $\lambda(x + y + z - 6) + 2x + 3y + z + 5 = 0$

$(\lambda + 2)x + (\lambda + 3)y + (\lambda + 1)z + 5 - 6\lambda = 0$

$\lambda + 1 = 0 \Rightarrow \lambda = -1$

P : x + 2y + 11 = 0

perpendicular distance $= \frac{11}{\sqrt{5}}$

25. Official Ans. by NTA (2)

Sol. $x + 3y + \lambda z - \mu = p\ (x + y + z - 5) + q\ (x + 2y + 2z - 6)$

on comparing the coefficient;

p + q = 1 and p + 2q = 3

$\Rightarrow$ (p,q) = (–1,2)

Hence $x + 3y + \lambda z - \mu = x + 3y + 3z - 7$

$\Rightarrow \lambda = 3,\ \mu = 7$

26. Official Ans. by NTA (3)

Sol. G is the centroid of Δ ABC

G $\equiv$ (2,4,2)

$\overrightarrow{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

$\overrightarrow{OA} = 3\hat{i} - \hat{k}$

$\cos(\angle GOA) = \dfrac{\overrightarrow{OG} \cdot \overrightarrow{OA}}{|\overrightarrow{OG}||\overrightarrow{OA}|} = \dfrac{1}{\sqrt{15}}$

27. Official Ans. by NTA (1)

Sol. One of the point on line is P(0, 1, –1) and given point is Q(β, 0, β).

So, $\overrightarrow{PQ} = \beta\hat{i} - \hat{j} + (\beta+1)\hat{k}$

Hence, $\beta^2 + 1 + (\beta+1)^2 - \dfrac{(\beta-\beta-1)^2}{2} = \dfrac{3}{2}$

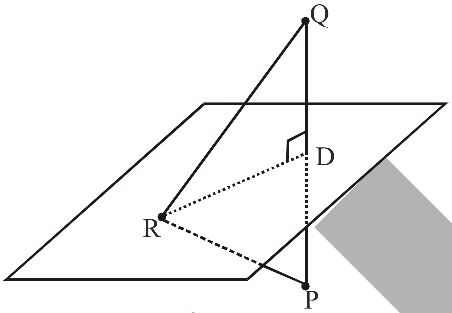
$\Rightarrow 2\beta^2 + 2\beta = 0$

$\Rightarrow \beta = 0, -1$

$\Rightarrow \beta = -1$ (as $\beta \neq 0$)

28. Official Ans. by NTA (4)

Sol. R lies on the plane.



$DQ = \dfrac{|1-12-2|}{\sqrt{9+1+16}} = \dfrac{13}{\sqrt{26}} = \sqrt{\dfrac{13}{2}}$

$\Rightarrow PQ = \sqrt{26}$

Now, $RQ = \sqrt{9+1} = \sqrt{10}$

$\Rightarrow RD = \sqrt{10 - \dfrac{13}{2}} = \sqrt{\dfrac{7}{2}}$

Hence, ar(ΔPQR) $= \dfrac{1}{2} \times \sqrt{26} \times \sqrt{\dfrac{7}{2}} = \dfrac{\sqrt{91}}{2}$.

29. Official Ans. by NTA (1)

Sol. Let point P on the line is $(2\lambda + 1, -\lambda - 1, \lambda)$ foot of perpendicular Q is given by

$\dfrac{x - 2\lambda - 1}{1} = \dfrac{y + \lambda + 1}{1} = \dfrac{z - \lambda}{1} = \dfrac{-(2\lambda - 3)}{3}$

$\because$ Q lies on x + y + z = 3 & x – y + z = 3

$\Rightarrow$ x + z = 3 & y = 0

$y = 0 \Rightarrow \lambda + 1 = \dfrac{-2\lambda + 3}{3} \Rightarrow \lambda = 0$

$\Rightarrow$ Q is (2, 0, 1)

30. Official Ans. by NTA (3)

Sol. 4x – 2y + 4z + 6 = 0

$\dfrac{|\lambda - 6|}{\sqrt{16+4+16}} = \left|\dfrac{\lambda - 6}{6}\right| = \dfrac{1}{3}$

$|\lambda - 6| = 2$

$\lambda = 8, 4$

$\dfrac{|\mu - 3|}{\sqrt{4+4+1}} = \dfrac{2}{3}$

$|\mu - 3| = 2$

$\mu = 5, 1$

$\therefore$ Maximum value of $(\mu + \lambda) = 13$.

31. Official Ans. by NTA (1)

Sol. $\dfrac{x-2}{3} = \dfrac{y+1}{2} = \dfrac{z-1}{-1} = \lambda$

$x = 3\lambda + 2,\ y = 2\lambda - 1,\ z = -\lambda + 1$

Intersection with plane 2x + 3y – z + 13 = 0

$2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$

$13\lambda + 13 = 0$ $\boxed{\lambda = -1}$

$\therefore$ P(–1, –3, 2)

Intersection with plane

3x + y + 4z = 16

$3(3\lambda+2) + (2\lambda-1) + 4(-\lambda+1) = 16$

$\lambda = 1$

Q(5, 1, 0)

$PQ = \sqrt{6^2 + 4^2 + 2^2} = \sqrt{56} = 2\sqrt{14}$

32. Official Ans. by NTA (1)

Sol. perpendicular vector to the plane

$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{i} + 3\hat{j} + 3\hat{k}$

Eq. of plane

– 3 (x – 1) + 3 (y – 1) + 3z = 0

$\Rightarrow$ x – y – z = 0

$d_{(2,1,4)} = \dfrac{|2-1-4|}{\sqrt{1^2+1^2+1^2}} = \sqrt{3}$

33. Official Ans. by NTA (2)

Sol. equation of bisector of angle

$$\frac{2x-y+2z-4}{3}=\pm\frac{x+2y+2z-2}{3}$$

(+) gives $x - 3y = 2$

(–) gives $3x + y + 4z = 6$

therefore option (ii) satisfy

PARABOLA

1. Ans. (3)

Let equation of tangent to the parabola $y^2 = 4x$ is

$y=mx+\frac{1}{m}$,

$\Rightarrow m^2x-ym+1 = 0$ is tangent to $x^2 + y^2 - 6x = 0$

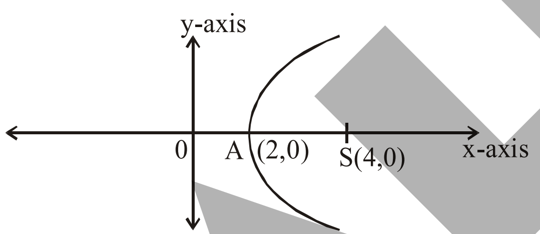
$$\Rightarrow \frac{|3m^2+1|}{\sqrt{m^4+m^2}}=3$$

$$m=\pm\frac{1}{\sqrt{3}}$$

$\Rightarrow$ tangent are $x+\sqrt{3}y+3=0$

and $x-\sqrt{3}y+3=0$

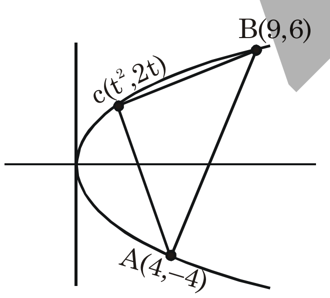
2. Ans. (3)



equation of parabola is $y^2 = 8(x - 2)$

(8, 6) does not lie on parabola.

3. Ans. (4)



Area $= 5|t^2 - t - 6| = 5\left|\left(t-\frac{1}{2}\right)^2-\frac{25}{4}\right|$

is maximum if $t=\frac{1}{2}$

4. Ans. (1,2,3,4)

Normal to these two curves are

$y = m(x - c) - 2bm - bm^3$,

$y = mx - 4am - 2am^3$

If they have a common normal

$(c + 2b) m + bm^3 = 4am + 2am^3$

Now $(4a - c - 2b) m = (b - 2a)m^3$

We get all options are correct for m = 0

(common normal x-axis)

Ans. (1), (2), (3), (4)

Remark :

If we consider question as

If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal other than x-axis, then which one of the following is a valid choice for the ordered triad (a, b, c) ?

When $m \neq 0$: $(4a - c - 2b) = (b - 2a)m^2$

$$m^2=\frac{c}{2a-b}-2>0\Rightarrow\frac{c}{2a-b}>2$$

Now according to options, option 4 is correct

5. Ans. (3)

$x^2 = 4y$

$x-\sqrt{2}y+4\sqrt{2}=0$

Solving together we get

$$x^2=4\left(\frac{x+4\sqrt{2}}{\sqrt{2}}\right)$$

$$\sqrt{2}x^2+4x+16\sqrt{2}$$

$$\sqrt{2}x^2-4x-16\sqrt{2}=0$$

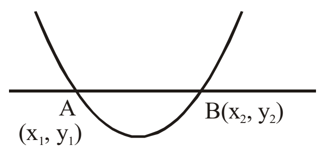
$x_1 + x_2 = 2\sqrt{2}$; $x_1x_2=\frac{-16\sqrt{2}}{\sqrt{2}}=-16$

Similarly,

$$\left(\sqrt{2}y-4\sqrt{2}\right)^2=4y$$

$2y^2 + 32 - 16y = 4y$

$2y^2 - 20y + 32 = 0$ → $y_1 + y_2 = 10$, $y_1 y_2 = 16$



$$\ell_{AB}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{\left(2\sqrt{2}\right)^2+64+(10)^2-4(16)}$$

$$=\sqrt{8+64+100-64}$$

$= \sqrt{108} = 6\sqrt{3}$

Option (3)

6. **Ans. (4)**

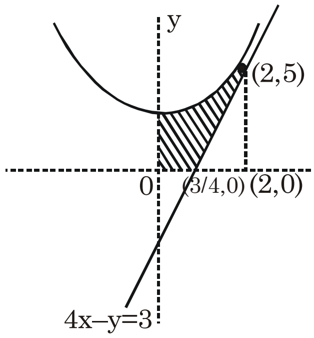
Vertex is $(a^2,0)$

$y^2 = -(x - a^2)$ and $x = 0 \Rightarrow (0, \pm 2a)$

Area of triangle is $= \frac{1}{2}.4a.\left(a^2\right) = 250$

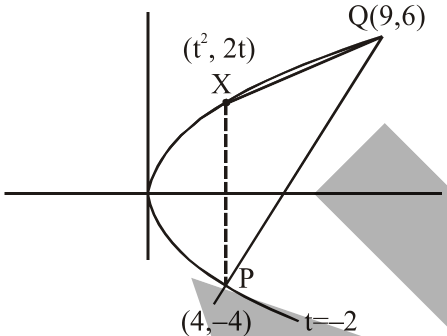
$\Rightarrow a^3 = 125$ or $a = 5$

7. **Ans. (3)**



Area $= \int_0^2 \left(x^2 + 1\right)dx - \frac{1}{2}\left(\frac{5}{4}\right)(5) = \frac{37}{24}$

8. **Ans. (1)**



$y^2 = 4x$

$2yy' = 4$

$y' = \frac{1}{t} = 2$, $t = \frac{1}{2}$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} \frac{1}{4} & 1 & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix} = \frac{125}{4}$$

9. **Ans (1)**

$x^2 = 8y$

$\Rightarrow \frac{dy}{dx} = \frac{x}{4} = \tan\theta$

$\therefore x_1 = 4\tan\theta$

$y_1 = 2\tan^2\theta$

Equation of tangent :-

$y - 2\tan^2\theta = \tan\theta\,(x - 4\tan\theta)$

$\Rightarrow x = y\cot\theta + 2\tan\theta$

10. **Official Ans. by NTA (3)**

Sol. Given $y^2 = 4x$ ...(1)

and $x^2 + y^2 = 5$ ...(2)

by (1) and (2)

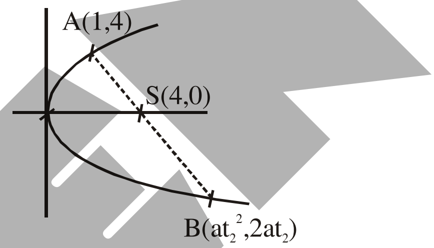
$\Rightarrow x = 1$ and $y = 2$

equation of tangent at (1,2) to $y^2 = 4x$

is $y = x + 1$

11. **Official Ans. by NTA (1)**

Sol.



$y^2 = 4ax = 16x \Rightarrow a = 4$

$A(1,4) \Rightarrow 2.4.t_1 = 4 \Rightarrow t_1 = \frac{1}{2}$

$\therefore$ length of focal chord $= a\left(t + \frac{1}{t}\right)^2$

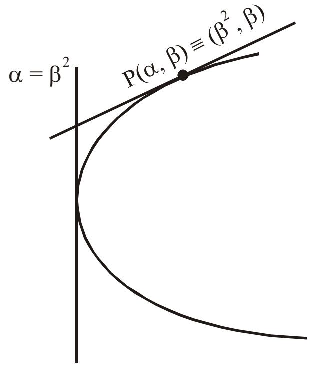
$= 4\left(\frac{1}{2} + 2\right)^2 = 4.\frac{25}{4} = 25$

12. **Official Ans. by NTA (3)**

Sol. $T: y(\beta) = \frac{1}{2}(x + \beta^2)$

$2y\beta = x + \beta^2$

$y = \left(\frac{1}{2\beta}\right)x + \frac{\beta}{2}$



$m = \frac{1}{2\beta}; C = \frac{\beta}{2}$

$\frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2} + \frac{1}{2}}$

$\frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$

$$\frac{\beta^2}{4}=\frac{1+2\beta^2}{4\beta^2}$$

$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$

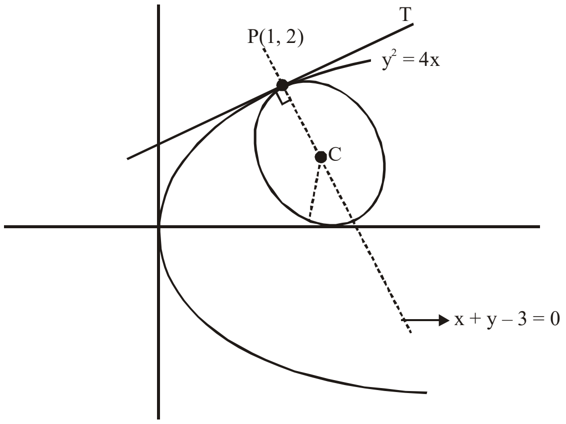
$(\beta^2 - 1)^2 = 2$

$\beta^2 - 1 = \sqrt{2}$

$\beta^2 = \sqrt{2}+1$

13. Official Ans. by NTA (2)

Sol.



Equation of circle is

$(x - 1)^2 + (y - 2)^2 + \lambda(x - y + 1) = 0$

$\Rightarrow x^2 + y^2 + x(\lambda - 2) + y(-4 - \lambda) + (5 + \lambda) = 0$

As cirlce touches x axis then $g^2 - c = 0$

$$\frac{(\lambda-2)^2}{4}=(5+\lambda)$$

$\lambda^2 + 4 - 4\lambda = 20 + 4\lambda$

$\lambda^2 - 8\lambda - 16 = 0$

$$\lambda=\frac{8\pm\sqrt{128}}{2}$$

$\lambda = 4 \pm 4\sqrt{2}$

Radius = $\left|\frac{(-4-\lambda)}{2}\right|$

Put λ and get least radius.

14. Official Ans. by NTA (3)

Sol. Tangent to $y^2 = 4\sqrt{2}\,x$ is $y = mx + \frac{\sqrt{2}}{m}$

it is also tangent to $x^2 + y^2 = 1$

$$\Rightarrow \left|\frac{\sqrt{2}/m}{\sqrt{1+m^2}}\right|=1 \quad \Rightarrow m = \pm 1$$

$\Rightarrow$ Tagent will be $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$

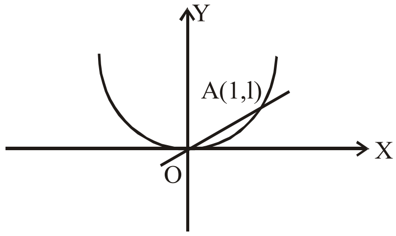
compare with $y = -ax + C$

$\Rightarrow a = \pm 1$ & $C = \pm\sqrt{2}$

15. Official Ans. by NTA (3)

Sol. Put $x - 2 = X$ & $y + 1 = Y$

$\therefore$ given curve becomes $Y = X^2$ and $Y = X$



tangent at origin is X-axis

and tangent at A(1,1) is $Y + 1 = 2X$

$\therefore$ there intersection is $\left(\frac{1}{2},0\right)$

$\therefore\ x-2=\frac{1}{2}$ & $y+1=0$

therefore $x=\frac{5}{2}, y=-1$

16. Official Ans. by NTA (4)

Sol. tangent to the parabola $y^2 = 16x$ is $y = mx + \frac{4}{m}$

solve it by curve $xy = -4$

i.e. $mx^2 + \frac{4}{m}x + 4 = 0$

condition of common tangent is $D = 0$

$\therefore m^3 = 1$

$\Rightarrow m = 1$

$\therefore$ equation of common tangent is $y = x + 4$

ELLIPSE

1. Ans. (3)

Equation of general tangent on ellipse

$$\frac{x}{a\sec\theta}+\frac{y}{b\,\mathrm{cosec}\theta}=1$$

$a = \sqrt{2}$, $b = 1$

$$\Rightarrow \frac{x}{\sqrt{2}\sec\theta}+\frac{y}{\mathrm{cosec}\theta}=1$$

Let the midpoint be (h, k)

$$h=\frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta=\frac{1}{\sqrt{2}h}$$

and $k = \dfrac{\text{cosec}\theta}{2} \Rightarrow \sin\theta = \dfrac{1}{2k}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \dfrac{1}{2h^2} + \dfrac{1}{4k^2} = 1$

$\Rightarrow \dfrac{1}{2x^2} + \dfrac{1}{4y^2} = 1$

2. Ans. (2)

$\dfrac{2b^2}{a} = 8$ and $2ae = 2b$

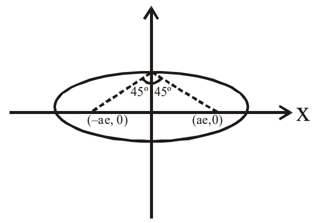
$\Rightarrow \dfrac{b}{a} = e$ and $1 - e^2 = e^2 \Rightarrow e = \dfrac{1}{\sqrt{2}}$

$\Rightarrow b = 4\sqrt{2}$ and $a = 8$

so equation of ellipse is $\dfrac{x^2}{64} + \dfrac{y^2}{32} = 1$

3. Ans. (3)

$m_{SB} \cdot m_{S'B} = -1$



$b^2 = a^2e^2$ .... (i)

$\dfrac{1}{2} S'B \cdot SB = 8$

$S'B \cdot SB = 16$

$a^2e^2 + b^2 = 16$ ..... (ii)

$b^2 = a^2(1 - e^2)$ ..... (iii)

using (i),(ii), (iii) $a = 4$

$b = 2\sqrt{2}$

$e = \dfrac{1}{\sqrt{2}}$

$\therefore \ell\,(\text{L.R}) = \dfrac{2b^2}{a} = 4$ $\boxed{\text{Ans.3}}$

4. Official Ans. by NTA (2)

Sol. $4a^2 + b^2 = 8$ ....(1)

also $\left.\dfrac{dy}{dx}\right)_{(1,2)} = -\dfrac{4x}{y} = -2$

$\Rightarrow -\dfrac{4a}{b} = \dfrac{1}{2}$

$b = -8a$

$\Rightarrow b^2 = 64a^2$

$68a^2 = 8$

$a^2 = \dfrac{2}{17}$

5. Official Ans. by NTA (3)

Sol. Let equation of ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$2a - 2b = 10$ ...(1)

$ae = 5\sqrt{3}$ ...(2)

$\dfrac{2b^2}{a} = ?$

$b^2 = a^2(1 - e^2)$

$b^2 = a^2 - a^2e^2$

$b^2 = a^2 - 25 \times 3$

$\Rightarrow b = 5$ and $a = 10$

$\therefore$ length of L.R. $= \dfrac{2(25)}{10} = 5$

6. Official Ans. by NTA (1)

Sol. Tangent at $\left(3, -\dfrac{9}{2}\right)$

$\dfrac{3x}{a^2} - \dfrac{9y}{2b^2} = 1$

Comparing this with $x - 2y = 12$

$\dfrac{3}{a^2} = \dfrac{9}{4b^2} = \dfrac{1}{12}$

we get $a = 6$ and $b = 3\sqrt{3}$

$L(LR) = \dfrac{2b^2}{a} = 9$

7. Official Ans. by NTA (3)

Sol. $3x^2 + 5y^2 = 32$

$\left.\dfrac{dy}{dx}\right|_{(2,2)} = -\dfrac{3}{5}$

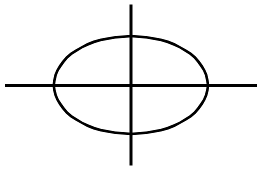
Tangent : $y - 2 = -\dfrac{3}{5}(x - 2) \Rightarrow Q\left(\dfrac{16}{3}, 0\right)$

Normal : $y - 2 = \frac{5}{3}(x - 2) \Rightarrow R\left(\frac{4}{5},0\right)$

Area is $= \frac{1}{2}(QR)\times 2 = QR = \frac{68}{15}$.

8. Official Ans. by NTA (4)

Sol. $3x^2 + 4y^2 = 12$



$\frac{x^2}{4}+\frac{y^2}{3}=1$

$x = 2\cos\theta,\ y = \sqrt{3}\sin\theta$

Let $P(2\cos\theta, \sqrt{3\sin\theta})$

Equation of normal is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

$2x\sin\theta-\sqrt{3}\cos\theta y=\sin\theta\cos\theta$

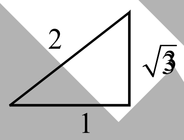
Slope $\frac{2}{\sqrt{3}}\tan\theta=-2 \quad \therefore \tan\theta=-\sqrt{3}$

Equation of tangent is

it passes through (4, 4)

$3x\cos\theta + 2\sqrt{3}\sin\theta\, y = 6$

$12\cos\theta + 8\sqrt{3}\sin\theta = 6$



$\cos\theta=-\frac{1}{2},\ \sin\theta=\frac{\sqrt{3}}{2} \quad \therefore \theta = 120°$

Hence point P is $(2\cos 120°, \sqrt{3}\sin 120°)$

$P\left(-1, \frac{3}{2}\right)$, Q (4, 4)

$PQ=\frac{5\sqrt{5}}{2}$

9. Official Ans. by NTA (4)

Sol. given that be = 2 and a = 2

(here a < b)

$\because a^2 = b^2(1 - e^2)$

$\therefore b^2 = 8$

$\therefore$ equation of ellipse $\frac{x^2}{4}+\frac{y^2}{8}=1$

HYPERBOLA

1. Ans. (2)

$e=\sqrt{1+\tan^2\theta}=\sec\theta$

As, $\sec\theta > 2 \Rightarrow \cos\theta < \frac{1}{2}$

$\Rightarrow \theta \in (60°, 90°)$

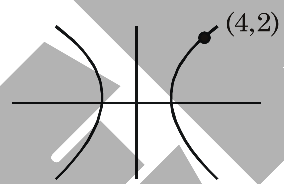
Now, $\ell(L{\cdot}R) = \frac{2b^2}{a}=2\frac{(1-\cos^2\theta)}{\cos\theta}$

$=2(\sec\theta - \cos\theta)$

Which is strictly increasing, so

ℓ (L.R) $\in(3,\infty)$.

2. Ans. (1)



$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$2a = 4 \Rightarrow a = 2$

$\frac{x^2}{4}-\frac{y^2}{b^2}=1$

Passes through (4,2)

$4-\frac{4}{b^2}=1\Rightarrow b^2=\frac{4}{3}\Rightarrow e=\frac{2}{\sqrt{3}}$

3. Ans. (3)

Hyperbola $\frac{x^2}{5}-\frac{y^2}{4}=1$

slope of tangent = 1

equation of tangent $y=x\pm\sqrt{5-4}$

$\Rightarrow y = x \pm 1$

$\Rightarrow y = x + 1$ or $y = x - 1$

4. Ans. (4)

$\frac{y^2}{1+r}-\frac{x^2}{1-r}=1$

for $r > 1$, $\frac{y^2}{1+r}+\frac{x^2}{r-1}=1$

$e=\sqrt{1-\left(\frac{r-1}{r+1}\right)}$

$= \sqrt{\dfrac{(r+1)-(r-1)}{(r+1)}}$

$= \sqrt{\dfrac{2}{r+1}} = \sqrt{\dfrac{2}{r+1}}$

Option (4)

5. Ans. (1)

Let the equation of tangent to parabola

$y^2 = 4x$ be $y = mx + \dfrac{1}{m}$

It is also a tangent to hyperbola $xy = 2$

$\Rightarrow \quad x\left(mx + \dfrac{1}{m}\right) = 2$

$\Rightarrow \quad x^2 m + \dfrac{x}{m} - 2 = 0$

$D = 0 \Rightarrow \; m = -\dfrac{1}{2}$

So tangent is $2y + x + 4 = 0$

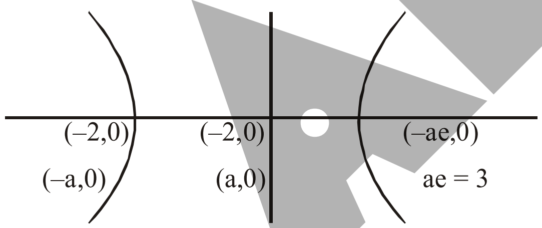
6. Ans. (4)

$2b = 5$ and $2ae = 13$

$b^2 = a^2(e^2 - 1) \;\Rightarrow\; \dfrac{25}{4} = \dfrac{169}{4} - a^2$

$\Rightarrow a = 6 \;\Rightarrow e = \dfrac{13}{12}$

7. Ans. (3)



$ae = 3,\; e = \dfrac{3}{2},\; b^2 = 4\left(\dfrac{9}{4} - 1\right),\; b^2 = 5$

$\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$

8. Official Ans. by NTA (1)

Sol. Let us Suppose equation of hyperbola is

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$e = 2 \Rightarrow b^2 = 3a^2$

passing through $(4,6) \Rightarrow a^2 = 4,\; b^2 = 12$

$\Rightarrow$ equaiton of tangent

$x - \dfrac{y}{2} = 1$

$\Rightarrow 2x - y - 2 = 0$

9. Official Ans. by NTA (3)

Sol. $\dfrac{x^2}{24} - \dfrac{y^2}{18} = 1 \Rightarrow a = \sqrt{24}; b = \sqrt{18}$

Parametric normal :

$\sqrt{24}\cos\theta.x + \sqrt{18}.y\cot\theta = 42$

At $x = 0$: $y = \dfrac{42}{\sqrt{18}}\tan\theta = 7\sqrt{3}$ (from given equation)

$\Rightarrow \tan\theta = \sqrt{\dfrac{3}{2}} \Rightarrow \sin\theta = \pm\sqrt{\dfrac{3}{5}}$

slope of parametric normal $= \dfrac{-\sqrt{24}\cos\theta}{\sqrt{18}\cot\theta} = m$

$\Rightarrow m = -\sqrt{\dfrac{4}{3}}\sin\theta = -\dfrac{2}{\sqrt{5}}$ or $\dfrac{2}{\sqrt{5}}$

10. Official Ans. by NTA (1)

Sol. Hyperbola is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\dfrac{a}{e} = \dfrac{4}{\sqrt{5}}$ and $\dfrac{16}{a^2} - \dfrac{12}{b^2} = 1$

$a^2 = \dfrac{16}{5}e^2$ ....(1) and $\dfrac{16}{a^2} - \dfrac{12}{a^2(e^2-1)} = 1$ ....(2)

From (1) & (2)

$16\left(\dfrac{5}{16e^2}\right) - \dfrac{12}{(e^2-1)}\left(\dfrac{5}{16e^2}\right) = 1$

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

11. Official Ans. by NTA (3)

Sol. $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

$a = 3,\; b = 4$ & $e = \sqrt{1 + \dfrac{16}{9}} = \dfrac{5}{3}$

corresponding focus will be $(-ae, 0)$ i.e., $(-5, 0)$.

12. Official Ans. by NTA (1)

Sol. Equation of tangents

$y^2 = 12x \quad \Rightarrow \quad y = 2x + \frac{3}{m}$

$\frac{x^2}{1} - \frac{y^2}{8} = 1 \Rightarrow y = mx \pm \sqrt{m^2 - 8}$

Since they are common tangent

$\therefore \frac{3}{m} = \pm\sqrt{m^2-8}$ | $\frac{x^2}{1} - \frac{y^2}{8} = 1$

$m^4 - 8m^2 - 9 = 0$ | $e = 3$

$m = \pm 3$ | $ae = 3$

$\therefore \begin{rcases} y = 3x + 1 \\ y = -3x - 1 \end{rcases} P\left(-\frac{1}{3}, 0\right)$, $S = (3, 0)$, $S' = (-3, 0)$

$\xleftarrow{\;8/3\;}\!\!\rightarrow\;\xleftarrow{\;10/3\;}\!\!\rightarrow$

S'(–3,0) $\left(-\frac{1}{3}, 0\right)$ S(3,0)

COMPLEX NUMBER

1. Ans. (2)

Given $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely img

so real part becomes zero.

$z = \left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right) \times \left(\frac{1+2i\sin\theta}{1+2i\sin\theta}\right)$

$z = \frac{(3-4\sin^2\theta)+i(8\sin\theta)}{i+4\sin^2\theta}$

Now $\operatorname{Re}(z) = 0$

$\frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = - -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

$\because \theta \in \left(-\frac{\pi}{2}, \pi\right)$

then sum of the elements in A is

$-\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}$

2. Ans. (1)

$z_0 = \omega$ or ω^2 (where ω is a non-real cube root of unity)

$z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93}$

$z = 3 + 3i$

$\Rightarrow \arg z = \frac{\pi}{4}$

3. Ans. (Bonus)

$3|z_1| = 4|z_2|$

$\Rightarrow \frac{|z_1|}{|z_2|} = \frac{4}{3}$

$\Rightarrow \frac{|3z_1|}{|2z_2|} = 2$

Let $\frac{3z_1}{2z_2} = a = 2\cos\theta + 2i\sin\theta$

$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = a + \frac{1}{a}$

$= \frac{5}{2}\cos\theta + \frac{3}{2}i\sin\theta$

Now all options are incorrect

Remark :

There is a misprint in the problem actual problem should be :

"Let z_1 and z_2 be any non-zero complex number such that $3|z_1| = \mathbf{2}|z_2|$.

If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$, then"

Given

$3|z_1| = 2|z_2|$

Now $\left|\frac{3z_1}{2z_2}\right| = 1$

Let $\frac{3z_1}{2z_2} = a = \cos\theta + i\sin\theta$

$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$

$= a + \frac{1}{a} = 2\cos\theta$

$\therefore \operatorname{Im}(z) = 0$

Now option (4) is correct.

4. Ans. (4)

$$z=\left(\frac{\sqrt{3}+i}{2}\right)^5+\left(\frac{\sqrt{3}-i}{2}\right)^5$$

$$z=\left(e^{i\pi/6}\right)^5+\left(e^{-i\pi/6}\right)^5$$

$$=e^{i5\pi/6}+e^{-i5\pi/6}$$

$$=\cos\frac{5\pi}{6}+i\frac{\sin 5\pi}{6}+\cos\left(\frac{-5\pi}{6}\right)+i\sin\left(\frac{-5\pi}{6}\right)$$

$$=2\cos\frac{5\pi}{6}<0$$

$I(z)=0$ and $Re(z)<0$

Option (4)

5. Ans. (4)

$$\left(-2-\frac{i}{3}\right)^3=-\frac{(6+i)^3}{27}$$

$$=\frac{-198-107i}{27}=\frac{x+iy}{27}$$

Hence, $y-x=198-107=91$

6. Ans. (4)

$|z|+z=3+i$

$z=3-|z|+i$

Let $3-|z|=a \Rightarrow |z|=(3-a)$

$\Rightarrow z=a+i \Rightarrow |z|=\sqrt{a^2+1}$

$\Rightarrow 9+a^2-6a=a^2+1 \Rightarrow a=\frac{8}{6}=\frac{4}{3}$

$\Rightarrow |z|=3-\frac{4}{3}=\frac{5}{3}$

7. Ans. (2)

$$\frac{z-\alpha}{z+\alpha}+\frac{\bar{z}-\alpha}{\bar{z}+\alpha}=0$$

$$z\bar{z}+z\alpha-\alpha\bar{z}-\alpha^2+z\bar{z}-z\alpha+\bar{z}\alpha-\alpha^2=0$$

$|z|^2=\alpha^2$, $a=\pm 2$

8. Ans. (1)

$|z_1|=9$, $|z_2-(3+4i)|=4$

C_1 (0, 0) radius $r_1=9$

$C_2(3,4)$, radius $r_2=4$

$C_1C_2=|r_1-r_2|=5$

$\therefore$ Circle touches internally

$\therefore |z_1-z_2|_{min}=0$

9. Official Ans. by NTA (1)

Sol. $z=\frac{\sqrt{3}}{2}+\frac{i}{2}=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$

$\Rightarrow z^5=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}=\frac{-\sqrt{3}+i}{2}$

and $z^8=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\left(\frac{1+i\sqrt{3}}{2}\right)$

$$\Rightarrow \left(1+iz+z^5+iz^8\right)^9=\left(1+\frac{i\sqrt{3}}{2}-\frac{1}{2}-\frac{\sqrt{3}}{2}+\frac{i}{2}-\frac{i}{2}+\frac{\sqrt{3}}{2}\right)^9$$

$$=\left(\frac{1+i\sqrt{3}}{2}\right)^9=\cos 3\pi+i\sin 3\pi=-1$$

10. Official Ans. by NTA (1)

Sol. Let $\frac{\alpha+i}{\alpha-i}=z$

$\Rightarrow \frac{|\alpha+i|}{|\alpha-i|}=|z|$

$\Rightarrow 1=|z|$

$\Rightarrow$ circle of radius 1

11. Official Ans. by NTA (1)

Sol. Roots of the equation $x^2+x+1=0$ are

$\alpha=\omega$ and $\beta=\omega^2$

where ω, ω^2 are complex cube roots of unity

$$\therefore \Delta=\begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

$$\Rightarrow \Delta=y\begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

Expanding along R_1, we get

$\Delta=y.y^2 \Rightarrow D=y^3$

12. Official Ans. by NTA (3)

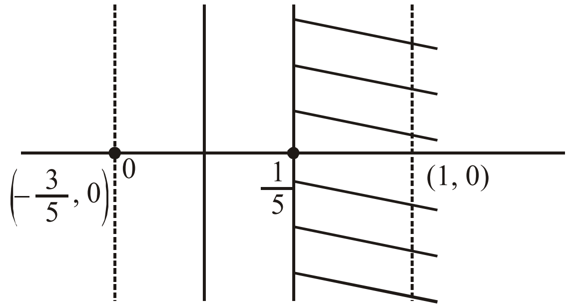
Sol. $|z|<1$

$5\omega(1-z)=5+3z$

$5\omega-5\omega z=5+3z$

$$z=\frac{5\omega-5}{3+5\omega}$$

$$|z|=5\left|\frac{\omega-1}{3+5\omega}\right|<1$$



$5|\omega - 1| < |3 + 5\omega|$

$5|\omega-1| < 5\left|\omega+\frac{3}{5}\right|$

$|\omega-1| < 5\left|\omega-\left(-\frac{3}{5}\right)\right|$

13. Official Ans. by NTA (3)

Sol. Given a > 0

$z = \frac{(1+i)^2}{a-i} = \frac{2i(a+i)}{a^2+1}$

Also $|z| = \sqrt{\frac{2}{5}} \Rightarrow \frac{2}{\sqrt{a^2+1}} = \sqrt{\frac{2}{5}} \Rightarrow a = 3$

So $\bar{z} = \frac{-2i(3-i)}{10} = \frac{-1-3i}{5}$

14. Official Ans. by NTA (2)

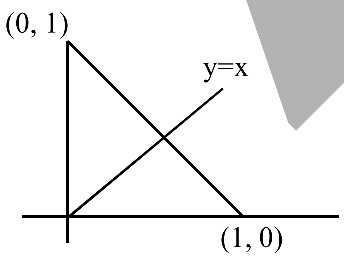
Sol. $|z|.|w| = 1 \quad z = re^{i(\theta+\pi/2)}$ and $w = \frac{1}{r}e^{i\theta}$

$\bar{z}.w = e^{-i(\theta+\pi/2)}.e^{i\theta} = e^{-i(\pi/2)} = -i$

$z.\bar{w} = e^{i(\theta+\pi/2)}.e^{-i\theta} = e^{i(\pi/2)} = i$

15. Official Ans. by NTA (4)

Sol.



$|z - i| = |z - 1|$

$y = x$

16. Official Ans. by NTA (3)

Sol. Put $z = x + 10i$

$\therefore 2(x + 10i) - n = (2i - 1) . [2(x+10i) + n]$

compare real and imginary coefficients

$x = -10, n = 40$

PROBABILITY

1. Ans. (2)

Two cards are drawn successively with replacement

4 Aces 48 Non Aces

$P(x=1) = \frac{^4C_1}{^{52}C_1} \times \frac{48C_1}{52C_1} + \frac{48C_1}{52C_1} \times \frac{4C_1}{52C_1} = \frac{24}{169}$

$P(x=2) = \frac{^4C_1}{^{52}C_1} \times \frac{^4C_1}{^{52}C_1} = \frac{1}{169}$

$P(x = 1) + P(x = 2) = \frac{25}{169}$

2. Ans. (2)

E_1 : Event of drawing a Red ball and placing a green ball in the bag

E_2 : Event of drawing a green ball and placing a red ball in the bag

E : Event of drawing a red ball in second draw

$P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$

$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49}$

3. Ans. (3)

$$\boxed{\text{Start}} \begin{cases} \xrightarrow{1/2} H \to \text{Sum } 7 \text{ or } 8 \Rightarrow \frac{11}{36} \\ \xrightarrow{1/2} T \to \text{Number is } 7 \text{ or } 8 = \frac{2}{9} \end{cases}$$

$P(A) = \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{19}{72}$

4. Ans. (2)

$1 - {^nC_0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\frac{1}{6} > \left(\frac{2}{3}\right)^n \Rightarrow 0.1666 > \left(\frac{2}{3}\right)^n$

$n_{min} = 5 \Rightarrow$ Option (2)

5. Ans. (1)

Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space

$= {^5C_2} + {^6C_2}$

so required probability $= \frac{^5C_2}{^5C_2 + {^6C_2}}$

6. Ans. (2)

7,

1,6

$\boxed{P = \frac{5}{2^{20}}}$

2,5

3,4

1,2,4

7. Ans. (3)

p (probability of getting white ball) = $\frac{30}{40}$

$q = \frac{1}{4}$ and n = 16

mean =np = $16.\frac{3}{4} = 12$

and standard diviation

$= \sqrt{npq} = \sqrt{16.\frac{3}{4}.\frac{1}{4}} = \sqrt{3}$

8. Ans. (2)

$\_\ \_\ \_\ \underline{4}\ \underline{4}$

$\frac{1}{6^2}\left(\frac{5^3}{6^3} + \frac{2C_1.5^2}{6^3}\right) = \frac{175}{6^5}$

9. Ans. (4)

No. of ways = $10C_3 = 120$

10. Ans. (3)

Expected Gain/ Loss

= w × 100 + Lw (–50 + 100) + L^2w (–50 –50 + 100) + L^3 (–150)

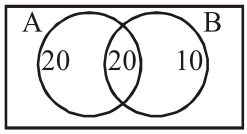
$= \frac{1}{3} \times 100 + \frac{2}{3}.\frac{1}{3}(50) + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)(0)$

$+ \left(\frac{2}{3}\right)^3(-150) = 0$

here w denotes probability that outcome 5 or 6 (w = $\frac{2}{6} = \frac{1}{3}$)

here L denotes probability that outcome 1,2,3,4 (L = $\frac{4}{6} = \frac{2}{3}$)

11. Ans. (2)



A → opted NCC

B → opted NSS

$\therefore$ P (neither A nor B) = $\frac{10}{60} = \frac{1}{6}$

12. Official Ans. by NTA (4)

Sol. $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$

$(\text{as } A \subset B \Rightarrow P(A \cap B) = P(A))$

$\Rightarrow \quad P(A|B) \geq P(A)$

13. Official Ans. by NTA (4)

Sol. Probability of observing at least one head out of n tosses

$= 1 - \left(\frac{1}{2}\right)^n \geq 0.9$

$\Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1$

$\Rightarrow n \geq 4$

$\Rightarrow$ minimum number of tosses = 4

14. Official Ans. by NTA (3)

Sol. Let persons be A,B,C,D

P(Hit) = 1 – P(none of them hits)

$= 1 - P\left(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}\right)$

$= 1 - P\left(\bar{A}\right).P\left(\bar{B}\right).P\left(\bar{C}\right).P\left(\bar{D}\right)$

$= 1 - \frac{1}{2}.\frac{2}{3}.\frac{3}{4}.\frac{7}{8}$

$= \frac{25}{32}$

15. Official Ans. by NTA (1)

Sol. P(B) = P(G) = 1/2

Required Proballity =

$$\frac{\text{all 4 girls}}{(\text{all 4 girls}) + (\text{exactly 3 girls} + 1\text{boy}) + (\text{exactly 2 girls} + 2\text{boys})}$$

$$= \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4 + {}^4C_2\left(\frac{1}{2}\right)^4} = \frac{1}{11}$$

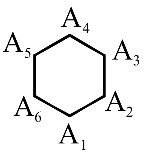
16. Official Ans. by NTA (3)

Sol. $1-\left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{100} \Rightarrow n = 7.$

17. Official Ans. by NTA (2)

Sol.



Only two equilateral tringles are possible $A_1 A_3 A_5$ and $A_2A_5A_6$

$\frac{2}{6_{C_3}} = \frac{2}{20} = \frac{1}{10}$

18. Official Ans. by NTA (4)

Sol. np = 8

npq = 4

$q = \frac{1}{2} \Rightarrow p = \frac{1}{2}$

n = 16

$p\,(x = r) = {}^{16}C_r\left(\frac{1}{2}\right)^{16}$

$p\,(x \le 2) = \frac{{}^{16}C_0 + {}^{16}C_1 + {}^{16}C_2}{2^{16}}$

$= \frac{137}{2^{16}}$

19. Official Ans. by NTA (2)

Sol. Let X be random varibale which denotes number of problems that candidate is unbale to solve

$\because\ p = \frac{1}{5}$ and $X < 2$

$\Rightarrow P(X<2) = P(X=0) + P(X=1)$

$= \left(\frac{4}{5}\right)^{50} + {}^{50}C_1 . \left(\frac{1}{5}\right) . \left(\frac{4}{5}\right)^{49}$

20. Official Ans. by NTA (2)

Sol. win Rs.15 $\rightarrow$ number of cases = 6

win Rs.12 $\rightarrow$ number of cases = 4

loss Rs.6 $\rightarrow$ number of cases = 26

p(expected gain/loss) = $15 \times \frac{6}{36} + 12 \times \frac{4}{36} - 6 \times \frac{26}{36} = -\frac{1}{2}$

STATISTICS

1. Ans. (2)

Given $\vec{x} = \frac{\Sigma x_i}{5} = 150$

$\Rightarrow \sum_{i=1}^{5} x_i = 750$ .....(i)

$\frac{\Sigma x_i^2}{5} - (\vec{x})^2 = 18$

$\frac{\Sigma x_i^2}{5} - (150)^2 = 18$

$\Sigma x_i^2 = 112590$ ....(ii)

Given height of new student

$x_6 = 156$

Now, $\vec{x}_{new} = \frac{\sum_{i=1}^{6} x_i}{6} = \frac{750+156}{6} = 151$

Also, New variance $= \frac{\sum_{i=1}^{6} x_i^2}{6} - (\bar{x}_{new})^2$

$= \frac{112590 + (156)^2}{6} - (151)^2$

$= 22821 - 22801 = 20$

2. Ans. (2)

$\sum (x_i + 1)^2 = 9n$ ...(1)

$\sum (x_i - 1)^2 = 5n$ ...(2)

(1) + (2) $\Rightarrow \sum (x_1^2 + 1) = 7n$

$\Rightarrow \frac{\sum x_i^2}{n} = 6$

(1) - (2) $\Rightarrow 4\Sigma x_i = 4n$

$\Rightarrow \Sigma x_i = n$

$\Rightarrow \frac{\Sigma x_i}{n} = 1$

$\Rightarrow$ variance = 6 – 1 = 5

$\Rightarrow$ Standard diviation $= \sqrt{5}$

3. Ans. (1)

Let two observations are x_1 & x_2

mean $= \frac{\sum x_i}{5} = 5 \Rightarrow 1 + 3 + 8 + x_1 + x_2 = 25$

$\Rightarrow x_1 + x_2 = 13$ ....(1)

variance $(\sigma^2) = \frac{\sum x_i^2}{5} - 25 = 9.20$

$\Rightarrow \sum x_i^2 = 171$

$\Rightarrow x_1^2 + x_2^2 = 97$ .....(2)

by (1) & (2)

$(x_1 + x_2)^2 - 2x_1x_2 = 97$

or $x_1x_2 = 36$

$\therefore x_1 : x_2 = 4 : 9$

4. Ans. (2)

$\bar{x} = 10 \Rightarrow \sum_{i=1}^{5} x_i = 50$

$$\text{S.D.} = \sqrt{\frac{\sum_{i=1}^{5} x_i^2}{5} - (\bar{x})^2} = 8$$

$\Rightarrow \sum_{i=1}^{5} (x_i)^2 = 109$

$$\text{variance} = \frac{\sum_{i=1}^{5}(x_i)^2 + (-50)^2}{6} - \left(\sum_{i=1}^{5} \frac{x_i - 50}{6}\right)$$

$= 507.5$

Option (2)

5. Ans. (4)

Variance is independent of origin. So we shift the given data by $\frac{1}{2}$.

so, $\frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

6. Ans. (4)

$\sum_{i=1}^{50} (x_i - 30) = 50$

$\Sigma x_i = 50 \times 30 = 50$

$\Sigma x_i = 50 + 50 + 30$

Mean $= \bar{x} = \frac{\Sigma x_i}{n} = \frac{50 \times 30 + 50}{50} = 30 + 1 = 31$

7. Ans. (3)

mean $\bar{x} = 4, \sigma^2 = 5.2, n = 5,. x_1 = 3\ x_2 = 4 = x_3$

$\sum x_i = 20$

$x_4 + x_5 = 9$ ....(i)

$\frac{\sum x_i^2}{x} - (\bar{x})^2 = \sigma^2 \Rightarrow \sum x_i^2 = 106$

$x_4^2 + x_5^2 = 65$ ....(ii)

Using (i) and (ii) $(x_4 - x_5)^2 = 49$

$|x_4 - x_5| = 7$

8. Official Ans. by NTA (3)

Sol. Let 7 observations be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$\bar{x} = 8 \Rightarrow \sum_{i=1}^{7} x_i = 56$ ......(1)

Also $\sigma^2 = 16$

$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - (\bar{x})^2$

$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64$

$\Rightarrow \left(\sum_{i=1}^{7} x_i^2\right) = 560$ .....(2)

Now, $x_1 = 2, x_2 = 4, x_3 = 10, x_4 = 12, x_5 = 14$

$\Rightarrow x_6 + x_7 = 14$ (from (1))

& $x_6^2 + x_7^2 = 100$ (from (2))

$\therefore x_6^2 + x_7^2 = (x_6 + x_7)^2 - 2x_6.x_7 \Rightarrow x_6.x_7 = 48$

9. Official Ans. by NTA (1)

Sol. Let x be the 6th observation

$\Rightarrow 45 + 54 + 41 + 57 + 43 + x = 48 \times 6 = 288$

$\Rightarrow x = 48$

variance $= \left(\frac{\Sigma x_i^2}{6} - (\bar{x})^2\right)$

$\Rightarrow$ variance $= \frac{14024}{6} - (48)^2$

$= \frac{100}{3}$

$\Rightarrow$ standard deviation $= \frac{10}{\sqrt{3}}$

10. Official Ans. by NTA (2)

Sol. $S.D = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$\bar{x} = \frac{\Sigma x}{4} = \frac{-1+0+1+k}{4} = \frac{k}{4}$

Now $\sqrt{5} = \sqrt{\frac{\left(-1-\frac{k}{4}\right)^2 + \left(0-\frac{k}{4}\right)^2 + \left(1-\frac{k}{4}\right)^2 + \left(k-\frac{k}{4}\right)^2}{4}}$

$\Rightarrow 5 \times 4 = 2\left(1+\frac{k}{16}\right)^2 + \frac{5k^2}{8}$

$\Rightarrow 18 = \frac{3k^2}{4}$

$\Rightarrow k^2 = 24$

$\Rightarrow k = 2\sqrt{6}$

11. Official Ans. by NTA (1)

Sol. $\frac{34+x}{2} = 35$

$x = 36$

$42 = \frac{10+22+26+29+34+36+42+67+70+y}{10}$

$420 - 336 = y \Rightarrow y = 84$

$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

12. Official Ans. by NTA (1)

Sol. $\sum f_i = 20 = 2x^2 + 2x - 4$

$\Rightarrow x^2 + 2x - 24 = 0$

$x = 3, -4$ (rejected)

$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = 2.8$

13. Official Ans. by NTA (4)

Sol. Mean $(\mu) = \frac{\sum x_i}{50} = 16$

standard deviation $(\sigma) = \sqrt{\frac{\sum x_i^2}{50} - (\mu)^2} = 16$

$\Rightarrow (256) \times 2 = \frac{\sum x_i^2}{50}$

$\Rightarrow$ New mean

$= \frac{\sum (x_i - 4)^2}{50} = \frac{\sum x_i^2 + 16 \times 50 - 8\sum x_i}{50}$

$= (256) \times 2 + 16 - 8 \times 16 = 400$

14. Official Ans. by NTA (2)

Sol. $x_1 + ... + x_4 = 44$

$x_5 + ... + x_{10} = 96$

$\bar{x} = 14$, $\Sigma x_i = 140$

Variance $= \frac{\Sigma x_i^2}{n} - \bar{x}^2 = 4$

Standard deviation = 2

REASONING

1. Ans. (1)

$(p \oplus q) \wedge (\sim p \odot q) \equiv p \wedge q$ (given)

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$\sim p \vee q$	$\sim p \wedge q$	$(p \wedge q) \wedge (\sim p \vee q)$
T	T	F	T	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	T	F	F

from truth table $(\oplus, \odot) = (\wedge, \vee)$

2. Ans. (1)

$s[\sim(\sim p \vee q) \wedge (p \wedge r)] \cap (\sim q \wedge r)$

$\equiv [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$\equiv [p \wedge (\sim q \vee r)] \wedge (\sim q \wedge r)$

$\equiv p \wedge (\sim q \wedge r)$

$\equiv (p \wedge r) \sim q$

3. Ans. (4)

It is obvious

$\therefore$ Option (4)

4. Ans. (4)

Given q is F and $(p \wedge q) \leftrightarrow r$ is T

$\Rightarrow p \wedge q$ is F which implies that r is F

$\Rightarrow$ q is F and r is F

$\Rightarrow (p \wedge r)$ is always F

$\Rightarrow (p \wedge r) \rightarrow (p \vee r)$ is tautology.

5. Ans. (1)

Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

6. Ans. (3)

7. Ans. (1)

p	q	~p	~p→q	~(~p→q)	(~p ^ ~q)
T	T	F	T	F	F
F	T	T	T	F	F
T	F	F	T	F	F
F	F	T	F	T	T

8. Official Ans. by NTA (2)

Sol. The contrapositive of statement
$p \to q$ is $\sim q \to \sim p$
Here, p : you are born in India.
q : you are citizen of India.
So, contrapositive of above statement is
" If you are not a citizen of India, then you are not born in India".

9. Official Ans. by NTA (4)

Sol. Tautology

(1)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Tautology

(2)

p	q	$p \wedge q$	$\sim p \vee q$	$(p \wedge q) \to (\sim p) \vee q$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Tautology

(3)

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Tautology

(4)

p	q	$p \vee q$	$\sim p$	$p \vee \sim q$	$p \vee q \to p \wedge (\sim q)$
T	T	T	F	F	F
T	F	T	F	T	T
F	T	T	T	F	T
F	F	F	T	F	T

10. Official Ans. by NTA (4)

Sol. $\sim(p \vee (\sim p \wedge q))$
$= \sim p \wedge \sim(\sim p \wedge q)$
$= \sim p \wedge (p \vee \sim q)$
$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$
$= c \vee (\sim p \wedge \sim q)$
$= (\sim p \wedge \sim q)$

11. Official Ans. by NTA (2)

Sol. $P \Rightarrow (q \vee r) : F$
$P : T \; q \vee r : F$
$P : T : q : F : r : F$

12. Official Ans. by NTA (4)

Sol. (1) $(p \vee q) \wedge (\sim p \vee \sim q) \equiv (p \vee q) \wedge \sim (p \wedge q)$
$\to$ Not tautology (Take both p and q as T)
(2) $(p \wedge q) \vee (p \wedge \sim q) \equiv p \wedge (q \vee \sim q) \equiv p \wedge t \equiv p$
(3) $(p \vee q) \wedge (p \vee \sim q) \equiv p \vee (q \wedge \sim q) \equiv p \vee c \equiv p$
(4) $(p \vee q) \vee (p \vee \sim q) \equiv p \vee (q \vee \sim q) \equiv p \vee t \equiv t$

13. Official Ans. by NTA (2)

Sol. $\sim(\sim s \vee (\sim r \wedge s))$
$s \wedge (r \vee \sim s)$
$(s \wedge r) \vee (s \wedge \sim s)$
$(s \wedge r) \vee (c)$
$(s \wedge r)$

14. Official Ans. by NTA (3)

Sol. $P \to (\sim q \vee r)$
$\sim p \vee (\sim q \vee r)$
$$\left.\begin{array}{l} \sim p \to F \\ \sim q \to F \\ r \to F \end{array}\right\} \Rightarrow \left.\begin{array}{l} p \to T \\ q \to T \\ r \to F \end{array}\right\}$$

15. Official Ans. by NTA (4)

Sol. $\sim(p \to (\sim q)) = \sim(\sim p \vee \sim q)$
$= p \wedge q$

MATHEMATICAL INDUCTION

1. Ans. (4)

P(n) : $n^2 - n + 41$ is prime
P(5) = 61 which is prime
P(3) = 47 which is also prime

Important Notes