LEADER & ENTHUSIAST COURSE
JEE-MAIN 2013

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) — Path to Success

MAJOR TEST # 02 | **DATE : 11 - 03 - 2013**

SYLLABUS : SECTION – 2

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	4	3	1	1	3	1	3	4	2	2	1	3	4	2	1	4	4	1	4	1
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	1	3	2	3	2	2	4	4	1	1	1	4	2	4	1	1	3	1	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	2	1	3	2	2	2	3	4	2	4	2	2	1	2	2	1	1	1	2
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	1	2	3	3	3	2	1	2	4	1	4	2	4	1	3	1	3	4	1
Que.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	3	1	1	1	4	3	2	3	3	4	1	3	3	2	4	1	3	2	3	4
Que.	101	102	103	104	105															
Ans.	2	3	2	1	2															

HINT – SHEET

1. $\dfrac{3}{\lambda x + i\mu y} - 2 = e^{i\theta}$

$\dfrac{3 - 2\lambda x - 2i\mu y}{\lambda x + i\mu y} = e^{i\theta}$

taking modulus both side

$(3 - 2\lambda x)^2 + 4\mu^2y^2 = \lambda^2x^2 + \mu^2y^2$

$3\lambda^2x^2 + 3\mu^2y^2 - 12\lambda x + 9 = 0$

at $\lambda = 1,\ \mu = 2$

$3x^2 - 12x + 12y^2 + 9 = 0$

$3(x^2 - 4x + 4) + 12y^2 + 9 - 12 = 0$

$3(x - 2)^2 + 12y^2 = 3$

$\dfrac{(x-2)^2}{1} + \dfrac{y^2}{\frac{1}{4}} = 1$ rep-ellipse

at $\lambda = 1,\ \mu = 1$ rep. circle

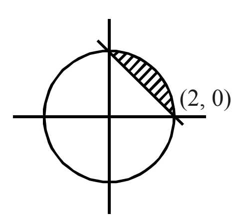
at $\lambda = 1,\ \mu = 0$

$3x^2 - 12x + 9 = 0$ rep pair of straight lines

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0;\quad x = 1, x = 3$

2.

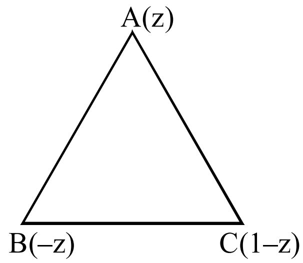


$x^2 + y^2 \le 4\ ;\ x + y \ge 2$

$\dfrac{1}{4} \times \pi \times 4 - \dfrac{1}{2} \times 2 \times 2 = \pi - 2$

4. AB = BC = CA

$|2z| = 1 = |2z - 1|;\ 2|z| = 1$



$(2z - 1)(2\bar{z} - 1) = 1$

$4|z|^2 - 2(z + \bar{z}) + 1 = 1$

$1 - 2(2x) = 0$

$x = \frac{1}{4}$ $\qquad$ $Re(z) = \frac{1}{4}$

6. $1 + \omega + 1 = 2 + \omega$

7. $\tan\theta/2 = b/a$

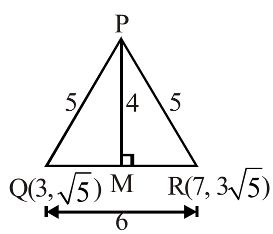
$\Rightarrow \cos\theta/2 = \frac{a}{\sqrt{a^2+b^2}}$

$\Rightarrow \cos\theta/2 = \frac{1}{\sqrt{1+\frac{b^2}{a^2}}} = \frac{1}{\sqrt{e^2}} = \frac{1}{e}$

8. For the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ equation of director circle is $x^2 + y^2 = 25$. This director circle will cut the ellipse $\frac{x^2}{50}+\frac{y^2}{20}=1$ at 4 points hence number of points = 4.

9. $A = \frac{1}{2} \times 6 \times PM$

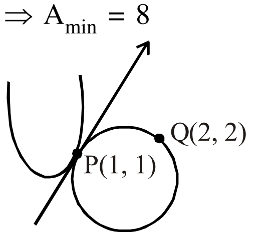
area will be maximum when PM is maximum and for PM to be maximum triangle will be isosceles. Hence PM = 4, A = 12.



10. $A = 2 \times 4\text{cosec}^2\theta = 8\text{cosec}^2\theta$

$\Rightarrow A_{min} = 8$

11.



Eq. of tangent to parabola $y = x^2$ at P(1, 1) is

$2x - y - 1 = 0$

Eq. of circle is

$(x - 1)^2 + (y - 1)^2 + \lambda(2x - y - 1) = 0$ ....(i)

Circle passes through Q(2, 2)

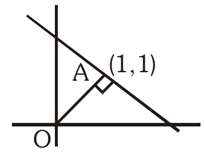
$\therefore (2 - 1)^2 + (2 - 1)^2 + \lambda(4 - 2 - 1) = 0$

$\Rightarrow \lambda = -2$

Put $\lambda = -2$ in (i)

$x^2 + y^2 - 6x + 4 = 0$

12. Minimum area in I quadrant will be corresponding to a line perpendicular to OA i.e. x + y = 1

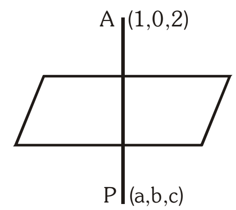


$\Rightarrow$ $A_{min} = 2$ (in I quadrant) in II & IV quadrant area can vary from $(0, \infty)$

$\Rightarrow$ Number of straight lines = 2

13. Let the image be (a, b, c)

Equation of line AP is



$\frac{x-1}{1} = \frac{y}{-1} = \frac{z-2}{1} = \lambda$

$\Rightarrow$ Let $a = \lambda + 1$,

$b = -\lambda$ & $c = \lambda + 2$

Mid-point of AP lies on the plane

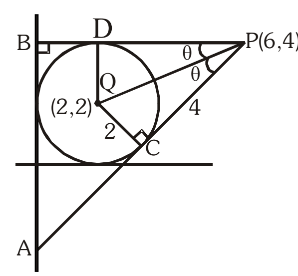
$\Rightarrow \frac{\lambda+2}{2}+\frac{\lambda}{2}+\frac{\lambda+4}{2}=0$

$\Rightarrow \lambda = -2 \quad \Rightarrow \quad P(-1,2,0)$

14. Circle is $(x - 2)^2 + (y - 2)^2 = 2^2$

BP = 6

$\Rightarrow$ CP = 4



$\tan\theta = \frac{1}{2}$

$\tan 2\theta = \frac{AB}{BP}$

$\Rightarrow$ AB = 6tan2θ

$$= \frac{12\tan\theta}{1-\tan^2\theta} = \frac{6}{1-\frac{1}{4}} = 8$$

15. Let the centre be (x, y), then its distance from (0, 1) is same as from y – x = 0

$\Rightarrow \sqrt{x^2+(y-1)^2} = \frac{|y-x|}{\sqrt{2}}$

$\Rightarrow 2x^2 + 2y^2 + 2 - 4y = x^2 + y^2 - 2xy$

$\Rightarrow x^2 + y^2 + 2xy = 4y - 2 \Rightarrow (x + y)^2 = 4y - 2$

16. $p_1 = \vec{r}.(\hat{i}+2\hat{j}-2\hat{k}) = 3$

$\Rightarrow$ x + 2y – 2z = 3 ....(i)

P_2 : 2x + 2y – 4z = 5

$\Rightarrow$ x + 2y – 2z = $\frac{5}{2}$ ....(ii)

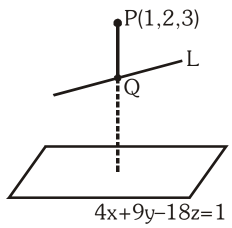
Distance between (i) & (ii) is equal to

$$\frac{\left|\frac{5}{2}-3\right|}{\sqrt{1^2+2^2+2^2}} = \frac{1}{6}$$

17. Any point on line is

$Q(1-2\lambda, -1+3\lambda, 5+4\lambda)$

$\therefore$ Direction ratios of PQ are $(-2\lambda, 3\lambda-3, 4\lambda+2)$



PQ is perpendicular to the plane

$\Rightarrow$ PQ is parallel to normal of the plane

$$\Rightarrow \frac{-2\lambda}{4} = \frac{3\lambda-3}{9} = \frac{4\lambda+2}{-18}$$

$$\Rightarrow \lambda = \frac{2}{5}$$

18. $(\vec{a}+3\vec{b}).(7\vec{a}-5\vec{b}) = 0$

$\Rightarrow$ $7|\vec{a}|^2 + 16\vec{a}.\vec{b} - 15|\vec{b}|^2 = 0$ ....(i)

$(\vec{a}-5\vec{b}).(7\vec{a}+3\vec{b}) = 0$

$\Rightarrow$ $7|\vec{a}|^2 - 32\vec{a}.\vec{b} - 15|\vec{b}|^2 = 0$ ....(ii)

(i) – (ii) $\Rightarrow$ $48\vec{a}.\vec{b} = 0$ or $\vec{a} \perp \vec{b}$

19. If lines $\vec{r} = (2+\lambda)\hat{i} + (1-2\lambda)\hat{j} + \hat{k}$

& $\vec{r} = \hat{i} + (1+\mu)\hat{j} + (-3+2\mu)\hat{k}$

intersects each other, then

$2 + \lambda = 1$, $1 - 2\lambda = 1 + \mu$ & $1 = -3 + 2\mu$

$\Rightarrow$ $\lambda = -1$, $\mu = 2$

20. Required plane is

$[r-(2\hat{i}+\hat{j}-3\hat{k})].(2\hat{i}+\hat{j}-3\hat{k}) = 0$

21. $\frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}] = 3$ (Given)

$[\vec{a}\,\vec{b}\,\vec{c}] = 18$

Vol. of parallelopiped= $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+a\,]$

$= 2[\vec{a}\ \vec{b}\ \vec{c}]$ = 2 × 18 = 36

22. $\hat{a}+\hat{b} = \left(\frac{7i-4j-4k}{9}\right) + \left(\frac{2\hat{i}-\hat{j}+2\hat{k}}{3}\right)$

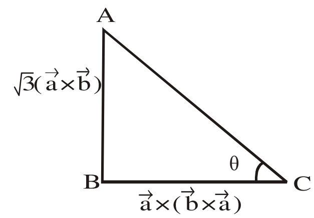
$= \frac{(7i-4j-4k)+(-6i-3j-6k)}{9} = \frac{i-7j+2k}{9}$

$\therefore$ Vector mag. $3\sqrt{6} = \frac{3\sqrt{6}(i-7j+2k)}{\sqrt{54}}$

23. $\vec{b}-(\vec{a}.\vec{b})\vec{a} = (\vec{a}.\vec{a})\vec{b}-(\vec{a}.\vec{b})\vec{a} = \vec{a}\times(\vec{b}\times\vec{a})$

here $\vec{a}\times\vec{b} \perp \vec{a}\times(\vec{b}\times\vec{a})$

$\angle ABC = 90°$



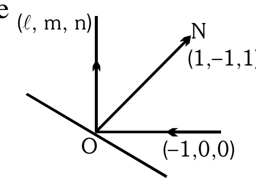
$$\tan\theta = \frac{\sqrt{3}\left|\vec{a}\times\vec{b}\right|}{|\vec{a}|\left|\vec{b}\times\vec{a}\right|} = \sqrt{3}$$

$\theta = \pi/3$

$\angle ABC = \pi/2$: $\angle BAC = \pi/6$

$\angle ACB = \pi/3$

24. The Dr's of incident ray are (–1, 0, 0). Let the Dr's of reflected ray be (ℓ, m, n). Then the Dr's of the normal to the plane of mirror will be (ℓ–1, m, n)



So, $\frac{\ell-1}{1} = \frac{m}{-1} = \frac{n}{1} = K(\text{say})$

$\ell = K + 1$, $m = -K$, $n = K$

$\ell^2 + m^2 + n^2 = 1 \Rightarrow K = -\frac{2}{3}$

So, $(\ell, m, n) \equiv \left(\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$

25. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a ΔABC, and let G be its centroid. Then,

Δ_1 = Area of ΔGBC

$\Rightarrow \Delta_1 = \frac{\Delta}{3}$, where Δ is the area of ΔABC

Δ_2 = Area of triangle formed by the mid-points of the sides

$\Rightarrow \Delta_2 = \frac{1}{4}\Delta$

$\therefore \Delta_1 : \Delta_2 = 4 : 3$

26. The equation of the chord of contact of tangents drawn from the origin to the circle

$x^2 + y^2 + 2gx + 2fy = c = 0$ is

$gx + fy + c = 0$ ...(i)

The required circle passes through the intersection of the given circle and line (i). Therefore, its equation is

$(x^2 + y^2 + 2gx + 2fy + c) + \lambda\,(gx + fy + c) = 0$...(ii)

this passes through (0, 0)

$\therefore\ c + \lambda c = 0 \Rightarrow \lambda = -1$

putting $\lambda = -1$ in (ii), the eq. of the req. circle is $x^2 + y^2 + gx + fy = 0$.

27. The equations of the lines are

$2x - y + 4 = 0$ and $-x + 2y + 1 = 0$

We have, $2 \times -1 + (-1) \times 2 < 0$ i.e. $a_1a_2+b_1b_2<0$

Therefore, the equation of the bisector of acute angles is

$$\frac{2x-y+4}{\sqrt{1+4}}=\frac{-x+2y+1}{\sqrt{(-1)^2+2^2}}$$

$\Rightarrow 2x - y + 4 = -x + 2y + 1$

$\Rightarrow 3x - 3y + 3 = 0$

$\Rightarrow x - y + 1 = 0$

28. Since, the equation of latus rectum and equation of tangent both are parallel and they lie in the same side of the origin

$$\therefore\ a = \left|\frac{-8+12}{\sqrt{1^2+1^2}}\right| = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$\therefore$ Length of latus rectum $= 4a = 4(2\sqrt{2})$

$= 8\sqrt{2}$

29. $\hat{a}.\hat{b}=0\ :\ \hat{b}.\hat{c}=\hat{a}.\hat{c}=\frac{1}{2}$

therefore $[\hat{a}\,\hat{b}\,\hat{c}]^2=\frac{1}{2}$

31. Eq. of focal chord is $y = m(x - 4)$.......(1)

(1) is tangent to $(x - 6)^2 + y^2 = 2$

$$\therefore\ \left|\frac{2m}{\sqrt{1+m^2}}\right| = \sqrt{2}$$

$\Rightarrow 4m^2 = 2 + 2m^2$

$\Rightarrow 2m^2 = 2$

$\Rightarrow m = \pm 1$.

34. $|\bar{a}\times\bar{b}|^2 = |a|^2|\bar{b}|^2 - |\bar{a}.\bar{b}|^2$

$= 64 - 64\cos\frac{\pi}{6} = 64 - 48 = 16$

35. D.r's of line are (2,1,3)

$\Rightarrow\ \parallel$ vector $= 2\hat{i}+\hat{j}+3\hat{k}=\vec{b}$ (say)

Normal to plane P is $\vec{n}=3\hat{i}-2\hat{k}$

$\vec{b}.\vec{n}=0$

$\Rightarrow$ L is $\parallel$ to P

Distance of L & P is distance of (1,0,2) on L from P

$$d=\frac{|3\times1-2\times2-1|}{\sqrt{3^2+2^2}}=\frac{2}{\sqrt{13}}$$

61. $F_m = qvB$ (directed radially outward)

$$N - mg\sin\theta - qvB = \frac{mv^2}{R}$$

or $$N = \frac{mv^2}{R} + mg\sin\theta + qvB$$

Hence, at $\theta=\frac{\pi}{2}$

$$N_{max}=\frac{2mgR}{R}+mg+qB\sqrt{2gR}$$

$$= 3mg + qB\sqrt{2gR}$$

66. Magnetic moment is a vector quantity. If the magnetic moments of the two magnets are M each then, the net magnetic moment when the magnets are placed perpendicular to each other, is

$$M_{eff} = \sqrt{M^2+M^2} = M\sqrt{2}$$

and the moment of inertia is 2I.

Thus, $$T = 2\pi\sqrt{\frac{2\ell}{M\sqrt{2}H}}$$

when one of the magnets is withdrawn, the time period is

$$T' = 2\pi\sqrt{\frac{I}{MH}}$$

$$\therefore\quad \frac{T'}{T}=\sqrt{\frac{1}{2^{1/2}}}\ \text{ or }\ T'=\frac{T}{2^{1/4}}=2^{\frac{5}{4}-\frac{1}{4}}=2\,\text{sec}$$

67. $\frac{mv^2}{r} = qvB\sin 90°$

or $r = \frac{mv}{qB}$

If the speed of electron is doubled and the magnetic field is halved, then

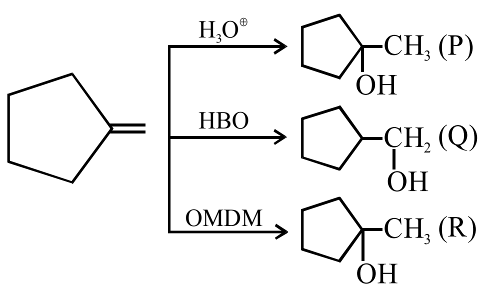
$$r' = \frac{m.2v}{q(B/2)} = 4 \times \frac{mv}{qB} = 4r$$

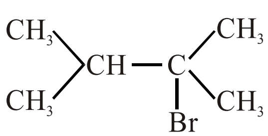
68. Since magnetic field at the centre of an arc is equal to

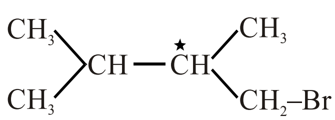
$$B = \frac{\mu_0 I}{4\pi r}\theta$$

hence, $\text{net } B = \frac{\mu_0 I}{4\pi}\left[\frac{1}{r} - \frac{1}{2r} + \frac{1}{3r}\right]\theta$

$$= \frac{5\mu_0 I\theta}{24\pi r}$$

77. 

78. P = 

Q = 

80. SN^1 reactivity ∝ stability of carbocation

81. $CH_3 - C{\equiv}CH \xrightarrow{CH_3MgBr}$

$CH_4 + CH_3 - C \equiv \overset{\ominus}{C}\overset{\oplus}{Mg}Br \xrightarrow[\text{(ii) } H_2O / H^{\oplus}]{\text{(i) } CO_2} CH_3 - C \equiv C - CO_2H$

83. $CH_3–CH_2OH \xrightarrow[H^{\oplus}]{KMnO_4/\overset{\ominus}{O}H,\Delta} CH_3–COOH \xrightarrow{SOCl_2 \,|\, NH_3/\Delta} CH_3–\overset{O}{\overset{\|}{C}}–NH_2 \xrightarrow{Br_2+KOH} CH_3–NH_2$

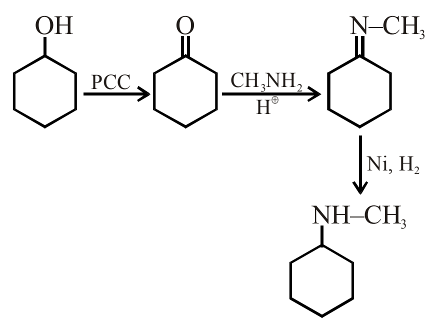
84. $CH_3–\overset{O}{\overset{\|}{C}}–O–CH_2–CH_3 + H_2\overset{18}{O} \longrightarrow CH_3–\overset{O}{\overset{\|}{C}}–\overset{18}{O}H + C_2H_5OH$

87. $Ph–\overset{O}{\overset{\|}{C}}–CH_3 \xrightarrow{Br_2 + KOH} Ph–\overset{O}{\overset{\|}{C}}–\overset{\ominus}{O}\ K^{\oplus} + CHBr_3 \xrightarrow{H^{\oplus}} Ph–\underset{O}{\underset{\|}{C}}–OH$

89. Stability of alkene $\propto \frac{1}{\text{H.O.H}}$

92. $CH_2 = CH–\ddot{C}l$ does not show SN^2 reaction under normal conditions.

94. $Ph–\ddot{O}⋮CH_2–CH_3 \xrightarrow[\Delta]{HI} Ph–OH + CH_3–CH_2–I$

96. 

97. Aldol condensation given by those carbonyl compound that has at least one α – H.

99. (1) $CH_3–CH_2–\underset{O}{\underset{\|}{C}}–CH_3 \xrightarrow{NH_2–NH_2/\overset{\ominus}{O}H} CH_3–CH_2–\underset{CH_3}{\underset{|}{CH_2}}$

(2) $CH_3–\overset{O}{\overset{\|}{C}}–CH_3 \xrightarrow{Zn–Hg/HCl} CH_3–CH_2–CH_3$

(3) $CH_3 – CH{=}O \xrightarrow{Ni, H_2} CH_3–CH_2–OH \xrightarrow{Red\ P + HI} CH_3–CH_3$

103. Due to leaving tendancy of groups

$$–X > –O–\underset{O}{\underset{\|}{C}}–R > –OR > –NH_2$$