MAJOR TEST # 03 | **DATE : 15 - 03 - 2013**

SYLLABUS : SECTION – 3

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	3	1	4	3	2	3	2	4	3	4	4	4	3	2	1	3	3	4	4
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	1	4	1	4	4	1	4	1	2	1	3	4	4	3	2	1	1	4	3	3
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	4	2	4	2	1	4	4	2	1	1	4	4	4	4	4	1	2	3	4
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	4	1	1	4	1	3	2	2	1	1	4	2	2	3	3	1	3	2	2	2
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	3	3	1	1	3	2	3	3	3	4										

HINT – SHEET

36. **Case – 1** : $4 - x^2 \ge 0$ and $[x] + 2 > 0$

$x^2 - 4 \le 0 \quad [x] > -2$

$x \in [-2, 2]$

$x \in (-1, 2)\ x \in [-1, \infty)$

Case – 2 : $4 - x^2 \le 0$ and $[x] + 2 < 0$

$x \in [-\infty, -2] \cup [2a) \quad [x] < -2$

$x \in (-\infty, -2)$

$x \in (-\infty, -2)$

$x \in [-\infty, -2] \cup [-1, 2]$

37. $f(x) = \dfrac{5\sin^3 x\cos x}{\sec^2 x} = 5\ \sin^3 x.\cos^3 x$

$f(x) = \dfrac{5}{8}\ (\sin^3 2x)$

$f_{max} = \dfrac{5}{8}$

39. $\lim\limits_{x\to -1} \dfrac{\cos 2 - \cos 2x}{x^2 - |x|} \quad \left[\dfrac{0}{0} \text{ form}\right]$

Use DL

$\lim\limits_{x\to -1} \dfrac{2\sin(2x)}{2x - \dfrac{|x|}{x}} = \dfrac{-2\sin 2}{-2+1} = 2\ \sin 2$

40. $\dfrac{x}{2} \in (2,3) \Rightarrow \left[\dfrac{x}{2}\right] = 2$

$f(x) = x + 2$

$y - 2 = x$

$f^{-1}(x) = x - 2$

41. $\lim\limits_{n\to\infty} \dfrac{[1^2x] + [2^2x] + \ldots. + [n^2x]}{n^3} + \dfrac{(1^2 + 2^2 + \ldots.n^2)}{n^3}$

$= \dfrac{x}{3} + \dfrac{1}{3}$

42. $y = x + \sin x.\cos x$

$y = x + \dfrac{\sin 2x}{2}$

$y' = \cos 2x + 1$

$y'(\pi/2) = 0$

$x = \pi/2$

43. $\text{LST} = \dfrac{y_1}{m}$

$x^m y^n = a^{m+n}$

$y^n = \dfrac{a^{m+n}}{x^m}$

$$n.y^{n-1}\ y^1 = a^{m+n}\left(\frac{-m}{x^{m+1}}\right)$$

$$y' = -\frac{-m}{n}\ a^{m+n}\left[\frac{1}{x^{m+1}.y^{n-1}}\right]$$

$$LST = \frac{y}{-\frac{m}{n}a^{m+n}} \times \frac{1}{\left[\frac{1}{x^{m+1}.y^{n-1}}\right]}$$

$$LST = \frac{-n}{m\,a^{m+n}} \times y^n.x^m.x$$

$$LST = \frac{-n}{m}x$$

$LST \propto x$

44. $$f(x) = \begin{cases} -\frac{(x-1)}{x^2}\ ;\ x\in(-\infty,0)\cup(0,1) \\ \frac{(x-1)}{x^2}\ ;\quad x\in(1,\infty)\end{cases}$$

$$f'(x) = \begin{cases} \frac{x(x-2)}{x^4}\ ;\ x\in(-\infty,0)\cup(0,1) \\ \frac{-x(x-2)}{x^4}\ ;\ x\in(1,\infty)\end{cases}$$

45. $f(x) = (1/x)^x$

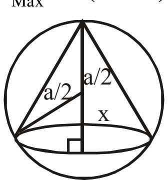
$\ell ny = x\ell n(1/x) = -\,x\ell nx$

$$\frac{1}{y}\frac{dy}{dx} = -\,[1 + \ell nx]$$

$$\frac{dy}{dx} = -\left(\frac{1}{x}\right)^x(\ell nx + 1)$$

+ | − (sign change at 1/e)

$f_{Max} = (1/1/e)^{1/e} = e^{1/e}$

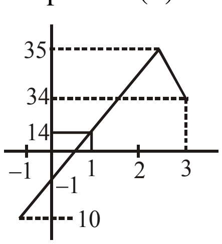
46. 

$$v(x) = \frac{1}{3}\pi\left(\frac{a^2}{x^2}-x^2\right)\left(\frac{a}{2}+x\right)$$

v(x) is max at $x = \frac{a}{6}$

$\therefore$ height $= \frac{a}{2}+\frac{a}{6}=\frac{2a}{3}$

47. Graph of f(x) is



48. $\phi'(x) = 0$

$\therefore\ \phi(x)$ is constant

49. $$y = \tan^{-1}\left[\frac{(x+1)-x}{1+x(x+1)}\right]+\tan^{-1}\left[\frac{(x+2)-(x+1)}{1+(x+1)(x+2)}\right]$$

$$+........+\tan^{-1}\left[\frac{(x+x)-(x+n-1)}{1+(x+n)(x+n-1)}\right]$$

$y = \tan^{-1}(x + n) - \tan^{-1}x$

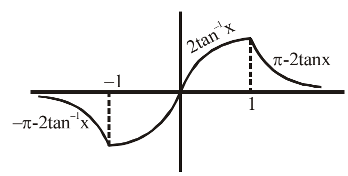
$$\frac{dy}{dx}=\frac{1}{1+(x+x)^2}-\frac{1}{1+x^2}$$

51. $x = 1 - x$

$$x = \frac{1}{2}$$

conti. at $x = \frac{1}{2}$

53. Graph of $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$



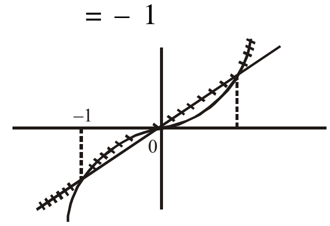
56. $$\lim_{x\to0}\frac{1+\frac{P}{2}x-\left(1-\frac{Px}{2}\right)}{x}$$

$$= \lim_{x\to0}\frac{2x+1}{x-2}$$

$$P = \frac{1}{-2}$$

57. Limit $= \lim_{x\to2}$ [x] + [–x]

$= -\,1$

60. 

$x = \{-1,0,1\}$

61. Let initial conditions = V,T

and final conditions = V',T'

By Charle's law $V \propto T$ [P remains constant]

$$\frac{V}{T}=\frac{V'}{T'}\Rightarrow\frac{V}{T}=\frac{V'}{1.2T'} \Rightarrow V' = 1.2\ V$$

But as per question, volume is reduced by 10% means V' = 0.9 V

So percentage of volume leaked out

$$= \frac{(1.2-0.9)V}{1.2V}\times 100 = 25\%$$

62. $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

$$\frac{1\times 30,000\times 10^{-6}}{300} = \frac{P_2\times 5.2\times 10^{-3}}{200}$$

P_2 = 3.86 atm.

63. Average kinetic energy per molecule per degree of freedom = 1/2 kT. Since both the gases are diatomic and at same temperature (300K), both will have the same number of rotational degree of freedom i.e. two. Therefore, both the gases will have the same average rotational kinetic energy per molecule

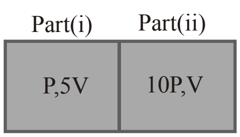
$\left(=2\times\dfrac{1}{2}kT = kT\right)$. Thus $\dfrac{E_1}{E_2} = \dfrac{1}{1}$

64. Mean kinetic energy for μ mole gas = $\mu.\dfrac{f}{2}RT$

$$\therefore E = \mu\ \frac{7}{2}RT = \left(\frac{m}{M}\right)\ \frac{7}{2}\ NkT = \frac{1}{44}\left(\frac{7}{2}\right)NkT$$

$= \dfrac{7}{88}$ NkT [As f = 7 and M = 44 for CO_2]

65.



When the piston is allowed to move the gases are kept separated but the pressure has to be equal.
($P_1 = P_2$) and final volume χ and (6V – χ), the no of moles are same in initial and final position at each parts.

$\because P_1 = P_2$ $\qquad$ $PV = n_1RT$

$\dfrac{n_1RT}{\chi} = \dfrac{n_2RT}{6V-\chi}$ $\qquad$ $n_1 = \dfrac{5PV}{RT}$

$\dfrac{n_1}{\chi} = \dfrac{n_2}{6V-\chi}$ $\qquad$ $n_2 = \dfrac{10PV}{RT}$

$$\Rightarrow \frac{5PV}{\chi RT} = \frac{10PV}{(6V-\chi)RT} \Rightarrow \frac{1}{\chi} = \frac{2}{6V-\chi}$$

$\Rightarrow 6V - \chi = 2\chi \Rightarrow \chi = 2V$ and $6V - \chi$
$\Rightarrow 6V - 2V = 4V$
$\therefore$ (2V, 4V)

66. Mean kinetic energy of gas molecule

$$E = \frac{f}{2}\ kT = \frac{f}{2}k(t+273) = \left(\frac{f}{2}k\right)t + \frac{f}{2}\times 273k;$$

Comparing it with standard equation of straight line y = mx + c. We get $m = \dfrac{f}{2}k$ and $c = \dfrac{f}{2}273k$

So the graph between E and t will be straight line with positive intercept on E-axis and positive slope with t-axis.

67. At critical temperature the horizontal portion in P-V curve almost vanishes as at temperature T_2. Hence the correct answer will be(2).

68. $$\gamma_{mix} = \frac{\dfrac{\mu_1\gamma_1}{\gamma_1-1}+\dfrac{\mu_2\gamma_2}{\gamma_2-1}}{\dfrac{\mu_1}{\gamma_1-1}+\dfrac{\mu_2}{\gamma_2-1}} = \frac{\dfrac{3\times 1.3}{(1.3-1)}+\dfrac{2\times 1.4}{(1.4-1)}}{\dfrac{3}{(1.3-1)}+\dfrac{2}{(1.4-1)}} = 1.33$$

69. $$\eta = \frac{T_1-T_2}{T_1} = \frac{(273+727)-(273+227)}{273+727}$$

$$= \frac{1000-500}{1000} = \frac{1}{2}$$

70. Fraction of supplied energy which increases the internal energy is given by

$$f = \frac{\Delta U}{(\Delta Q)_P} = \frac{(\Delta Q)_V}{(\Delta Q)_P} = \frac{\mu C_V\Delta T}{\mu C_p\Delta T} = \frac{1}{\gamma}$$

For diatomic gas $\gamma = \dfrac{7}{5} \Rightarrow f = \dfrac{5}{7}$

71. In both cylinders A and B the gases are diatomic (γ = 1.4). Piston A is free to move i.e., it is isobaric process. Piston B is fixed i.e., it is isochoric process. If same amount of heat ΔQ is given to both then

$(\Delta Q)_{isobaric} = (\Delta Q)_{isochoric} \Rightarrow \mu C_p(\Delta T)_A = \mu C_v(\Delta T)_B$

$$\Rightarrow (\Delta T)_B = \frac{C_p}{C_v}\ (\Delta T)_A = \gamma\ (\Delta T)_A = 1.4\times 30 = 42\text{ K}$$

73. The cyclic process 1 is clockwise where as process 2 is anticlockwise. Clockwise area represents positive work and anticlockwise area represents negative work. Since negative area (2) > positive area (1), hence net work done is negative.

74. From the given VT diagram,
In process AB, $V \propto T \Rightarrow$ Pressure is constant (As quantity of the gas remains same)
In process Bc, V = Constant and in process CA, T = constant.
$\therefore$ These processes are correctly represented on PV diagram by graph (3).

75. $W_{BCOB} = -$ Area of triangle BCO $= -\dfrac{P_0V_0}{2}$

$W_{AODA} = +$ Area of triangle AOD $= +\dfrac{P_0V_0}{2}$

85. $\dfrac{E_0}{B_0} = C \Rightarrow B_0 = \dfrac{E_0}{C}$

$= \dfrac{750}{3\times10^8} = 2.5\mu T$

86. $I_{av} = C\bar{U} = \dfrac{C\varepsilon_0 E_0^2}{2}$

$= \dfrac{3\times10^8\times8.85\times10^{-12}\times(36)^2}{2} = 1.72\ W/m^2$

88. For stationary waves, two waves must go in opposite direction,
$y = y_1 + y_2$
$= 2\cos(4x - \pi t) - 2\cos(4x + \pi t)$
$= 2.2 \sin 4x \sin \pi t$
At x = 0 node is formed i.e. y = 0

89. Beats $= n_1 - n_2$

$= \dfrac{v}{\lambda_1} - \dfrac{v}{\lambda_2}$

$\dfrac{10}{3} = \dfrac{v}{1} - \dfrac{v}{1.01}$

$= v\left[\dfrac{1.01-1}{1.01}\right]$

$v = \dfrac{10}{3}\times\dfrac{1.01}{0.01} = \dfrac{1010}{3} = 336.6$

90. For closed pipe — For open pipe

$n = \dfrac{xv}{4L}$; x = 1,3,5 — $n = \dfrac{xv}{2L}$; x = 1,2....

$= \dfrac{x\times340}{4\times2}$ — $= \dfrac{x\times340}{2\times2}$

$= 42.5\ x$ — $= 85\ x$

No common values from the option is possible.