MAJOR TEST # 05 | **DATE : 21 - 03 - 2013**

FULL SYLLABUS

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	1	2	3	2	4	2	4	1	3	3	4	2	1	4	1	1	4	3	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	1	1	1	3	4	4	1	3	3	1	3	2	3	3	3	1	3	2	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	4	2	3	1	3	1	2	1	2	1	2	2	1	3	2	4	4	4	4
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	1	3	3	3	4	3	4	2	2	3	2	4	3	3	3	3	2	3	2
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	1	1	4	2	3	3	1	3	3	3										

HINT – SHEET

1. $^4C_0{}^4C_4 + {}^4C_1{}^4C_3 + {}^4C_2{}^4C_2$

$1 + 16 + 36 = 53$

2. $1 + |e^x - 1| = e^{2x} - 2x + 1 - 1$

$1 + |e^x - 1| = |e^x - 1|^2 - 1$

$|e^x - 1|^2 - |e^x - 1| - 2 = 0$

let $|e^x - 1| = t$

$t^2 - t - 2 = 0$

$(t - 2)(t + 1) = 0$

$t = 2, -1$

$|e^x - 1| = 2$

$e^x - 1 = \pm 2$ $\quad\because e^x = -1$ is not possible

$e^x = 3$

$x = \ell n3$

Only one solution.

3. $x^2 - 13x - 30 \le 0$

$(x - 15)(x + 2) \le 0$

$-2 \le x \le 15$ $\quad$ but $x \in N$

so $x \in [1, 15]$

Probability $= \dfrac{15}{100} = \dfrac{3}{20}$

4. $Z = \dfrac{\sqrt{3}+i}{2} = \dfrac{-1+\sqrt{3}i}{2i}$

$Z = \dfrac{\omega}{i}$

$(Z^{101} + i^{103})^{105}$

$\left(\dfrac{\omega^{101}}{i^{101}} + i^{103}\right)^{105}$

$\left(\dfrac{\omega^2}{i} - i\right)^{105} = \left(\dfrac{\omega^2+1}{i}\right)^{105}$

$= \left(\dfrac{-\omega}{i}\right)^{105} = \dfrac{-1}{i} = i = Z^3$

6. Let the first 5 terms of AP are $a - 2d$, $a-d$, a, $a+d$, $a + 2d$

Now $a_1 + a_3 + a_5 = -12$

$\Rightarrow \quad 3a = -12 \quad \Rightarrow \quad a = -4$

Also, $a_1 . a_2 . a_3 = 8$

$\Rightarrow \quad (a - 2d)(a - d)\, a = 8$

$\Rightarrow (-4 - 2d)(-4 - d)(-4) = 8 \Rightarrow d = -3$

Hence the AP is 2, –1, –4, –7, –10, –13,...

Hence $a_2 + a_4 + a_6 = -21$

7. Let 3n terms of G.P. are a, ar, ar^2,.....,ar^{3n-1}

Then $S_1 = a + ar + ar^2 + ... + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}$

$S_2 = ar^n + ar^{n+1} + ar^{n+2} + ... + ar^{2n-1} = \dfrac{ar^n(1-r^n)}{1-r}$

$S_3 = ar^{2n} + ar^{2n+1} + ar^{2n+2} + ... + ar^{3n-1} = \dfrac{ar^{2n}(1-r^n)}{1-r}$

So, $S_2^2 = S_1S_3$. Hence S_1, S_2, S_3 are in G.P.

IInd method : Put the value of n.

9. Clearly, L = 0 is the perpendicular bisector of the segment joining (–2, 6) and (4, 2). The equation of which is

$y - 4 = \dfrac{3}{2}(x - 1) \Rightarrow 3x - 2y + 5 = 0$

$\therefore L = 3x - 2y + 5$

10. Put z = 0 in line

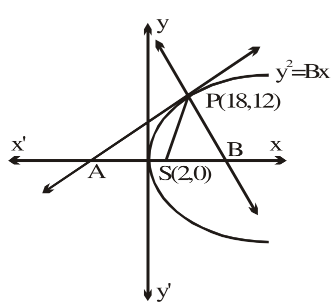
$\therefore x = 5 : y = 1$ put is curve

$c = \pm\sqrt{5}$

11. We know that

PS = AS = SB

$\Rightarrow$ S is the circum-centre of ΔPAB



$\therefore$ Equation of the required circle is

$(x - 2)^2 + (y - 0)^2 = (2 - 18)^2 + (0 - 12)^2$

$\Rightarrow x^2 + y^2 - 4x - 396 = 0$

12. $\vec{r}.\vec{a} = \mu[\vec{a}\,\vec{b}\,\vec{c}]$

$\vec{r}.\vec{b} = \nu[\vec{a}\,\vec{b}\,\vec{c}]$

$\vec{r}.\vec{c} = \lambda[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow \vec{r}.[\vec{a}+\vec{b}+\vec{c}] = (\lambda+\mu+\nu)[\vec{a}\ \vec{b}\ \vec{c}]$

$\Rightarrow 8 = (\lambda + \mu + \nu)\dfrac{1}{8}$

$\Rightarrow \lambda + \mu + \nu = 64$

14. Since, the given line touches the given circle, the length of the perpendicular from the centre (2, 4) of the circle to the line 3x – 4y – k = 0 is equal to the radius $\sqrt{4+16+5} = 5$ of the circle.

$\therefore \dfrac{3\times 2 - 4\times 4 - k}{\sqrt{9+16}} = \pm 5$

$\Rightarrow k = 15 \qquad [\because k > 0]$

hence equation of tangent is

$3x - 4y - 15 = 0$ ..... (1)

Let equation of normal to circle

$4x + 3y = \lambda$

It passes through centre (2, 4)

$\Rightarrow \lambda = 20$

hence equation of normal is

$4x + 3y = 20$ .... (2)

Solve (1) & (2)

a = 5, b = 0

k + a + b = 15 + 5 + 0 = 20

21. f(0) = 0

$\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{x}{\sqrt{x+1}-\sqrt{x}}$

$= \lim\limits_{x\to 0} x(\sqrt{x+1}+\sqrt{x})$

= 0

f(x) is conti. at x = 0

$\text{LHD} = \lim\limits_{h\to 0} \dfrac{\dfrac{-h}{\sqrt{-h+1}-\sqrt{-h}} - 0}{-h} = 1$

$\text{RHD} = \lim\limits_{h\to 0} \dfrac{\dfrac{h}{\sqrt{h+1}-\sqrt{-h}} - 0}{h} = 1$

22. $\sqrt{3-2\sin^2 x} + \cos y.y' = 0$

$y' = \sqrt{3}$

23. $f'(x) = 2(x - 1)(x - 2)^3 + 3(x - 1)^2(x - 2)^2$

$f'(x) = (x - 1)(x - 2)^2[2(x - 2) + 3(x - 1)]$

$f'(x) = (x - 1)(x - 2)^2(5x - 7)$

24. $\dfrac{1}{f(x)} = |x| - \{x\}$

$|x| < \{x\}$

$x \in \left(\dfrac{-1}{2}, 0\right)$

25. $p \to q \equiv \sim p \vee q$

$\equiv \sim q \to \sim p$

26. $\dfrac{^{15}C_2 + {}^{16}C_2}{^{31}C_2} = \dfrac{15}{31}$

St.–1, 2 Both are true and St. 2 is a correct explanation of St.–1.

27. The number of ways of selecting committee of r persons among 40 women and 60 men = $^{100}C_r$. This will assume greatest value at r = 50.

30. $\sin^{-1}\dfrac{2x}{1+x^2} = \cos^{-1}\dfrac{1-x^2}{1+x^2} = 2\tan^{-1} x$

$= 2\tan^{-1}x$

$\forall\ x \in (0, 1)$

$\therefore$ St. 1 is correct but

St. 2 is not correct.

32. Mean kinetic energy of molecule depends upon temperaturee only. For O_2 it is same as that of H_2 at the same temperature of –73ºC

33. In first case $\eta_1 = \dfrac{T_1 - T_2}{T_1}$

In second case $\eta_2 = \dfrac{2T_1 - 2T_2}{2T_1} = \dfrac{T_1 - T_2}{T_1} = \eta$

34. A is compressed isothermally, hence

$P_1V = P_2\dfrac{V}{2} \Rightarrow P_2 = 2P_1$

and B is compressed adiabatically, hence

$P_1'\ V^\gamma = P'_2\left(\dfrac{V}{2}\right)^\gamma \Rightarrow P'_2 = (2)^\gamma P'_1$

Since $\gamma > 1$, hence $P_2' > P_2$ or $P_2 < P_2'$

45. q —r— 2q —r— 4q

Force on 4q due to q, $F_1 = \dfrac{kq4q}{(2r)^2} = \dfrac{kq^2}{r^2}$

net force on 2q $F_2 = F_{4q} - F_q$

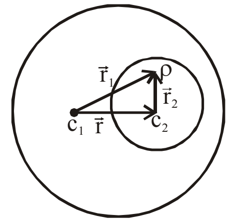
$= \dfrac{k4q \times 2q}{r^2} - \dfrac{kq \times 2q}{r^2} = \dfrac{6kq^2}{r^2}$

46. Electric field due to solid sphere $= \dfrac{\rho r}{3 \in_0}$

Now electric field at any point inside cavity

$\vec{E}_p = \vec{E}_{large} - \vec{E}_{small}$

$= \dfrac{\rho\vec{r}_1}{3\in_0} - \dfrac{\rho\vec{r}_2}{3\in_0}$



$= \dfrac{\rho}{3\in_0}(\vec{r}_1 - \vec{r}_2)$

$= \dfrac{\rho\vec{r}}{3\in_0}$ $\qquad \{\vec{r}_2 + \vec{r} = \vec{r}_1\}$

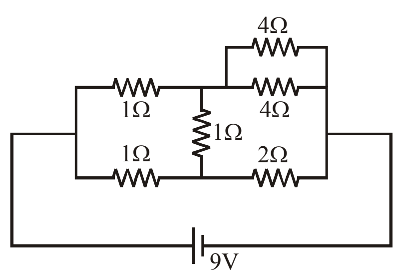
So $\vec{E}$ at any point inside cavity is along the line joining centrs c_1 and c_2 i.e. +x direction.

47. $V_{CB} = \dfrac{\frac{4}{1}+\frac{4}{1}-\frac{2}{1}}{\frac{1}{1}+\frac{1}{1}+\frac{1}{1}} = 2$

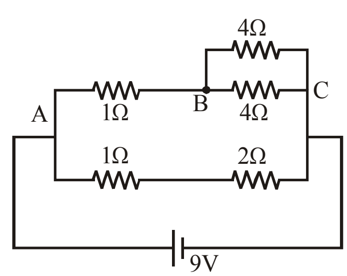
So $V_A - 1 - 2 = V_B$

$V_A - V_B = 3V$

48. CKt can be reduced as



Which is balanced WSB So



$V_{BC} = \dfrac{2}{3} \times 9$

$= 6V$

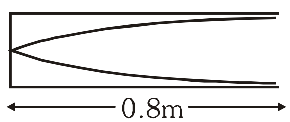
$Vi_{4\Omega} = \dfrac{6}{4} = \dfrac{3}{2}A$

51. $$f' = \left(\frac{v}{v - v_s}\right)\left(\frac{v + v_0}{v}\right) f$$

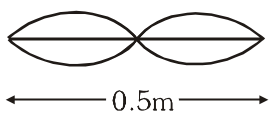
$$\Rightarrow f' = \left(\frac{320}{320 - 10}\right)\left(\frac{320 + 10}{320}\right) \times 8$$

$$\Rightarrow f' \approx 8.50 \text{ kHz}$$

52.



$$\frac{\lambda_p}{4} = 0.8$$



$\lambda_S = 0.5$

$$f_p = \frac{v}{\lambda_p} = 100\text{Hz}, \quad f_S = \frac{1}{0.5}\sqrt{\frac{50}{\mu}}$$

$$\because f_p = f_S \;\&\; \mu = \frac{M}{\ell} \quad \therefore M = 10 \text{ g}$$

53. L = 20 cm
m = 1 gm

$$\mu = \frac{m}{L} = \frac{1}{20}\text{gm/cm} = \frac{1}{20} \times \frac{10^{-3}}{10^{-2}}\text{g/m}$$

$$\mu = \left(\frac{1}{200}\right)\text{kg/m}$$

T = 0.5 N $\qquad v = \sqrt{\dfrac{0.5}{\left(\dfrac{1}{200}\right)}}$ = 10 m/s

f = 100 Hz

$$\lambda = \frac{v}{f} = \frac{10}{100} = \frac{1}{10}\text{m}$$

$$\frac{\lambda}{2} = \frac{1}{20}\text{m} = 5 \text{ cm}$$

54. By using $\dfrac{hc}{\lambda} = W_0 = \dfrac{1}{2}mv^2$

$$\Rightarrow \frac{hc}{400 \times 10^{-9}} = W_0 + \frac{1}{2}mv^2 \quad .....(i)$$

$$\text{and } \frac{hc}{250 \times 10^{-9}} = W_0 + \frac{1}{2}m(2v)^2 \quad .....(ii)$$

On solving (i) and (ii)

$$\frac{1}{2}mv^2 = \frac{hc}{3}\left[\frac{1}{250 \times 10^{-9}} - \frac{1}{400 \times 10^{-9}}\right] \quad .....(iii)$$

From equation (i) and (iii) $W_0 = 2hc \times 10^6$ J

55. $$\frac{C_{14}}{C_{12}} = \frac{1}{4} = \left(\frac{1}{2}\right)^{t/5700} \Rightarrow \frac{t}{5700} = 2$$

$\Rightarrow$ t = 11400 years

56. $$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{20} = 0.03465$$

Now time of decay $t = \dfrac{2.303}{\lambda}\log\dfrac{N_0}{N}$

$$\Rightarrow t_1 = \frac{2.303}{0.03465}\log\frac{100}{67} = 11.6\text{min}$$

$$\text{and } t_2 = \frac{2.303}{0.03465}\log\frac{100}{33} = 32\text{min}$$

Thus time difference between points of time $= t_1 - t_2 = 32 - 11.6 = 20.4$ min $\approx$ 20 min

57. $\sigma = ne(\mu_e + \mu_h)$
$= 2 \times 10^{19} \times 1.6 \times 10^{-19}(0.36 + 0.14)$
$= 1.6(\Omega\text{-m})^{-1}$

$$R = \rho\frac{1}{A} = \frac{1}{\sigma A} = \frac{0.5 \times 10^{-3}}{1.6 \times 10^{-4}} = \frac{25}{8}\Omega$$

$$\therefore \quad i = \frac{V}{R} = \frac{2}{25/8} = \frac{16}{25}A = 0.64A$$

61. $E^\circ_{cell} = (E^\circ_{RP})_{cathode} - (E^\circ_{RP})_{anode}$

$E^\circ_{cell} = 0.3435 - (-0.453)$

$E^\circ_{cell} = 0.3435 + 0.453$
$= 0.7965$ volt

$$E^\circ_{cell} = \frac{0.059}{n}\log K_{eq}$$

$$\log K_{eq} = \frac{0.7965 \times 2}{0.059} = 27 \Rightarrow K_{eq} = 10^{27}$$

63. $$\Delta x \times \Delta P = \frac{h}{4\pi}$$

$\because \Delta x = \Delta p$ (given)

$$\therefore (\Delta P)^2 = \frac{h}{4\pi} \Rightarrow \Delta P = \sqrt{\frac{h}{4\pi}}$$

$$\Rightarrow m\,\Delta v = \frac{1}{2}\sqrt{\frac{h}{\pi}}$$

65. $X \rightleftharpoons 2Y$

t_0 1 0

t_{eq} $1-\alpha$ 2α

$$K_{p_1} = \frac{4\alpha^2 p_1}{1-\alpha^2}$$

$Z \rightleftharpoons P + Q$

1 0 0

$1-\alpha$ α α

$$K_{p_2} = \frac{\alpha^2 p_2}{1-\alpha^2}$$

$$\frac{K_{p_1}}{K_{p_2}} = \frac{1}{9} = \frac{4p_1}{p_2} \text{ or } \frac{p_1}{p_2} = \frac{1}{36}$$

$$\sqrt{\frac{p_2}{p_1}} = 6$$

66. $HCl + NaOH \longrightarrow NaCl + H_2O$

M_1V_1 M_2V_2

	M_1V_1	M_2V_2	
(A)	10	10	Complete neutralization pH = 7
(B)	5.5	4.5	$[H^+] = \frac{5.5-4.5}{100}$ pH = 2
(C)	1	9	$[OH^-] = \frac{9-1}{100}$ pH = 12.9
(D)	15	5	$[H^+] = \frac{15-5}{100}$ pH = 1

67. $\Delta T_f = i\, K_f m$

$$i = \frac{0.372}{1.86 \times 0.1} = 2$$

68. Nearest distance $= \frac{\sqrt{3}\,a}{2}$

$$0.368 = \frac{\sqrt{3}\,a}{2}$$

$a = 0.425$ nm

69. $$\frac{\text{No. of atom}}{N_A} = \frac{\text{Weight}}{\text{Atomic wt}}$$

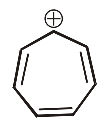
$$\frac{4.6 \times 10^{22}}{6.023 \times 10^{23}} = \frac{13.8}{\text{Atomic wt.}}$$

70. $\log \frac{x}{m} = \log K + 1/x \log P$

$\log K = \log 2$

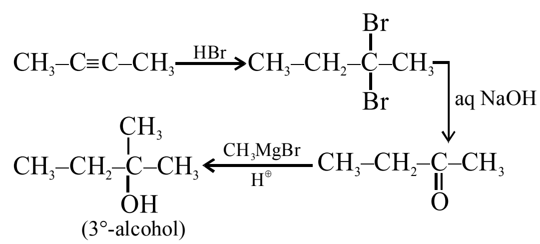
$1/n = 1$

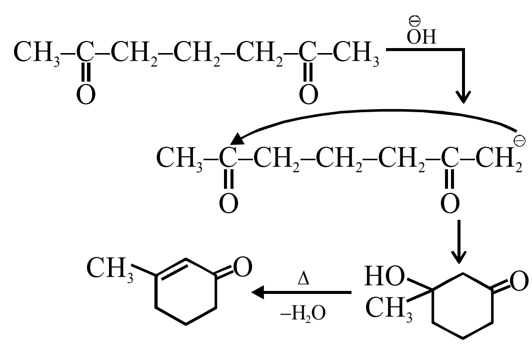
$x/m = K\,(P)^{1/n} = 4$

84.  is most stable among following due to aromaticity.

85. $HCCl_3 \xrightarrow{-H^{\oplus}} \overset{\ominus}{C}Cl_3$

Here $\ominus$ charge is stabilized by d-orbital resonance as Cl has vacant d-orbital.

86. 

87. 

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