LEADER & ENTHUSIAST COURSE

JEE-MAIN 2013

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) — Path to Success

MAJOR TEST # 06 | **DATE : 25 - 03 - 2013**

FULL SYLLABUS

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	3	1	1	2	1	2	3	1	3	2	1	3	1	2	2	3	2	4	4
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	4	4	3	1	3	1	1	4	1	2	3	3	3	1	3	1	2	1	3
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	3	2	2	1	1	2	3	1	4	3	1	4	1	3	3	3	1	4	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	3	3	4	4	1	3	1	2	4	4	4	2	3	2	3	3	1	4	1
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	3	3	3	2	4	4	1	2	1										

HINT – SHEET

1. Orbital angular momentum = $\sqrt{\ell(\ell+1)}\dfrac{h}{2\pi}$

2. (Λ_m^∞) of $CaCl_2$ $= (\Lambda_m^\infty)_{Ca^{+2}} + 2(\Lambda_m^\infty)_{Cl^-}$

$= 119 + 2 \times 76$

$= 119 + 152$

$= 271\ s\ cm^2\ mol^{-1}$

$(\Lambda_m^\infty)_{MgSO_4} = (\Lambda_m^\infty)_{Mg^{+2}} + (\Lambda_m^\infty)_{SO_4^{-2}}$

$= 106 + 160 = 266\ s\ cm^2\ mol^{-1}$

4. $2.303 \log_{10} K = 2.303 \log_{10} A = \dfrac{E_a}{RT}$

$\therefore \quad \dfrac{E_a}{2.303R} = 1.25 \times 10^4$

$\therefore \quad E_a = 1.25 \times 10^4 \times 2.303 \times 8.314$

$= 239.34 \times 10^3$ J

$\log A = 14 \therefore A = 1 \times 10^{14}$

5. $NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$

$x + x = 1.12$ atm

$$K_p = \frac{\left(\frac{x}{2x}\times 1.12\right)\left(\frac{x}{2x}\times 1.12\right)}{1}$$

$K_p = 0.3136\ atm^2$

6. For hydrolysis of WAWB type salt

$$h = \sqrt{\frac{K_w}{K_a.K_b}} = 0.2$$

7. $\Delta E = q_v = -321.3$ kJ/mol

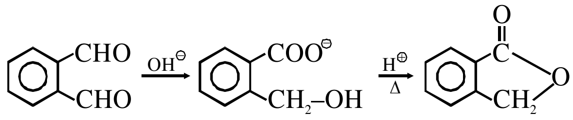
$\Delta H = q_p = \Delta E + (\Delta ng)RT$

$$C_6H_5COOH(l) + \frac{15}{2}O_2(g) \longrightarrow 7CO_2(g) + 3H_2O(l)$$

$\Delta ng = 7 - \dfrac{15}{2} = \dfrac{-1}{2}$

$\Delta H = -321.3 + \left(\dfrac{-1}{2}\right)(8.314)(300) \times 10^{-3}$

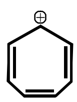
$= -322.54$ kJ

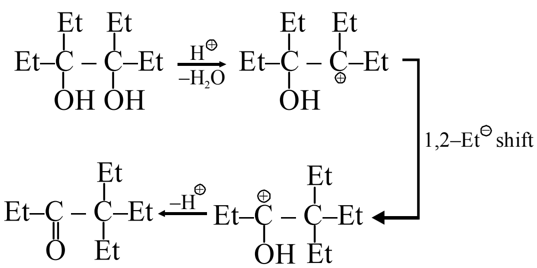
23. 

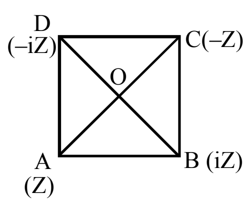
24. –Cl is o/p directing due to +M & weak deactivating group deu to –I.

25. Acidic strength $\uparrow \propto$ Ka$\uparrow \propto \frac{1}{pk_a \downarrow}$

26. Stability of alkene $\propto \frac{1}{\text{H.O.H.}}$.

27.  is aromatic in nature (highly stable).

28. 

31. 

Centroid = $\frac{-Z+iZ-iZ}{3} = \frac{-Z}{3}$

32. ${}^{2n}C_2 - n = \frac{2n(2n-1)}{2} - n = 2n(n-1)$

33. P(A) = 0.59, P(B) = 0.30

$P(A \cap B) = 0.21$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$= 1 - \{P(A) + P(B) - P(A \cap B)\}$

$= 1 - \{0.59 + 0.30 - 0.21\}$

$= 0.32$

34. $\alpha + \beta + \gamma = 2$

$\alpha\beta + \beta\gamma + \gamma\alpha = 3$

$\alpha\beta\gamma = 2$

$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\gamma+\alpha+\beta}{\alpha\beta\gamma} = \frac{2}{2} = 1$$

35. $\left(\frac{\underline{|4}}{\underline{|1}\,\underline{|1}\,\underline{|2}\,\underline{|2}} \times \underline{|3}\right) \times \left({}^{2+3-1}C_{3-1}\right)$

$36 \times 6 = 216$

36. Let the first instalment be a and common difference of AP be d.

Given, 3600 = sum of 40 terms

$= \frac{40}{2}[2a + (40-1)d]$

$\Rightarrow$ 2a + 39d = 180 ....(i)

After 30 instalments one-third of the loan is unpaid.

Now $\frac{3600}{3} = 1200$ is unpaid and 2400 is paid

So, 2400 = $\frac{30}{2}[2a + (30-1)d]$

$\Rightarrow$ 2a + 29d = 160 .....(ii)

Solving eq. (i) & (ii) a = 51, d = 2

So 8^{th} instalment = a + (8 – 1) d = 65

37. $(1 - x)^5 (1 + x)^4 (1 + x^2)^4$

$\Rightarrow (1 - x^2)^4 (1 + x^2)^4 (1 - x) = (1 - x^4)^4 (1 - x)$

$\therefore$ Coefficient of x^{13} in

$\{(1 - x)(1 - 4x^4 + 6x^8 - 4x^{12} ...)\} = 4$

38. A.(Adj. A) = |A| I

|A| = xyz – 8x – 3(z – 8) + 2(2 – 2y)

= xyz – (8x + 3z + 4y) + 28

= 50 – 30 + 28 = 48

$\therefore$ A.(Adj A) = 48 I = $\begin{pmatrix} 48 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 48 \end{pmatrix}$

39. Radius of the circle = Perpendicular distance of (1, 1) from y – 3x = 0

$= \frac{2}{\sqrt{10}}$

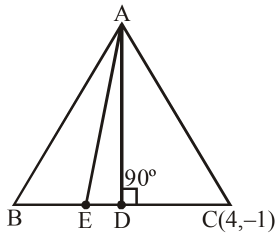
Let the other tangent be y – kx = 0

$\Rightarrow$ Perpendicular distance from (1, 1) = radius

$\Rightarrow \frac{1-k}{\sqrt{1+k^2}} = \frac{2}{\sqrt{10}} \qquad \Rightarrow \quad k = 3, \frac{1}{3}$

Therefore other tangent is $y = \frac{1}{3}x$

40.



AD and AE meet at A therefore intersection of these two lines will give the coordinates of A which is (–3, 2).

Let coordinates of B is (h, k)

$\Rightarrow$ Slope of BC × Slope of AD = –1

$$\Rightarrow \quad \frac{k+1}{h-4} = \frac{-3}{2}$$

$\Rightarrow$ 3h + 2k = 10 ...(i)

Also mid point of BC lies on AE

$$\Rightarrow \quad \left(\frac{4+h}{2}, \frac{-1+k}{2}\right) \text{ satisfies } 2x + 3y = 0$$

$\Rightarrow$ 2h + 3k = –5 ...(ii)

Solving (i) and (ii) we get h = 8 and k = –7

Therefore, A = (–3, 2), B = (8, –7) and C = (4, –1)

$$\therefore \quad \text{Slope } AB = \frac{-7-2}{8+3} = \frac{-9}{11}$$

41. Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where $b^2 = a^2(e^2 - 1)$

Then from the given facts

$2ae = 2 \times 10 \Rightarrow ae = 10$

Also $\frac{2a}{e} = 2a'e'$

$$\Rightarrow \quad \frac{2a}{e} = 2 \times 10 \times \frac{3}{5} \quad \Rightarrow \quad \frac{a}{e} = 6$$

($\frac{3}{5}$ is the eccentricity of the ellipse)

We easily get $a^2 = 60$, $e^2 = \frac{100}{60}$

Hence $b^2 = a^2(e^2 - 1)$

$$= 60\left(\frac{100}{60} - 1\right) = 40$$

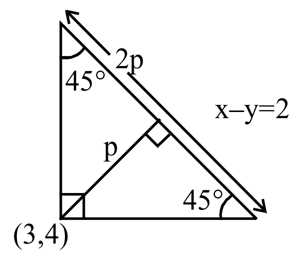
$\Rightarrow$ Equation of hyperbola is

$$\frac{x^2}{60} - \frac{y^2}{40} = 1$$

43. Required equation is $(\bar{r} - \bar{a}).\bar{b} \times \bar{c} = 0$

(Passing through $A(\bar{a}), B(\bar{b}), C(\bar{c})$)

44.



$$p = \left|\frac{3-4-2}{\sqrt{1+1}}\right| = \frac{3}{\sqrt{2}}$$

$$\text{Area of triangle } = \left|\frac{1}{2}(p)(2p)\right|$$

$$= p^2 = \frac{9}{2} \text{ sq. unit}$$

45. $f''(x) = \sec^2 x - 1$

Integrating $f'(x) = \tan x - x + c_1$

But $f'(0) = 0 - 0 + c_1 \Rightarrow c_1 = 0$

$\Rightarrow f'(x) = \tan x - x$

Again Integrating

$f(x) = \log \sec x - \frac{x^2}{2} + c_2$ but $f'(0) = 0$

$\Rightarrow c_2 = 0$

$\Rightarrow f(x) = \log \sec x - \frac{x^2}{2}$.

46. Putting $t = \frac{1}{y}$

$$= \int_{1/e}^{\tan x} \frac{t}{1+t^2}\, dt + \int_{e}^{\tan x} \frac{y}{\left(1+\frac{1}{y^2}\right)} \times \frac{-1}{y^2}\, dy$$

$$= \int_{1/e}^{\tan x} \frac{t}{1+t^2}\, dt + \int_{\tan x}^{e} \frac{y}{1+y^2}\, dy$$

$$= \int_{1/e}^{\tan x} \frac{t}{1+t^2}\, dt + \int_{\tan x}^{e} \frac{t}{1+t^2}\, dt \text{ (By prop (1))}$$

$$= \int_{1/e}^{e} \frac{t}{1+t^2}\, dt = \frac{1}{2}\left[\ell n(1+t^2)\right]_{1/e}^{e} = 1.$$

47. For $x \in [-2, 2]$; $-\frac{1}{2} \le \frac{x}{4} \le \frac{1}{2}$

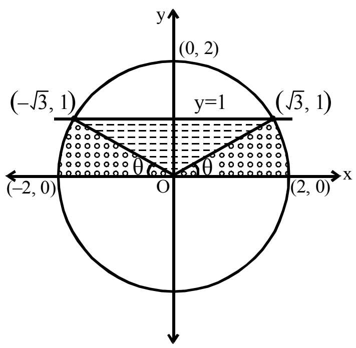
$\cos\frac{x}{4} \ge \cos^2\frac{x}{4}$

i.e. $\left(\sin^2\frac{x}{4}+\cos\frac{x}{4}\right) \ge 1$

Also, $\left(\sin^2\frac{x}{4}+\cos\frac{x}{4}\right) < 2$

$\therefore \left[\sin^2\frac{x}{4}+\cos\frac{x}{4}\right] = 1$

$\therefore y = 1$



Required Area = 2(Area of sector) + Area of Δ

$= 2\left(\frac{1}{2}r^2\theta\right) + \frac{1}{2} \times 2\sqrt{3} \times 1$

$= 4\frac{\pi}{6} + \sqrt{3}$

$= \left(\frac{2\pi}{3}+\sqrt{3}\right)$ square unit.

48. $\cot y \operatorname{cosec} y \frac{dy}{dx} + \operatorname{cosec} y \frac{1}{x} = \frac{1}{x^2}$ ...(i)

Put $\operatorname{cosec} y = z$

$-\operatorname{cosec} y \cot y \frac{dy}{dx} = \frac{dz}{dx}$

$-\frac{dz}{dx} + z \cdot \frac{1}{x} = \frac{1}{x^2}$

$\Rightarrow \frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$ $\therefore$ I.F. $= e^{\int -\frac{1}{x}dx} = \frac{1}{x}$

$\therefore$ Solution is $z \cdot \frac{1}{x} = -\int \frac{1}{x^3}dx + K$

or $\frac{1}{x}\operatorname{cosec} y = \frac{1}{2x^2} + K$.

57. **Statement–1 :** The inverse of adiagonal matrix is always a diagonal matrix.

Statement–2 :

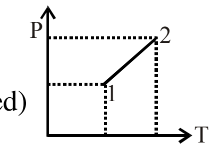
$|AB^{-1}| = |A| \,.\, |B^{-1}| = |A|\,|B^{-1}| = (-6)\left(\frac{1}{2}\right) = -3$

So statement–1 is true & statement–2 is false.

58. Lines are inter secting.

61. PV = μRT

$\Rightarrow V \propto \frac{T}{P}$ ($\because$ μ and R are fixed)



Since, T increases rapidly and P increases slowly thus volume of the gas increases.

62. From ideal gas equation PV = RT .....(i)

or PΔV = RΔT

Dividing equation (ii) by (i) we get

$\frac{\Delta V}{V} = \frac{\Delta T}{T} \Rightarrow \frac{\Delta V}{V\Delta T} = \frac{1}{T} = \delta$ (given)

$\therefore \delta = \frac{1}{T}$. So the graph between δ and T will be rectangular hyperbola

63. $h = \frac{T_1 - T_2}{T_1} = \frac{W}{Q} \Rightarrow W = \frac{Q(T_1 - T_2)}{T_1}$

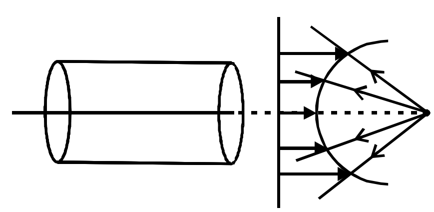
$= \frac{6\times10^4[(227+273)-(273+127)]}{(227+273)}$

$= \frac{6\times10^4\times100}{500} = 1.2 \times 10^4$

64. $W_{BCOB} = -$ Area of triangle BCO $= \frac{-P_0V_0}{2}$

$W_{AODA} = +$ Area of triangle AOD $= +\frac{+P_0V_0}{2}$

80.



Parallel beam comes from infinity i.e. source is at infinity hence it is a plane wavefront.

81. $\mu(I) = \mu_0 + \mu_2 I$
$r\uparrow\ I\downarrow\ \mu\downarrow$

$v = \frac{c}{\mu}$ $v\uparrow$ (minimum at the axis)

82. Converging

83. $E = W_0 + eV_0 \Rightarrow 4\text{ eV} = 2\text{ eV} + eV_0$

$\Rightarrow V_0 = 2$ volt

$$\Rightarrow \lambda = \sqrt{\frac{150}{2}} = \sqrt{75}\,\text{Å} = 5\sqrt{3}\,\text{Å}$$

84. Let us assume that the intial total number of radioactive substances are the same.

Smaple-1 : Half life $T_{1/2} = 1$ year

Smaple-1 : Half life $T_{1/2} = 2$ year

		1Yr	2Yr	3Yr	4Yr	5Yr	6Yr
Sample-1	N	$N_0/2$	$N_0/4$	$N_0/8$	$N_0/16$	$N_0/32$	$N_0/64$
Sample-2	N		$N_0/2$		$N_0/4$		$N_0/8$

$$\frac{dN_1}{dt} = -\lambda_1 . \frac{N_0}{64} \Rightarrow \frac{dN_1}{dt} = -\frac{0.694}{1\text{ yr}} . \frac{N_0}{64}$$

$$\frac{dN_2}{dt} = -\lambda_2 . \frac{N_0}{8} \Rightarrow \frac{dN_2}{dt} = -\frac{0.694}{2\text{ yr}} . \frac{N_0}{8}$$

85. Let ground state energy (in eV) be E_1.
Then from the given condition

$$E_{2n} - E_1 = 204\text{eV} \text{ or } \frac{E_1}{4n^2} - E_1 = 204\text{eV}$$

$$\Rightarrow E_1\left(\frac{1}{4n^2} - 1\right) = 204 \text{ eV} \qquad .....(i)$$

and $E_{2n} - E_n = 40.8$ eV

$$\Rightarrow \frac{E_1}{4n^2} - \frac{E_1}{n^2} = E_1\left(-\frac{3}{4n^2}\right) = 40.8 \text{ eV}$$

From equation (i) and (ii),

$$\frac{1 - \frac{1}{4n^2}}{\frac{3}{4n^2}} = 5 \Rightarrow \qquad n = 2$$

86. In irreversible process there always occurs some loss of energy. This is because energy spent in working against the dissipative force is not recovered back. Some irreversible process occurs in nature such as friction where extra work is done to cancel the effect of friction. Salt dissolves in water but a salt does not separate by itself into pure salt and pure water.

90. The energy gap between valence band and conduction band in germanium is 0.76 eV and the energy gap between valence band and conduction band in silicond is 1.1 eV. Also it is true that thermal energy produces fewer minority carriers in silicon that in germanium.