JEE-MAIN	TEST DATE	ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)
TARGET : JEE 2013	28 - 03 - 2013	Path to success — Kota | Chandigarh | Ahmedabad

FULL SYLLABUS

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	2	4	1	2	3	1	3	2	1	2	2	1	3	4	4	3	1	3	2
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	1	4	2	4	3	2	2	1	1	3	4	4	4	2	3	1	4	2	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	4	2	2	2	4	4	3	2	1	2	2	2	4	2	4	1	2	4	2
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	2	3	1	4	1	2	3	1	4	1	3	2	1	2	4	4	4	1	1	1
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	3	3	3	4	2	1	4	4	3	3										

HINT – SHEET

1. $\mu = \mu_1 + \mu_2$

$$\frac{P(2V)}{RT_1} = \frac{PV}{RT_1} + \frac{PV}{RT_2}$$

$$\Rightarrow \frac{2P}{RT_1} = \frac{P'}{R}\left[\frac{T_2+T_1}{T_1T_2}\right]$$

$$P' = \frac{2PT_2}{(T_1+T_2)} = \frac{2\times1\times600}{(300+600)} = \frac{4}{3}\text{atm}$$

2. $VP^3 = \text{constant} = k \Rightarrow P = \frac{k}{V^{1/3}}$

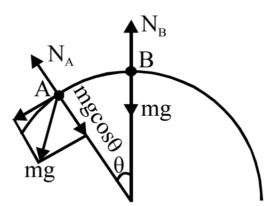
Also $PV = \mu RT \Rightarrow \frac{k}{V^{1/3}} . V = \mu RT$

$$\Rightarrow V^{2/3} = \frac{\mu RT}{k} \text{ Hence } \left(\frac{V_1}{V_2}\right)^{2/3} = \frac{T_1}{T_2}$$

$$\Rightarrow \left(\frac{V}{27V}\right)^{2/3} = \frac{T}{T_2} \Rightarrow T_2 = 9\ T$$

3. $\eta = 1 - \frac{T_2}{T_1}$; for η to be max. ratio $\frac{T_2}{T_1}$ should be min.

4.



$$mg\cos\theta - N_A = \frac{mv^2}{R}$$

$$N_A = mg\cos\theta - \frac{mv^2}{R}$$

$$mg - N_B = \frac{mv^2}{R}$$

$$N_B = mg - \frac{mv^2}{R}$$

$N_B > N_A$

5. On platform

$2T - N - 40g = 40 \times 2$ ...(1)

On man

$T + N - 60g = 60 \times 2$ ...(2)

Solving (1) & (2)

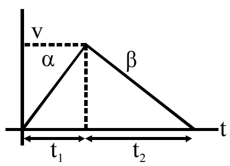
$T = 400N$

6. $m(v_f - v_i)$

$2\left[0 - \left(-\sqrt{2g \times 5}\right)\right]$

$= 20$ N-s

7. $\frac{v}{t_1} = \alpha, \frac{v}{t_2} = \beta$



$t_1 + t_2 = t$

$\frac{v}{\alpha} + \frac{v}{\beta} = t$

$v = \frac{\alpha\beta t}{\alpha + \beta}$

$v = \frac{\alpha\beta t}{\alpha + \beta}$

18. $\Delta x = t(\mu - 1)$

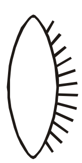
$\Delta x_1 = t(1.5 - 1) = 0.5t$

$\Delta x_2 = 2t(4/3 - 1) = 2t \times 1/3 = t/3 = 0.33t$

$\Delta x_1 > \Delta x_2$

Shifts in +y axis direction

19. $f' = \frac{R}{4\mu - 2}$



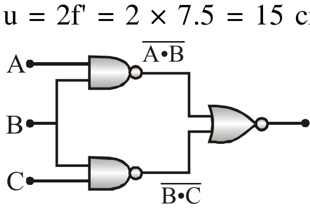
$= \frac{30}{4 \times 1.5 - 2}$

$= 7.5$ cm

Concave mirror

$u = 2f' = 2 \times 7.5 = 15$ cm

20.



$Y = \overline{\overline{A \bullet B} + \overline{B \bullet C}}$

$= A \bullet B \bullet B \bullet C$

$= A \bullet B \bullet C$

21. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{128} = \left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^n \Rightarrow n = 7$

After 7 half lives intensity emitted will be safe

$\therefore$ Total time taken $= 7 \times 2 = 14$ hrs.

22. As we know current density $J = nqv$

$\Rightarrow J_e = n_e q v_e$ and $J_h = n_h q v_h$

$\Rightarrow \frac{J_e}{J_h} = \frac{n_e}{n_h} \times \frac{v_e}{n_h} \Rightarrow \frac{3/4}{1/4} = \frac{n_e}{n_h} \times \frac{5}{2} \Rightarrow \frac{n_e}{n_h} = \frac{6}{5}$

23. Adiabatic expansion produces cooling.

25. Process CD is isochoric as volume is constant, process DA is isothermal as temperature constant and process AB is isobaric as pressure is constant.

26. $M_1 = M$

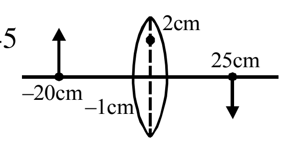
$M_2 = \frac{M}{a^2} \times \frac{\pi a^2}{16} = \frac{\pi M}{16}$

$X_{cm} = \frac{M_1 X_1 - M_2 X_2}{M_1 - M_2}$

$X_{cm} = \frac{M \times O - \frac{\pi M}{16} \times \frac{a}{4}}{M - \frac{\pi M}{16}}$

$X_{cm} = \frac{\pi a}{64 - \pi}$

29. $m = \frac{I}{0} = \frac{1}{2} = \frac{V}{u} \Rightarrow u = 2v$



Distance $= u + v = 45$

$2v + v = 45$

$v = 15$ cm

Position of lens $x = +10$ cm

30. Number of photoelectrons emitted up to $t = 10$ sec are

$n = \frac{(\text{Number of photons per unit area per unit time}) \times (\text{Area} \times \text{Time})}{10^6}$

$= \frac{1}{10^6}[(10)^{16} \times (5 \times 10^{-4}) \times (10)] = 5 \times 10^7$

At time t = 10s

Charge on plate A $= q_A = +ne$

$= 5 \times 10^7 \times 1.6 \times 10^{-19}$

$= 8 \times 10^{-12}C = 8pC$

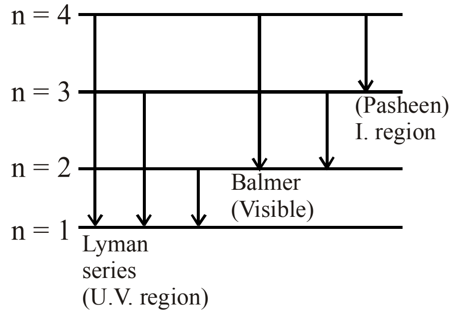
and charge on plate B; $q_s = 33.7 - 8 = 25.7$ pc

Electric field batween the plates

$E = \frac{(q_B - q_A)}{2\varepsilon_0 A}$

$= \frac{(25.7 - 8) \times 10^{-12}}{2 \times 8.85 \times 10^{-12} \times 5 \times 10^{-4}} = 2 \times 10^3 \frac{N}{C}$

31.



33. $X(g) \rightleftharpoons 2Y(g)$

a

$a - a\alpha \qquad 2a\alpha$

$\frac{M_{Th.}}{M_{obs.}} = 1 + \alpha \Rightarrow \frac{100}{75} = 1 + \alpha \Rightarrow \alpha = \frac{1}{3}$

$K_P = \frac{4\alpha^2 P_T}{(1-\alpha^2)} = 1$

34. $Ag_2CrO_4 \longrightarrow 2Ag^+ + CrO_4^{-2}$

In water $\qquad 2s_1 \qquad s_1$

$K_{sp} = (2s_1)^2 s_1 \Rightarrow s_1 = 1.31 \times 10^{-4}$

In 0.1 M $\qquad 0.1 \qquad s_2$

$AgNO_3 \quad K_{sp} = (0.1)^2 s_2 \Rightarrow s_2 = 9 \times 10^{-10}$

In 0.1 M $\qquad 2s_3 \qquad 0.1$

$Na_2CrO_4 \quad K_{sp} = (2s_3)^2(0.1) \quad s_3 = 1.5 \times 10^{-5.5}$

so order $s_2 < s_3 < s_1$.

35. $N_2H_4(\ell) \longrightarrow N_2H_4(g) \qquad \Delta H = 18$ kJ

$2NH_3(g) \longrightarrow N_2H_4(\ell) + H_2(g) \qquad \Delta H = 142$ kJ

$2NH_3(g) \longrightarrow N_2H_4(g) + H_2(g) \qquad \Delta_r H = 160$ kJ

$160 = 2(3N-H) - \begin{bmatrix} 1\ N-N \\ + \\ 4\ N-H \\ + \\ 1\ H-H \end{bmatrix}$

$160 = 2\ (N-H) - 1\ (N-N) - 1\ (H-H)$

$160 = 2\ (393) - (N-N) - 436$

$E_{N-N} = 190$ kJ

37. $BaCl_2 + H_2SO_4 \rightarrow BaSO_4 + 2HCl$

0.5 mol $\qquad$ 1 mol

0 $\qquad$ 0.5 mol $\qquad$ 0.5 mol

38. $P_S = P_B^0 X_B + P_T^0 X_T$

$= 120 \times \left(\frac{2}{2+2}\right) + 80\left(\frac{2}{2+2}\right)$

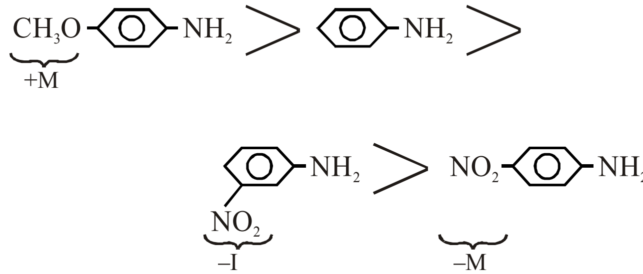
$= 100$

$X'_T = \frac{P_T^0 X_T}{P_S} = \frac{40}{100} = 0.4$

48. $\overset{7}{C}H_3 - \overset{6}{C}H = \overset{5}{C}H - \overset{4}{C}H_2 - \overset{3}{C}H(NH_2) - \overset{2}{C}H_2 - \overset{1}{C}OOH$

3-Aminohept-5-enoic acid

49. Basic strength order



50.

$CH_3-C(CH_3)_2-CH_2-Cl \xrightarrow[C_2H_5OH]{C_2H_5O^{\ominus}\overset{\oplus}{Na}} CH_3-C(CH_3)_2-CH_2-O-CH_2-CH_3$

Reaction followed by SN^2 Path.

51.

$CH_3=C(CH_3)-CH=CH_2 \xrightarrow[Zn + H_2O]{O_3} 2CH_2=O + CH_3-C(=O)-CHO$

52. $CH_3-C(=O)-$ and $CH_3-CH(OH)-$ group containing compound gives iodoform test

55. $K_{sp} = [Cu^{+2}][OH^-]^2$

$\therefore\ P^H = 14 \Rightarrow P^{OH} = 0$

$\therefore\ (OH^-) = 1$ M

$\therefore\ 1.0 \times 10^{-19} = [Cu^{+2}]\ [1]^2$

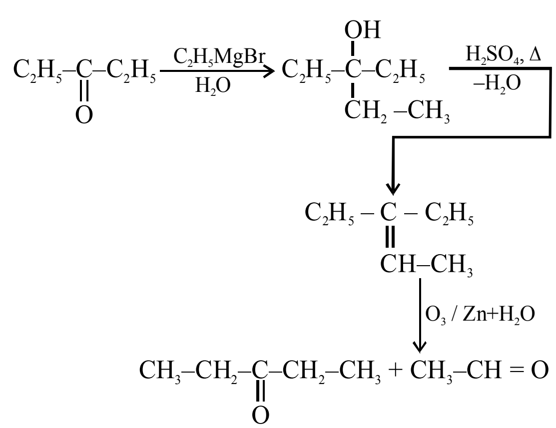
$[Cu^{+2}] = 10^{-19}$ M

$E_{red} = E^o_{red} + \frac{0.0591}{2} \log [Cu^{+2}]$

$= 0.34 + \frac{0.0591}{2} \log 10^{-19}$

$= -\ 0.221$ volt

60.



61. $10^{100} = 2^{100}, 5^{100}$

For divisor which are divisibly by $10^{90} = 2^{90}$, 5^{90} atleast 90 two's and 90 five's are to be compulsory.

So $2^{10}, 5^{10}$

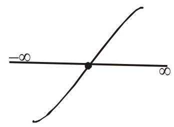
$(10 + 1)(10 + 1) = 121$

62. $f'(x) > 0\ \forall\ x \in R$

So function $f|x|$ is increasing in whole domain R and

$x \to -\infty\ f(x) < 0$

$x \to \infty\ f(x) > 0$



so exactly one real root lie in $(-\infty, \infty)$

63. $P_1 = \dfrac{|\underline{6}\,|\underline{6} \times 2}{|\underline{12}}$

$P_2 = \dfrac{|\underline{5} \times |\underline{6}}{|\underline{11}}$

$\dfrac{P_1}{P_2} = 1$

64. $\sum_{r=0}^{\infty} r^n = S \Rightarrow 1 + r + r^2 + ...\infty = S$

$\Rightarrow \dfrac{1}{1-r} = S \Rightarrow r = \dfrac{S-1}{S}$

Now $\sum_{r=0}^{\infty} r^{2n} = 1 + r^2 + r^4 + ...\infty$

$= \dfrac{1}{1-r^2} = \dfrac{1}{1-\left(\dfrac{S-1}{S}\right)^2} = \dfrac{S^2}{2S-1}$

65. Let the number are a & b.

Then a, A_1, A_2, b are in A.P. $\Rightarrow A_1 + A_2 = a + b$

a, G_1, G_2, b are in GP $\Rightarrow G_1G_2 =$ ab

a, H_1, H_2, b are in HP

$\Rightarrow \dfrac{1}{H_1} + \dfrac{1}{H_2} = \dfrac{1}{a} + \dfrac{1}{b} \Rightarrow \dfrac{H_1 + H_2}{H_1H_2} = \dfrac{a+b}{ab}$

$\therefore \quad \dfrac{H_1 + H_2}{H_1H_2} = \dfrac{A_1 + A_2}{G_1G_2}$

$\Rightarrow \dfrac{G_1G_2}{H_1H_2} = \dfrac{A_1 + A_2}{H_1 + H_2}$

66. $3^{37} = 3.3^{36} = 3.(81)^9 = 3(80 + 1)^9$

$= 3({}^9C_0 80^9 + {}^9C_1.80^8 + ... + {}^9C_8 \,.\, 80^1 + {}^9C_9)$

So required remainder is 3.

67. Since $d_1 = d_2 = d_3 = 0 \Rightarrow D_1 = D_2 = D_3 = 0$

So for non-zero solution

$$\begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & a+1 & a+2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$= -\begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a+2 \\ a^3 & (a+1)^3 & (a+2)^3 \end{vmatrix} = 0 \Rightarrow a = -1$$

68. $\vec{\alpha}.\vec{\beta} = |\vec{\alpha}||\vec{\beta}|\cos\dfrac{\pi}{4}$

$\sqrt{2}a + 3\sqrt{2}b + 4c = I.6.\dfrac{1}{\sqrt{2}}$

$a + b + 2\sqrt{2}\,C = I.3$

$\Rightarrow$ LHS is integer if c = 0

then $\vec{\alpha} = a\hat{i} + b\hat{j}$ which lies in xy plane

69. Since a, b, c are the roots of the equation $x^3 - 3x^2 + x + \lambda = 0$

$\Rightarrow a + b + c = 3 \qquad \Rightarrow a + b = 3 - c$

Now area of the triangle will be

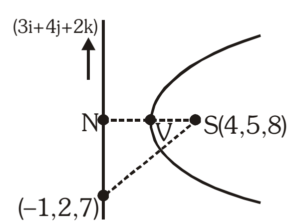
$A = \dfrac{1}{2} \times \dfrac{1}{a+b} \times 1 = \dfrac{1}{2(a+b)} = \dfrac{1}{2(3-c)}$

$\Rightarrow \dfrac{dA}{dc} = \dfrac{1}{2(3-c)^2} > 0$

As A is an increasing function & $c \in [1,2]$

$\therefore \quad A_{max} = \dfrac{1}{2}$ sq. units.

70. Locus of variable point 'P' is a parabola with given line as directrix and given point as focus. Vertex is the mid point of S and foot of perpendicular 'N'.



Let N be $(-1 + 3\lambda, 2+4\lambda, 7+2\lambda)$

$\overline{SN} = (5 - 3\lambda)\hat{i} + (3 - 4\lambda)\hat{j} + (1 - 2\lambda)\hat{k}$

$\overline{SN}.(3\hat{i} + 4\hat{j} + 2\hat{k}) = 0 \Rightarrow \lambda = 1$

$N \equiv (2,6,9)$

$\Rightarrow \quad V \equiv \left(3, \frac{11}{2}, \frac{17}{2}\right)$

71. $\dfrac{\left(\dfrac{2x+3y-5}{\sqrt{13}}\right)^2}{4} + \dfrac{\left(\dfrac{-3x+2y+1}{\sqrt{13}}\right)^2}{1} = 1$

which is equivalent ot $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with a = 2, b=1

$\therefore$ Area of ellipse = π.2.1 = 2π

72. Let P(h, k) be the point from which two tangents are drawn to $y^2 = 4x$. Any tangent to the parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$

If it passes through P(h, k), then

$k = mh + \frac{1}{m} \Rightarrow m^2 h - mk + 1 = 0$

Let m_1, m_2 be the roots of this equation. Then,

$m_1 + m_2 = \frac{k}{h}$ and $m_1m_2 = \frac{1}{h}$

$\Rightarrow 3m_2 = \frac{k}{h}$ and $2m_2^2 = \frac{1}{h}$ [$\because$ $m_1 = 2m_2$(given)]

$\Rightarrow 2\left(\frac{k}{3h}\right)^2 = \frac{1}{h} \Rightarrow 2k^2 = 9h$

Hence, P(h, k) lies on $2y^2 = 9x$

73. $I = \int_0^{\pi} \frac{x\tan x}{\sec x + \cos x}\,dx$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x)+\cos(\pi - x)}\,dx$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \cos x}\,dx$

$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\tan x\,dx}{\sec x + \cos x} = 2\pi\int_0^{\pi/2} \frac{\tan x\,dx}{\sec x + \tan x}$

$\Rightarrow I = \pi\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$

$\Rightarrow I = -\pi\int_1^{0} \frac{dt}{1+t^2} = \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$

74. $\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1} = (x+1) + \frac{y-3}{x+1}$

Putting x + 1 = X, y – 3 = Y, $\frac{dy}{dx} = \frac{dY}{dX}$

the equation becomes

$\frac{dY}{dX} = X + \frac{Y}{X}$ or $\frac{dY}{dX} - \frac{1}{X}.Y = X$ [L.D.E.]

I.F. = $e^{\int(-1/X)dX} = e^{-\log X} = \frac{1}{X}$

$\therefore$ the solution is

$Y.\left(\frac{1}{X}\right) = c + \int X\left(\frac{1}{X}\right)dx = c + X$

or $\frac{(y-3)}{(x+1)} = c + x + 1$

x = 2, y = 0 $\Rightarrow \frac{0-3}{2+1} = c + 2 + 1$

$\Rightarrow$ c = –4

$\therefore$ the quation of the curve is

$\frac{y-3}{x+1} = x - 3$ or $y = x^2 - 2x$.

78. USE DL

$L = a^a (1 + \ell na)$ and $M = a^a$

$1 + \ell na = 2$

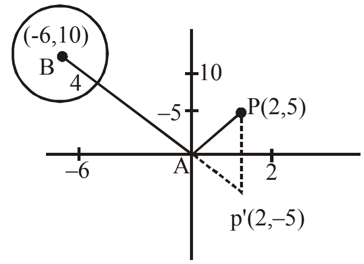
a = e

79. LHD = $\lim_{h\to 0} \frac{f(-h)-f(0)}{-h} = \lim_{h\to 0} \frac{-h+1-1}{-h} = 1$

RHD = $\lim_{h\to 0} \frac{f(h)-f(0)}{h} = \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$

Not differentiable but continuous.

80.



BP'=17

85. $\left|\dfrac{Z_1 - 2Z_2}{2 - Z_1\bar{Z}_2}\right| = 1$

so $\left(\dfrac{Z_1 - 2Z_2}{2 - Z_1\bar{Z}_2}\right)\left(\dfrac{\bar{Z}_1 - 2\bar{Z}_2}{2 - \bar{Z}_1 Z_2}\right) = 1$

$|Z_1|^2 \ -2(Z_1\bar{Z}_2 + \bar{Z}_1 Z_2) \ + 4|Z_2|^2$

$= 4 - 2 \ (Z_1\bar{Z}_2 + \bar{Z}_1 Z_2) \ + |Z_1|^2 \ |Z_2|^2$

$|Z_1|^2 (1 - |Z_2|^2) - 4 (1 - |Z_2|^2) = 0$

$(1\text{-}|Z_2|^2) (|Z_1|^2 - 4) - 4) = 0$

$\because \ |Z_2| \neq 1$ so $|Z_1| = 2$

86. $ax + by = 0$

$cx + dy = 0$

homogenesous system

It always have atleast one solution so St–2 is correct.

For unique solution

$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

these are 6 Δ possible probability $= \dfrac{6}{2^4} = \dfrac{6}{16} = \dfrac{3}{8}$

so St-1 is also true but St-2 is not a correct explanation of St-1.

88. Statement-2 is clearly true.

Since $\overline{OM} = \lambda\overline{OD} = \lambda\vec{d}$

Now points A, B, C and M are coplanar

$\Rightarrow \quad [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{m}\ \vec{a}\ \vec{b}] + [\vec{m}\ \vec{b}\ \vec{c}] + [\vec{m}\ \vec{c}\ \vec{a}]$

$= [\lambda\vec{d}\ \vec{a}\ \vec{b}] + [\lambda\vec{d}\ \vec{b}\ \vec{c}] + [\lambda\vec{d}\ \vec{c}\ \vec{a}]$

$\Rightarrow \quad \lambda = \dfrac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{d}\ \vec{a}\ \vec{b}] + [\vec{d}\ \vec{b}\ \vec{c}] + [\vec{d}\ \vec{c}\ \vec{a}]}$

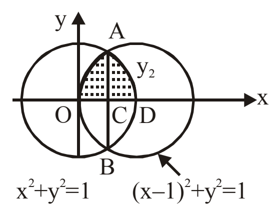
and therefore Statement-1 is also true.

89. $x^2 + y^2 = 1$

and $(x - 1)^2 + y^2 = 1$

are symmetrical about x-axis

Solving these,



C is (1/2,0), also D is (1, 0).

$\therefore$ Required area

$= 2\left\{\int_0^{1/2}\sqrt{1-(x-1)^2}\,dx + \int_{1/2}^{1}\sqrt{1-x^2}\,dx\right\}$

$= \left(\dfrac{2\pi}{3} - \dfrac{\sqrt{3}}{2}\right)$ sq. units.

90. Statement-1 is correct statement-2 is false.