JEE-MAIN | TARGET : JEE 2013

TEST DATE | 31 - 03 - 2013

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) — Path to success — Kota | Chandigarh | Ahmedabad

FULL SYLLABUS

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	3	4	4	4	1	3	2	3	2	2	4	4	4	3	2	3	4	2	1
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	2	1	2	3	2	1	4	4	2	4	2	3	2	1	4	4	2	2	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	2	2	2	2	1	1	1	2	2	1	1	2	1	2	4	2	2	4	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	2	1	3	3	1	3	1	2	2	4	4	4	1	3	4	3	2	2	2
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	2	1	4	1	4	3	2	2	2										

HINT – SHEET

1. $|z_1 + z_2| = |z_1| - |z_2|$

$|z_1|^2 + |z_2|^2 + 2|z_1|\,|z_2| \cos(\theta_1 - \theta_2)$

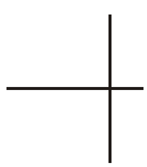
$= |z_1|^2 + |z_2|^2 - 2|z_1|\,|z_2|$

$\cos(\theta_1 - \theta_2) = -1$

$\theta_1 - \theta_2 = \pi$

$\arg(z_1) - \arg(z_2) = \pi$

$\arg\left(\frac{z_1}{z_2}\right) = \pi$



$\frac{z_1}{z_2} \rightarrow$ purely negative real number

2. $|z - a^2| + |z + 2a| = 3$

for an ellipse

$3 > |a^2 + 2a|$

$|a^2 + 2a| < 3$

$-3 < a^2 + 2a < 3$

$a^2 + 2a - 3 < 0$ and $a^2 + 2a + 3 > 0$

$(a + 3)(a - 1) < 0$ and $a \in R$

$a \in (-3, 1)$ and $a \in R$

So $a \in (-3, 1)$

3. Four couples = 4 Men, 4 women

Possible cases

(i) All 4 men = ${}^4C_4 = 1$

(ii) 3 Men, 1 Woman = ${}^4C_3 \times {}^1C_1 = 4$

(iii) 2 Men, 2 Women = ${}^4C_2 \times {}^2C_2 = 6$

(iv) 1 Man, 3 Women = ${}^4C_1 \times {}^3C_3 = 4$

(v) All 4 Women = ${}^4C_4 = 1$

16

4. Let Both A & B are having daughters = x

So required probability $= \frac{{}^xC_3}{{}^{2x}C_3} = \frac{1}{20}$

$\Rightarrow x = 0, 3$ but $x \neq 0$

So x = 3.

5. $\tan^2\theta + p\tan\theta + q = 0$

$\tan\alpha + \tan\beta = -p$

$\tan\alpha \tan\beta = q$

$$\tan(\alpha+\beta) = \frac{-p}{1-q} \Rightarrow \frac{p}{q-1}$$

6. $S_1 = \frac{n_1}{2}[2a+(n_1-1)d] \Rightarrow \frac{2S_1}{n_1} = 2a+(n_1-1)d$

$S_2 = \frac{n_2}{2}[2a+(n_2-1)d] \Rightarrow \frac{2S_2}{n_2} = 2a+(n_2-1)d$

$S_3 = \frac{n_3}{2}[2a+(n_3-1)d] \Rightarrow \frac{2S_3}{n_3} = 2a+(n_3-1)d$

$\therefore \frac{S_1}{n_1}(n_2-n_3)+\frac{S_2}{n_2}(n_3-n_1)+\frac{S_3}{n_3}(n_1-n_2)=0$

7. ${}^{n-1}C_0.{}^nC_2 + {}^{n-1}C_1.{}^nC_3 + \ldots + {}^{n-1}C_{n-2}.{}^nC_n$

$\Rightarrow {}^{n-1}C_0.{}^nC_{n-2} + {}^{n-1}C_1.{}^nC_{n-3} + \ldots + {}^{n-1}C_{n-2}.{}^nC_0$

So we have to find coefficient of x^{n-2} in expansion of $(1 + x)^{n-1}.(x + 1)^n$ i.e. $(1 + x)^{2n-1} = {}^{2n-1}C_{n-2}$

8. We have

$[A(A + B)^{-1}B]^{-1} = B^{-1}[(A + B)^{-1}]^{-1}A^{-1}$

$= B^{-1}(A + B) A^{-1} = (B^{-1}A + I) A^{-1}$

$\Rightarrow B^{-1}I + IA^{-1} = A^{-1} + B^{-1}$.

9. $$\lambda\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = (a+b+c)(b+c-a)(c+a-b)(a+b-c)$$

$\Rightarrow 4\lambda\, \Delta^2 = 16\, s(s - a)(s - b)(s - c)$

$\Rightarrow 4\lambda\, \Delta^2 = 16\, \Delta^2$

$\Rightarrow 4\lambda = 16 \quad \Rightarrow \lambda = 4$

10. Eq of any tangent to $y^2 = 4x$ in terms of m is

$$y = mx + \frac{1}{m} \quad ....(i)$$

Eq of any normal to $x^2 = 4by$ in terms of m is

$$y = mx + 2b + \frac{b}{m^2} \quad ....(ii)$$

Equation (i) and (ii) represent the same line

$\therefore \frac{1}{m} = 2b + \frac{b}{m^2}$

or $2bm^2 - m + b = 0$

For two distinct values of m

$D = (-1)^2 - 4(2b)(b) > 0$

$\Rightarrow 1 - 8b^2 > 0$

$\Rightarrow b^2 < \frac{1}{8} \quad \Rightarrow |b| < \frac{1}{2\sqrt{2}}$

11. $\vec{a}.\vec{b} > 0$

$\Rightarrow x^2 + 2x - 1 + a > 0$

$\Rightarrow x^2 + 2x + (a - 1) > 0$

$\Rightarrow D = 4 - 4(a - 1) < 0$

$\Rightarrow a > 2$

$\Rightarrow a \in (2, \infty)$

12. Eq of will is given by

$(x - 3)^2 + (y - 2)^2 + \lambda(3x + 8y - 25) = 0$ ...(i)

It passes through (8, –3)

$\therefore \lambda = 2$

Put $\lambda = 2$ in (i)

$x^2 + y^2 + 12y - 37 = 0$

$r = \sqrt{36+37} = \sqrt{73}$.

14. Let ABCD be the given tetrahedron. Then,

$\overrightarrow{AB} = \hat{j}$, $\overrightarrow{AC} = \hat{j}+\hat{k}$ and $\overrightarrow{AD} = \hat{i}+3\hat{j}+\lambda\hat{k}$

Now, volume $= \frac{1}{6}$

$\Rightarrow \frac{1}{6}[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = \frac{1}{6}$

$\Rightarrow [\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 1$

$\Rightarrow (\overrightarrow{AB}\times\overrightarrow{AC}).\overrightarrow{AD} = 1$

$\Rightarrow \hat{i}.(\hat{i}+3\hat{j}+\lambda\hat{k}) = 1$, which is true for all values of λ.

17. $$\int \frac{y}{\sqrt{1-y^2}}dy = x + c$$

$\Rightarrow -\sqrt{1-y^2} = x + c$

$\Rightarrow 1 - y^2 = (x + c)^2$

$\Rightarrow (x + c)^2 + y^2 = 1$

which represents a family of circle with fixed radius 1 and variable centres along the x-axis.

23. $\lim\limits_{x\to\infty} \dfrac{\cot^{-1}\dfrac{x+1-x}{\sqrt{x+1}+\sqrt{x}}}{\sec^{-1}\{(2)^{\infty}\}} = \dfrac{\cot^{-1}(0)}{\sec^{-1}(\infty)}$

$= \dfrac{\pi/2}{\pi/2}$

$= 1.$

26. St.-1 : $(n + 1)^7 - n^7 - 1$

${}^7C_0 n^7 + {}^7C_1 n^6 + ... + {}^7C_6 n^1 + {}^7C_7 - n^7 - 1$

${}^7C_1 n^6 + ... + {}^7C_6 n$ which are multiple of 7. So St.-1 is true

St.-2 : $f(n) = n^7 - n$ by P.M.I.

$n = 0 \Rightarrow f(0) = 0 \rightarrow$ Div. by 7

$n = k \Rightarrow f(k) = k^7 - k = 7I$ (let)

$n = k + 1 \Rightarrow f(k + 1) = (k + 1)^7 - (k + 1)$

$= (k + 1)^7 - (k + 1) - k^7 + k^7$

$= \underbrace{(k+1)^7 - k^7 - 1}_{} + k^7 - k$

(St.-1) + 7I

So f(k + 1) is also divisible by 7

St.-1, 2 both are true and St.-2 is a correct explanation of St.-1

27. **Statement-1 :** $|A_r| = r^2 - (r - 1)^2 = 2r - 1$

$\Rightarrow |A_1| + |A_2| + ... + |A_{2013}|$

$= \sum\limits_{r=1}^{2013} (2r-1) = (2013)^2$

Statement-2 : Using properties

$|\text{Adj } A| = |A|^{n-1}$

$\therefore |\text{Adj } A| = |A|^2 = 2^2$

$\therefore$ Statement–1 is true & Statement–2 is true but not correct explanation.

31. $\rho = \dfrac{PM}{RT}$

Density ρ remains constant when P/T or volume remains constant.

In graph (i) Pressure is increasing at constant temperature hence volume is decreasing so density is increasing. Graphs (ii) and (iii) volume is increasing hence, density is decreasing. Note that volume would had been constant in case the straight line in graph (iii) had passed through origin.

32. Vander Waal's, gas equation for μ mole of real gas

$\left(P + \dfrac{\mu^2 a}{V^2}\right)(V - \mu b) = \mu RT \Rightarrow P = \dfrac{\mu RT}{V - \mu b} - \dfrac{\mu^2 a}{V^2}$

on comparing the given equation with this standard equation we get $\mu = \dfrac{1}{2}$.

Hence $\mu = \dfrac{m}{M} \Rightarrow$ mass of gas

$m = \mu M = \dfrac{1}{2} \times 44 = 22$ g

33. $(C_V)_{max} = \dfrac{\mu_1 C_{v_1} + \mu_2 C_{v_2}}{\mu_1 + \mu_2} = \dfrac{1 \times \frac{3}{2}R + 1 \times \frac{5}{2}R}{1+1}$

$= 2R \left((C_V)_{mono} = \dfrac{3}{2}R, (C_V)_{di} = \dfrac{5}{2}R\right)$

34. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ $\qquad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

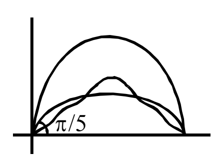
$\Rightarrow \dfrac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\dfrac{T_1}{T_2}}$ $\qquad \Rightarrow \dfrac{T_1}{T_2} = \left(\dfrac{V_2}{V_1}\right)^{\gamma-1}$

$\Rightarrow \dfrac{(v_{rms})_1}{(v_{rms})_2} = \left(\dfrac{V_2}{V_1}\right)^{\frac{\gamma-1}{2}}$

$\Rightarrow \dfrac{v}{\frac{v}{2}} = \left(\dfrac{V_2}{V_1}\right)^{\frac{\frac{7}{5}-1}{2}} \Rightarrow 2 = \left(\dfrac{V_2}{V_1}\right)^{\frac{2}{5}\times\frac{1}{2}} = \left(\dfrac{V_2}{V_1}\right)^{1/5}$

$\Rightarrow \left(\dfrac{V_2}{V_1}\right) = 2^5 = 32$

35.



$\theta = \dfrac{\pi}{2} - \dfrac{\pi}{5}$

$= \dfrac{3\pi}{10}$

36. $\vec{\alpha}$ = constant

$\therefore \vec{\theta} = \vec{\omega}_1 t + \dfrac{1}{2}\vec{\alpha}t^2$

$\therefore \vec{\omega}_2 + \vec{\omega}_1 + \vec{\alpha}t$

$|\vec{v}|^2 - |\vec{u}|^2 = 2|\vec{\alpha}_t| r\theta$

$(r\omega_2)^2 - (r\omega_1)^2 = 2r\alpha \cdot r\theta$

$\omega_2^{\,2} - \omega_1^{\,2} = 2\alpha\theta$ constant

37. A stops for purchasing.

38. $\vec{p}_{initial} = \vec{p}_{final}$

$0 = 0 + (-m\vec{v}_B) + m\vec{v}_C$

$\vec{v}_C = \vec{v}_B$

$|\vec{v}_C| = |\vec{v}_B| = v$

39. Released gradually

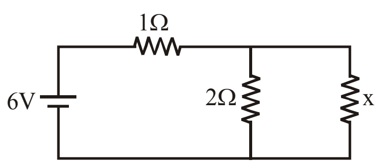
$kd = mg \Rightarrow d = \frac{mg}{k}$

realeased suddenly

$mgx = \frac{1}{2}kx^2$

$\therefore \quad x = \frac{2mg}{k} = 2d$

45.



$\frac{x+2}{2+x} + 1 = x$

on solving $x = 2\Omega$

$i = \frac{6}{1+1} = 3A$

as $x = 2$ so $i_{2\Omega} = \frac{3}{2} = 1.5$ A

51. Reflection from deuser medium causes a phae change of π.
Since rays reflected from p and q undergo a phase change so $n_3 > n_2 > n_1$

52. For reflected rays

$2n_2t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$

$2n_2t \cos r = n\lambda$

53. For transmitted rays

$2n_2t \cos r - \frac{\lambda}{2} = n\lambda$

$2n_2 t \cos r = (2n+1)\frac{\lambda}{2}$ or $(2n-1)\frac{\lambda}{2}$

where value of n may change from different values.

54. $E = W_0 + K_{max}$. From the given data E is 6.78 eV (for λ = 1824 Å) or 10.17 eV (for λ = 1216 Å)

$\therefore \quad W_0 = E - K_{max} = 6.78 - 5.3 = 1.48$ eV

or $\quad W_0 = 10.17 - 8.7 = 1.47$ eV.

55. Here $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{1/3}$

where n = Number of half lives $= \frac{1}{3}$

$\Rightarrow \frac{N}{N_0} = \frac{1}{1.26} \Rightarrow \frac{N_U}{N_{Pb} + N_U} = \frac{1}{1.26}$

$\Rightarrow N_{Pb} = 0.26\, N_U \Rightarrow \frac{N_{Pb}}{N_U} = 0.26$

56. for 'NAND' gate $C = \overline{A.B}$

i.e. $\overline{0.0} = \bar{0} = 1$, $\overline{0.1} = \bar{0} = 1$

$\overline{1.0} = \bar{0} = 1$, $\overline{1.1} = \bar{1} = 0$

57. $i_e = i_b + i_c \Rightarrow i_c = i_e - i_b$

58. We can change the temperature of a body without giving (or taking) heat to (or from) it. For example in an adiabatic compression temperature rises and in an adiabatic expansion temperature falls althrough no heat is given or taken from the system in the respective changes.

61. According Bohr's theory

angular momentum (mvr) $= \frac{nh}{2\pi}$

where n = 1, 2, 3, ...

62. $Ni_{(s)} + 2Ag^+ \longrightarrow Ni^{+2} + 2Ag(s)$

$Q = \frac{[Ni^{+2}]}{[Ag^+]^2} = \frac{0.16}{(0.002)^2}$

$= \frac{0.16}{(2\times10^{-3})^2} = \frac{0.16}{4\times10^{-6}}$

$Q = \frac{0.16\times10^6}{4} = \frac{16\times10^4}{4} = 4\times10^4$

$E_{cell} = E^\circ_{cell} - \frac{0.059}{n} \log Q$ (From nernst equation)

$$E_{cell} = 1.05 - \frac{0.06}{2}\log(4 \times 10^4)$$
$$= 1.05 - 0.03\ \{\log 4 + \log 10^4\}$$
$$= 1.05 - 0.03\ \{0.6 + 4\log 10\}$$
$$= 1.05 - 0.03\ \{4.6\} = 0.912\text{V}$$

63. $\Lambda_{eq} = \frac{1}{R} \times \left(\frac{\ell}{A}\right) \times \frac{1000}{N}$

$$= \frac{1}{200} \times 2 \times \frac{1000}{0.1} = 100\text{s cm}^2\text{eq}^{-1}$$

65. According to Le Chatelier's principle by doubling the volume of container, pressure decreases, so reaction goes in backward direction, number of moles of Cl_2 increases.

66. For precipitation

$[Ca^{+2}]\ [SO_4^{-2}] > K_{sp}$

67. $A_xB_y \longrightarrow xA^{y+} + yB^{x-}$

$\because$ $i = 1 + (n - 1)\alpha$

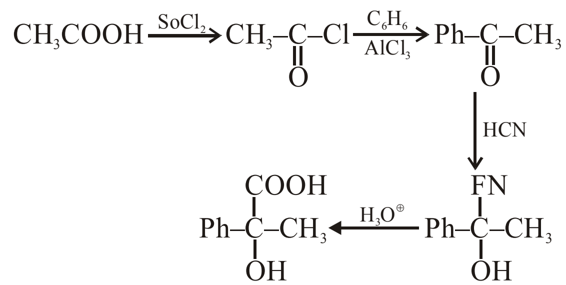
here, $n = x + y$

68. $r_{cs^+} + r_{Br^-} = \frac{\sqrt{3}\,a}{2}$

70. $d = \frac{n \times M}{N_A \times a^3}$

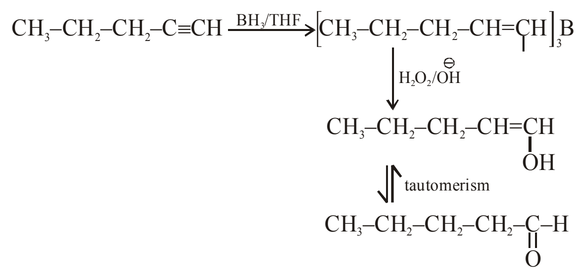
71. Enthalpy of ionisation of NH_4OH $= 55.90 - 51.46 = 4.44\ \text{kJmol}^{-1}$

83.



84. HCl does not show peroxide effect even in presence of peroxide.

85.



86. Terminal alkynes [Ethyne, Propyne, 1-butyne] gives white ppt with ammonical silver nitrate solution.

88.