SOLUTION OF TRIANGLE

1. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$

(3) $\frac{5}{4}$ (4) $\frac{7}{4}$

2. With the usual notation, in ΔABC, if $\angle A + \angle B = 120°$, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio $\angle A : \angle B$, is :

(1) 7 : 1 (2) 5 : 3 (3) 9 : 7 (4) 3 : 1

3. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is :

(1) $\frac{y}{\sqrt{3}}$ (2) $\frac{c}{\sqrt{3}}$

(3) $\frac{c}{3}$ (4) $\frac{3}{2}y$

4. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value :-

(1) (3, 4, 5) (2) (19, 7, 25)

(3) (7, 19, 25) (4) (5, 12, 13)

5. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is :

(1) 5 : 9 : 13 (2) 5 : 6 : 7

(3) 4 : 5 : 6 (4) 3 : 4 : 5

6. The angles A, B and C of a triangle ABC are in A.P. and a : b = 1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is :

(1) $4\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$

(3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

SOLUTION

1. Ans. (4)

r = 1 is obviously true.

Let $0 < r < 1$

$\Rightarrow \quad r + r^2 > 1$

$\Rightarrow r^2 + r - 1 > 0$

$$\left(r - \frac{-1-\sqrt{5}}{2}\right)\left(r - \left(\frac{-1+\sqrt{5}}{2}\right)\right)$$

$$\Rightarrow r - \frac{-1-\sqrt{5}}{2} \text{ or } r > \frac{-1+\sqrt{5}}{2}$$

$$r \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$$

$$\frac{\sqrt{5}-1}{2} < r < 1$$

When $r > 1$

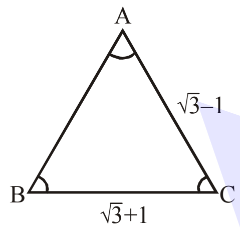
$$\Rightarrow \frac{\sqrt{5}+1}{2} > \frac{1}{r} > 1$$

$$\Rightarrow r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$$

Now check options

2. Ans. (1)

$A + B = 120°$



$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})}\cot(30°) = \frac{1}{\sqrt{3}}.\sqrt{3} = 1$$

$$\frac{A-B}{2} = 45° \qquad \Rightarrow A - B = 90°$$

$A + B = 120°$

$2A = 210°$

$A = 105°$

$B = 15°$

$\therefore$ Option (1)

3. Ans. (2)

Given $a + b = x$ and $ab = y$

If $x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = -\frac{1}{2}$$

$$\Rightarrow \cos C = -\frac{1}{2}$$

$$\Rightarrow \angle C = \frac{2\pi}{3}$$

$$R = \frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$$

4. Ans. (3)

$b + c = 11\lambda$, $c + a = 12\lambda$, $a + b = 13\lambda$

$\Rightarrow a = 7\lambda$, $b = 6\lambda$, $c = 5\lambda$

(using cosine formula)

$$\cos A = \frac{1}{5}, \cos B = \frac{19}{35}, \cos C = \frac{5}{7}$$

$\alpha : \beta : \gamma \Rightarrow 7 : 19 : 25$

5. Official Ans. by NTA (3)

Sol. $a < b < c$ are in A.P.

$\angle C = 2\angle A$ (Given)

$\Rightarrow \sin C = \sin 2A$

$\Rightarrow \sin C = 2\sin A.\cos A$

$$\Rightarrow \frac{\sin C}{\sin A} = 2\cos A$$

$$\Rightarrow \frac{c}{a} = 2\frac{b^2+c^2-a^2}{2bc}$$

put $a = b - \lambda$, $c = b + \lambda$, $\lambda > 0$

$$\Rightarrow \lambda = \frac{b}{5}$$

$$\Rightarrow a = b - \frac{b}{5} = \frac{4}{5}b, c = b + \frac{b}{5} = \frac{6b}{5}$$

$\Rightarrow$ required ratio = 4 : 5 : 6

6. Official Ans. by NTA (3)

Sol. $\angle B = \frac{\pi}{3}$, by sine Rule

$$\sin A = \frac{1}{2}$$

$\Rightarrow A = 30°$, $a = 2$, $b = 2\sqrt{3}$, $c = 4$

$$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3} \text{ sq. cm}$$