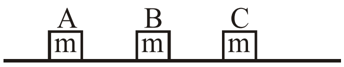
COM & COLLISION

1. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an brutal speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$ th of the initial kinetic energy is lost in whole process. What is value of M/m ?



(1) 4 (2) 5

(3) 3 (4) 2

2. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : ($g = 10 ms^{-2}$)

(1) 30 m (2) 10 m

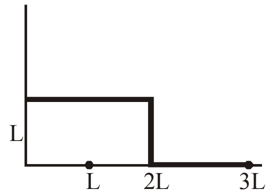
(3) 40 m (4) 20 m

3. A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by :

(1) $\frac{m}{2}\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$ (2) $\frac{m}{2}\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$

(3) $m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (4) $m\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$

4. The position vector of the centre of mass $\vec{r}$ cm of an symmetric uniform bar of negligible area of cross-section as shown in figure is :



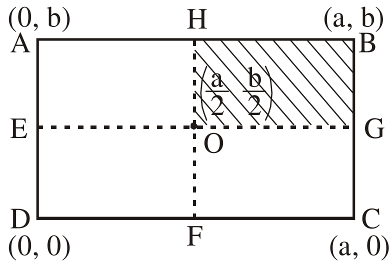
(1) $\vec{r}\,cm = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

(2) $\vec{r}\,cm = \frac{11}{8}L\,\hat{x} + \frac{3}{8}L\hat{y}$

(3) $\vec{r}\,cm = \frac{3}{8}L\,\hat{x} + \frac{11}{8}L\hat{y}$

(4) $\vec{r}\,cm = \frac{5}{8}L\,\hat{x} + \frac{13}{8}L\hat{y}$

5. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be :-



(1) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (2) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

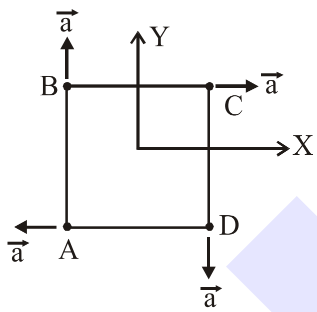
(3) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (4) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

6. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is :-

(1) $v_4 - \frac{v_2}{4}$ (2) $v_4 - \frac{v_2}{2}$

(3) $v_4 - v_2$ (4) $v_4 + v_2$

7. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is :



(1) $\frac{a}{5}(\hat{i} - \hat{j})$ (2) $\frac{a}{5}(\hat{i} + \hat{j})$

(3) Zero (4) $a(\hat{i} + \hat{j})$

8. A particle of mass 'm' is moving with speed '2v' and collides with a mass '2m' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the origianl direction. The speed of each of the moving particle will be :-

(1) $v/(2\sqrt{2})$ (2) $2\sqrt{2}v$

(3) $\sqrt{2}v$ (4) $v/\sqrt{2}$

9. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by :-

(1) $\frac{2v^2}{7g}$ (2) $\frac{v^2}{g}$

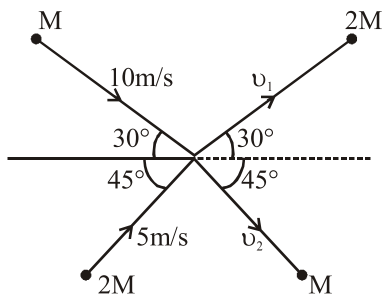
(3) $\frac{2v^2}{5g}$ (4) $\frac{v^2}{2g}$

10. A body of mass 2 kg makes an eleastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body ?

(1) 1.8 kg (2) 1.2 kg

(3) 1.5 kg (4) 1.0 kg

11. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds υ_1 and υ_2, respectively. The values of υ_1 and υ_2 are nearly :



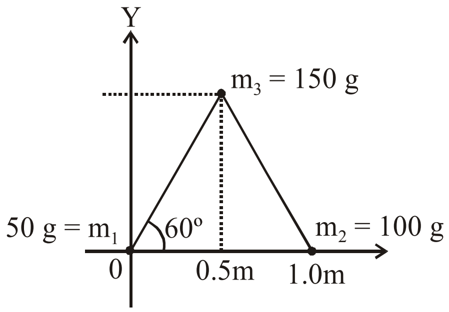
(1) 3.2 m/s and 6.3 m/s

(2) 3.2 m/s and 12.6 m/s

(3) 6.5 m/s and 6.3 m/s

(4) 6.5 m/s and 3.2 m/s

12. Three particles of masses 50 g, 100 g and 150g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :



(1) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$ (2) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$

(3) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$ (4) $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$

13. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is :

(1) 0.20 ms^{-1}

(2) 0.14 ms^{-1}

(3) 0.47 ms^{-1}

(4) 0.28 ms^{-1}

SOLUTION

1. Ans. (1)

$$k_i = \frac{1}{2}mv_0^2$$

From linear momentum conservation

$mv_0 = (2m + M)\ v_f$

$$\Rightarrow v_f = \frac{mv_0}{2m+M}$$

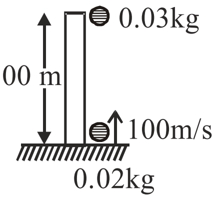
$$\frac{k_i}{k_f} = 6$$

$$\Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m+M)\left(\frac{mv_0}{2m+M}\right)^2} = 6$$

$$\Rightarrow \frac{2m+M}{m} = 6$$

$$\Rightarrow \frac{M}{m} = 4$$

2. Ans. (3)



Time taken for the particles to collide,

$$t = \frac{d}{V_{rel}} = \frac{100}{100} = 1\,sec$$

Speed of wood just before collision = gt = 10 m/s

& speed of bullet just before collision v-gt

= 100 – 10 = 90 m/s

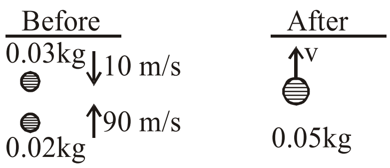
Now, conservation of linear momentum just before and after the collision -

–(0.02) (1v) + (0.02) (9v) = (0.05)v

⇒ 150 = 5v

⇒ v = 30 m/s

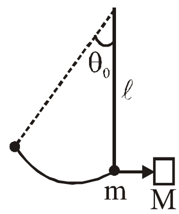
Max. height reached by body $h = \frac{v^2}{2g}$



$$h = \frac{30\times 30}{2\times 10} = 45m$$

∴ Height above tower = 40 m

3. Ans. (3)



Before colision | After collision

$v = \sqrt{2g\ell(1-\cos\theta_0)}$ $\quad v_1 = \sqrt{2g\ell(1-\cos\theta_1)}$

By momentum conservation

$$m\sqrt{2g\ell(1-\cos\theta_0)} = MV_m - m\sqrt{2gl(1-\cos\theta}$$

$$\Rightarrow m\sqrt{2g\ell}\left\{\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}\right\} = MV_m$$

and $e = 1 = \dfrac{V_m + \sqrt{2g\ell(1-\cos\theta_1)}}{\sqrt{2g\ell(1-\cos\theta_0)}}$

$$\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}\right) = V_m \quad ..(I)$$

$$m\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}\right) = MV_M \quad ..(II)$$

Dividing

$$\frac{\left(\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}\right)}{\left(\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}\right)} = \frac{M}{m}$$

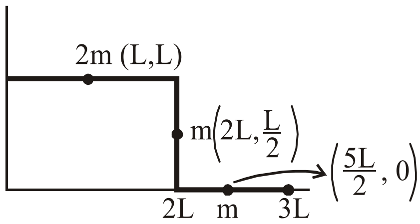
By componendo divided

$$\frac{m-M}{m+M} = \frac{\sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0}} = \frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}$$

$$\Rightarrow \frac{M}{m} = \frac{\theta_0-\theta_1}{\theta_0+\theta_1} \Rightarrow M = m\frac{\theta_0-\theta_1}{\theta_0+\theta_1}$$

4. Ans. (1)

Sol.

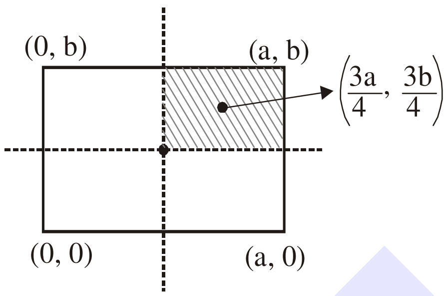


$$X_{cm} = \frac{2mL + 2mL + \frac{5mL}{2}}{4m} = \frac{13}{8}L$$

$$Y_{cm} = \frac{2m \times L + m \times \left(\frac{L}{2}\right) + m \times 0}{4m} = \frac{5L}{8}$$

5. Ans. (4)

Sol.



$$x = \frac{M\frac{a}{2} - \frac{M}{4} \times \frac{3a}{4}}{M - \frac{M}{4}}$$

$$= \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{\frac{5a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

$$y = \frac{M\frac{b}{2} - \frac{M}{4} \times \frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

6. Ans. (3)

Sol. Applying linear momentum conservation

$m_1v_1\hat{i} + m_2v_2\hat{i} = m_1v_3\hat{i} + m_2v_4\hat{i}$

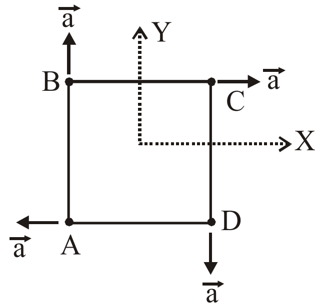
$m_1v_1 + 0.5\, m_1v_2 = m_1(0.5\, v_1) + 0.5\, m_1v_4$

$0.5\, m_1v_1 = 0.5\, m_1(v_4 - v_2)$

$v_1 = v_4 - v_2$

7. Ans. (1)

Sol.



$$\vec{a}_A = -a\hat{i}$$

$$\vec{a}_B = a\hat{j}$$

$$\vec{a}_C = a\hat{i}$$

$$\vec{a}_D = -a\hat{j}$$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2ma\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

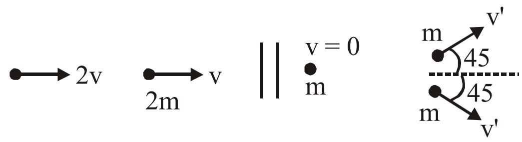
$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m}$$

$$= \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j}$$

$$= \frac{a}{5}\left(\hat{i} - \hat{j}\right)$$

8. Ans. (2)

Sol.



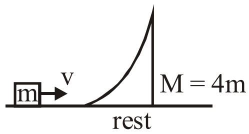
Linear momentum conservation

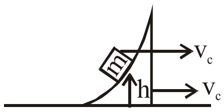
$$m\,2v + 2m\,v = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$$

$$v' = 2\sqrt{2}\,v.$$

9. **Ans. (3)**

Sol.





Applying Linear momentum conservation

$mv = (m + M)v_c$

$v_c = \frac{v}{5}$

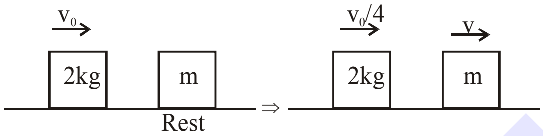
applying work energy theorem

$-mgh = \frac{1}{2}(m + M)v_c^2 - \frac{1}{2}mv^2$

solve, $h = \frac{2v^2}{5g}$

10. **Ans. (2)**

Sol.



By conservation of linear momentum :-

$2v_0 = 2\left(\frac{v_0}{4}\right) + mv \Rightarrow 2v_0 = \frac{v_0}{2} + mv$

$\Rightarrow \frac{3v_0}{2} = mv$ ....(1)

Since collision is elastic →

$V_{separation} = v_{approch}$

$\Rightarrow v - \frac{v_0}{4} = v_0 \Rightarrow \frac{5v_0}{4} = v$ .....(2)

equating (2) and (1)

$\frac{3v_0}{2} = m\left(\frac{5v_0}{4}\right) \Rightarrow m = \frac{6}{5} = 1.2$ kg

Option (2)

11. **Ans. (3)**

Sol. $M \times 10 \cos 30° + 2M \times 5 \cos 45°$

$= 2M \times v_1 \cos 30° + M v_2 \cos 45°$

$5\sqrt{3} + 5\sqrt{2} = 2v_1\frac{\sqrt{3}}{2} + \frac{v_2}{\sqrt{2}}$

$10 \times M \sin 30° - 2M \times 5 \sin 45°$

$= M v_2 \sin 45° - 2M v_1 \sin 30°$

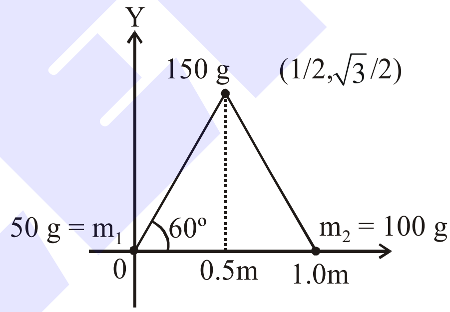
$5 - 5\sqrt{2} = \frac{v_2}{\sqrt{2}} - v_1$

Solving $v_1 = \frac{17.5}{2.7} \simeq 6.5$ m / s

$v_2 \approx 6.3$ m/s

12. **Ans. (3)**

Sol.



The co-ordinates of the centre of mass

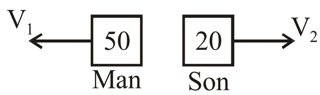
$$\vec{r}_{cm} = \frac{0 + 150 \times \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) + 100 \times \hat{i}}{300}$$

$$\vec{r}_{cm} = \frac{7}{12}\hat{i} + \frac{\sqrt{3}}{4}\hat{j}$$

$\therefore$ Co-ordinate $\left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)$ m

13. **Ans. (1)**

Sol.



$\Rightarrow 0 = 50V_1 - 20V_2$ and $V_1 + V_2 = 0.7$

$\Rightarrow V_1 = 0.2$