HYPERBOLA

1. If a hyperbola passes through the point $P(10,16)$ and it has vertices at $(\pm 6,0)$, then the equation of the normal to it at P is

(1) $x + 2y = 42$ (2) $3x + 4y = 94$

(3) $2x + 5y = 100$ (4) $x + 3y = 58$

2. If e_1 and e_2 are the eccentricities of the ellipse, $\dfrac{x^2}{18} + \dfrac{y^2}{4} = 1$ and the hyperbola, $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to :

(1) 15   (2) 14

(3) 17   (4) 16

3. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to :

(1) 5   (2) 6

(3) 8   (4) 10

4. For some $\theta \in \left(0, \dfrac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is :

(1) $\sqrt{30}$ (2) $\dfrac{4\sqrt{5}}{3}$

(3) $2\sqrt{6}$ (4) $\dfrac{2\sqrt{5}}{3}$

5. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points ?

(1) $\left(1, -\dfrac{1}{\sqrt{2}}\right)$ (2) $\left(\sqrt{\dfrac{3}{2}}, \dfrac{1}{\sqrt{2}}\right)$

(3) $\left(\dfrac{1}{\sqrt{2}}, 0\right)$ (4) $\left(-\sqrt{\dfrac{3}{2}}, 1\right)$

6. Let $P(3, 3)$ be a point on the hyperbola, $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. If the normal to it at P intesects the x-axis at $(9, 0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :

(1) $\left(\dfrac{9}{2}, 3\right)$ (2) $\left(\dfrac{9}{2}, 2\right)$

(3) $\left(\dfrac{3}{2}, 2\right)$ (4) $(9, 3)$

7. If the line $y = mx + c$ is a common tangent to the hyperbola $\dfrac{x^2}{100} - \dfrac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?

(1) $5m = 4$ (2) $4c^2 = 369$

(3) $c^2 = 369$ (4) $8m + 5 = 0$

SOLUTION

1. NTA Ans. (3)

Sol. $\dfrac{x^2}{36} - \dfrac{y^2}{b^2} = 1$

...(i)

P(10,16) lies on (i) get $b^2 = 144$

$\dfrac{x^2}{36} - \dfrac{y^2}{144} = 1$

Equation of normal is

$\dfrac{a^2x}{x_1} + \dfrac{b^2y}{y_1} = a^2e^2$

$2x + 5y = 100$

(3) Option

2. NTA Ans. (4)

Sol. $e_1 = \sqrt{1 - \dfrac{4}{18}} = \dfrac{\sqrt{7}}{3}$

$e_2 = \sqrt{1 + \dfrac{4}{9}} = \dfrac{\sqrt{13}}{3}$

$\because$ (e_1, e_2) lies on $15x^2 + 3y^2 = k$

$\Rightarrow 15e_1^2 + 3e_2^2 = k$

$\Rightarrow k = 16$

3. Official Ans. by NTA (2)

Sol. Slope of tangent is 2, Tangent of hyperbola

$\dfrac{x^2}{4} - \dfrac{y^2}{2} = 1$ at the point (x_1, y_1) is

$\dfrac{xx_1}{4} - \dfrac{yy_1}{2} = 1$ (T = 0)

Slope : $\dfrac{1}{2}\dfrac{x_1}{y_1} = 2 \Rightarrow \boxed{x_1 = 4y_1}$ …(1)

(x_1, y_1) lies on hyperbola

$\Rightarrow \boxed{\dfrac{x_1^2}{4} - \dfrac{y_1^2}{2} = 1}$ …(2)

From (1) & (2)

$\dfrac{(4y_1)^2}{4} - \dfrac{y_1^2}{2} = 1 \Rightarrow 4y_1^2 - \dfrac{y_1^2}{2} = 1$

$\Rightarrow 7y_1^2 = 2 \Rightarrow \boxed{y_1^2 = 2/7}$

Now $x_1^2 + 5y_1^2 = (4y_1)^2 + 5y_1^2$

$= (21)y_1^2 = 21 \times \dfrac{2}{7} = 6$

4. Official Ans. by NTA (2)

Sol. Given $\theta \in \left(0, \dfrac{\pi}{2}\right)$

equation of hyperbola $\Rightarrow x^2 - y^2\sec^2\theta = 10$

$\Rightarrow \dfrac{x^2}{10} - \dfrac{y^2}{10\cos^2\theta} = 1$

Hence eccentricity of hyperbola

$(e_H) = \sqrt{1 + \dfrac{10\cos^2\theta}{10}}$ …(1)

$\left\{ e = \sqrt{1 + \dfrac{b^2}{a^2}} \right\}$

Now equation of ellipse $\Rightarrow x^2\sec^2\theta + y^2 = 5$

$\Rightarrow \dfrac{x^2}{5\cos^2\theta} + \dfrac{y^2}{5} = 1$ $\left\{ e = \sqrt{1 - \dfrac{a^2}{b^2}} \right\}$

Hence eccenticity of ellipse

$(e_E) = \sqrt{1 - \dfrac{5\cos^2\theta}{5}}$

$(e_E) = \sqrt{1 - \cos^2\theta} = |\sin\theta| = \sin\theta$ …(2)

$\left\{ \because \theta \in \left(0, \dfrac{\pi}{2}\right) \right\}$

given $\Rightarrow e_H = \sqrt{5}\, e_e$

Hence $1 + \cos^2\theta = 5\sin^2\theta$

$1 + \cos^2\theta = 5(1 - \cos^2\theta)$

$1 + \cos^2\theta = 5 - 5\cos^2\theta$

$6\cos^2\theta = 4$

$\cos^2\theta = \dfrac{2}{3}$ …(3)

Now length of latus rectum of ellipse

$= \dfrac{2a^2}{b} = \dfrac{10\cos^2\theta}{\sqrt{5}} = \dfrac{20}{3\sqrt{5}} = \dfrac{4\sqrt{5}}{3}$

5. **Official Ans. by NTA (2)**

Sol. Ellipse : $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$

eccentricity = $\sqrt{1 - \dfrac{3}{4}} = \dfrac{1}{2}$

$\therefore$ foci = $(\pm 1, 0)$

for hyperbola, given $2a = \sqrt{2} \Rightarrow a = \dfrac{1}{\sqrt{2}}$

$\therefore$ hyperbola will be

$\dfrac{x^2}{1/2} - \dfrac{y^2}{b^2} = 1$

eccentricity = $\sqrt{1 + 2b^2}$

$\therefore$ foci = $\left(\pm \sqrt{\dfrac{1 + 2b^2}{2}}, 0 \right)$

$\because$ Ellipse and hyperbola have same foci

$\Rightarrow \sqrt{\dfrac{1 + 2b^2}{2}} = 1$

$\Rightarrow b^2 = \dfrac{1}{2}$

$\therefore$ Equation of hyperbola : $\dfrac{x^2}{1/2} - \dfrac{y^2}{1/2} = 1$

$\Rightarrow x^2 - y^2 = \dfrac{1}{2}$

Clearly $\left(\sqrt{\dfrac{3}{2}}, \dfrac{1}{\sqrt{2}} \right)$ does not lie on it.

6. **Official Ans. by NTA (1)**

Sol. Since, $(3, 3)$ lies on $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\dfrac{9}{a^2} - \dfrac{9}{b^2} = 1$
....(1)

Now, normal at $(3, 3)$ is $y - 3 = -\dfrac{a^2}{b^2}(x - 3)$,

which passes through $(9, 0) \Rightarrow b^2 = 2a^2$
....(2)

So, $e^2 = 1 + \dfrac{b^2}{a^2} = 3$

Also, $a^2 = \dfrac{9}{2}$ (from (i) & (ii))

Thus, $(a^2, e^2) = \left(\dfrac{9}{2}, 3 \right)$

7. **Official Ans. by NTA (2)**

Sol. $y = mx + c$ is tangent to

$\dfrac{x^2}{100} - \dfrac{y^2}{64} = 1$ and $x^2 + y^2 = 36$

$c^2 = 100 m^2 - 64 \mid c^2 = 36(1 + m^2)$
$\Rightarrow 100 m^2 - 64 = 36 + 36m^2$

$m^2 = \dfrac{100}{64} \Rightarrow m = \pm \dfrac{10}{8}$

$c^2 = 36\left(1 + \dfrac{100}{64} \right) = \dfrac{36 \times 164}{64}$

$4c^2 = 369$