# COMPLEX NUMBER

**1.** If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then arg $\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is : (Here arg(z) denotes the principal argument of complex number z)

(1) $\frac{\pi}{4}$ (2) $-\frac{3\pi}{4}$ (3) $-\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

**2.** If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral $\int_0^{\theta} \sin x\, dx$ is equal to :

(1) 1 (2) 2 (3) –1 (4) 0

**3.** Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to

(1) 1 (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) 2

**4.** Let

$$S = \left\{ n \in \mathbf{N} \,\middle|\, \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a,b,c,d \in \mathbf{R} \right\},$$

where $i = \sqrt{-1}$. Then the number of 2–digit numbers in the set S is ______.

**5.** The equation of a circle is $\text{Re}(z^2) + 2(\text{Im}(z))^2 + 2\text{Re}(z) = 0$, where $z = x + iy$. A line which passes through the center of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to ____.

**6.** Let C be the set of all complex numbers. Let
$S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\}$,
$S_2 = \{z \in C \mid \text{Re}(z) \geq 5\}$ and
$S_3 = \{z \in C \mid |z - \bar{z}| \geq 8\}$.

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to

(1) 1 (2) 0
(3) 2 (4) Infinite

**7.** Let $\mathbb{C}$ be the set of all complex numbers. Let $S_1 = \{z \in \mathbb{C} : |z - 2| \leq 1\}$ and $S_2 = \{z \in \mathbb{C} : z(1+i) + \bar{z}(1-i) \geq 4\}$.

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to :

(1) $\frac{3+2\sqrt{2}}{4}$ (2) $\frac{5+2\sqrt{2}}{2}$

(3) $\frac{3+2\sqrt{2}}{2}$ (4) $\frac{5+2\sqrt{2}}{4}$

**8.** If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}$, $\theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to______.

**9.** The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with:

(1) centre at (0, –1) and radius $\sqrt{2}$
(2) centre at (0, 1) and radius $\sqrt{2}$
(3) centre at (0, 0) and radius $\sqrt{2}$
(4) centre at (0, 1) and radius 2

**10.** Let $z = \frac{1-i\sqrt{3}}{2}$, $i = \sqrt{-1}$. Then the value of

$$21 + \left(z+\frac{1}{z}\right)^3 + \left(z^2+\frac{1}{z^2}\right)^3 + \left(z^3+\frac{1}{z^3}\right)^3 + \ldots + \left(z^{21}+\frac{1}{z^{21}}\right)^3$$

is _______.

**11.** If $(\sqrt{3}+i)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation :

(1) $x^2 - (\sqrt{3}-1)x - \sqrt{3} = 0$
(2) $x^2 + (\sqrt{3}+1)x + \sqrt{3} = 0$
(3) $x^2 + (\sqrt{3}-1)x - \sqrt{3} = 0$
(4) $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

**12.** The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i=\sqrt{-1}$ is a positive integer, is______.

**13.** If $S=\left\{z\in\mathbb{C}:\frac{z-i}{z+2i}\in\mathbb{R}\right\}$, then :

(1) S contains exactly two elements

(2) S contains only one element

(3) S is a circle in the complex plane

(4) S is a straight line in the complex plane

**14.** Let $z_1$ and $z_2$ be two complex numbers such that arg $(z_1 - z_2) = \frac{\pi}{4}$ and $z_1$, $z_2$ satisfy the equation $|z - 3| = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to _________.

**15.** A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$, then the minimum value of $\left|z-9\sqrt{2}-2i\right|^2$ is equal to _________.

**16.** If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is :

(1) $2\sqrt{2} - 1$ (2) $3\sqrt{2}$

(3) $6\sqrt{2}$ (4) $2\sqrt{2}$

**17.** If for the complex numbers z satisfying $|z-2-2i| \le 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then a + b is equal to ________.

**18.** Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$, and $n = [|k|]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to_______.

**19.** If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in \mathbb{C}$ and $i=\sqrt{-1}$) has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ____

**20.** Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz+\bar{z}+1+i = 0$ is tangent to this circle C, then its radius is:

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $3\sqrt{2}$ (4) $\frac{3}{2\sqrt{2}}$

**21.** Let z be those complex numbers which satisfy $|z + 5| \le 4$ and $z(1+i)+\bar{z}(1-i)\ge -10, i=\sqrt{-1}$. If the maximum value of $|z + 1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is _______.

**22.** The sum of 162[th] power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is

**23.** The least value of |z| where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right)\ge \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|,$$

$i=\sqrt{-1}$, is equal to :

(1) 3 (2) $\sqrt{5}$ (3) 2 (4) 8

**24.** Let a complex number z, $|z| \ne 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right)\le 2$. Then, the largest value of |z| is equal to _____ .

(1) 8 (2) 7 (3) 6 (4) 5

**25.** Let

$$P=\begin{bmatrix}-30 & 20 & 56\\ 90 & 140 & 112\\ 120 & 60 & 14\end{bmatrix} \text{ and } A=\begin{bmatrix}2 & 7 & \omega^2\\ -1 & -\omega & 1\\ 0 & -\omega & -\omega+1\end{bmatrix}$$

Where $\omega=\frac{-1+i\sqrt{3}}{2}$, and $I_3$ be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to ______ .

**26.** Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2$, $\left|\frac{z+i}{z-3i}\right| = 1$ and Re(w) has minimum value. Then, the minimum value of $n \in \mathbb{N}$ for which $w^n$ is real, is equal to _____.

**27.** Let $S_1$, $S_2$ and $S_3$ be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z-1| \le \sqrt{2}\}$

$S_2 = \{z \in \mathbb{C} : \text{Re}((1-i)z) \ge 1\}$

$S_3 = \{z \in \mathbb{C} : \text{Im}(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$

(1) is a singleton

(2) has exactly two elements

(3) has infinitely many elements

(4) has exactly three elements

**28.** The area of the triangle with vertices A(z), B(iz) and C (z + iz) is :

(1) 1 (2) $\frac{1}{2}|z|^2$

(3) $\frac{1}{2}$ (4) $\frac{1}{2}|z+iz|^2$

**29.** If the equation $a|z|^2 + \overline{\bar{\alpha}z + \alpha\bar{z}} + d = 0$ represents a circle where a,d are real constants then which of the following condition is correct?

(1) $|\alpha|^2 - ad \ne 0$

(2) $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

(3) $|\alpha|^2 - ad \ge 0$ and $a \in R$

(4) $\alpha = 0$, $a,d \in R^+$

**30.** Let $z_1$, $z_2$ be the roots of the equation $z^2 + az + 12 = 0$ and $z_1$,$z_2$ form an equilateral triangle with origin. Then, the value of |a| is

**31.** Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that |zw| = 1 and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

(1) 4 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

**32.** If $f(x)$ and g(x) are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then P(1) is equal to______.

**SOLUTION**

**1. Official Ans. by NTA (3)**

**ALLEN Ans. (2)**

**Sol.** As $|z\omega| = 1$

$\Rightarrow$ If $|z| = r$, then $|\omega| = \frac{1}{r}$

Let $\arg(z) = \theta$

$\therefore \arg(\omega) = \left(\theta - \frac{3\pi}{2}\right)$

So, $z = re^{i\theta}$

$\Rightarrow \bar{z} = re^{i(-\theta)}$

$\omega = \frac{1}{r}e^{i\left(\theta - \frac{3\pi}{2}\right)}$

Now, consider

$$\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega} = \frac{1-2e^{i\left(-\frac{3\pi}{2}\right)}}{1+3e^{i\left(-\frac{3\pi}{2}\right)}} = \left(\frac{1-2i}{1+3i}\right)$$

$$= \frac{(1-2i)(1-3i)}{(1+3i)(1-3i)} = -\frac{1}{2}(1+i)$$

$\therefore$ prin arg $\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

= prin arg $\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

$$= \left(-\frac{1}{2}(1+i)\right)$$

$$= -\left(\pi - \frac{\pi}{4}\right) = \frac{-3\pi}{4}$$

So, option (2) is correct.

**2. Official Ans. by NTA (1)**

**Sol.** $z = \frac{1}{1-\cos\theta + 2i\sin\theta}$

$$= \frac{2\sin^2\frac{\theta}{2} - 2i\sin\theta}{(1-\cos\theta)^2 + 4\sin^2\theta}$$

$$= \frac{\sin\frac{\theta}{2} - 2i\cos\frac{\theta}{2}}{4\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)}$$

$$\text{Re}(z) = \frac{1}{2\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)} = \frac{1}{5}$$

$$\sin\frac{2\theta}{2} + 4\cos^2\frac{\theta}{2} = \frac{5}{2}$$

$$1 - \cos^2\frac{\theta}{2} + 4\cos\frac{\theta}{2} = \frac{5}{2}$$

$$3\cos^2\frac{\theta}{2} = \frac{3}{2}$$

$$\cos^2\frac{\theta}{2} = \frac{1}{2}$$

$$\frac{\theta}{2} = n\pi \pm \frac{\pi}{4}$$

$$\theta = 2n\pi \pm \frac{\pi}{2}$$

$$\theta = 2n\pi \pm \frac{\pi}{2}$$

$$\theta \in (0, \pi)$$

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta - [-\cos\theta]_0^{\frac{\pi}{2}}$$

$= -(0 - 1)$

$= 1$

**3. Official Ans. by NTA (2)**

**Sol.** $z^2 + 3\overline{z} = 0$

Put z = x + iy

$\Rightarrow x^2 - y^2 + 2ixy + 3(x - iy) = 0$

$\Rightarrow (x^2 - y^2 + 3x) + i(2xy - 3y) = 0 + i0$

$\therefore \; x^2 - y^2 + 3x = 0$ .....(1)

$2xy - 3y = 0$ .....(2)

$x = \frac{3}{2}, y = 0$

Put $x = \frac{3}{2}$ in equation (1)

$\frac{9}{4} - y^2 + \frac{9}{2} = 0$

$y^2 = \frac{27}{4} \Rightarrow y = \pm\frac{3\sqrt{3}}{2}$

$\therefore \; (x, y) = \left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right), \left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$

Put y = 0 ⇒ $x^2 - 0 + 3x = 0$

x = 0, –3

∴ (x, y) = (0, 0), (–3, 0)

∴ No of solutions = n = 4

$$\sum_{K=0}^{\infty}\left(\frac{1}{n^k}\right) = \sum_{K=0}^{\infty}\left(\frac{1}{4^k}\right)$$

$$= \frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\ldots$$

$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

**4. Official Ans. by NTA (11)**

**Sol.** Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ & $A = \begin{pmatrix} o & i \\ 1 & 0 \end{pmatrix}^n$

⇒ AX = IX

⇒ A = I

$\Rightarrow \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n = I$

$\Rightarrow A^8 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

⇒ n is multiple of 8

So number of 2 digit numbers in the set

S = 11 (16, 24, 32, .......,96)

**5. Official Ans. by NTA (1)**

**Sol.** Equation of circle is $(x^2 - y^2) + 2y^2 + 2x = 0$

$x^2 + y^2 + 2x = 0$

Centre : (–1, 0)

Parabola : $x^2 - 6x - y + 13 = 0$

$(x - 3)^2 = y - 4$

Vertex : (3, 4)

Equation of line ≡ $y - 0 = \frac{4 - 0}{3 + 1}(x + 1)$

y = x + 1

y-intercept = 1

**6. Official Ans. by NTA (1)**

**Sol.** $S_1 : |z-3-2i|^2 = 8$

$|z-3-2i| = 2\sqrt{2}$

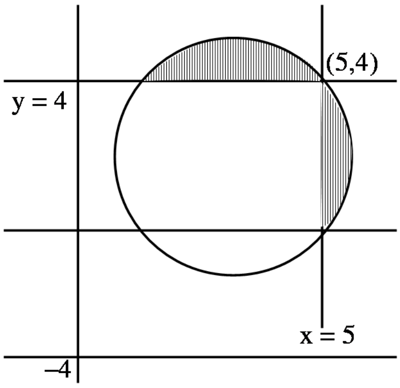
$(x-3)^2 + (y-2)^2 = \left(2\sqrt{2}\right)^2$

$S_2 : x \geq 5$

$S_3 : |z-\bar{z}| \geq 8$

$|2iy| \geq 8$

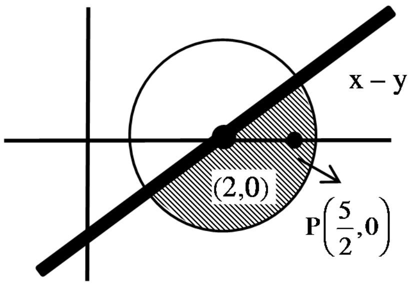
$2|y| \geq 8 \therefore y \geq 4, y \leq -4$




$n\left(S_1 \cap S_2 \cap S_3\right) = 1$

**7. Official Ans. by NTA (4)**

**Sol.** $|t-2| \leq 1$ Put $t = x + iy$




$(x-2)^2 + y^2 \leq 1$

Also, $t(1+i) + \bar{t}(1-i) \geq 4$

Gives $x - y \geq 2$

Let point on circle be $A(2 + \cos\theta, \sin\theta)$

$\theta \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

$(AP)^2 = \left(2 + \cos\theta - \frac{5}{2}\right)^2 + \sin^2\theta$

$= \cos^2\theta - \cos\theta + \frac{1}{4} + \sin^2\theta$

$= \frac{5}{4} - \cos\theta$

For $(AP)^2$ maximum $\theta = -\frac{3\pi}{4}$

$(AP)^2 = \frac{5}{4} + \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}+4}{4\sqrt{2}}$

**8. Official Ans. by NTA (1)**

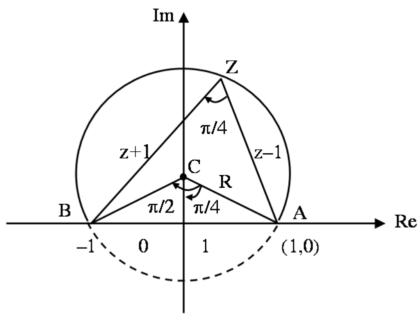
**Sol.** Re (z) $= \frac{3 - 6\cos^2\theta}{1 + 9\cos^2\theta} = 0$

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, $\sin^2 3\theta + \cos^2\theta = 1$.

**9. Official Ans. by NTA (2)**

**Sol.**




In $\Delta OAC$

$\sin\left(\frac{\pi}{4}\right) = \frac{1}{AC}$

$\Rightarrow AC = \sqrt{2}$

Also, $\tan\frac{\pi}{4} = \frac{OA}{OC} = \frac{1}{OC}$

$\Rightarrow OC = 1$

$\therefore$ centre (0, 1) ; Radius = $\sqrt{2}$

**10. Official Ans. by NTA (13)**

**Sol.** $Z = \frac{1-\sqrt{3}i}{2} = e^{-i\frac{\pi}{3}}$

$$z^r + \frac{1}{z^r} = 2\cos\left(-\frac{\pi}{3}\right)r = 2\cos\frac{r\pi}{3}$$

$$\Rightarrow 21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3 = 8\left(\cos^3\frac{r\pi}{3}\right) = 2\left(\cos r\pi + 3\cos\frac{r\pi}{3}\right)$$

$$\Rightarrow 21 + \left(z + \frac{1}{2}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \ldots\ldots\left(z^{21} + \frac{1}{z^{21}}\right)^3$$

$$= 21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3$$

$$= 21 + \sum_{r=1}^{21}\left(2\cos r\pi + 6\cos\frac{r\pi}{3}\right)$$

$$= 21 - 2 - 6$$

$$= 13$$

**11. Official Ans. by NTA (1)**

**Sol.** $\left(2e^{i\pi/6}\right)^{100} = 2^{99}(p+iq)$

$$2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right) = 2^{99}(p+iq)$$

$$p + iq = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$p = -1,\ q = \sqrt{3}$

$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0.$

**12. Official Ans. by NTA (6)**

**Sol.** $\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(-2i)^{\frac{n-2}{2}}}$

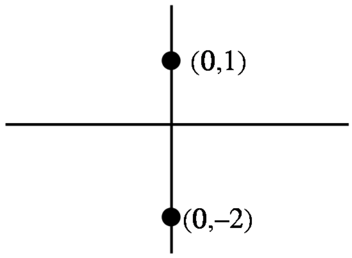
$$= \frac{(2i)^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}} = \frac{2^{\frac{n+2}{2}} i^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}}$$

This is positive integer for n = 6

**13. Official Ans. by NTA (4)**

**Sol.** Given $\frac{z-i}{z+2i} \in R$

Then $\arg\left(\frac{z-i}{z+2i}\right)$ is 0 or $\Pi$




$\Rightarrow$ S is straight line in complex

**14. Official Ans. by NTA (6)**

**Sol.** $|z - 3| = \text{Re}(z)$

let $Z = x = iy$

$\Rightarrow (x - 3)^2 + y^2 = x^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = x^2$

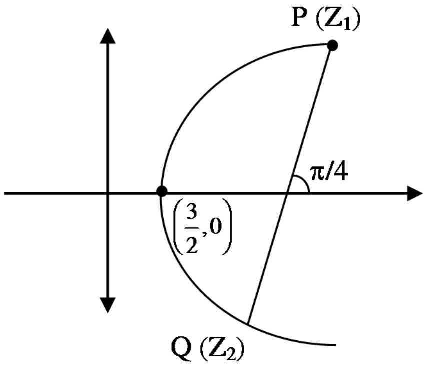
$\Rightarrow y^2 = 6x - 9$

$\Rightarrow y^2 = 6\left(x - \frac{3}{2}\right)$

$\Rightarrow z_1$ and $z_2$ lie on the parabola mentioned in eq.(1)

$\arg(z_1 - z_2) = \frac{\pi}{4}$

$\Rightarrow$ Slope of PQ = 1.




Let $P\left(\frac{3}{2} + \frac{3}{2}t_1^2, 3t_1\right)$ and $Q\left(\frac{3}{2} + \frac{3}{2}t_2^2, 3t_2\right)$

Slope of PQ = $\frac{3(t_2 - t_1)}{\frac{3}{2}(t_1^2 - t_1^2)} = 1$

$\Rightarrow \frac{2}{t_2 + t_2} = 1$

$\Rightarrow t_2 + t_1 = 2$

$\text{Im}(z_1 + z_2) = 3t_1 + 3t_2 = 3(t_1 + t_2) = 3\ (2)$

Ans. 6.00

**Aliter :**

Let $z_1 = x_1 + iy_1$ ; $z_2 = x_2 + iy_2$

$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$\therefore\ \arg(z_1 - z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{4}$

$y_1 - y_2 = x_1 - x_2$ ______(1)

$|z_1 - 3| = \text{Re}(z_1) \Rightarrow (x_1 - 3)^2 + y_1^2 = x_1^2$ __ (2)

$|z_2 - 3| = \text{Re}(z_2) \Rightarrow (x_2 - 3)^2 + y_2^2 = x_2^2$ __(2)

sub (2) & (3)

$(x_1 - 3)^2 - (x_2 - 3)^2 + y_1^2 - y_2^2 = x_1^2 - x_2^2$

$(x_1 - x_2)(x_1 + x_2 - 6) + (y_1 - y_2)(y_1 + y_2)$

$= (x_1 - x_2)(x_1 + x_2)$

$x_1 + x_2 - 6 + y_1 + y_2 = x_1 + x_2 \Rightarrow y_1 + y_2 = 6.$

**15. Official Ans. by NTA (98)**

**Sol.** Let $z = x + iy$

$\arg\left(\frac{x - 2 + iy}{x + 2 + iy}\right) = \frac{\pi}{4}$

$\arg(x - 2 + iy) - \arg(x + 2 + iy) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$

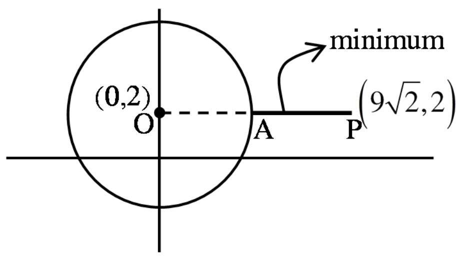
$\frac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \left(\frac{y}{x-2}\right)\cdot\left(\frac{y}{x+2}\right)} = \tan\frac{\pi}{4} = 1$

$\frac{xy + 2y - xy + 2y}{x^2 - 4 + y^2} = 1$

$4y = x^2 - 4 + y^2$

$x^2 + y^2 - 4y - 4 = 0$

locus is a circle with center (0, 2) & radius = $2\sqrt{2}$




min. value = $(AP)^2 = (OP - OA)^2$

$= \left(9\sqrt{2} - 2\sqrt{2}\right)^2$

$= \left(7\sqrt{2}\right)^2 = 98$

**16. Official Ans. by NTA (4)**

**Sol.** $\frac{z-i}{z-1}$ is purely Imaginary number

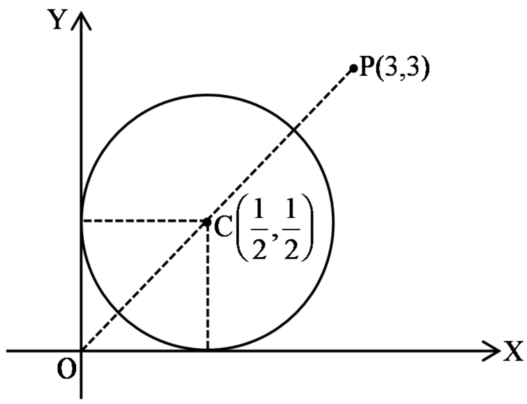
Let z = x + iy

$$\therefore \frac{x+i(y-1)}{(x-1)+i(y)} \times \frac{(x-1)-iy}{(x-1)-iy}$$

$$\Rightarrow \frac{x(x-1)+y(y-1)+i(-y-x+1)}{(x-1)^2+y^2}$$ is purely Imaginary number

$\Rightarrow x(x-1)+y(y-1)=0$

$$\Rightarrow \left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{2}$$




$$\therefore |z-(3+3i)|_{min} = |PC| - \frac{1}{\sqrt{2}}$$

$$= \frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\sqrt{2}$$

**17. Official Ans. by NTA (5)**

**Sol.** $|z-2-2i| \le 1$

$|x+iy-2-2i| \le 1$

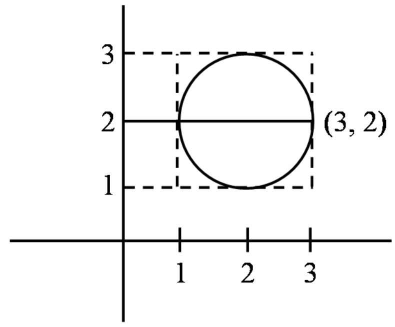
$|(x-2)+i(y-2)| \le 1$

$(x-2)^2+(y-2)^2 \le 1$

$|3iz+6|_{max}$ at a + ib

$$|3i|\left|z+\frac{6}{3i}\right|$$

$3|z-2i|_{max}$




From Figure maximum distance at 3 + 2i

a + ib = 3 + 2i = a + b = 3 + 2 = 5 **Ans.**

**18. Official Ans. by NTA (310)**

**Sol.** $$K = \frac{1}{2^9}\left[\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^{24}} + \frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^{24}}\right]$$

$$K = \frac{1}{512}\left[\frac{\left(e^{i\frac{2\pi}{3}}\right)^{21}}{\left(e^{-\frac{i\pi}{4}}\right)^{24}} + \frac{\left(e^{\frac{i\pi}{3}}\right)^{21}}{\left(e^{\frac{i\pi}{4}}\right)^{24}}\right]$$

$$K = \frac{1}{512}\left[e^{i(14\pi+6\pi)} + e^{i(7\pi-6\pi)}\right]$$

$$K = \frac{1}{512}\left[e^{20\pi i} + e^{\pi i}\right]$$

$$K = \frac{1}{512}[1+(-1)] = 0$$

n = [|k|] = 0

$$\sum_{j=0}^{5}(j+5)^2 - \sum_{j=0}^{5}(j+5)$$

$$\sum_{j=0}^{5}(j^2+25+10j-j-5)$$

$$\sum_{j=0}^{5}(j^2+9j+20)$$

$$\sum_{j=0}^{5}j^2 + 9\sum_{j=0}^{5}j + 20\sum_{j=0}^{5}1$$

$$\frac{5\times6\times11}{6} + 9\left(\frac{5\times6}{2}\right) + 20\times6$$

= 55 + 135 + 120

= 310

**19. Official Ans. by NTA (10)**

**Sol.** Put z = x + iy

$x + iy + \alpha|x + iy - 1| + 2i = 0$

$\Rightarrow x + \alpha\sqrt{(x-1)^2 + y^2} + i(y+2) = 0 + 0i$

$\Rightarrow y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$

$\Rightarrow y = -2$ and $\alpha^2 = \dfrac{x^2}{x^2 - 2x + 5}$

Now $\dfrac{x^2}{x^2 - 2x + 5} \in \left[0, \dfrac{5}{4}\right]$

$\therefore \alpha^2 \in \left[0, \dfrac{5}{4}\right] \Rightarrow \alpha \in \left[-\dfrac{\sqrt{5}}{2}, \dfrac{\sqrt{5}}{2}\right]$

$\therefore p = -\dfrac{\sqrt{5}}{2}; q = \dfrac{\sqrt{5}}{2}$

$\Rightarrow 4(p^2 + q^2) = 4\left(\dfrac{5}{4} + \dfrac{5}{4}\right) = 10$

**20. Official Ans. by NTA (4)**

**Sol.** (i) $(2 - i)z = (2 + i)\,\overline{z}$

$\boxed{y = \dfrac{x}{2}}$

(ii) $(2 + i)z + (i - 2)\,\overline{z} - 4i = 0$

$\boxed{x + 2y = 2}$

(iii) $iz + \overline{z} + 1 + i = 0$

Eq$^n$ of tangent $\boxed{x - y + 1 = 0}$

Solving (i) and (ii)

$x = 1, y = \dfrac{1}{2}$

Now, $p = r \Rightarrow \left|\dfrac{1 - \frac{1}{2} + 1}{\sqrt{2}}\right| = r$

$\Rightarrow r = \dfrac{3}{2\sqrt{2}}$

**21. Official Ans. by NTA (48)**

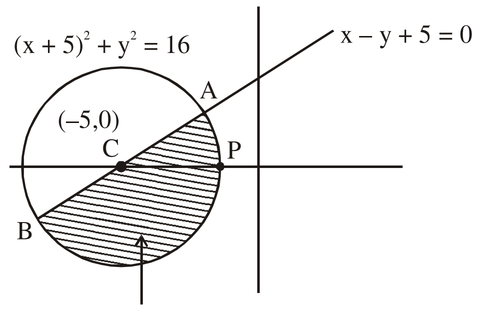
**Sol.** $|z + 5| \le 4$

$(x + 5)^2 + y^2 \le 16$ …(1)

$z(1+i) + \overline{z}(1-i) \ge -10$

$(z + \overline{z}) + i(z - \overline{z}) \ge -10$

$x - y + 5 \ge 0$ …(2)




Region bounded by inequalities (1) & (2)

$|z + 1|^2 = |z - (-1)|^2$

Let P(–1, 0)

$\boxed{|z+1|^2_{Max.} = PB^2}$ (where B is in 3[rd] quadrant)

for point of intersection

$\left.\begin{matrix}(x+5)^2 + y^2 = 16 \\ x - y + 5 = 0\end{matrix}\right\rangle y = \pm 2\sqrt{2}$

$A\left(2\sqrt{2} - 5, 2\sqrt{2}\right)$ $B\left(-2\sqrt{2} - 5, -2\sqrt{2}\right)$

$PB^2 = \left(+2\sqrt{2} + 4\right)^2 + \left(2\sqrt{2}\right)^2$

$|z+1|^2 = 8 + 16 + 16\sqrt{2} + 8$

$\alpha + \beta\sqrt{2} = 32 + 16\sqrt{2}$

$\alpha = 32, \beta = 16 \Rightarrow \alpha + \beta = 48$

**22. Official Ans. by NTA (3)**

**Sol.** $x^3 - 2x^2 + 2x - 1 = 0$

x = 1 satisfying the equation

$\therefore$ x– 1 is factor of

$x^3 - 2x^2 + 2x - 1$

$= (x - 1)(x^2 - x + 1) = 0$

$x = 1, \dfrac{1+i\sqrt{3}}{2}, \dfrac{1-i\sqrt{3}}{2}$

$x = 1, -\omega^2, -\omega$

sum of 162[th] power of roots

$= (1)^{162} + (-\omega^2)^{162} + (-\omega)^{162}$

$= 1 + (\omega)^{324} + (\omega)^{162}$

$= 1 + 1 + 1 = 3$

n + m = 45

**23. Official Ans by NTA (1)**

**Sol.** $\exp\left(\dfrac{(|z|+3)(|z|-1)}{||z|+1|}\ell n\, 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$

$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq \log_{\sqrt{2}}(16)$

$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3$

$\Rightarrow \dfrac{(|z|+3)(|z|-1)}{(|z|+1)} \geq 3$

$\Rightarrow (|z| + 3)(|z| - 1) \geq 3(|z| + 1)$

$|z|^2 + 2|z| - 3 \geq 3|z| + 3$

$\Rightarrow |z|^2 + |z| - 6 \geq 0$

$\Rightarrow (|z| - 3)(|z| + 2) \geq 0 \Rightarrow |z| - 3 \geq 0$

$\Rightarrow |z| \geq 3 \quad \Rightarrow |z|_{min} = 3$

**24. Official Ans. by NTA (2)**

**Sol.** $\log_{\frac{1}{\sqrt{2}}}\left(\dfrac{|z|+11}{(|z|-1)^2}\right) \leq 2$

$\dfrac{|z|+11}{(|z|-1)^2} \geq \dfrac{1}{2}$

$2|z| + 22 \geq (|z| - 1)^2$

$2|z| + 22 \geq |z|^2 + 1 - 2|z|$

$|z|^2 - 4|z| - 21 \leq 0$

$\Rightarrow \quad |z| \leq 7$

$\therefore$ Largest value of $|z|$ is 7

**25. Official Ans. by NTA (36)**

**Sol.** Let $M = (P^{-1}AP - I)^2$

$= (P^{-1}AP)^2 - 2P^{-1}AP + I$

$= P^{-1}A^2P - 2P^{-1}AP + I$

$PM = A^2P - 2AP + P$

$= (A^2 - 2A.I + I^2)P$

$\Rightarrow$ Det(PM) = $\text{Det}((A - I)^2 \times P)$

$\Rightarrow$ DetP.DetM = $\text{Det}(A - I)^2 \times \text{Det}(P)$

$\Rightarrow$ Det M = $(\text{Det}(A - I))^2$

Now $A - I = \begin{bmatrix} 1 & 7 & w^2 \\ -1 & -w-1 & 1 \\ 0 & -w & -w \end{bmatrix}$

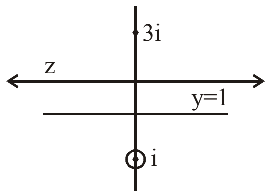
$\text{Det}(A - I) = (w^2 + w + w) + 7(-w) + w^3 = -6w$

$\text{Det}((A - I))^2 = 36w^2$

$\Rightarrow \quad \alpha = 36$

**26. Official Ans. by NTA (4)**

**Sol.** $\omega = z\bar{z} - 2z + 2$




$\left|\dfrac{z+i}{z-3i}\right| = 1$

$\Rightarrow \quad |z+i| = |z-3i|$

$\Rightarrow \quad z = x + i, \ x \in \mathbb{R}$

$\omega = (x + i)(x - i) - 2(x + i) + 2$

$= x^2 + 1 - 2x - 2i + 2$

$\text{Re}(\omega) = x^2 - 2x + 3$

For min $(\text{Re}(\omega))$, x = 1

$\Rightarrow \quad \omega = 2 - 2i = 2(1 - i) = 2\sqrt{2}\, e^{-i\frac{\pi}{4}}$

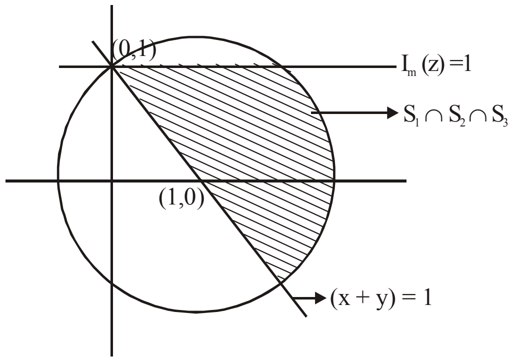
$\omega^n = (2\sqrt{2})^n e^{-i\frac{n\pi}{4}}$

For real & minimum value of n,

n = 4

**27. Official Ans. by NTA (3)**

**Sol.** For $|z-1|\le\sqrt{2}$, z lies on and inside the circle of radius $\sqrt{2}$ units and centre (1, 0).




**For $S_2$**

Let z = x + iy

Now, (1 – i) (z) = (1 – i) (x + iy)

Re((1 – i)z) = x + y

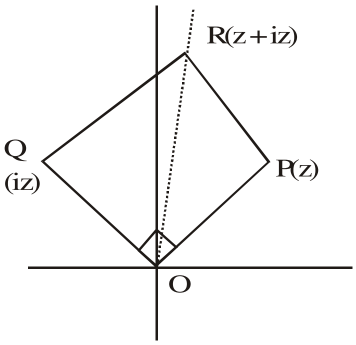
$\Rightarrow x + y \ge 1$

$\Rightarrow S_1 \cap S_2 \cap S_3$ has infinity many elements

**Ans. (3)**

**28. Official Ans. by NTA (2)**

**Sol.**




$A=\frac{1}{2}|z|\,|iz|$

$=\frac{|z|^2}{2}$

**29. Official Ans. by NTA (2)**

**Sol.** $az\bar{z}+\alpha\bar{z}+\bar{\alpha}z+d=0 \rightarrow$ Circle

centre = $\frac{-\alpha}{a}$ $\quad 2=\sqrt{\frac{\alpha\bar{\alpha}}{a^2}-\frac{d}{a}}=\sqrt{\frac{\alpha\bar{\alpha}-ad}{a^2}}$

So $|\alpha|^2 - ad > 0$ & $a \in R - \{0\}$

**30. Official Ans. by NTA (6)**

**Sol.** If $0, z, z_2$ are vertices of equilateral triangles

$\Rightarrow a^2 + z_1^2 + z_2^2 = 0\ (z_1 + z_2) + z_1z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$

$\Rightarrow a^2 = 3 \times 12$

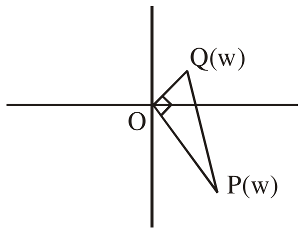
$\Rightarrow |a| = 6$

**31. Official Ans. by NTA (2)**

**Sol.** $w = 1 - \sqrt{3}.i \Rightarrow |w| = 2$

Now, $|z| = \frac{1}{|w|} \Rightarrow |z| = \frac{1}{2}$

and $amp(z) = \frac{\pi}{2} + amp(w)$




$\Rightarrow$ Area of triangle = $\frac{1}{2}.OP.OQ$

$=\frac{1}{2}.2.\frac{1}{2}=\frac{1}{2}$

**32. Official Ans. by NTA (0)**

**Sol.** $P(x) = f(x^3) + xg(x^3)$

$P(1) = f(1) + g(1)$ ...(1)

Now P(x) is divisible by $x^2 + x + 1$

$\Rightarrow P(x) = Q(x)(x^2 + x + 1)$

$P(w) = 0 = P(w^2)$ where w, $w^2$ are non-real cube roots of units

$P(x) = f(x^3) + xg(x^3)$

$P(w) = f(w^3) + wg(w^3) = 0$

$f(1) + wg(1) = 2$ ...(2)

$P(w^2) = f(w^6) + w^2g(w^6) = 0$

$f(1) + w^2g(1) = 0$ ...(3)

(2) + (3)

$\Rightarrow 2f(1) + (w + w^2)g(1) = 0$

$2f(1) = g(1)$ ...(4)

(2) – (3)

$\Rightarrow (w - w^2)g(1) = 0$

$g(1) = 0 = f(1)$ from (4)

from (1) $P(1) = f(1) + g(1) = 0$