ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)
path to success

Pre Nurture & Career Foundation Division

For Class 6th to 10th, Olympiads & Board

ANSWER KEY (Paper Code : 63)

NATIONAL STANDARD EXAMINATION in PHYSICS

NSEP-2024 [24-11-2024]

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	a	c	b	d	NA	b	c	a	d	NA	b	b	b	a	b
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	d	c	c	d	b	c	a	d	d	d	a	c	a or b	b	a
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	b	a	b	b	d	a	b	c	b	b	a	c	b	d	d
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	c	b	d	a,b,c,d	a,b,c	b,c	a,c,d	a,b,d	b,c,d	a,c,d	a,b,c	a,b,d	b,c	b,c,d	a,b,c

NA = Options are Not Correct

Date: 24/11/2024

NSEP-2024 (NSEP STAGE-I)

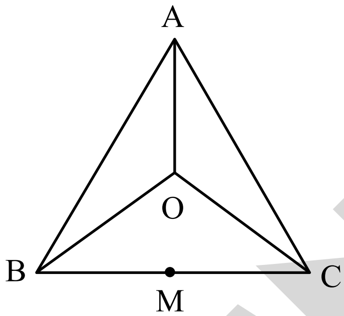
Date of examination : 24th Nov, 2024

PAPER CODE - 63

Max. Marks: 216 **Time allowed: 2 hours**

SOLUTIONS

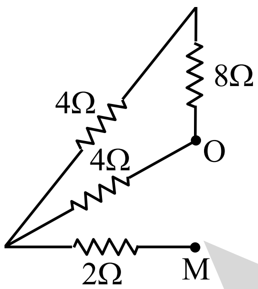
1. OABC is a regular tetrahedron, each side of which is made of uniform wire of resistance 4Ω/m. The length of each side is 2 m. The point M is the midpoint of the side BC. The resistance between O and M is



(a) 5Ω (b) 4Ω (c) 10Ω (d) 15Ω

Ans. (a)

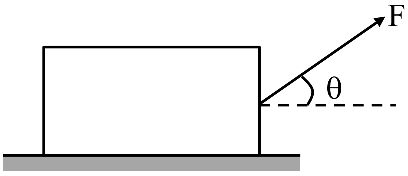
Sol. By folding symmetry the circuit can be reduced to



$\therefore R_{OM} = 5\Omega$

Resistance of each side = 2 × 4 = 8Ω

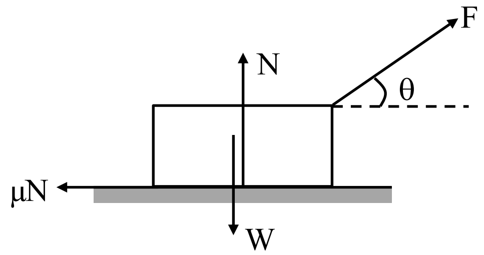
2. A box weighing W is placed on a rough horizontal floor. The coefficient of static friction between the box and the floor is μ. To move the box, a pulling force F is applied along the rope joined to the box at an angle θ with horizontal. By suitable choice of θ, the minimum value of F that can make the box move is



(a) μW (b) $\frac{W}{\sqrt{1+\mu^2}}$ (c) $\frac{\mu W}{\sqrt{1+\mu^2}}$ (d) $\frac{W\sqrt{1-\mu^2}}{\mu}$

Ans. (c)

Sol.



For block to move

$F\cos\theta - \mu N \geq 0$

$F\sin\theta + N - W = 0$

$\therefore\ N = W - F\sin\theta$

$\therefore\ F\cos\theta - \mu(W - F\sin\theta) \geq 0$

For F_{min}

$F(\cos\theta + \mu\sin\theta) = \mu W$

$\therefore F_{min} = \dfrac{\mu W}{\sqrt{1+\mu^2}}$ [Maximum value of $\sqrt{a\sin\theta + b\cos\theta} = \sqrt{a^2+b^2}$]

3. The external wall of a room measuring 2 m × 3 m consists of a layer of white pine of thickness d_{pine} = 2.0 cm and a layer of rock wool in succession. The external temperature is 36 K below the indoor temperature (Given the thermal conductivity coefficient of white pine K_P = 0.10 Wm^{-1} $kelvin^{-1}$ and that of rock wool K_w = 0.04 Wm^{-1} $kelvin^{-1}$). The thickness of the layer of the rock wool, so that the thermal conduction rate $P_{cond} = \dfrac{dQ}{dt}$ across the wall does not exceed 120 watt (assuming no loss of heat during conduction and no other way of heat transfer other than conduction), is

(a) 7.2 cm (b) 6.4 cm (c) 4.8 cm (d) 0.8 cm

Ans. (b)

Sol. Let the thickness of wool be t.

$$\therefore R_{pine} = \frac{\ell}{KA} = \frac{2\times10^{-2}}{2\times3\times0.1}\frac{K}{W}$$

$$R_{wool} = \frac{t}{KA} = \frac{t}{2\times3\times0.04}\frac{K}{W}$$

$$\therefore \frac{dQ}{dt} = \frac{\Delta T}{R_{net}} = \frac{36}{\dfrac{2\times10^{-2}}{0.6}+\dfrac{t}{0.24}}$$

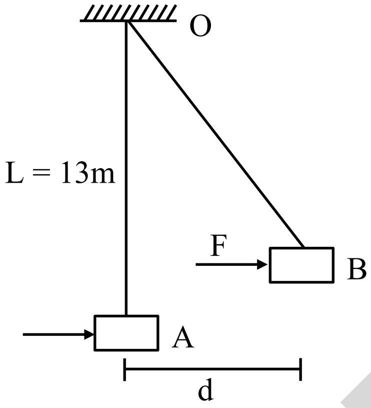
Given $\left.\dfrac{dQ}{dt}\right|_{max}$ = 120 Watt

$$\therefore \frac{36}{\dfrac{2\times10^{-2}}{0.6}+\dfrac{t}{0.24}} = 120 \Rightarrow t = 0.064\text{ m} = 6.4\text{ cm}$$

4. A 240 kg block is suspended from a fixed point O, at the end of a long (L = 13 m) massless rope. A horizontal force F slowly pushes the block to move it a horizontal distance d = 5 m sideways, to a position B where it remains stationary as shown in the figure.

Statement-1 : Force F at position B is 980 N.

Statement-2 : Work done by the force to bring the box from A to B is 2352 J



(a) Only statement 1 is correct

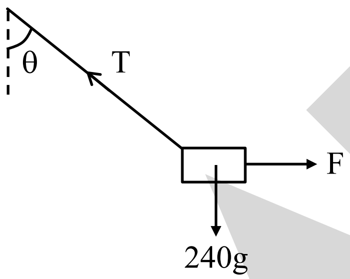
(b) Only statement 2 is correct

(c) Both statements 1 and 2 arc wrong

(d) Both statements 1 and 2 are correct

Ans. (d)

Sol. At B, block is in equilibrium



$F - T\sin\theta = 0$

$T\cos\theta - 240\,g = 0$

$$\therefore \tan\theta = \frac{F}{240g}$$

From geometry, $\sin\theta = \dfrac{5}{13}$

$$\therefore \frac{5}{12} = \frac{F}{240g} = F = 980\text{N}$$

From work-energy theorem, since block is moved slowly $\Rightarrow \Delta KE = 0$

$\therefore\ W_{mg} + \cancelto{0}{W_T} + W_F = 0$ [T ⊥ displacement]

$\therefore\ W_F = -W_{mg}$

$\therefore\ W_F = -[-240g\ 1(1-\cos\theta)] = 240 \times 9.8 \times 1\ \text{J} = 2352\ \text{J}.$

5. The number of radioactive nuclei N of a radioactive sample is experimentally measured as a function of time t. At t = 0, N(t = 0) = 50,000 and at t = 10s, N(t = 10s) = 5000 ± 100. The half-life of the sample is estimated from these measurement. The error in the estimation of the half-life is approximately [note that for small values of x, Lt $\to$ 0, $\ell n(1 + x) \approx x$]

(a) 0.26 s (b) 0.15 s (c) 0.05 s (d) 0.10 s

Ans. (NA)

Sol. $N = N_0 e^{-\lambda t}$

$$5000 = 50000 e^{-\frac{\ell n2}{T_{1/2}}(10)}$$

$$\frac{1}{10} = e^{-\frac{10\ell n2}{T_{1/2}}}$$

$$\ell n10 = \frac{10\ell n2}{T_{1/2}}$$

$$T_{1/2} = \frac{10\ell n2}{\ell n10} = \frac{10\times0.7}{2.30} = 3.04\,\text{sec}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\ell nN - \ell nN_0 = -\lambda t$$

$$\frac{1}{N}.\Delta N = +(\Delta\lambda)(10) = +\frac{\ell n2}{(T_{1/2})^2}\times\Delta T_{1/2}(10)$$

$$\frac{100}{5000} = +\frac{\ell n2}{(3.04)^2}\times10\times\Delta T_{1/2}$$

$$\Delta T_{1/2} = \frac{100}{5000}\times\frac{(3.04)^2}{\ell n2}\times\frac{1}{10}$$

= 0.026 sec

6. An amount of heat equal to 10.61 J is given to an ideal gas at constant pressure of one atm (1.01×10^5 Pa). As a result the volume of the gas increases by 30.0 cm^3. The gas is

(a) mono atomic

(b) diatomic

(c) tri-atomic

(d) mixture of monoatomic and diatomic

Ans. (b)

Sol. Since pressure is constant.

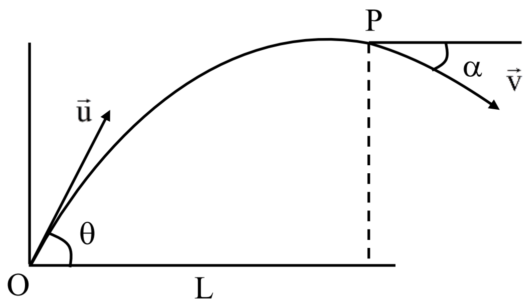
$$\therefore dQ = nC_P dT = (ndT)C_p = \frac{(PdV)C_P}{R} \quad [PdV = nRdT]$$

$$\therefore 10.61 = \frac{1.01\times10^5\times30\times10^{-6}}{R}\cdot\left(\frac{f}{2}+1\right)R$$

$\therefore$ f = 5

Thus gas is diatomic

7. A ball is projected from point O on the ground with a certain velocity $\vec{u}$ at angle θ from horizontal. When it reaches point P located at a horizontal distance L from O and is at a height h above the ground, the angle α subtended by the velocity vector $\vec{v}$ with horizontal at this point P is expressed as



(a) $\tan\alpha = \frac{h}{L} - \tan\theta$

(b) $\tan\alpha = \frac{2h}{L} + \tan\theta$

(c) $\tan\alpha = \tan\theta - \frac{2h}{L}$

(d) $\tan\alpha = \frac{2h}{L} - \tan\theta$

Ans. (c)

Sol. $y = x\tan\theta - \frac{g}{2u^2}x^2\sec^2\theta$

$$\frac{dy}{dx} = \tan\theta - \frac{g}{2u^2}\cdot 2x\cdot\sec^2\theta$$

$$-\tan\alpha = \tan\theta - \frac{g}{2u^2}\cdot 2L\sec^2\theta$$

$$-\tan\alpha - \tan\theta = -\frac{g}{u^2}\cdot L\sec^2\theta$$

$$\tan\alpha = \frac{gL}{u^2}\sec^2\theta - \tan\theta$$

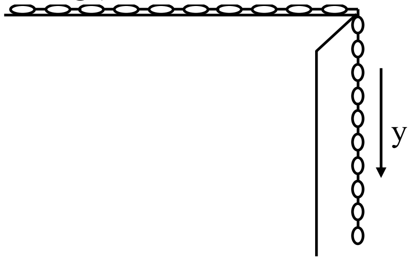
$$h = L\tan\theta - \frac{g}{2u^2}L^2\sec^2\theta$$

$$\frac{h}{L} = \tan\theta - \frac{g}{2u^2}L\sec^2\theta$$

$$\frac{g}{2u^2}L\sec^2\theta = \tan\theta - \frac{h}{L}$$

$$\tan\alpha = 2\left(\tan\theta - \frac{h}{L}\right) - \tan\theta = \tan\theta - \frac{2h}{L}$$

8. A flexible chain, of length L and uniform mass per unit length λ, slides off the edge of a frictionless table (sec figure). Initially a length $y = y_0$ of the chain hangs over the edge, with the chain held at rest. Now the chain is let free. The velocity, of the chain when the chain becomes completely vertical (i.e. when the chain is just to leave the edge) is

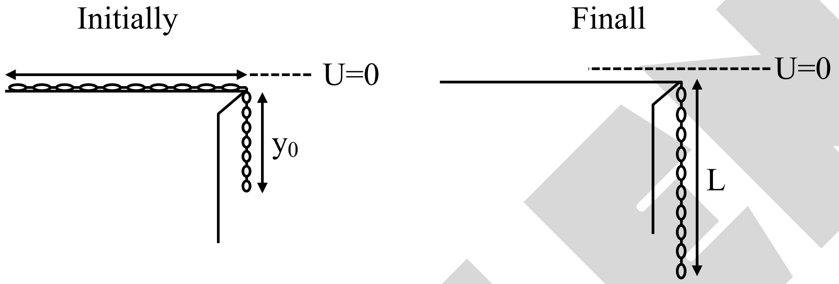


(a) $v = \sqrt{g\left(L - \frac{y_0^2}{L}\right)}$ (b) $v = \sqrt{2g\left(L - \frac{y_0^2}{L}\right)}$

(c) $v = \sqrt{g(L - y_0)}$ (d) $v = \sqrt{2g(L - y_0)}$

Ans. (a)

Sol.



$\therefore$ Using energy conservation,

$U_i + K_i = U_f + K_f$

$$\Rightarrow -\lambda y_0 g \frac{y_0}{2} + 0 = -\lambda L g \frac{L}{2} + \frac{1}{2}\lambda L v^2$$

$$\Rightarrow v = \sqrt{g\left(L - \frac{y_0^2}{L}\right)}$$

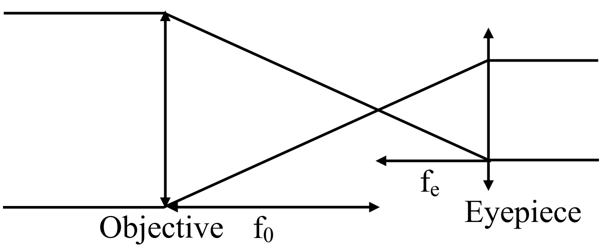
9. A Keplerian telescope is adjusted in its normal setting for parallel rays. Mounting of the objective has diameter D and the diameter of the image of that mounting formed by the eye piece of the telescope is d. Magnifying power of the telescope is

(a) $\frac{D}{d} - 1$ (b) $\frac{D}{d} + 1$ (c) $\frac{D+d}{D-d}$ (d) $\frac{D}{d}$

Ans. (d)

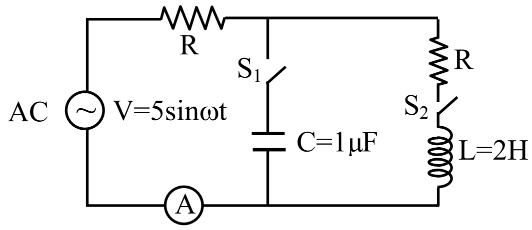
Sol. $m = \frac{f_0}{f_e}$

Since the telescope is in normal setting $\Rightarrow$ Distance between objective & eyepiece = $f_0 + f_e$.



$$\therefore m = \frac{f_0}{f_e} = \frac{D}{d}$$

10. In the circuit shown below, the resistance $R=\sqrt{3}\times10^3\Omega$, the inductance L = 2 H and the capacitance C = 1 μF have been connected to an AC supply of the peak voltage of V_{max} = 5 volt at a frequency ω. Either the switch S_1 or the switch S_2 is closed at a time. In either case, same maximum current (i_{max}) is recorded in the circuit. The frequency of the AC source is nearly



(a) 1.5 kHz (b) 500 Hz (c) 326 Hz (d) 126 Hz

Ans. (NA)

Sol. $z_1 = z_2$

$$R^2+X_C^2=(2R)^2+X_L^2$$

$$R^2+\left(\frac{1}{\omega C}\right)^2=4R^2+(\omega L)^2$$

$$\left(\frac{1}{\omega C}\right)^2-(\omega L)^2=3R^2$$

$$\left(\frac{10^{+6}}{\omega}\right)^2-[\omega(2)]^2=3(3\times10^6)=9\times10^6$$

$$\frac{10^{12}}{\omega^2}-4\omega^4=9\times10^6$$

$10^{12}-4\omega^4=9\times10^6\omega^2$

$4\omega^4+9\times10^6\omega^2-10^{12}=0$

$4(2\pi f)^4+9\times10^6(2\pi f)^2-10^{12}=0$

$4\times16\times100f^4+9\times10^6\times4\times10f^2-10^{12}=0$

$6400f^4+36\times10^7f^2-10^{12}=0$

$$f^4+56250f^2-\frac{10^{12}}{6400}=0$$

$$f^2=\frac{-56250\pm\sqrt{3789062500}}{2}$$

$$f^2=\frac{61555.36-56250}{2}=2652.68$$

f = 51.50 Hz

11. There are two given physical quantities $A=\frac{F}{\rho v^2 D}$ and $B=\frac{\rho v D}{\eta}$ where F is force, ρ is density, D is diameter, v is velocity and η is the Coefficient of viscosity. Which of the two A and B is/are dimensionless ?

(a) A (b) B (c) neither A nor B (d) both A and B

Ans. (b)

Sol. $[F]=MLT^{-2}$

$[\rho] = ML^{-3}$

$[v] = LT^{-1}$

$[D] = L$

$[\eta] = ML^{-1}T^{-1}$

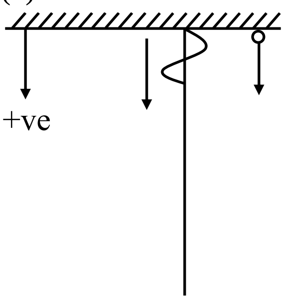
$$\therefore [A] = \frac{MLT^{-2}}{ML^{-3}L^{2}T^{-2}L} = L$$

$$[B] = \frac{ML^{-3}LT^{-1}L}{ML^{-1}T^{-1}} = M^{0}L^{0}T^{0} \rightarrow \text{Dimensionless.}$$

12. A uniform inextensible string of length L and mass M is suspended vertically from a rigid support. A transverse pulse is allowed to propagate down through the string from the support. At the same time, a ball of mass m is dropped from the rigid support. The ball will pass the pulse at a distance of (from the top of the string i.e. from the support)

(a) $\frac{15}{16}L$ (b) $\frac{8}{9}L$ (c) $\frac{L}{2}$ (d) $\frac{3L}{4}$

Ans. (b)



Sol.

Position of pulse at any time, t is given by

$$x = t\sqrt{Lg} - \frac{1}{2}\frac{g}{2}t^{2}$$

Position of ball at any time t is given by

$$x = \frac{1}{2}gt^{2}$$

$\therefore$ They will cross each other if

$$t\sqrt{Lg} - \frac{1}{4}gt^{2} = \frac{1}{2}gt^{2}$$

$$\sqrt{Lg} = \frac{3}{4}gt \Rightarrow t = \frac{4}{3}\sqrt{\frac{L}{g}}$$

$$\therefore x = \frac{1}{2}g\frac{16}{9}\frac{L}{g} = \frac{8}{9}L$$

13. In a positron decay, the radionuclide ^{11}C decays according to $^{11}C \rightarrow {}^{11}B + e^{+} + \upsilon$ [the respective atomic masses are ^{11}C = 11.01142 u, ^{11}B = 11.00931u and the mass of positron = the mass of electron = m_e = 0.00055 u]. The disintegration energy (i.e. the Q value) is approximately

(a) 1.45 MeV (b) 0.94 MeV (c) 0.43 MeV (d) – 1.45 MeV

Ans. (b)

Sol. $^{11}C \rightarrow {}^{11}B + e^{+} + \upsilon$

$\therefore$ Δm = (11.01142 –11.00931 – 2(0.00055)u

= 0.00101u

$\therefore$ θ = 0.00101 × 931.5 MeV ≈ 0.94 MeV

14. Two mechanical waves, given by y_1 = A sin (8x – 50t) and y_2 = A sin $\left(8x + 50t + \frac{\pi}{3}\right)$ travelling in opposite directions along x-axis, superpose. The position of the node (for x > 0) nearest to the origin is (the displacements y_1 and y_2 are in meter)

(a) 32.7 cm (b) 16.35 cm (c) 6.54 cm (d) 5.45 cm

Ans. (a)

Sol. y_1 = Asin (8x – 50t)

y_2 = Asin (8x + 50t + $\frac{\pi}{3}$)

$$\therefore y_R = y_1 + y_2 = 2A\sin\left(8x + \frac{\pi}{6}\right)\cos\left(50t + \frac{\pi}{6}\right)$$

$\therefore$ For node, $\sin\left(8x + \frac{\pi}{6}\right) = 0$

$$\therefore 8x + \frac{\pi}{6} = n\pi$$

$$\Rightarrow 8x = n\pi - \frac{\pi}{6}$$

$\therefore x = \dfrac{n\pi - \frac{\pi}{6}}{8} = 0.327\text{m} = 32.7\text{cm}$ [n = 1 for minimum distance from origin]

15. At a certain location on the Earth surface, the intensity of sunlight is 1.00 kW/m^2. A perfectly reflecting concave mirror, of radius of curvature R and aperture radius r, is facing the Sun to produce the light intensity of 100 kW/m^2 at the image. Knowing that the disc of the Sun subtends an angle of 0.01 radian at the Earth surface, the relation between R and r is

(a) R = 10r (b) R = 20r (c) R = 40r (d) r = 20R

Ans. (b)

Sol. Total energy incident on the mirror = $I_0\pi r^2$

Radius of image formed = $f\theta = \frac{R}{2}\theta$

$\therefore$ Given,

$$\frac{I_0\pi r^2}{\pi(f\theta)^2} = 100 \Rightarrow \frac{4r^2}{R^2\theta^2} = 100$$

$$\therefore \frac{2r}{R} = 100 \Rightarrow \frac{r}{R} = \frac{10\times 0.01}{2} = \frac{1}{20}$$

$\therefore$ R = 20r

16. A charge Q is uniformly distributed throughout the volume of a non-conducting sphere of radius R. The total electrostatic energy of electric field inside the sphere is $U = \alpha \times \frac{Q^2}{4\pi\varepsilon_0 R}$. The value of α is:

(a) Zero (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{10}$

Ans. (d)

Sol. $U_M = U_{solid} - U_{hollow}$

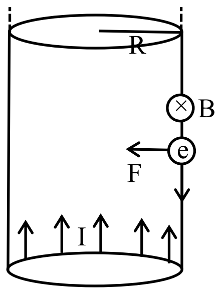
$$= \frac{3}{5}\frac{KQ^2}{R} - \frac{1}{2}\frac{KQ^2}{R} = \frac{KQ^2}{10R} = \frac{1}{10}\frac{Q^2}{4\pi\varepsilon_0 R}$$

17. A long straight vertical wire of circular cross section of radius R contains n conduction electrons per unit volume. A current I flows upward in the wire. The expression for magnetic force on an electron at the surface of the wire is [Assume all the conduction electrons are moving with drift velocity]

(a) $\frac{\mu_0}{4\pi^2}\frac{I^2}{nR^3}$ outward (b) $\frac{\mu_0}{2\pi}\frac{I^2}{nR^3}$ inward (c) $\frac{\mu_0}{2\pi^2}\frac{I^2}{nR^3}$ inward (d) $\frac{\mu_0}{2\pi^2}\frac{I^2}{nR^3}$ outward

Ans. (c)

Sol.



$I = nAeVd$

$\therefore V_d = \frac{I}{neA} = \frac{I}{\pi neR^2}$

$B = \frac{\mu_0 R}{2\pi R}$

$F = eV_dB = e\frac{\mu_0 I}{2\pi R} \times \frac{I}{\pi neR^2}$

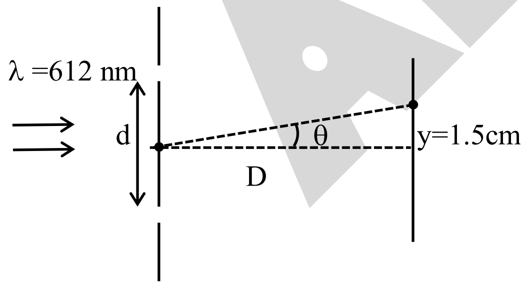
$= \frac{\mu_0 I^2}{2\pi^2 R^3}$ inward

18. In Young's double slit experiment, a bright Fringe is observed at y = 1.5 cm from the center of the fringe pattern when monochromatic light of wavelength 612 nm is used. The screen is at 1.4 m from the plane of the two slits, whose separation is 0.4 mm. The number of dark fringes between the center and the said bright fringe at y = 1.5 cm is:

(a) 13 (b) 8 (c) 7 (d) 6

Ans. (c)

Sol.



d = 0.4 mm

$\tan\theta = \frac{1.5\times10^{-2}}{1.4} = \frac{15}{14}\times10^{-2} \approx \sin\theta$

For maxima.

$\Delta x = d\sin\theta = n\lambda$

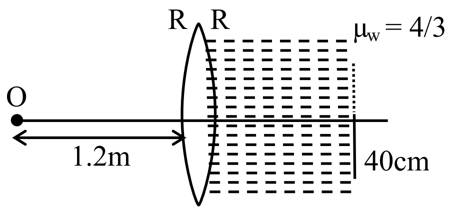
$n = \frac{4\times10^{-4}\times\frac{15}{14}\times10^{-2}}{612\times10^{-9}} = 7$

$\therefore$ 7 dark fringes

19. An illuminated point object is placed in front of an equi-convex lens of refractive $\mu = 1.5$ and focal length f = 40 cm, at a distance of u = 1.20 m in front of the lens on its principal axis. Water $\left(\mu = \frac{4}{3}\right)$ fills the space behind the lens up to a distance of 40 cm from the lens. The final image is formed on the principal axis beyond the lens at a distance v from the lens. The value of v and the nature of image are

(a) 60 cm, virtual (b) 80 cm, virtual (c) 110 cm, real (d) 130 cm, real

Ans. (d)



Sol. $\mu = 1.5$, f = 40cm

$$\frac{1}{40} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R} - \frac{1}{R}\right) = \frac{1}{R}$$

$\therefore$ R = 40 cm

I_{st} Surface

$$\frac{1.5}{V_1} - \frac{1}{(-1.2)} = \frac{1.5 - 1}{0.4} = \frac{1}{0.8}$$

$$\frac{1.5}{V_1} - \frac{1}{0.8} = \frac{1}{1.2} = \frac{1}{2.4}$$

$\therefore$ V_1 = 3.6 m

2^{nd} surface

u = +3.6m

$$\frac{\frac{4}{3}}{v_2} - \frac{1.5}{3.6} = \frac{\frac{4}{3} - 1.5}{-0.4} = +\frac{1}{2.4}$$

$$\frac{4}{3v_2} = \frac{1}{2.4} + \frac{1.5}{3.6} = \frac{1}{1.2}$$

$$\therefore\ v_2 = \frac{4}{3} \times 1.2 = 1.6\text{m}$$

Last plane surface

S_O = 1.6 – 0.4 = 1.2 m

$$\frac{S_i}{1.2} = \frac{1}{\frac{4}{3}}$$

$\therefore S_i = \frac{3}{4} \times 1.2$ = 0.9m = 90 cm

$\therefore$ 130 cm behind lens, real image.

20. A paramagnetic gas at room temperature 27 °C is paced in an external uniform magnetic field of magnitude B =1.5 tesla. The atoms of the gas have magnetic dipole moment μ = 1.0 μ_B where $1\mu_B = \frac{eh}{4\pi m}$ is Bohr magneton, and m is the mass of electron. The energy difference ΔU_B between the parallel alignment and the antiparallel alignment of the atom's magnetic dipole moment, with respect to the external field B, is x $\times 10^{-4}$ eV, where the value of x is:

(a) 17.4 (b) 1.74 (c) 34.8 (d) 8.7

Ans. (b)

Sol. Paramagnetic gas

T = 300 K

B = 1.5 T

$U = -MB(\cos\theta_2 - \cos\theta_1)$, M = magnetic moment, B = Magnetic field

θ_2 = 180°, θ_1 = 0°

Then U = 2MB = $\frac{2ehB}{4\pi m}$

$$U = \frac{2\times 1.6\times 10^{-19}\times 6.62\times 10^{-34}\times 1.5}{4\times 3.14\times 9.1\times 10^{-31}} = 2.78\times 10^{-23}\text{ J}$$

$$U = \frac{2.78\times 10^{-23}}{1.6\times 10^{-19}} = 1.74\times 10^{-4}\text{ eV}$$

21. A glass capillary with inner diameter of 0.40 mm is vertically submerged in water so that the length of its part protruding above the water is h = 25 mm. Surface tension of water is T = 0.073 Nm^{-1}. Since the water wets the glass completely, it may be concluded that the:

(a) length of capillary above the water surface is insufficient so the water will flow out as a fountain.

(b) water will rise up to the brim forming a meniscus of radius 1.2 mm at the top.

(c) water will rise up to the brim forming a meniscus of radius 0.6 mm at the top.

(d) water will rise up to the brim forming a meniscus of radius equal to radius of the capillary.

Ans. (c)

Sol. d = 0.4 mm, r = 0.2 mm, h = 25 mm, T = 0.073 N/m

Free rise

$$h' = \frac{2T}{r\rho g} = \frac{2\times 0.073}{0.2\times 10^{-3}\times 1000\times 10} = 73\text{ mm}$$

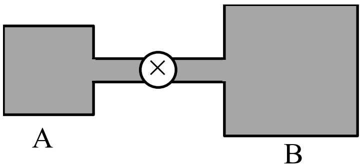
h' > 25mm

$$\rho g h = \frac{2T}{R}$$

$$10^3\times 10\times 25\times 10^{-3} = = \frac{2\times 73\times 10^{-3}}{R}$$

$$R = \frac{2\times 73}{25\times 10^4} = 5.84\times 10^{-4}\text{ m} = 0.6\text{ mm}$$

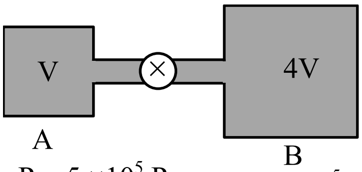
22. The container A in the figure holds an ideal gas at a pressure of 5.0 × 10^5 Pa at 27 °C. It is connected to the container B by a thin tube fitted with a closed valve. Container B with volume four times the volume of A holds the same ideal gas at a pressure 1.0 × 105 Pa at 127 °C The valve is now opened to allow the pressure to equalize, but the temperature of each container is maintained as before. The common pressure in the two containers (in kPa) is



(a) 200 (b) 300 (c) 320 (d) 180

Ans. (a)

Sol. Initial Condition



$P = 5 \times 10^5$ Pa
T = 300K

$P = 1 \times 10^5$ Pa
T = 400K

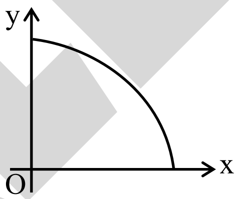
$$n_1 + n_2 = n'_1 + n'_2$$

$$\frac{5\times10^5 \times V}{R\times300} + \frac{1\times10^5\times4V}{R\times400} = \frac{PV}{R\times300} + \frac{P(4V)}{R\times400}$$

On Solving

$P = 2 \times 10^5$ Pa = 200 kPa

23. A thin plastic disc, which is a quarter of a circle of radius R = 0.6 m, lies in the first quadrant of x-y plane, with the center of curvature at the origin O as shown. It is charged uniformly on one side (one face) with surface charge density σ. Electric potential at point P (0, 0, 0.8 m) is:



(a) $\frac{\sigma}{8\in_0}$ (b) $\frac{\sigma}{8\pi\in_0}$ (c) $\frac{\sigma}{20\in_0}$ (d) $\frac{\sigma}{40\in_0}$

Ans. (d)

Sol. Potential by quarter disc= $\frac{1}{4}$ potential of disc

$$V = \frac{\frac{\sigma}{2\varepsilon_0}[\sqrt{R^2+z^2} - z]}{4}$$

$$= \frac{\sigma}{8\varepsilon_0}[\sqrt{0.6^2+0.8^2} - 0.8] = \frac{\sigma}{40\varepsilon_0}$$

24. A spring is compressed under the action of a constant force F. The compression of the spring is ξ. Suppose the direction of the force is reversed suddenly as well as its magnitude is doubled. The maximum extension of the spring beyond its natural length will now be (the spring obey's Hook's law).

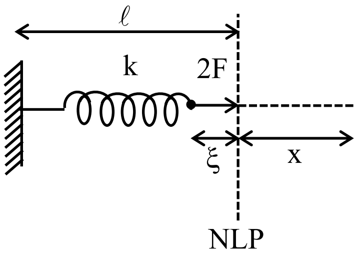
(a) 2ξ (b) 3ξ (c) 4ξ (d) 5ξ

Ans. (d)

Sol. When force F is applied then at equilibrium position

F = kξ

Work energy theorem



$W_F + W_{SP} = 0$

$$2F(x+\xi) - \frac{1}{2}k(x^2-\xi^2) = 0$$

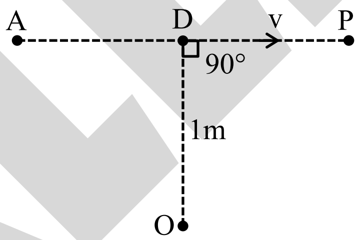
$$2F(x+\xi) = \frac{1}{2}k(x^2-\xi^2)$$

$$2F = \frac{1}{2}\frac{F}{\xi}(x-\xi)$$

$4\xi = x - \xi$

$\therefore\ x = 5\xi$

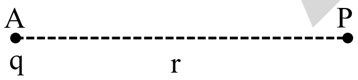
25. The charge on a small point object at A produces an electric potential V = 3 volt at a point P. Because of an unavoidable situation, leakage of the charge from the object at starts at t = 0 at a constant rate of 3 μCs^{-1}. To maintain the potential of 3 volt at the point P, the object is made to move towards P with a certain velocity v. When the point object crosses the point D shown in the figure, it is found to have a charge of 10 μC and the direction of its velocity is perpendicular to OD. The resulting magnetic field B, at this instant, at the loacation O (such that OD = 1.00 m) is:



(a) 8.1 nT (b) $\frac{1}{9}\mu T$ (c) 9.9 μT (d) 9.0 nT

Ans. (d)

Sol.



Potential at point P is given by

$$V_P = \frac{Kq}{r} = \text{constant} = 3$$

$$\frac{dq}{dt}r - \frac{q\,dr}{dt} = 0$$

$$\therefore -\frac{dq}{dt}r = -q\frac{dr}{dt}$$

$$V = -\frac{dr}{dt} = \frac{r}{q}\left(-\frac{dq}{dt}\right) = \frac{k}{V_P}\left(-\frac{dq}{dt}\right)$$

$$= \frac{9\times10^9}{3}\times3\times10^{-6} = 9\times10^3\ \text{m/s}$$

$$B = \frac{\mu_0}{4\pi}\frac{qv}{r^2} = \frac{10^{-7}\times 10\times 10^{-6}\times 9\times 10^3}{1^2} = 9nT$$

26. In the Bohr model of hydrogen atom, the force between the nucleus and the electron is modified as $F = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2}+\frac{\delta}{r^3}\right)$, where δ is a small constant. Using the Bohr radius the radius $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$, the radius of n^{th} orbit is:

(a) $a_0 n^2 - \delta$ (b) $a_0 n^2 + \delta$ (c) $a_0 (n - \delta)^2$ (d) $a_0 (n + \delta)^2$

Ans. (a)

Sol. $F = \frac{e^2}{4\pi \in_0}\left(\frac{1}{r^2}+\frac{\delta}{r^3}\right) = \frac{mv^2}{r}$

$$mv^2 = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r}+\frac{\delta}{r^2}\right) \,\&\, mvr = \frac{nh}{2\pi}$$

$$m\times\frac{n^2h^2}{4\pi^2m^2r^2} = \frac{e^2}{4\pi\in_0}\left(\frac{1}{r}+\frac{\delta}{r^2}\right)$$

$$\frac{n^2h^2}{\pi m r^2}\times\frac{\in_0}{e^2} = \frac{1}{r}+\frac{\delta}{r^2}$$

$$\frac{n^2a_0}{r^2} = \frac{1}{r}+\frac{\delta}{r^2}$$

$$\frac{\left(n^2a_0-\delta\right)}{r^2} = \frac{1}{r} \quad \therefore\ r = (n^2a_0 - \delta)$$

27. An infinite number of conducting rings having increasing radius r_0, r_1, r_2, r_3 and so no, such that $r_0 = r$, $r_1 = 2r$, $r_2 = 2^2r$, $r_3 = 2^3r$ and so on ………. Upto ∞, have been placed concentrically on a plane. All the rings carry the same current I but the current in consecutive rings is in opposite direction shown. The magnetic field produced at the common centre of the rings is:

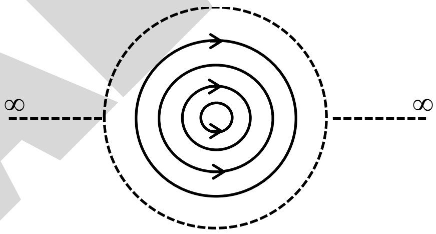


Diagram not to scale

(a) zero (b) $\frac{\mu_0 i}{4r}$ (c) $\frac{\mu_0 i}{3r}$ (d) $\frac{\mu_0 i}{2r}$

Ans. (c)

Sol. We know that magnetic field due to circular loop at its centre is given by $\frac{\mu_0 i}{2R}$

$$B = \frac{\mu_0 i}{2r}\left[1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+.........\right]$$

$$= \frac{\mu_0 i}{2r}\frac{1}{\left(1-\left(-\frac{1}{2}\right)\right)} = \frac{\mu_0 i}{2r}\cdot\frac{2}{3} = \frac{\mu_0 i}{3r}$$

28. One mole of an ideal monoatomic gas, initially at temperature T, is heated such a way that its molar heat capacity during the process of heating is C = 2R. The volume of the gas gets tripled (at constant pressure) during the process. The final temperature attained by the gas is:

(a) 3T (b) 3^2 T (c) $\frac{1}{3}$T (d) 3^3 T

Ans. (a or b)

Sol. $C = C_v + \frac{R}{1-x}$

$2R = \frac{3}{2}R + \frac{R}{1-x}$

$\therefore$ x = – 1

PV^{-1} = constant

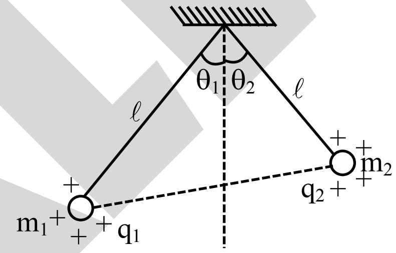
$\therefore P \propto V$

$V_f = 3V$

As per heat capacity: $P_f = 3P$, $\therefore T_f = 9T$

At constant pressure: $T_f = 3T$

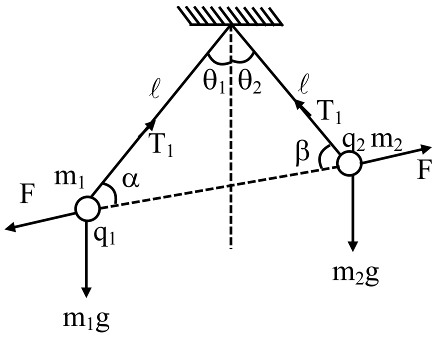
29. Two small positively charged spherical balls are suspended from a common point at the ceiling by non-conducting massless string of equal length ℓ. The first ball has mass m_1 and charge q_1 while the second ball has mass m_2 and charge q_2. If the two strings subtend angles θ_1 and θ_2 with the vertical as shown then



(a) $\frac{\sin\theta_1}{\sin\theta_2} = \frac{q_2}{q_1}$ (b) $\frac{\sin\theta_1}{\sin\theta_2} = \frac{m_2}{m_1}$ (c) $\frac{\tan\theta_1}{\tan\theta_2} = \frac{q_1}{q_2} \times \frac{m_2}{m_1}$ (d) $\frac{\sin\theta_1}{\sin\theta_2} = 1$

Ans. (b)

Sol.



$$\frac{F}{\sin(\pi-\theta_1)} = \frac{m_1 g}{\sin(\pi-\alpha)}$$

$$\frac{F}{\sin\theta_1} = \frac{m_1 g}{\sin\alpha} \quad \text{..... (i)}$$

$$\frac{F}{\sin(\pi - \theta_2)} = \frac{m_2 g}{\sin(\pi - \beta)}$$

$$\frac{F}{\sin\theta_2} = \frac{m_2 g}{\sin\beta} \quad \text{..... (ii)}$$

(i)/(ii)

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{m_1}{m_2} \cdot \frac{\sin\beta}{\sin\alpha}$$

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{m_1}{m_2} \quad \text{as } \alpha = \beta$$

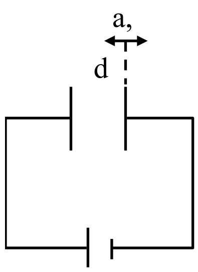
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{m_1}{m_2}$$

30. An air filled parallel plate capacitor, with plate area A and plate separation d, is connected to a battery of emf V volt having negligible internal resistance. One of the plates of the capacitor vibrates with amplitude 'a' (a << d) and angular frequency ω. If the instantaneous current in the circuit reaches a maximum value I_0, the amplitude of the vibrations is 'a' equal to:

(a) $\frac{I_0 d^2}{\varepsilon_0 \omega V A}$ (b) $\frac{2I_0 d}{\varepsilon_0 \omega V A}$ (c) $\frac{2I_0 d^2}{\varepsilon_0 \omega V A}$ (d) $\frac{I_0 d^2}{2\varepsilon_0 \omega V A}$

Ans. (a)

Sol.



$$C = \frac{A \in_0}{d + x}$$

$$q = CV = \frac{A \in_0 V}{d + x}$$

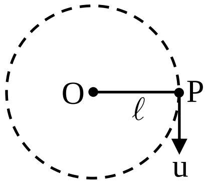
$$i = \frac{dq}{dt} = \frac{-A \in_0 V}{(d + x)^2} \cdot \frac{dx}{dt}$$

$$i = \frac{-A \in_0 V}{(d + x)^2} v$$

$$i_{max} = \frac{A \in_0 V}{d^2}(a\omega) = \frac{A \in_0 a\omega V}{d^2} = I_0$$

$$\therefore a = \frac{I_0 d^2}{A \in_0 \omega V}$$

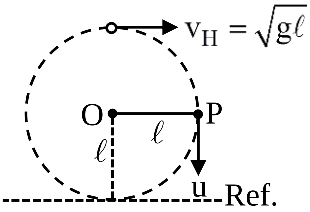
31. A small ball of mass m is attached to one end of a massless un-stretchable string of length ℓ and is held at the point P. The other end of the string is fixed to a support at O such that OP is horizontal. The minimum downward speed u, that should be imparted to the ball at the point P so that the ball can complete the vertical circle without any slack in the string is



(a) $\sqrt{2g\ell}$ (b) $\sqrt{3g\ell}$ (c) $\sqrt{4g\ell}$ (d) $\sqrt{5g\ell}$

Ans. (b)

Sol.



$$\frac{1}{2}mu^2 + mg\ell = \frac{1}{2}mv_H^2 + mg(2\ell)$$

$$\frac{1}{2}mu^2 = \frac{3mg\ell}{2}$$

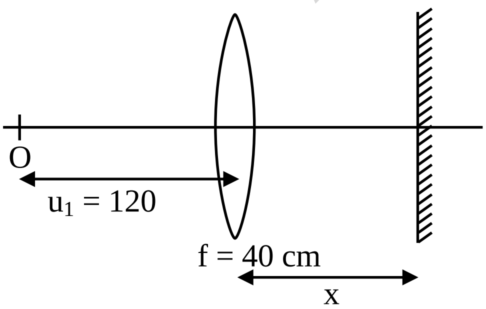
$$u = \sqrt{3g\ell}$$

32. An illuminated point object is placed on the principal axis, in front of an equi-convex glass lens of focal length f = 40 cm, at a distance of u = 1.20 m from the lens. A reflecting plane mirror has been placed behind the lens perpendicular to the principal axis and facing the lens. The nature of the final image and the distance of the plane mirror from the lens so as to form the final image at the plane mirror itself, is :-

(a) virtual image, 40 cm (b) real image, 40 cm (c) virtual image, 50 cm (d) real image, 20 cm

Ans. (a)

Sol.



for 1st refraction from lens

$$\frac{1}{v} - \frac{1}{-120} = \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{40} - \frac{1}{120} = \frac{2}{120}$$

v = 60 cm

Now reflection from plane mirror

$u_2 = -(x - 60)$

$v_2 = x - 60$ (Behind mirror)

Now final refraction from lens.

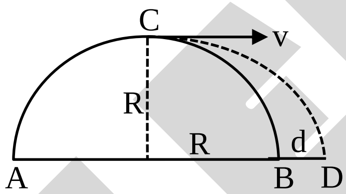
$u_3 = -(2x - 60)$ f = 40 $v_3 = -40$

$$\frac{1}{-40} + \frac{1}{+(2x-60)} = \frac{1}{40}$$

$$\frac{1}{2x-60} = \frac{1}{20}$$

x = 40 cm

33. A ball is kicked horizontally from the top C of a hemispherical rock ACB of radius R on a horizontal ground, with a velocity v, so as not to hit the rock at any point during its flight. Choose the correct statement (see the figure)



(a) The ball will just strike at B

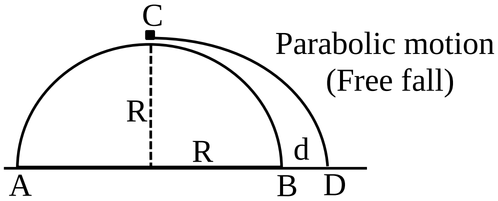
(b) The ball will strike at D where d = BD = $\left(\sqrt{2}-1\right)R$

(c) The ball will strike at D such that d = BD = $\left(2-\sqrt{2}\right)R$

(d) The speed v of the ball should always be greater than the critical speed v_0 where v_0 is $\sqrt{\frac{gR}{2}}$.

Ans. (b)

Sol.

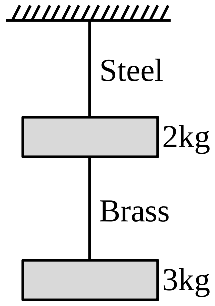


$t = \sqrt{\frac{2R}{g}}$ $v_{min} = \sqrt{gR}$

$R_{min} = \sqrt{2}R$

$BD = \sqrt{2}R - R = \left(\sqrt{2}-1\right)R$

34. The ratio of length, radii and Young's moduls of steel to brass wires in the figure are α, β and γ, respectively. The corresponding ratio of the increase in their length is :-



(a) $\frac{2\alpha}{3\beta^2\gamma}$ (b) $\frac{5\alpha}{3\beta^2\gamma}$ (c) $\frac{2\alpha\beta^2}{3\gamma}$ (d) $\frac{5\alpha\beta^2}{3\gamma}$

Ans. (b)

Sol. $T_B = 3g$

$$\frac{3g}{\pi R_B^2} = Y_B \frac{\Delta\ell_B}{\ell_B} \text{ ....(1)}$$

$$\frac{5g}{\pi R_S^2} = Y_S \frac{\Delta\ell_S}{\ell_S} \text{ ....(2)}$$

eq. (1) divides by eq. **(2)**

$$\frac{3R_S^2}{5R_B^2} = \frac{Y_3}{Y_5}\frac{\Delta\ell_B}{\Delta\ell_S}\cdot\frac{\ell_S}{\ell_B}$$

$$\frac{3}{5}\beta^2 = \frac{1}{\gamma}\frac{\Delta\ell_B}{\Delta\ell_S}.\alpha$$

$$\frac{\Delta\ell_S}{\Delta\ell_B} = \frac{5\alpha}{3\gamma\beta^2}$$

35. A uniform beam of light of intensity 60 mW/m^2 is incident on a totally absorbing sphere of radius 2.0 μm. The density of the material of the sphere is $\rho = 5.0 \times 10^3$ kg/m^3. The sphere is placed in a region of space whre gravitational force can be neglected/ignored. The magnitude of acceleration of the sphere due to the incidence of the light is :-

(a) 1.5 ms^{-2} (b) 3.0 ms^{-2} (c) 7.5 ms^{-2} (d) Zero

Ans. (d)

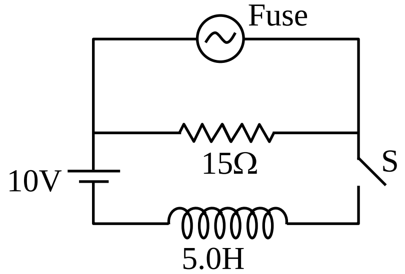
Sol. **Radiation force =** $\frac{I}{C}\pi R^2 = ma$

$$\frac{I}{C}.\pi R^2 = \rho.\frac{4}{3}\pi R^3 a$$

$$a = \frac{3I}{4R\rho C} = \frac{3\times 60\times 10^{-3}}{4\left(2\times 10^{-6}\right)5\times 10^3\times 3\times 10^8}$$

$$a = \frac{180\times 10^{-3}}{12\times 10^6} = 1.5\times 10^{-8}\,\text{m/s}^2 \approx 0$$

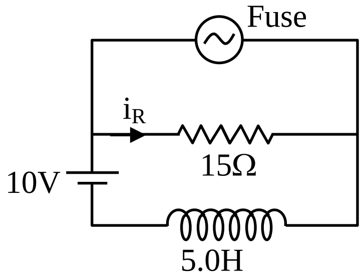
36. The fuse, in the upper branch of the circuit shown, is an ideal 4.0 A fuse. The fuse has zero resistance as long as current through it remains less than 4.0 A. The fuse blows out when the current reaches 4.0 A. Needless to say that the resistance becomes infinite thereafter. Switch S is closed at time t = 0. The fuse blows out at time :-



(a) t = 2 sec (b) t = 4 sec (c) t = 8 sec (d) It won't blow

Ans. (a)

Sol.



initially $i_R = 0$ (as fuse is resistance less)

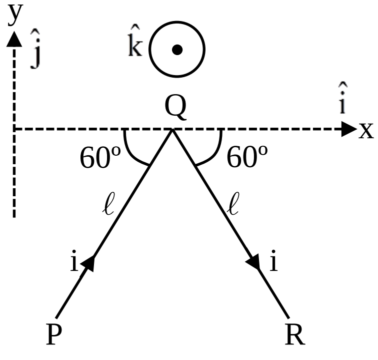
$$V - L\frac{di}{dt} = 0$$

$$i = \frac{V}{L}.t$$

$$4 = \frac{10}{5}t$$

t = 2 sec

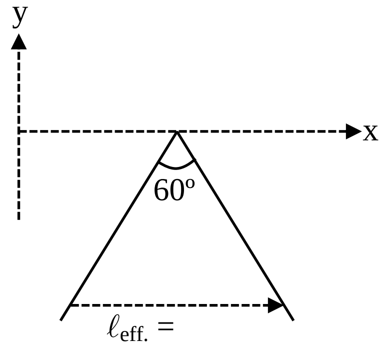
37. The bent wire PQR shown in the figure lies in a uniform magnetic field $\vec{B} = 3.0\hat{i} + 4.0\hat{k}T$. The direction $\hat{k}$ being normal to the plane of the paper and directed towards the viewer. The two straight sections PQ and QR of the wire each have length 2.0 m and the wire carries a current of 2.5 A. Net force on the wire due to magnetic field $\vec{B}$ is :-



(a) $40\hat{j}N$ (b) $-20\hat{j}N$ (c) $-40\hat{j}N$ (d) $-20\sqrt{3}\hat{j}N$

Ans. (b)

Sol.



i = 2.5 A

$$\vec{F} = i\,\vec{d\ell}\times\vec{B} = 5\hat{i}\times\left(3\hat{i}\times 4\hat{k}\right)$$

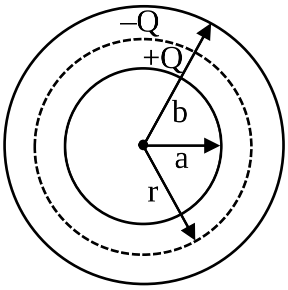
$$\vec{F} = -20\hat{j}.N.$$

38. A charged spherical capacitor consists of two concentric spherical shells of radii a and b (b > a). Half of the stored electrical energy of this system lies within a spherical region of radius r if :-

(a) $r = \sqrt{ab}$ (b) $r = \frac{a+b}{2}$ (c) $r = \frac{2ab}{a+b}$ (d) $r = \frac{a^2+b^2}{a+b}$

Ans. (c)

Sol.



$$E = \frac{kQ}{r^2}$$

Energy stored in capactor $E_1 = \int \frac{1}{2}\epsilon_0\, E^2.dV$

$$E_1 = \int \frac{1}{2}\epsilon_0 \frac{k^2Q^2}{r^4}.4\pi r^2 dr$$

$$E_1 = \frac{kQ^2}{2}.\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$E_2 = \frac{1}{2}E_1 = \frac{kQ^2}{2}.\int_a^R r^{-2}dr$$

$$\frac{kQ^2}{4}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{kQ^2}{2}\left(\frac{1}{a}-\frac{1}{r}\right)$$

$$\frac{1}{2a}-\frac{1}{2b} = \frac{1}{a}-\frac{1}{r}$$

$$\frac{1}{r} = \frac{1}{2a}+\frac{1}{2b}$$

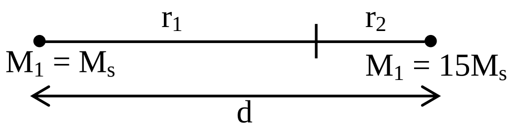
$$r = \frac{2ab}{a+b}$$

39. Two stars of masses $M_1 = M_s$ and $M_2 = 15\ M_s$ (where M_s = mass of the Sun) form a binary system. The stars are revolving round each other, always being at a separation of d = 4 AU between them (1 AU is distance of the Earth from the Sun), move in circular orbits about their center of mass. The period of revolution of each star is :-

(a) 1 year (b) 2 years (c) 4 years (d) 8 years

Ans. (b)

Sol.



r = 1Au

$$\omega = \sqrt{\frac{G(M_1 + M_2)}{d^3}} = \sqrt{\frac{G(16Ms)}{64r^3}}$$

$$\omega = \frac{1}{2}\sqrt{\frac{GMs}{r^3}} = \frac{\omega_{earth}}{2}$$

$T = 2T_{earth}$ = 2years.

40. The total energy released in α decay of a stationary Radium nucleus ^{226}Ra (mass 116 u) is Q = 4.9 MeV (mass of α particle m_α = 4 u) then the :-

(a) energy of the recoiled daughter nucleus is nearly 8.7 keV

(b) energy of the recoiled daughter nucleus is nearly 4.81 MeV.

(c) recoil speed of the daughter nucleus is $2.74 \times 10^5\ ms^{-1}$.

(d) speed of emitted α particle is nearly $1.5 \times 10^6\ ms^{-1}$

Ans. (b)

Sol. $$k_\alpha = \left(\frac{A-4}{A}\right)Q = \left(\frac{226-4}{226}\right)4.9MeV$$

$$= \frac{222}{226}(4.9MeV) = 4.81MeV$$

41. A listener at rest (with respect to the air and the ground) hears a sound of frequency f_1 from a source moving towards him with a velocity of 15 ms^{-1}, towards East. If the listener now moves towards the approaching source with a velocity of 25 ms^{-1}, towards West, he hears a frequency f_2 that differs from f_1 by 40 Hz. The frequency f of the sound produced by the source is (speed of sound in the air is 340 ms^{-1})

(a) 520 Hz (b) 450 Hz (c) 480 Hz (d) 550 Hz

Ans. (a)

Sol. **Case-1**

$$f_1 = \left(\frac{v}{v - v_1}\right)f_0$$

Case-2

$$f_2 = \left(\frac{v+v_2}{v-v_2}\right)f_0$$

$$f_2 - f_1 = \left(\frac{v+v_2}{v-v_1}\right)f_0 - \left(\frac{v}{v-v_1}\right)f_0 = \left(\frac{v_2}{v-v_1}\right)f_0$$

$$40 = \left(\frac{25}{340-15}\right)f_0$$

$$f_0 = \frac{40\times 325}{25}$$

f_0 = 520 Hz

42. A uniform rod of length ℓ swings from a pivot as a physical pendulum. The position of the pivot can be varied along the length of the rod. The minimum time period with which the rod can oscillate with an appropriate position of the pivot is T = $2\pi\sqrt{\frac{L}{g}}$ where L is equal to

(a) $\frac{\ell}{2}$ (b) $\frac{\ell}{\sqrt{2}}$ (c) $\frac{\ell}{\sqrt{3}}$ (d) $\frac{\ell}{2\sqrt{3}}$

Ans. (c)

Sol. for minimum time period in physical pendulum.

ℓ = k (k = Radius of Gyration)

so for uniform rod.

$$K = \frac{L}{\sqrt{12}} = \frac{L}{2\sqrt{3}}$$

$$\ell_{\text{eff.}} = \frac{K^2}{\ell} + \ell = \frac{\ell^2}{\ell} + \ell \Rightarrow \ell_{\text{eff}} = 2\ell = \frac{\ell}{\sqrt{3}}$$

43. The number density of conduction electrons in pure silicon at room temperature is about $10^{16}\ m^{-3}$. The number density of conduction electrons is increased by a factor of 10^6 by doping the silicon lattice with phosphorus. Assume that at room temperature every phosphorus atom contributes one electron to the conduction band. The fraction of silicon atoms replaced by phosphorus atoms is (Given that the density of silicon is 2.33 gm/cm^3 and that the molar mass of silicon M = 28.1 gm)

(a) 1.0×10^{-7} (b) 2.0×10^{-7} (c) 4.0×10^{-7} (d) 5.0×10^{-7}

Ans. (b)

Sol. $n_0 = 10^{16}$

$10^6 n_0 = n_0 + n_p$

$(10^6 - 1)n_0 = n_p$

$n_p = 10^{22}$

No. of silicon in 1gm is

$$N_{si} = \frac{m_{si}N_A}{M_{si}} = \frac{1\times 6.023\times 10^{23}}{28.1}$$

$$N_{Si} = 2.143\times 10^{22}\ \text{atoms / gm}$$

No. of phosphorous required is = 10^6 atoms/m^3

No. of si atoms/m^3 = Nsi × ρ

= $2.143 \times 10^{22} \times 2.33 \times 10^{+6}$

= $4.99 \times 10^{28} \approx 5 \times 10^{28}$

$$\frac{n_p}{n_{si}} = \frac{10^{22}}{5\times 10^{28}} = 2\times 10^{-7}$$

44. A soap bubble 10 cm in radius, with a film thickness of $\frac{10}{3}\times10^{-6}\,\text{cm}$, is charged to a potential of 80 V. The bubble bursts and converts into a single spherical drop. Assuming that the soap solution is a good conductor, the potential at the surface of the drop is :-

(a) 2 kV (b) 4 kV (c) 6 kV (d) 8 kV

Ans. (d)

Sol. $v = \frac{kQ}{R}$

$Qk = vR$

Volume conservation

$4\pi R^2 t = \frac{4}{3}\pi r^3$

$r = 10^{-3}$ m

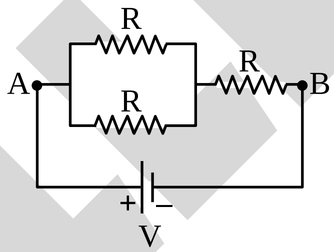
$v = \frac{kQ}{r} = \frac{vR}{r} = 80\left(\frac{10^{-1}}{10^{-3}}\right)$

= 8000 v = 8 kv

45. Three resistance, each of R = 4Ω rated 16 W, are connected across A and B as shown. Potential difference of V volt is applied between points A and B.

Statement 1 : Maximum potential difference V that can be applied is 12 volt.

Statement 2 : Maximum power that can be dissipated is 24 watt.



(a) Statements 1 and 2 both are wrong

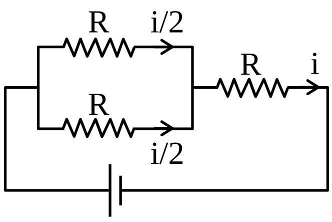
(b) Statement 1 is correct but statement 2 is wrong.

(c) Statement 1 is wrong but statement 2 is correct.

(d) Statements 1 and 2 both are correct.

Ans. (d)

Sol.



I_{max}

$i_{max}^2 R = 16$

$i_{max}^2 = 4$

i_{max} = 2 amp.

$v_{max} = \frac{iR}{2} + iR$

$= i\left(\frac{3R}{2}\right) = 12V$

$P_{max} = \frac{i^2}{4}R + \frac{i^2}{4}R + i^2R$

$= \frac{3i^2R}{2} = \frac{3}{2}(4).(4)^2 = 24$

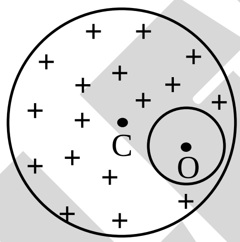
46. Water is filled in a vertical cylinder up to a certain height h. The cylinder is made to rotate with an angular velocity ω about a vertical axis coinciding with the axis of the cylinder. The water surface seen from top appears as a/an

(a) ellipsoid (b) hemisphere (c) paraboloid (d) hyperboloid

Ans. (c)

Sol. Surface of water is seen as rotating parabola so it is paraboloid $\left(y = \frac{\omega_0^2}{2g}x^2\right)$

47. A uniformly charged non-conducting sphere with its center at C carries positive charge with uniform charge density +ρ, except in a spherical cavity (inside the sphere) with center O. The electric field E at any point inside the cavity is :-



(a) zero (b) uniform

(c) directed radially outward (d) directed radially inward

Ans. (b)

Sol. $\vec{F} = \frac{\ell}{3\in_0}\overrightarrow{CO}$ (uniform electric field in the cavity)

48. A cylindrical tank with base area A = 0.05 m^2 is filled with water up to a height H = 50 cm. There is a small hole of area a = 0.001 m^2 (a << A) in the bottom of the tank. It takes time t to empty the tank up to a height $\frac{H}{2}$ (i.e. to empty half of the water volume). The additional time required to empty the tank completely is :-

(a) t (b) $t\sqrt{2}$ (c) $t\left(\sqrt{2}-1\right)$ (d) $t\left(\sqrt{2}+1\right)$

Ans. (d)

Sol. $T = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left(\sqrt{H}-\sqrt{h}\right)$ (H = initial height, h = final height)

$h = \frac{H}{2}$

$t = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left(\sqrt{H}-\sqrt{\frac{H}{2}}\right)$

h = 0, $H = \frac{H}{2}$

$$t_1 = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left(\sqrt{\frac{H}{2}} - 0\right)$$

$$\frac{t}{t_1} = \frac{\sqrt{H} - \sqrt{\frac{H}{2}}}{\sqrt{\frac{H}{2}}}$$

$$\frac{t}{t_1} = \sqrt{2} - 1$$

$$t_1 = \frac{t}{\sqrt{2} - 1}$$

$$t_1 = t\left(\sqrt{2} + 1\right)$$

49. According to Bohr theory, in the ground state of hydrogen atom, an electron revolves in circular orbit of radius r with velocity v and circulation frequency f. The magnetic dipole moment of the electronic orbit is p_m. The magnetic field produced by the circulating electron at the center of the atom is B. Then for a He^+ ion in the state n = 2 (electron in the 2^{nd} orbit)

(a) the radius of the orbit is 2r

(b) the magnetic dipole moment of the orbit is $2p_m$

(c) the frequency of circulartion is $\frac{f}{2}$.

(d) the magnetic field at the center is $\frac{B}{4}$.

Ans. (a,b,c,d)

Sol. **(a)** $r = \frac{n^2h^2}{\pi m Z e^2} = 2r_0$

(b) $M = \frac{q}{2m} \times L = \frac{q}{2m} \times \frac{2h}{2\pi} = 2\mu_0$

(c) $f = \frac{v}{r} = \frac{\frac{Ze^2}{2\in_0 nh}}{\frac{n^2h^2}{\pi m Z e^2}} \propto \frac{Z^2}{n^3} \Rightarrow \frac{f_0}{2}$

(d) $B = \frac{\mu_0 q \omega}{4\pi r} \propto \frac{f}{r} = \frac{B_0}{4}$

50. A body of mass m = 0.25 kg is moving along x axis under the action of a conservative force. Its potential energy as a function of position x is given by U(x) = $-\frac{100x}{x^2+4}$ J (x in m). Then

(a) force F(x) acting on the body at x = 0 is 25 N

(b) there is stable equilibrium at x = 2m.

(c) there is unstable equilibrium at x = –2m.

(d) the body executes (small) oscillations with angular frequency $\omega = 0.5\ rads^{-1}$

Ans. (a,b,c)

Sol. $F = -\frac{dU}{dx} = 100\frac{\left[\left(x^2+4\right)-x\times 2x\right]}{\left(x^2+4\right)^2}$

$= 100\frac{\left[4-x^2\right]}{\left(x^2+4\right)^2} = 0 \Rightarrow x = \pm 2$ (equilibrium position)

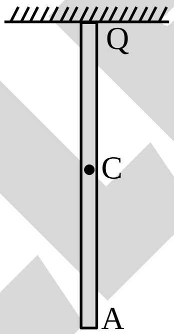
$F_0 = \frac{400}{16} = 25N$

$\frac{d^2U}{dx^2} = -100\frac{\left[-2x\left(x^2+4\right)^2-\left(4-x^2\right)\times 2\left(x^2+4\right)\times 2x\right]}{\left(x^2+4\right)^4}$

$= \frac{-100\left[-4\times 64\right]}{64^2}$ = +ve for x = 2 $\Rightarrow$ stable unstable for x = –2

$0.25\ a = F = 100\left[\frac{4-\left(2+\Delta\right)^2}{\left(2+4\right)^2}\right] \Rightarrow a = \frac{400\times 4}{64} \Rightarrow \omega = 5$ rad/sec

51. A uniform rod OA of length L is freely pivoted at one end O. Let C be the midpoint (i.e. OC = $\frac{L}{2}$ = CA). Choose the correct statement (s) :-



(a) The rod can perform small angular oscillations with time period $T = 2\pi\sqrt{\frac{L}{2g}}$

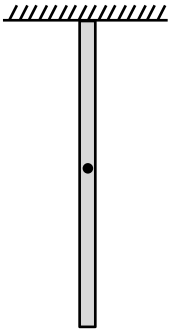
(b) The rod is brought to the horizontal position and then released from rest. The angular velocity of the rod at the instant when it is vertical is $\omega = \sqrt{\frac{3g}{L}}$.

(c) When the oscillating rod comes to the vertical position, it breaks at midpoing C without generating any impulsive force, the largest angle from vertical reached by the uper part OC of the rod is 60°.

(d) After breaking, the power part just falls vertically rotating about its center.

Ans. (b,c)

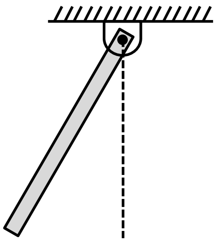
Sol.



$$T = 2\pi\sqrt{\frac{I}{mg\ell/2}} = 2\pi\sqrt{\frac{\frac{m\ell^2}{3}}{\frac{m\ell}{2}}} = 2\pi\sqrt{\frac{2\ell}{3g}}$$

$$mg\frac{L}{2} = \frac{1}{2}\times\frac{mL^2\omega^2}{3}$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{L}}$$



$$-\frac{mg}{2}\times\frac{L}{4}\left(1-\cos\theta\right) = \mathbf{0} - \frac{1}{2}\times\frac{m}{2}\frac{\left(L/2\right)^2}{3}\times\frac{3g}{L}$$

$$-\frac{mgL\left(1-\cos\theta\right)}{8} = -\frac{mL^2}{48}\times\frac{3g}{L}$$

$$1 - \cos\theta = \frac{1}{2}$$

$\theta = 60°$

Lower part has vertical circular motion.

52. A boy stands on a stationary ice boat on a frictionless horizontal flat iceberg. The boy and the boat have a combined mass of M = 60 kg. Two balls of masses m_1 = 10 kg and m_2 = 20 kg are placed on the boat. In order to get the boat moving, the boy throws the balls backward horizontally either in succession or both together. In each case the balls are thrown backward with a certain speed v_{rel} = 6 ms^{-1} relative to the boat just when the ball is being thrown. The resulting speed of the system of the boat and the boy is

(a) V = 2.00 ms^{-1} when both the balls m_1 and m_2 are thrown together.

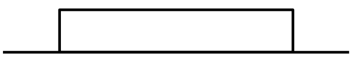
(b) V = 3.00 ms^{-1} when both the balls m_1 and m_2 are thrown together.

(c) V = 2.17 ms^{-1} if the balls are thrown one after the other, first m_1 and then m_2.

(d) V = 2.19 ms^{-1} if the balls are thrown one after the other, first m_2 and then m_1.

Ans. (a,c,d)

Sol.



$$60v_1 = P_{1C} = \mu v_{rel} = \frac{60\times30}{90}\times6$$

v_1 = 2 m/s

$$v'_1 = \frac{80\times10}{90}\times6$$

$$60\left(v_1 - \frac{2}{3}\right) = \frac{60\times20}{80}\times6$$

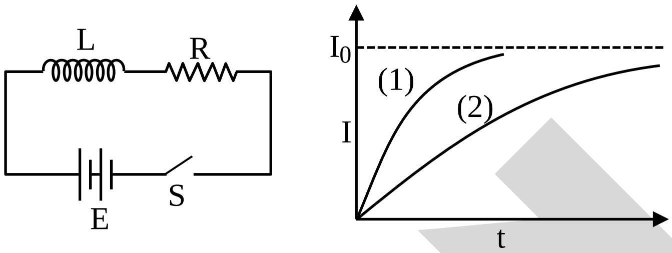
$$v_1 = \frac{2}{3}+\frac{3}{2} = \frac{13}{6} = 2.17\,m/s$$

$$(60+10)v'_1 = \frac{70\times 20}{90}\times 6$$

$$60\left(v_1 - \frac{4}{3}\right) = \frac{60\times 10}{70}\times 6$$

$$v_1 = \frac{4}{3}+\frac{6}{7}=\frac{46}{21}=2.19\text{m / s}$$

53. An electric circuit consists of a battery of emf E, an inductance L and a resistance R in series. The switch S is closed at t = 0. The current in the circuit grows exponentially with time as depicted by curve (1). The values of the circuit parameters (E, L or R) are now somehow changed. The circuit is closed second time, the growth of current I follows curve (2). The following conclusions(s) may be drawn :-



(a) E and R are unchanged but L has increased.

(b) $\frac{E}{R}$ is unchanged but $\frac{L}{R}$ has increased.

(c) $\frac{E}{R}$ is unchanged but $\frac{L}{R}$ has decreased.

(d) Stored magnetic energy has increased.

Ans. (a,b,d)

Sol. $I_0 = \frac{E}{R} = \frac{E'}{R'}$

$$I = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\frac{dI}{dt} = \frac{E}{R}\times\frac{R}{L}e^{-\frac{Rt}{L}}$$

$$= \frac{E}{L}e^{-\frac{Rt}{L}}$$

$L_2 > L_1$

54. The Earth is revolving around the Sun (mass M) in a circular orbit of radius r with a period of revolution T = 1 year. Suppose that during the revolution of the Earth, at any instant, the mass of the Sun instantaneously becomes double i.e. 2M. The correct alternative (s) is/are [Assume that the Sun and the Earth are point masses].

(a) The period of revolution of the Earth around the Sun now becomes $\frac{1}{\sqrt{2}}$ year.

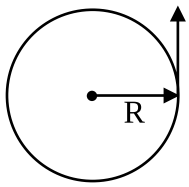
(b) There is no change in angular momentum of the Earth around the Sun.

(c) Minimum distance of the Earth from Sun during revolution is now $\frac{r}{3}$.

(d) Maximum speed of the Earth during revolution around the Sun is $3\sqrt{\frac{GM}{r}}$.

Ans. (b,c,d)

Sol.



$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$

$$T' = 2\pi\sqrt{\frac{a^3}{G \times 2M}}$$

R = a (1 + e)

$$E = \frac{1}{2} m \times v^2 - \frac{GMm}{R}$$

$$-\frac{2GMm}{2a} = \frac{1}{2} m \times \frac{GM}{R} - \frac{2GMm}{R}$$

$$\frac{1}{a} = \frac{3}{2}$$

$$a = \frac{2}{3} R$$

R = a(1 + e)

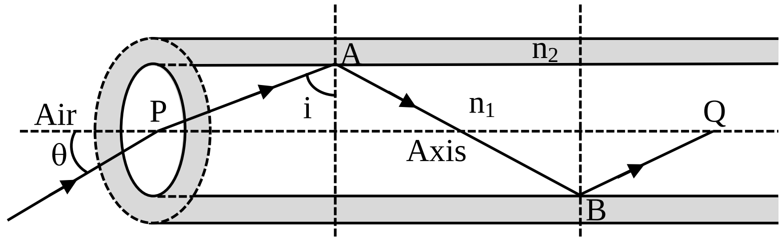
$$= \frac{2R}{3}(1 + e)$$

$$e = \frac{1}{2}$$

R_{min} = a (1 – e) = $\frac{2R}{3} \times \frac{1}{2} = \frac{R}{3}$

$$v_{max} = \sqrt{\frac{G \times 2M}{\frac{2R}{3}} \frac{\left(1 + \frac{1}{2}\right)}{(1 - e)}} = \sqrt{\frac{2GM \times \frac{3}{2}}{\frac{2R}{3}\frac{1}{2}}} = 3\sqrt{\frac{GM}{R}}$$

55. An optical fibre consists of a glass core (refractive index n_1) surrounded by a cladding (refractive index $n_2 < n_1$). A beam of light enters one end of the fibre at P, from air at angle θ, with the axis of fibre as shown in the figure. Choose the correct option(s) :-



(a) Maximum value of θ for which a ray can travel down the fibre is $\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$.

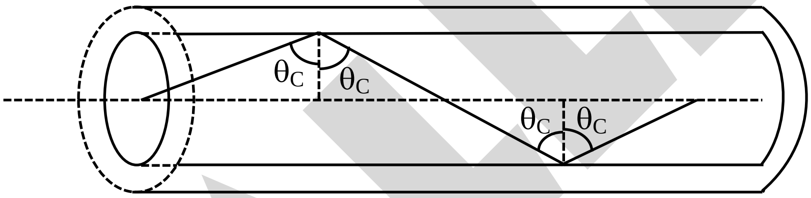
(b) Maximum value of θ for which a ray can travel down the fibre is $\theta = \cos^{-1}\sqrt{n_1^2 - n_2^2}$.

(c) If θ = 30° (in air) and $n_1 = 1.50$, then for reflection just at the critical angle, the value of n_2 is $\sqrt{2}$.

(d) A ray entering at θ = 0 travels a distance L from P to Q, inside the fibre directly along the fibre axis. Another ray travelling through fibre is repeatedly reflected at the critical angle, at interface of glass core and surrounding layer/cladding. Both the rays travel from point P to point Q on the axis. If the two rays started from point P at the same time, the difference Δt in the time taken to reach the point Q by the two rays is $\Delta t = \frac{n_1}{n_2} \times \frac{L}{c}\left(n_1 - n_2\right)$ (here c is the speed of ligth in vaccum)

Ans. (a,c,d)

Sol.



$v = \frac{c}{n_1} = \frac{L}{t_1} \Rightarrow t_1 = \frac{n_1 L}{c}$

$n_1 \sin i = n_2 \sin 90°$

$\sin i = \frac{n_2}{n_1}$

$1 \sin\theta = n_1 \sin(90 - i)$

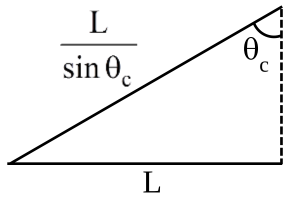
$= n_1\sqrt{1 - \frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$

$\frac{1}{2} = \sqrt{\frac{9}{4} - n_2^2}$

$\frac{1}{4} = \frac{9}{4} - n_2^2$

$n_2 = \sqrt{2}$

(d)



$$\frac{c}{n_1} = \frac{L}{\sin\theta_c t_2}$$

$$t_2 = \frac{n_1^2 L}{c \times n_2}$$

$$\Delta t = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right)$$

56. A series LCR circuit fed with AC has resonant angular frequency of $\omega = 2.0 \times 10^4$ rad/s. When the same circuit is driven at an angular frequency of 2.5×10^4 rad/s, it has an impedance of 1.0 kΩ and phase constant of $\phi = 45°$. The value of L, C and R for this circuit may be :-

(a) $R = 707\ \Omega$ (b) $L = 78.6$ mH (c) $C = 31.8$ nf (d) $R = 506\ \Omega$

Ans. (a,b,c)

Sol. $\omega_0 = 2 \times 10^4 = \frac{1}{\sqrt{LC}}$

$\omega > \omega_{res} \Rightarrow$ Inductive

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} = +1$$

$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$$

$$= \sqrt{2}R = 10^3$$

$$R = \frac{1000}{\sqrt{2}} = 707\Omega$$

$$\left(\omega L - \frac{1}{\omega C}\right) = +\frac{1000}{\sqrt{2}}$$

$$LC = \frac{1}{4 \times 10^8}$$

$$\frac{2.5 \times 10^4}{4 \times 10^8 C} - \frac{1}{2.5 \times 10^4 C} = \frac{1000}{\sqrt{2}}$$

$$C = \frac{\sqrt{2}}{1000}\left[\frac{5}{8} - \frac{2}{5}\right] \times 10^{-4}$$

$$= \sqrt{2}\left[\frac{4}{40}\right] \times 10^{-8} = 31.8\text{nF}$$

$$L = \frac{10^9}{4 \times 10^8 \times 31.8} = 78.61\text{mH}$$

57. A long straight wire, having a radius greater than 4.0 mm, carries a current that is uniformly distributed over its cross-section. The magnitude of magnetic field B due to the current is 0.28 mT at r = 4.0 mm and 0.20 mT at r = 10.0 mm, respectively, from the axis of the wire then the :-

(a) magnitude of the magnetic field B at a distance r = 2.0 mm from axis is 0.14 mT.

(b) magnitude of the magnetic field B at r = 5.0 mm is greater than that at r = 6.0 mm.

(c) current flowing in the wire is I = 20 A.

(d) current density in the wire is $J \approx 1.1 \times 10^5$ A/m^2.

Ans. (a,b,d)

Sol. $B = \mu_0 \frac{Jx}{2}$

$$0.28\times10^{-3} = 4\pi\times10^{-7}\times J\times\frac{4\times10^{-3}}{2}$$

$$J = \frac{0.28}{8\pi}\times10^{7} = 1.11\times10^{5}\,A/m^{2}$$

if r > 10 mm

$$0.2 \times 10^{-3} = 4\pi\times10^{-6}\times\frac{0.28}{8\pi}\times10^{3}\times\frac{10^{-2}}{2}$$

does not match

$\Rightarrow$ r < 10 mm

$$0.2 \times 10^{-3} = \frac{\mu_0 i}{2\pi\times0.01} = \frac{2\times10^{-7}\times i}{10^{-2}}$$

i = 10 A

$$\frac{0.28\times10^{7}}{8\pi} = \frac{10}{\pi x^{2}}$$

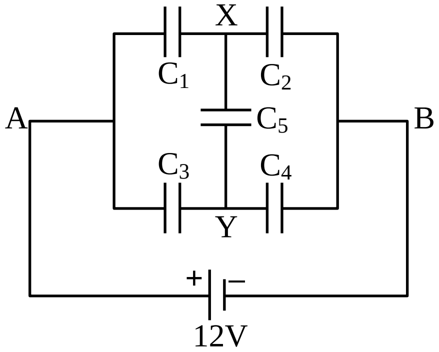
$$x^{2} = \frac{8\times10^{-6}}{0.28}$$

x = 5.34 mm

$$B_5 = \frac{\mu_0\times1.11\times10^{5}\times5\times10^{-3}}{2} = 277.5\mu_0$$

$$B_6 = \frac{\mu_0\times10}{2\pi\times6\times10^{-3}} = 265.25\mu_0$$

58. In the given combination of capacitors $C_1 = 4\mu F$, $C_2 = 2\ \mu F$, $C_3 = 2\ \mu F$, $C_4 = 4\ \mu F$ and $C_5 = 3\ \mu F$, a source of 12 volt is connected across the points A and B. Then the :-



(a) equivalent capacity between A and B is $\frac{8}{3}\mu F$.

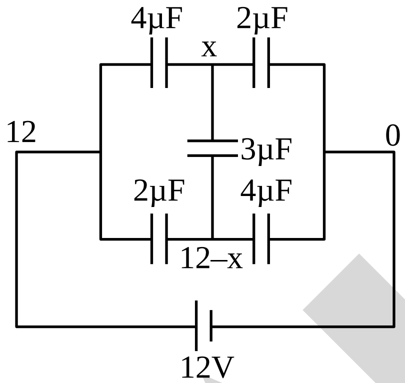
(b) stored electrical energy of the system is 204 μJ.

(c) potential difference between X and Y is $V_{xy} = 2$ volt.

(d) potential difference across the capacitor C_2 is 8 volt.

Ans. (b,c)

Sol.



$4(x - 12) + 2(x - 0) + 3(2x - 12) = 0$

$12x = 84$

$x = 7$

$Q = 20 + 14 = C_{eq} \times 12$

$C_{eq} = \frac{34}{12} = \frac{17}{6}$

$U = \frac{1}{2} \times \frac{17}{6} \times 12^2 = 204$

59. One mole of an ideal monoatomic gas, initially at temperature T, is compressed to $\frac{1}{8}$ of its voluem by a piston in a cylinder such that the heat dissipated into the environment is equal to the change in the internal energy of the gas. Then the

(a) molar heat capacity of the gas is $C = \frac{3R}{2}$.

(b) final temperature of the gas is 2T.

(c) work done on the gas is 3RT.

(d) equation of state of the process is $PV^{4/3}$ = constant.

Ans. (b,c,d)

Sol. **dQ = +dU =** $+\frac{3}{2}nRdT = nCdT$

$\Rightarrow$ **C =** $+\frac{3}{2}R$

but it is not isochoric

$\Rightarrow$ dQ = – dU $\Rightarrow$ C = $-\frac{3}{2}R$

$$-\frac{3R}{2} = C = \frac{3}{2}R - \frac{R}{m-1}$$

$$3R = \frac{R}{m-1}$$

m = $\frac{4}{3}$

$$TV^{1/3} = T_0V_0^{1/3}$$

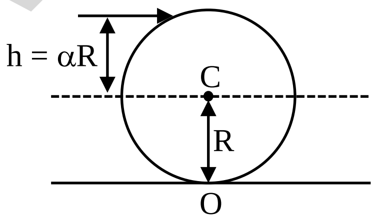
$$T\left(\frac{V_0}{8}\right)^{1/3} = T_0V_0^{1/3}$$

T = $2T_0$

W = Q – ΔU

$$= -\frac{3}{2}nR\times T - \frac{3}{2}nR\times T = -3nRT$$

60. A solid ball of mass M and radius R (see figure) is lying on a horizontal table. The ball experiences a short horizontal impulse which imparts a momentum p to the ball. The height of point of impact above the center line is h = αR ($0 \le \alpha \le 1$). Choose the correct option/option(s).



(a) Translational energy of the ball is $\frac{p^2}{2M}$.

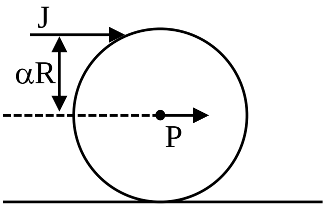
(b) Energy of pure rotational motion of the ball is $\frac{5}{4}\frac{p^2\alpha^2}{M}$.

(c) For $\alpha = \frac{2}{5}$, the ball rolls without sliding.

(d) For α = 0, pure rotational motion will be observed.

Ans. (a,b,c)

Sol.



$$K = \frac{p^2}{2m}$$

$$\alpha R \times \phi = \frac{2}{5} mR^2 \times \omega$$

$$\omega = \frac{\alpha p R}{I}$$

$$K = \frac{1}{2} \times \frac{2}{5} mR^2 \times \frac{\alpha^2 p^2 R^2}{\frac{4}{25} m^2 R^4}$$

$$= \frac{5}{4} \frac{\alpha^2 p^2}{m}$$

$$\omega = \frac{p}{mR} = \frac{\alpha p R}{\frac{2}{5} mR^2} \Rightarrow \alpha = \frac{2}{5}$$