ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
path to success

JEE (MAIN)
JANUARY & SEPTEMBER 2020

E

TOPICWISE TEST PAPERS

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

Chapter Contents 01

JEE (MAIN) TOPICWISE TEST PAPERS JANUARY & SEPTEMBER 2020

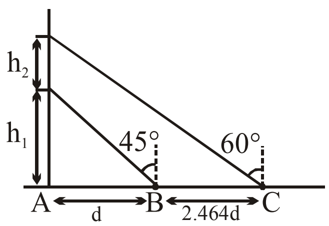
PHYSICS

JANUARY & SEPTEMBER 2020 ATTEMPT (PHYSICS)

BASIC MATHS & VECTOR

1. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, ____ .

2. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30° = 0.5774) :

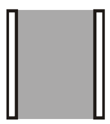


(1) d (2) 0.732d

(3) 1.464d (4) 0.464d

CAPACITOR

1. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as k(x) = K(1 + αx) where 'x' is the distance measured from one of the plates. If (αd) <<1, the total capacitance of the system is best given by the expression :



(1) $\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$ (2) $\frac{A\varepsilon_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

(3) $\frac{A\varepsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$ (4) $\frac{AK\varepsilon_0}{d}(1+\alpha d)$

2. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capactior is parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) ____.

3. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be :

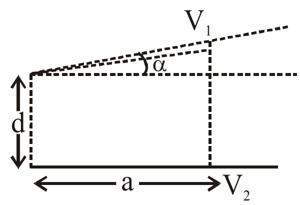
(1) 3.2 μF

(2) 8.4 μF

(3) 1.6 μF

(4) 4.2 μF

4. A capacitor is made of two square plates each of side 'a' making a very small angle α between them, as shown in figure. The capacitance will be close to :



(1) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$ (2) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$

(3) $\frac{\varepsilon_0 a^2}{d}\left(1+\frac{\alpha a}{d}\right)$ (4) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$

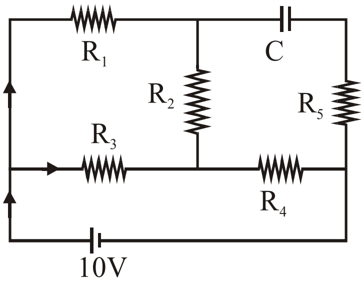
5. A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$J then value of X to the nearest integer is________.

6. A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is:

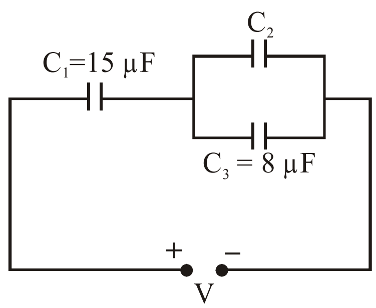
(1) 10 μF (2) 15 μF

(3) 20 μF (4) 30 μF

7. An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is 2 Ω. The potential difference (in V) across the capacitor when it is fully charged is _______.



8. In the circuit shown in the figure, the total charge in 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is :



(1) 590 μC (2) 450 μC

(3) 650 μC (4) 160 μC

9. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is :

(1) $\frac{1}{6}CV_0^2$ (2) $\frac{1}{2}CV_0^2$

(3) $\frac{1}{3}CV_0^2$ (4) $\frac{1}{4}CV_0^2$

10. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is:

(1) $\frac{9}{2}CV^2$

(2) $\frac{25}{6}CV^2$

(3) zero

(4) $\frac{3}{2}CV^2$

11. A parallel plate capacitor has plate of length 'l', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant k = 4 is being inserted between the plates of the capacitor. At what length of the slab inside plates, will be energy stored in the capacitor be two times the initial energy stored?

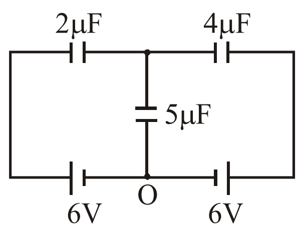
(1) l / 4

(2) l / 2

(3) l / 3

(4) 2l / 3

12. In the circuit shown, charge on the 5 μF capacitor is :



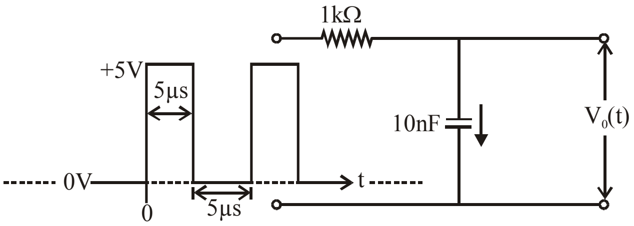
(1) 5.45 μC

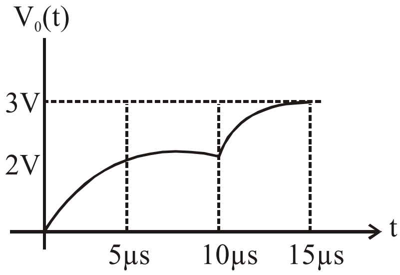
(2) 16.36 μC

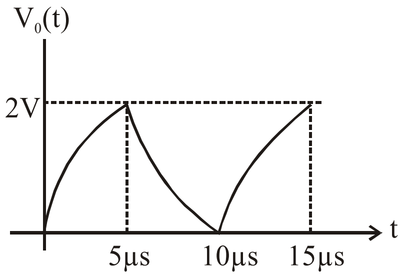
(3) 10.90 μC

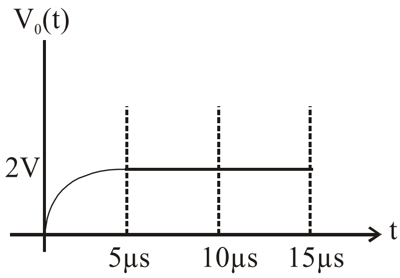
(4) 18.00 μC

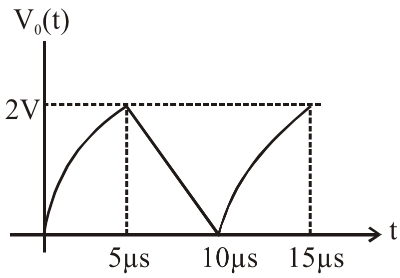
13. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_D(t)$, across the capacitor is correctly depicted by:



(1)


(2)


(3)


(4)


CIRCULAR MOTION

1. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take g = 10 ms^{-2} at the north pole and the radius of the earth = 6400 km):

(1) 195.66 N (2) 194.66 N
(3) 194.32 N (4) 195.32 N

2. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length ℓ. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is :

(1) $\frac{m\ell\omega^2}{k+m\omega^2}$ (2) $\frac{m\ell\omega^2}{k-m\omega^2}$

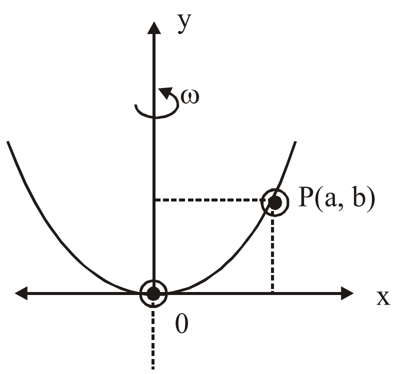
(3) $\frac{m\ell\omega^2}{k-\omega m}$ (4) $\frac{m\ell\omega^2}{k+m\omega}$

3. A spring mass system (mass m, spring constant k and natural length l) rest in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, ($k >> m\omega^2$) the relative change in the length of the spring is best given by the option :

(1) $\frac{2m\omega^2}{k}$ (2) $\frac{m\omega^2}{3k}$

(3) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ (4) $\frac{m\omega^2}{k}$

4. A bead of mass m stays at point P(a, b) on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) :



(1) $\sqrt{\frac{2gC}{ab}}$ (2) $2\sqrt{2gC}$

(3) $\sqrt{\frac{2g}{C}}$ (4) $2\sqrt{gC}$

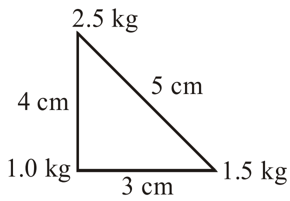
5. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of :

(1) 10^{-3} (2) 10^{-2}

(3) 10^{-4} (4) 10^{-1}

CENTRE OF MASS & COLLISION

1. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:



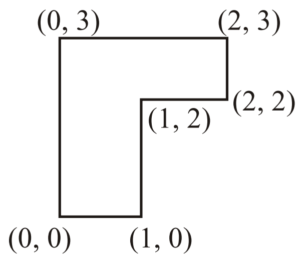
(1) 1.5 cm right and 1.2 cm above 1 kg mass

(2) 0.9 cm right and 2.0 cm above 1 kg mass

(3) 0.6 cm right and 2.0 cm above 1 kg mass

(4) 2.0 cm right and 0.9 cm above 1 kg mass

2. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are :

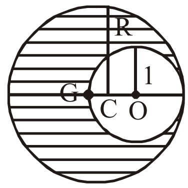


(1) (1.25m, 1.50m) (2) (1m, 1.75m)

(3) (0.75m, 0.75m) (4) (0.75m, 1.75m)

3. A body A, of mass m = 0.1 kg has an initial velocity of $3\hat{i}\,ms^{-1}$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}\,ms^{-1}$. After collision, A moves with a velocity $\vec{v}=4(\hat{i}+\hat{j})$. The energy of B after collision is written as $\frac{x}{10}J$. The value of x is ___________.

4. As shown in figure, when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e, on the surface of the cavity. R can be detemined by the equation :



(1) $(R^2 - R + 1)(2 - R) = 1$

(2) $(R^2 + R - 1)(2 - R) = 1$

(3) $(R^2 + R + 1)(2 - R) = 1$

(4) $(R^2 - R - 1)(2 - R) = 1$

5. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is :

(1) $\frac{1}{2}$ (2) $\sqrt{\frac{1}{2}}$

(3) $\sqrt{\frac{3}{4}}$ (4) $\sqrt{\frac{3}{2}}$

6. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is :

(1) $\frac{3}{4}mu^2$ (2) $\frac{1}{8}mu^2$

(3) $\sqrt{\frac{2}{3}}mu^2$ (4) $\frac{1}{3}mu^2$

7. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at :

(1) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ (2) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

(3) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ (4) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

8. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is:

(1) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$ (2) $2\sqrt{2}\frac{u^2}{g}$

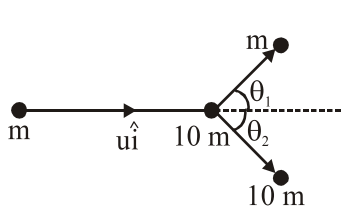
(3) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (4) $\frac{5}{8}\frac{u^2}{g}$

9. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3m at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by :

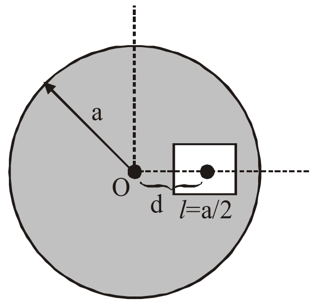
(1) $v = \sqrt{\frac{2}{3}}u$ (2) $v = \frac{1}{\sqrt{6}}u$

(3) $v = \frac{u}{\sqrt{3}}$ (4) $v = \frac{u}{\sqrt{2}}$

10. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$ then value of n is _______.



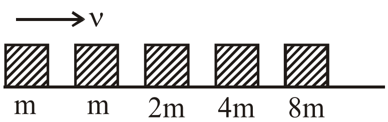
11. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is _____.



12. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take $g = 10\ m/s^2$. Assume there is no rotational motion and loss of energy after the collision is negligable.]

(1) 21 J (2) 23 J (3) 19 J (4) 20 J

13. Blocks of masses m, 2m, 4m and 8m are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8m starts moving the total energy loss is p% of the original energy. Value of 'p' is close to :



(1) 77 (2) 37 (3) 87 (4) 94

14. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where v(t) is its instantaneous velocity. The instantaneous acceleration of the satellite is:

(1) $-\frac{2bv^3}{M(t)}$ (2) $-\frac{bv^3}{2M(t)}$

(3) $-bv^3(t)$ (4) $-\frac{bv^3}{M(t)}$

15. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is.

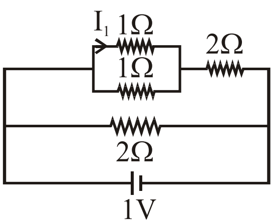
16. Particle A of mass m_1 moving with velocity $(\sqrt{3}\hat{i} + \hat{j})ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V}_1$ and $\vec{V}_2$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V}_1 = (\hat{i} + \sqrt{3}\hat{j})ms^{-1}$, the angle between $\vec{V}_1$ and $\vec{V}_2$ is :

(1) 60° (2) 15° (3) –45° (4) 105°

17. The centre of mass of a solid hemisphere of radius 8 cm is X cm from the centre of the flat surface. Then value of x is ________ .

CURRENT ELECTRICITY

1. The current I_1 (in A) flowing through 1 Ω resistor in the following circuit is :



(1) 0.5 (2) 0.2 (3) 0.25 (4) 0.4

2. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be:

(1) 10 A (2) 25 A

(3) 15 A (4) 20 A

3. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10 Ω is connected in parallel to the cell, the balancing length changes by 60cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer then value of N is____ .

4. The length of a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5V and internal resistance of 20Ω, the null point on it is found to be a 1000cm. The resistance of whole wire is :

(1) 120Ω (2) 60Ω

(3) 80Ω (4) 100Ω

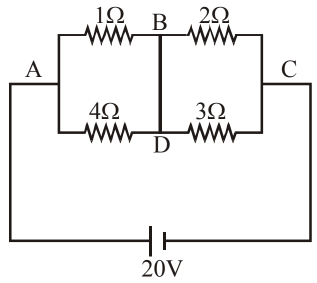
5. Four resistances of 15Ω, 12Ω, 4Ω and 10Ω respectively in cyclic order to form Wheatstone's network. The resistance that is to be connected in parallel with the resistance of 10Ω to balance the network is ________ Ω.

6. A galvanometer having a coil resistance 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V?

(1) 9.9 kΩ (2) 8.9 kΩ (3) 7.9 kΩ (4) 10 kΩ

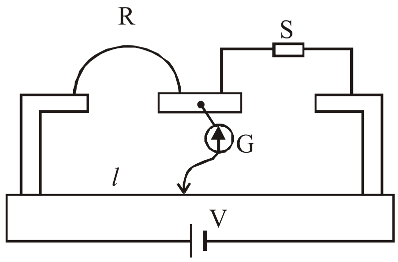
7. The series combination of two batteries, both of the same emf 10 V, but different internal resistance of 20Ω and 5Ω, is connected to the parallel combination of two resistors 30 Ω and R Ω. The voltage difference across the battery of internal resistance 20Ω is zero, the value of R (in Ω) is : _______

8. In the given circuit diagram, a wire is joining points B and D. The current in this wire is :



(1) 4A (2) 2A (3) 0.4A (4) Zero

9. In a meter bridge experiment S is a standard resistance. R is a resistance wire. It Is found that balancing length is l = 25 cm. If R is replaced by a wire of half length and half diameter that of R of same material, then the balancing distance l' (in cm) will now be________.

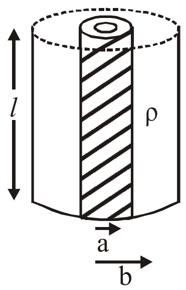


10. Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity $\rho_C > \rho_T > \rho_M$ and ρ_A respectively. Then:

(1) $\rho_A > \rho_T > \rho_C$ (2) $\rho_C > \rho_A > \rho_T$

(3) $\rho_A > \rho_M > \rho_C$ (4) $\rho_M > \rho_A > \rho_C$

11. Model a torch battery of length l to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance of value R, the maximum Joule heating in R will take place for:-



(1) $R = \frac{2\rho}{\pi l} ln\left(\frac{b}{a}\right)$ (2) $R = \frac{\rho}{\pi l} ln\left(\frac{b}{a}\right)$

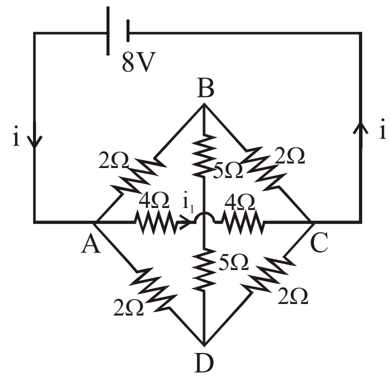
(3) $R = \frac{\rho}{2\pi l}\left(\frac{b}{a}\right)$ (4) $R = \frac{\rho}{2\pi l} ln\left(\frac{b}{a}\right)$

12. A battery of 3.0 V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5 V, the power dissipated within the internal resistance is :

(1) 0.50 W (2) 0.125 W

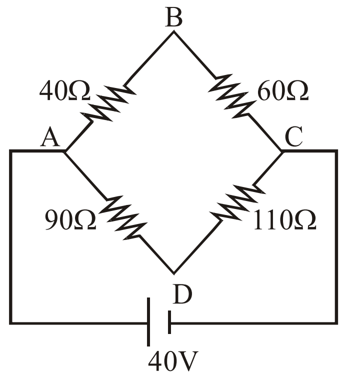
(3) 0.072 W (4) 0.10 W

13. The value of current i_1 flowing from A to C in the circuit diagram is :



(1) 5A (2) 2A (3) 4A (4) 1A

14.



Four resistances 40Ω, 60Ω, 90Ω and 110Ω make the arms of a quadrilateral ABCD. Across AC is a battery of emf 40V and internal resistance negligible. The potential difference across BD is V is _______.

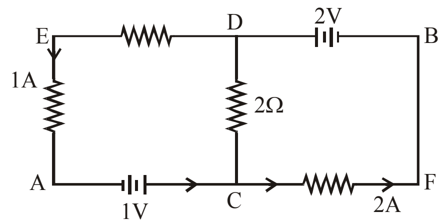
15. An electrical power line, having a total resistance of 2Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately:

(1) 72% (2) 96% (3) 91% (4) 85%

16. A galvanometer of resistance G is converted into a voltmeter of range 0 – 1V by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to 0 – 2V will be :

(1) R_1 (2) $R_1 + G$
(3) $R_1 - G$ (4) G

17. In the circuit, given in the figure currents in different branches and value of one resistor are shown. Then potential at point B with respect to the point A is :



(1) +1V (2) – 1V (3) – 2V (4) +2V

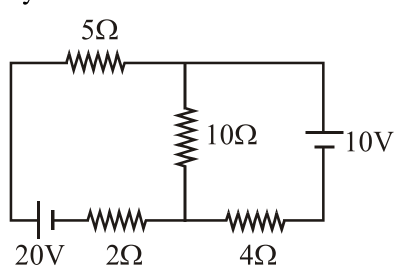
18. A galvanometer is used in laboratory for detecting the null point in electrical experiments. If, on passing a current of 6mA it produces a deflection of 2°, its figure of merit is close to :

(1) 3×10^{-3} A/div. (2) 333° A/div.
(3) 6×10^{-3} A/div. (4) 666° A/div.

19. A circuit to verify Ohm's law uses ammeter and voltmeter in series or parallel connected correctly to the resistor. In the circuit :

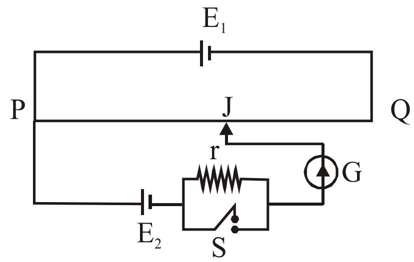
(1) ammeter is always connected series and voltmeter in parallel.
(2) Both, ammeter and voltmeter mast be connected in series.
(3) Both ammeter and voltmeter must be connected in parallel.
(4) ammeter is always used in parallel and voltmeter is series.

20. In the figure shown, the current in the 10 V battery is close to :



(1) 0.36 A from negative to positive terminal.
(2) 0.71 A from positive to negative terminal.
(3) 0.21 A from positive to negative terminal.
(4) 0.42 A from positive to negative terminal.

21. A potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is :



(1) 0.02 V/cm (2) 0.04 V/cm
(3) 0.01 V/cm (4) 0.03 V/cm

ELASTICITY

1. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (Breaking stress of wire = 4.8×10^7 Nm^{-2} and area of cross-section of the wire = 10^{-2} cm^2) is:

2. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is:

(1) $1:\sqrt{2}$ (2) 1 : 2
(3) 2 : 1 (4) $\sqrt{2}:1$

3. In a Young's double slit experiment 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then the value of λ is (in nm) ______.

4. A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to :

(Given bulk modulus of metal, $B = 8 \times 10^{10}$ Pa)

(1) 0.6 (2) 1.67
(3) 5 (4) 20

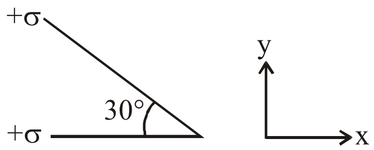
5. An object of mass m is suspended at the end of a massless wire of length L and area of cross-section, A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is:

(1) $f=\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ (2) $f=\frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

(3) $f=\frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ (4) $f=\frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$

ELECTROSTATICS

1. Two infinite planes each with uniform surface charge density + σ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by:



(1) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

(2) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

(3) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

(4) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

2. In finding the electric field using Gauss Law the formula $|\vec{E}|=\frac{q_{enc}}{\varepsilon_0|A|}$ is applicable. In the formula ε_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charge enclosed by the Gaussian surface. The equation can be used in which of the following situation?

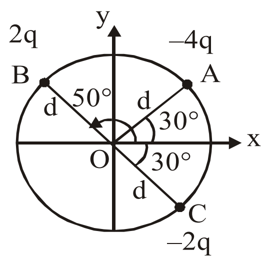
(1) Only when the Gaussian surface is an equipotential surface.

(2) Only when $|\vec{E}|$ = constant on the surface.

(3) For any choice of Gaussian surface.

(4) Only when the Gaussian surface is an equipotential surface and $|\vec{E}|$ is constant on the surface.

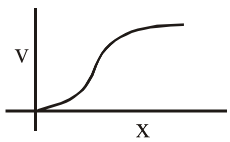
3. Three charged particle A, B and C with charges –4q, 2q and –2q are present on the circumference of a circle of radius d. the charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is :

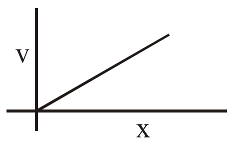


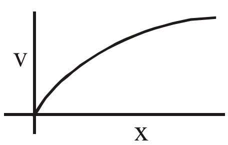
(1) $\frac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$ (2) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

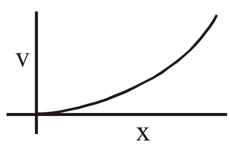
(3) $\frac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ (4) $\frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$

4. A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale)

(1) 

(2) 

(3) 

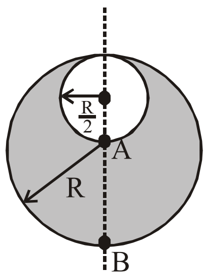
(4) 

5. Consider two charged metallic spheres S_1 and S_2 of radii R_1 and R_2, respectively. The electric fields E_1 (on S_1) and E_2 (on S_2) on their surfaces are such that $E_1/E_2 = R_1/R_2$. Then the ratio V_1 (on S_1) / V_2 (on S_2) of the electrostatic potentials on each sphere is :

(1) (R_2/R_1) (2) $\left(\frac{R_1}{R_2}\right)^3$

(3) R_1/R_2 (4) $(R_1/R_2)^2$

6. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown, the ratio $\frac{|\vec{E}_A|}{|\vec{E}_B|}$ of magnitude of electric field $\vec{E}_A$ and $\vec{E}_B$, respectively, at points A and B due to the remaining portion is :



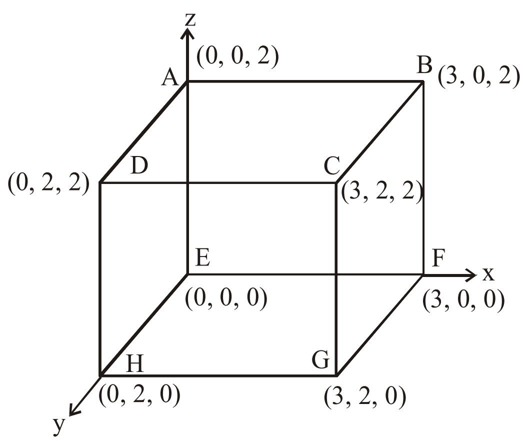
(1) $\frac{18}{54}$ (2) $\frac{21}{34}$ (3) $\frac{17}{54}$ (4) $\frac{18}{34}$

7. An electric dipole of moment $\vec{p} = (-\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29}$ C .m is at the origin (0, 0, 0). The electric field due to this dipole at $\vec{r} = +\hat{i} + 3\hat{j} + 5\hat{k}$ (note that $\vec{r}.\vec{p} = 0$) is parallel to :

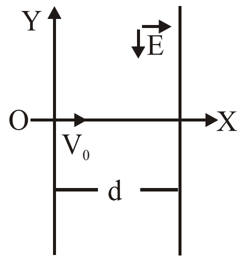
(1) $(-\hat{i} + 3\hat{j} - 2\hat{k})$ (2) $(+\hat{i} - 3\hat{j} - 2\hat{k})$

(3) $(+\hat{i} + 3\hat{j} - 2\hat{k})$ (4) $(-\hat{i} - 3\hat{j} + 2\hat{k})$

8. An electric field $\vec{E} = 4x\hat{i} - (y^2 + 1)\hat{j}$ N/C passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as ϕ_I and ϕ_{II} respectively. The difference between $(\phi_I - \phi_{II})$ is (in Nm²/C) ___________.



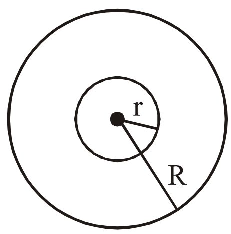
9. A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d. Equation of path of electron in the region x > d is :



(1) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$ (2) $y = \frac{qEd}{mV_0^2}(x - d)$

(3) $y = \frac{qEd}{mV_0^2}x$ (4) $y = \frac{qEd^2}{mV_0^2}x$

10. A charge Q is distributed over two concentric conducting thin spherical shells radii r and R (R > r). If the surface charge densities on the two shells are equal, the electric potential at the common centre is :



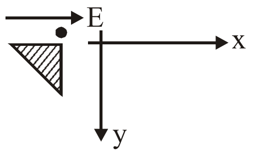
(1) $\frac{1}{4\pi\varepsilon_0}\frac{(R + 2r)Q}{2(R^2 + r^2)}$

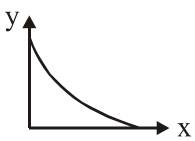
(2) $\frac{1}{4\pi\varepsilon_0}\frac{(R + r)}{2(R^2 + r^2)}Q$

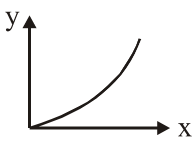
(3) $\frac{1}{4\pi\varepsilon_0}\frac{(R + r)}{(R^2 + r^2)}Q$

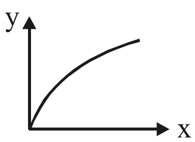
(4) $\frac{1}{4\pi\varepsilon_0}\frac{(2R + r)}{(R^2 + r^2)}Q$

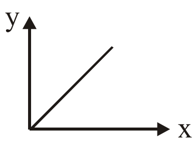
11. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass ? (Curves are drawn schematically and are not to scale).



(1) 

(2) 

(3) 

(4) 

12. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and –3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively :

(1) 6 μC and 3 μC (2) +4.5 μC and –4.5 μC
(3) 3 μC and 6 μC (4) 4.5 μC on both

13. Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V(R) – V(4R) is:

(1) $\frac{3Q_1}{16\pi\varepsilon_0 R}$

(2) $\frac{Q_2}{4\pi\varepsilon_0 R}$

(3) $\frac{3Q_1}{4\pi\varepsilon_0 R}$

(4) $\frac{3Q_2}{4\pi\varepsilon_0 R}$

14. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed?

(1) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.

(2) Multimeter shows a deflection,accompanied by a splash of light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED.

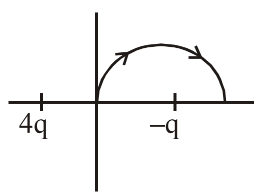
(3) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen compoenent is metal wire.

(4) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor.

15. Two resistors 400Ω and 800Ω are connected in series across a 6 V battery. The potential difference measured by a voltmeter of 10 kΩ across 400 Ω resistor is close to:

(1) 2 V (2) 1.95V (3) 2.05 V (4) 1.8 V

16. A two point charges 4q and –q are fixed on the x-axis at $x = -\frac{d}{2}$ and $x = \frac{d}{2}$, respectively. If a third point charge 'q' is taken from the origin to x = d along the semicircle as shown in the figure, the energy of the charge will :



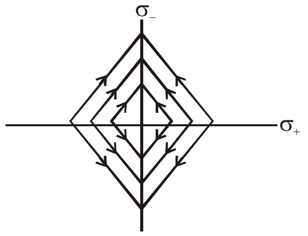
(1) increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$

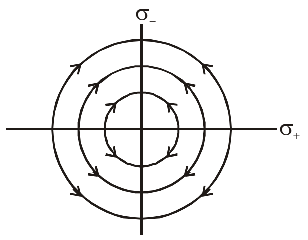
(2) increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$

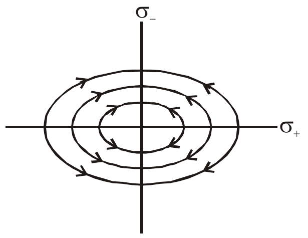
(3) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$

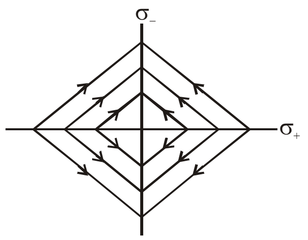
(4) decrease by $\frac{q^2}{4\pi\varepsilon_0 d}$

17. Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_- where $|\sigma_+| > |\sigma_-|$ intersect at right angle. Which of the following best represents the electric field lines for this system :

(1) 

(2) 

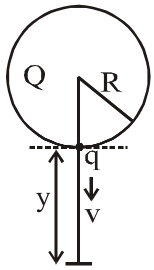
(3) 

(4) 

18. A particle of charge q and mass m is subjected to an electric field $E = E_0 (1 - ax^2)$ in the x-direction, where a and E_0 are constants. Initially the particle was at rest at x = 0. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is :

(1) $\sqrt{\frac{2}{a}}$ (2) $\sqrt{\frac{1}{a}}$ (3) a (4) $\sqrt{\frac{3}{a}}$

19. A solid sphere of radius R carries a charge (Q + q) distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then : (assume the remaining portion to be spherical).



(1) $v^2 = 2y\left[\frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g\right]$

(2) $v^2 = y\left[\frac{qQ}{4\pi\epsilon_0 R^2 ym} + g\right]$

(3) $v^2 = 2y\left[\frac{qQR}{4\pi\epsilon_0 (R+y)^3 m} + g\right]$

(4) $v^2 = y\left[\frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g\right]$

20. Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge (+q) each, while 2, 4, 6, 8, 10 have charge (–q) each. The potential V and the electric field E at the centre of the circle are respectively:
(Take V = 0 at infinity)

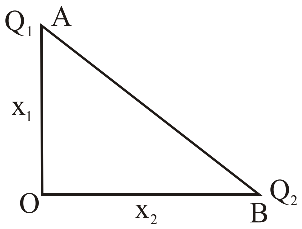
(1) $V = \frac{10q}{4\pi\epsilon_0 R}; E = \frac{10q}{4\pi\epsilon_0 R^2}$

(2) $V = 0, E = \frac{10q}{4\pi\epsilon_0 R^2}$

(3) V = 0, E = 0

(4) $V = \frac{10q}{4\pi\epsilon_0 R}; E = 0$

21. Charges Q_1 and Q_2 arc at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to :



(1) $\frac{x_2^2}{x_1^2}$ (2) $\frac{x_1^3}{x_2^3}$ (3) $\frac{x_1}{x_2}$ (4) $\frac{x_2}{x_1}$

22. Consider the force F on a charge 'q' due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statements is true for F, if 'q' is placed at distance r from the centre of the shell ?

(1) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ for r > R

(2) $\frac{1}{4\pi\varepsilon_0}\frac{qQ}{R^2} > F > 0$ for r < R

(3) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ for all r

(4) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for r < R

23. Two identical electric point dipoles have dipole moments $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$ and are held on the x axis at distance 'a' from each other. When released, they move along the x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is'm', their speed when they arc infinitely far apart is:

(1) $\frac{p}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$ (2) $\frac{p}{a}\sqrt{\frac{3}{2\pi\varepsilon_0 ma}}$

(3) $\frac{p}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$ (4) $\frac{p}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$

EM WAVE

1. If the magnetic field in a plane electromagnetic wave is given by $\vec{B} = 3\times10^{-8}\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{j}\,T$, then what will be expression for electric field?

(1) $\vec{E} = \left(9\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{k}\ V/m\right)$

(2) $\vec{E} = \left(3\times10^{-8}\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{i}\ V/m\right)$

(3) $\vec{E} = \left(60\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{k}\ V/m\right)$

(4) $\vec{E} = \left(3\times10^{-8}\sin\left(1.6\times10^3 x + 48\times10^{10} t\right)\hat{j}\ V/m\right)$

2. The electric field of a plane electromagnetic wave is given by $\vec{E} = E_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ cos (kz + ωt)

At t = 0, a positively charged particle is at the point (x, y, z) = $\left(0, 0, \frac{\pi}{k}\right)$. If its instantaneous velocity at (t = 0) is $\upsilon_0\,\hat{k}$, the force acting on it due to the wave is :

(1) zero (2) parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(3) antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (4) parallel to $\hat{k}$

3. A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\vec{B} = 5\times10^{-8}\hat{j}\,T$. The corresponding electric field $\vec{E}$ is (speed of light c = 3 × 10^8 ms^{-1})

(1) $1.66\times10^{-16}\ \hat{i}\ V/m$

(2) $15\ \hat{i}\ V/m$

(3) $-1.66\times10^{-16}\ \hat{i}\ V/m$

(4) $-15\ \hat{i}\ V/m$

4. The electric fields of two plane electromagnetic plane waves in vacuum are given by

$\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx)$ and

$\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$

At t = 0, a particle of charge q is at origin with a velocity $\vec{v} = 0.8c\hat{j}$ (c is the speed of light in vacuum). The instantaneous force experienced by the particle is :

5. A plane electromagnetic wave, has frequency of 2.0×10^{10} Hz and its energy density is 1.02×10^{-8} J/ m^3 in vacuum. The amplitude of the magnetic field of the wave is close to ($\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{Nm^2}{C^2}$ and speed of light $= 3 \times 10^8$ ms^{-1}) :

(1) 180 nT (2) 160 nT

(3) 150 nT (4) 190 nT

6. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i} - 2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave.

(1) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (2) $\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$

(3) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$ (4) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$

7. The magnetic field of a plane electromagnetic wave is

$\vec{B} = 3\times10^{-8}\sin[200\pi(y+ct)]\hat{i}$ T

Where $c = 3 \times 10^8$ ms^{-1} is the speed of light. The corresponding electric field is :

(1) $\vec{E} = -10^{-6}\sin[200\pi(y+ct)]\hat{k}$ V/m

(2) $\vec{E} = -9\sin[200\pi(y+ct)]\hat{k}$ V/m

(3) $\vec{E} = 9\sin[200\pi(y+ct)]\hat{k}$ V/m

(4) $\vec{E} = 3\times10^{-8}\sin[200\pi(y+ct)]\hat{k}$ V/m

8. The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is $\vec{E} = E_0\hat{j}\cos(\omega t - kx)$. The magnetic field $\vec{B}$, at the moment t = 0 is :

(1) $\vec{B} = E_0\sqrt{\mu_0 \epsilon_0}\cos(kx)\hat{j}$

(2) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}}\cos(kx)\hat{k}$

(3) $\vec{B} = E_0\sqrt{\mu_0 \epsilon_0}\cos(kx)\hat{k}$

(4) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}}\cos(kx)\hat{j}$

9. Choose the correct option relating wavelengths of differnet parts of electromagnetic wave spectrum :

(1) $\lambda_{\text{x-rays}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$

(2) $\lambda_{\text{visible}} > \lambda_{\text{x-rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{micro waves}}$

(3) $\lambda_{\text{radio waves}} > \lambda_{\text{micro waves}} > \lambda_{\text{visible}} > \lambda_{\text{x-rays}}$

(4) $\lambda_{\text{visible}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{x-rays}}$

10. The electric field of a plane electromagnetic wave is given by

$\vec{E} = E_0(\hat{x}+\hat{y})\sin(kz-\omega t)$

Its magnetic field will be given by :

(1) $\frac{E_0}{c}(\hat{x}-\hat{y})\cos(kz-\omega t)$

(2) $\frac{E_0}{c}(-\hat{x}+\hat{y})\sin(kz-\omega t)$

(3) $\frac{E_0}{c}(\hat{x}-\hat{y})\sin(kz-\omega t)$

(4) $\frac{E_0}{c}(\hat{x}+\hat{y})\sin(kz-\omega t)$

11. An electron is constrained to move along the y-axis with a speed of 0.1 c (c is the speed of light) in the presence of electromagnetic wave, whose electric field is

$\vec{E} = 30\hat{j}\ \sin(1.5\times10^7 t - 5\times10^{-2}x)$V/m.

The maximum magnetic force experienced by the electron will be :

(given $c = 3 \times 10^8$ ms^{-1} and electron charge $= 1.6 \times 10^{-19}$ C)

(1) 1.6×10^{-19} N (2) 4.8×10^{-19} N

(3) 3.2×10^{-18} N (4) 2.4×10^{-18} N

12. The correct match between the entries in column I and column II are :

I **Radiation**	II **Wavelength**
(a) Microwave	(i) 100m
(b) Gamma rays	(ii) 10^{-15} m
(c) A.M. radio waves	(iii) 10^{-10} m
(d) X-rays	(iv) 10^{-3} m

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(2) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
(3) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)
(4) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

13. Suppose that intensity of a laser is $\left(\frac{315}{\pi}\right)W/m^2$. The rms electric field, in units of V/m associated with this source is close to the nearest integer is
($\in_0 = 8.86 \times 10^{-12}\ C^2\ Nm^{-2}$; $c = 3 \times 10^8\ ms^{-1}$)

14. For a plane electromagnetic wave, the magnetic field at a point x and time t is

$$\vec{B}(x,t) = \left[1.2\times10^{-7}\sin(0.5\times10^3x+1.5\times10^{11}t)\hat{k}\right]T$$

The instantaneous electric field $\vec{E}$ corresponding to $\vec{B}$ is : (speed of light $c = 3 \times 10^8\ ms^{-1}$)

(1) $\vec{E}(x,t) = \left[36\sin(0.5\times10^3x+1.5\times10^{11}t)\hat{k}\right]\frac{V}{m}$

(2) $\vec{E}(x,t) = \left[-36\sin(0.5\times10^3x+1.5\times10^{11}t)\hat{j}\right]\frac{V}{m}$

(3) $\vec{E}(x,t) = \left[36\sin(1\times10^3x+0.5\times10^{11}t)\hat{j}\right]\frac{V}{m}$

(4) $\vec{E}(x,t) = \left[36\sin(1\times10^3x+1.5\times10^{11}t)\hat{j}\right]\frac{V}{m}$

EMI & AC

1. A long solenoid of radius R carries a time (t)-dependent current $I(t) = I_0t(1-t)$. A ring of radius 2R is placed coaxially near its middle. During the time interval $0 \le t \le 1$, the induced current (I_R) and the induced EMF(V_R) in the ring change as :
(1) At t = 0.5 direction of I_R reverses and V_R is zero
(2) Direction of I_R remains unchanged and V_R is zero at t = 0.25
(3) Direction of I_R remains unchanged and V_R is maximum at t = 0.5
(4) At t = 0.25 direction of I_R reverses and V_R is maximum

2. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be:

(1) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

(2) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

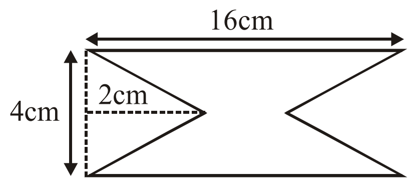
(3) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(4) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

3. An emf of 20 V is applied at time t=0 to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time t = ∞ and at t = 40 s is close to : (Take $e^2 = 7.389$)
(1) 1.06 (2) 1.15
(3) 1.46 (4) 0.84

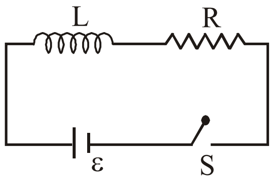
4. A planar loop of wire rotates in a uniform magnetic field. Initially, at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at :
(1) 2.5 s and 7.5 s (2) 5.0 s and 7.5 s
(3) 5.0 s and 10.0 s (4) 2.5 s and 5.0 s

5. At time t = 0 magnetic field of 100 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5s, then induced EMF in the loop is :



(1) 36 μV (2) 48 μV
(3) 56 μV (4) 28 μV

6. A shown in the figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at t = 0. The total charge that flows from the battery, between t = 0 and $t = t_c$ (t_c is the time constant of the circuit) is :



(1) $\frac{\varepsilon L}{R^2}\left(1-\frac{1}{e}\right)$ (2) $\frac{\varepsilon R}{eL^2}$

(3) $\frac{\varepsilon L}{R^2}$ (4) $\frac{\varepsilon L}{eR^2}$

7. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be __________ .

8. In LC circuit the inductance L = 40 mH and capacitance C = 100 μF. If a voltage V(t) = 10sin(314 t) is applied to the circuit, the current in the circuit is given as :

(1) 0.52 cos 314 t (2) 0.52 sin 314 t

(3) 10 cos 314 t (4) 5.2 cos 314 t

9. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2s. The maximum value of EMF induced (in μV) in the coil will be close to the integer ________.

10. An inductance coil has a reactance of 100 Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is :

(1) 1.1×10^{-2} H (2) 1.1×10^{-1} H

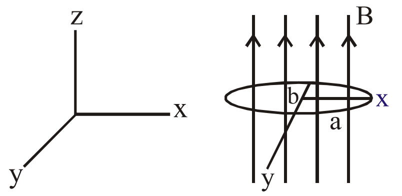
(3) 5.5×10^{-5} H (4) 6.7×10^{-7} H

11. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity 2J/°C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to :

(1) 418 s (2) 245 s

(3) 348 s (4) 365 s

12. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to Joule heating is :



(1) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$

(2) Zero

(3) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$

(4) $\frac{\pi a b B \omega}{R}$

13. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate dB/dt = 0.032 Ts^{-1}.The induced current in the loop is close to

(Resistivity of the metal wire is 1.23×10^{-8} Ωm)

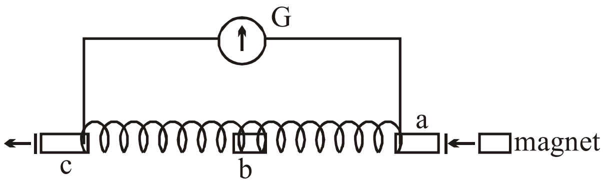
(1) 0.61 A

(2) 0.34 A

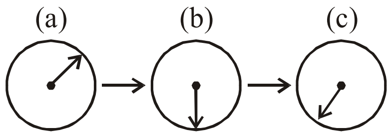
(3) 0.43 A

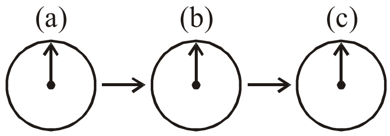
(4) 0.53 A

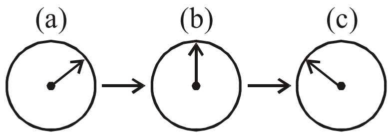
14. A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations wil be seen on the galvanometer G attached across the coil ?

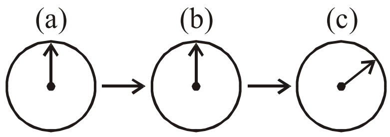


Three positions shown describe : (a) the magnet's entry (b) magnet is completely inside and (c) magnet's exit.

(1)


(2)


(3)


(4)


15. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at t = 0, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is :

(1) $\frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$ (2) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$

(3) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$ (4) $\frac{L}{R}\ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$

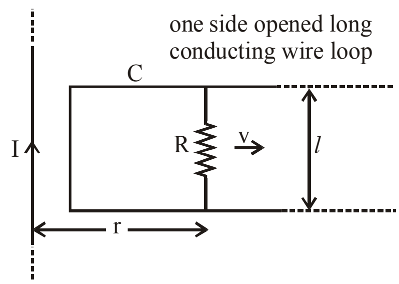
16. A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 Am^2. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4T is applied along the vertical, it starts rotating around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be :

(1) 10 rad s^{-1} (2) 20 π rad s^{-1}

(3) 10 π rad s^{-1} (4) 20 rad s^{-1}

17. Two concentric circular coils, C_1 and C_2, are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3)$ A where t is in s. The emf induced in C_1 (in mV), at the instant t = 1s is $\frac{4}{x}$. The value of x is ____.

18. An infinitely long straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length l and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight wire is :



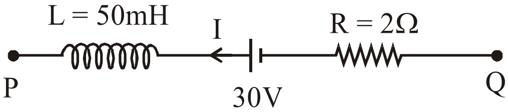
(1) $\frac{\mu_0}{\pi}\frac{Ivl}{Rr}$ (2) $\frac{\mu_0}{2\pi}\frac{Ivl}{Rr}$

(3) $\frac{2\mu_0}{\pi}\frac{Ivl}{Rr}$ (4) $\frac{\mu_0}{4\pi}\frac{Ivl}{Rr}$

19. An AC circuit has R = 100 Ω, C = 2 μF and L = 80 mH, connected in series. The quality factor of the circuit is :

(1) 0.5 (2) 2 (3) 20 (4) 400

20. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at a rate of 10^2A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is.



21. In a scries LR circuit, power of 400 W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value of C as $\left(\frac{n}{3\pi}\right)\mu F$, then value of n is ________ .

ERROR & MEASUREMENT

1. A simple pendulum is being used to determine th value of gravitational acceleration g at a certain place. Th length of the pendulum is 25.0 cm and a stop watch with 1s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is :

(1) 3.40% (2) 5.40% (3) 4.40% (4) 2.40%

2. If the screw on a screw-gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is :

(1) 0.001 mm (2) 0.001 cm

(3) 0.02 mm (4) 0.01 cm

3. For the four sets of three measured physical quantities as given below. Which of the following options is correct ?

(i) $A_1 = 24.36$, $B_1 = 0.0724$, $C_1 = 256.2$

(ii) $A_2 = 24.44$, $B_2 = 16.082$, $C_2 = 240.2$

(iii) $A_3 = 25.2$, $B_3 = 19.2812$, $C_3 = 236.183$

(iv) $A_4 = 25$, $B_4 = 236.191$, $C_4 = 19.5$

(1) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2$

(2) $A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2 < A_4 + B_4 + C_4$

(3) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3 = A_4 + B_4 + C_4$

(4) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$

4. The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4th VSD coincides with a main scale division. The length of the cylinder is : (VSD is vernier scale division)

(1) 3.21 cm (2) 2.99 cm

(3) 3.2 cm (4) 3.07 cm

5. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as :

(1) 2.123 cm (2) 2.125 cm

(3) 2.121 cm (4) 2.124 cm

6. A physical quantity z depends on four observables a, b, c and d, as $z = \frac{a^2 b^{\frac{2}{3}}}{\sqrt{c}\, d^3}$. The percentage of error in the measurement of a, b, c and d 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is:

(1) 12.25% (2) 14.5%

(3) 16.5% (4) 13.5%

7. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved, and the least count of the screw gauge, are respectively :

(1) Negative, 2 μm (2) Positive, 10 μm

(3) Positive, 0.1 μm (4) Positive, 0.1 mm

8. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)\%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is .

9. A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5.50 mm, 5.55 mm, 5.45 mm; 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter of the pencil should therefore be recorded as:

(1) (5.5375 ± 0.0739) mm
(2) (5.538 ± 0.074) mm
(3) (5.54 ± 0.07) mm
(4) (5.5375 ± 0.0740) mm

FLUIDS

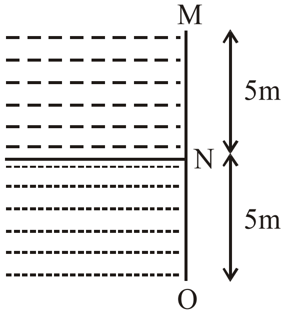
1. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{3}{4}$ (3) $\frac{81}{256}$ (4) $\frac{9}{16}$

2. Consider a solid sphere of radius R and mass density $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$, $0<r\le R$. The minimum density of a liquid in which it will float is :

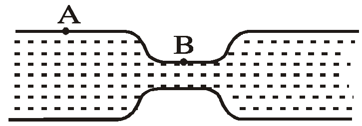
(1) $\frac{\rho_0}{5}$ (2) $\frac{\rho_0}{3}$ (3) $\frac{2\rho_0}{3}$ (4) $\frac{2\rho_0}{5}$

3. Two liquids of densities ρ_1 an ρ_2 ($\rho_2 = 2\rho_1$) are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing)



(1) 1/4 (2) 2/3 (3) 1/3 (4) 1/2

4. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube. (density of water = 1000 kgm^{-3})



(Fig.)

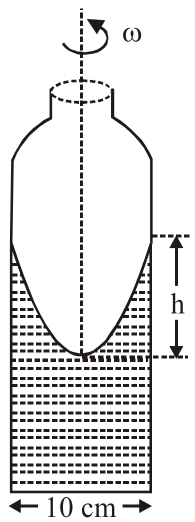
(1) 1810 cm^3/s (2) 3020 cm^3/s
(3) 2720 cm^3/s (4) 2420 cm^3/s

5. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet) :

(1) $r=\sqrt{\frac{2T}{3(d+\rho)g}}$ (2) $r=\sqrt{\frac{3T}{(2d-\rho)g}}$

(3) $r=\sqrt{\frac{T}{(d-\rho)g}}$ (4) $r=\sqrt{\frac{T}{(d+\rho)g}}$

6. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height, h(in cm) of liquid at the centre of vessel and at the side will be:



(1) $\frac{25\omega^2}{2g}$ (2) $\frac{2\omega^2}{5g}$ (3) $\frac{5\omega^2}{2g}$ (4) $\frac{2\omega^2}{25g}$

7. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to (g = 10 ms^{-2}).

(1) 0.137 m (2) 0.172 m
(3) 0.087 m (4) 0.049 m

8. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :

(1) 8 : 1 (2) 0.8 : 1 (3) 2 : 1 (4) 4 : 1

9. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in milliNewton m^{-1}, is [$\rho_{(liquid)}$ = 900 kgm^{-3}, g = 10 ms^{-2}] (Give answer in closest integer)________.

10. A air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is (g = 980 cm/s^2)

(1) 3.15 gm (2) 4.51 gm
(3) 4.15 gm (4) 1.52 gm

11. Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is :

(1) $gdS\ (x_2 + x_1)^2$ (2) $\frac{3}{4}gdS(x_2 - x_1)^2$

(3) $\frac{1}{4}gdS(x_2 - x_1)^2$ (4) $gdS(x_2^2 + x_1^2)$

12. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t. water, the value of r is :

(1) $\frac{4}{9}R$ (2) $\frac{8}{9}R$ (3) $\frac{1}{3}R$ (4) $\frac{2}{3}R$

13. In an experiment to verify Stokes law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to :
(ignore viscosity of air)

(1) r (2) r^4 (3) r^3 (4) r^2

14. A fluid is flowing through a horizontal pipe of varying cross-section, with speed v ms^{-1} at a point where the pressure is P Pascal. P At another point where pressure is $\frac{P}{2}$ Pascal its speed is V ms^{-1}. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to :

(1) $\sqrt{\frac{P}{2\rho} + v^2}$ (2) $\sqrt{\frac{P}{\rho} + v^2}$

(3) $\sqrt{\frac{2P}{\rho} + v^2}$ (4) $\sqrt{\frac{P}{\rho} + v}$

GEOMETRICAL OPTICS

1. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to :

(1) 22 mm (2) 12 mm
(3) 33 mm (4) 2 mm

2. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_1, then the ratio f_1/f is closest to the integer :

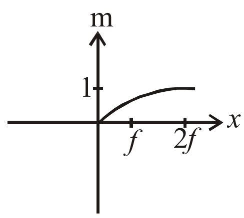
(1) 1 (2) 5 (3) 9 (4) 17

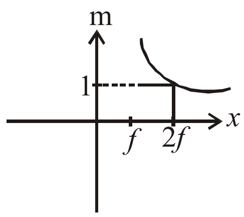
3. The magnifying power of a telescope with tube 60 cm is 5. What is the focal length of its eye piece ?
(1) 30 cm (2) 40 cm (3) 20 cm (4) 10 cm

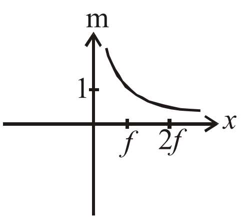
4. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be :
(1) 60° (2) 15° (3) 45° (4) 30°

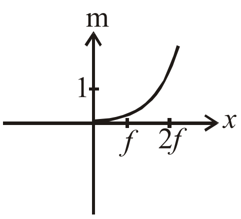
5. A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is ----.

6. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by :
(Graphs are drawn schematically and are not to scale)

(1) 

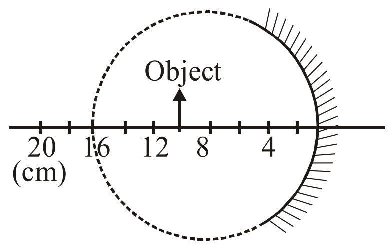
(2) 

(3) 

(4) 

7. A vessel of depth 2h is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be :
(1) $\frac{h}{\sqrt{2}}$ (2) $\frac{3}{4}h\sqrt{2}$ (3) $\frac{h}{2(\sqrt{2}+1)}$ (4) $\frac{h}{3\sqrt{2}}$

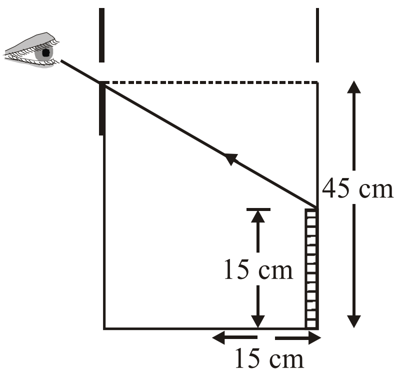
8. There is a small source of light at some depth below the surface of water (refractive index = $\frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly) :
[Use the fact that surface area of a spherical cap of height h and radius of curvature r is $2\pi rh$]:
(1) 17% (2) 21%
(3) 34% (4) 50%

9. 

A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the mirror, what will be the nature and magnification of the image of the object ? (Figure drawn as schematic and not to scale)
(1) Inverted, real and magnified
(2) Erect, virtual and magnified
(3) Erect, virtual and unmagnified
(4) Inverted, real and unmagnified

10. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is______.

11. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid N/100, where N is an integer, the value of N is___________.

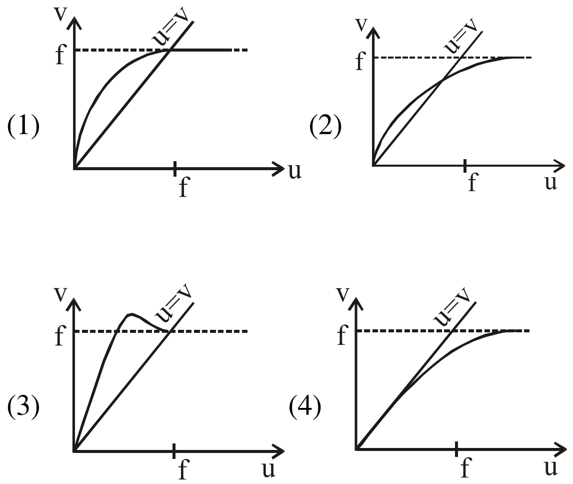


12. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at that instant is _______.

13. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is ________.

14. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)$D where N is an integer, the value of N is ______.

15. For a concave lens of focal length f, the relation between object and image distance u and v, respectively, from its pole can best be represented by (u = v is the reference line):



16. A compound microscope consists of an objective lens of focal length 1cm and an eye piece of focal length 5 cm with a separation of 10 cm.
The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is ____.

17. A prism of angle A = 1° has a refractive index μ = 1.5. A good estimate for the minimum angle of deviation (in degrees) is close to N/10. Value of N is ______.

18. A point like object is placed at a distance of 1m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is :
(1) 1 m from the mirror, virtual
(2) 1 m from the mirror, real
(3) 2.6 m from the mirror, real
(4) 2.6 m from the mirror, virtual

19. A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power 1.5 P is:
(1) $\frac{R}{2}$ (2) 2R (3) $\frac{3R}{2}$ (4) $\frac{R}{3}$

GRAVITATION

1. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth):

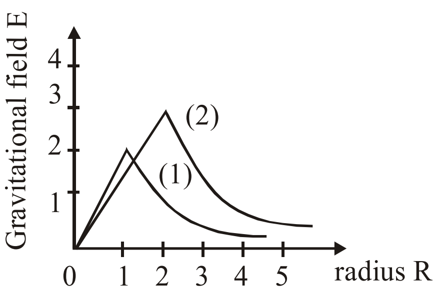
(1) $\frac{m}{20}\left(u-\sqrt{\frac{2GM}{3R}}\right)^2$

(2) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$

(3) $\frac{3m}{8}\left(u+\sqrt{\frac{5GM}{6R}}\right)^2$

(4) $\frac{m}{20}\left(u^2+\frac{113}{200}\frac{GM}{R}\right)$

2. Consider two solid spheres of radii R_1 = 1m, R_2 = 2m and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is :



(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{1}{6}$

3. An asteroid is moving directly towards the centre of the earth. When at a distance of 10R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s) ? Give your answer to the nearest integer in kilometer/s _____.

4. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body :

(1) starts moving in an elliptical orbit around the planet.

(2) continues to move in a circular orbit

(3) Falls vertically downwards towards the planet

(4) Escapes from the Planet's Gravitational field.

5. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are ν_A and ν_B, respectively, then $\frac{\nu_A}{\nu_B}=\frac{n}{4}$. The value of n is :

(1) 4 (2) 1 (3) 2 (4) 3

6. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as :

(1) $T \propto R$ (2) $T^2 \propto \frac{1}{R^3}$

(3) $T^2 \propto R$ (4) $T^2 \propto R^3$

7. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected):

(1) $\frac{\sqrt{5}R-R}{2}$ (2) $\frac{\sqrt{5}}{2}R-R$

(3) $\frac{R}{2}$ (4) $\frac{\sqrt{3}R-R}{2}$

8. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that is become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R, value of R is :

(1) $4R_e$ (2) $3R_e$ (3) $2R_e$ (4) $2.5R_e$

9. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at:

(1) $r = \frac{1}{\sqrt{3}}R$ (2) $r = \sqrt{\frac{5}{9}}R$

(3) $r = \sqrt{\frac{3}{4}}R$ (4) $r = R$

10. On the x-axis and a dsitance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is :

(1) $\frac{A}{(x^2+a^2)^{1/2}}$

(2) $\frac{A}{(x^2+a^2)^{3/2}}$

(3) $A(x^2 + a^2)^{3/2}$

(4) $A(x^2 + a^2)^{1/2}$

11. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is :

(1) 1 (2) 2 (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

12. The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals :

(1) $\frac{7}{9}$ (2) $\frac{4}{9}$ (3) $\frac{1}{3}$ (4) $\frac{5}{9}$

13. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is : (h<<R, where R is the radius of the earth)

(1) $\frac{R^2\omega^2}{8g}$ (2) $\frac{R^2\omega^2}{4g}$

(3) $\frac{R^2\omega^2}{g}$ (4) $\frac{R^2\omega^2}{2g}$

14. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is :

(1) 1 : 6 (2) 3 : 4 (3) 1 : 3 (4) 1 : 2

15. Two planets have masses M and 16 M and their radii are a and 2a, respectively. The separation between the centres of the planets is 10a. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is :

(1) $\sqrt{\frac{GM^2}{ma}}$ (2) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

(3) $4\sqrt{\frac{GM}{a}}$ (4) $2\sqrt{\frac{GM}{a}}$

HEAT & THERMODYNAMICS

1. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If $\gamma = 1.40$, the work done by air is : ($3^{1.4} = 4.6555$) [Take air to be an ideal gas]

(1) 90.5 J (2) 48 J

(3) 60.7 J (4) 100.8 J

2. Two moles of an ideal gas with $\frac{C_P}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_P}{C_V} = \frac{4}{3}$. The value of $\frac{C_P}{C_V}$ for the mixture is:

(1) 1.50 (2) 1.42

(3) 1.45 (4) 1.47

3. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is____.

4. A non-isotropic solid metal cube has coefficients of linear expansion as : $5 \times 10^{-5}/°C$ along the x-axis and $5 \times 10^{-6}/°C$ along the y and the z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-16}/°C$ then the value of C is ______.

5. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work ?

(1) $T = \frac{2T_1T_2}{T_1 + T_2}$ (2) $T = \sqrt{T_1T_2}$

(3) $T = \frac{T_1 + T_2}{2}$ (4) T = 0

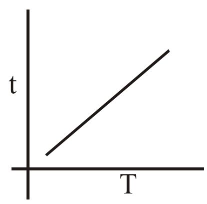
6. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_p}{C_v} = \gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by :

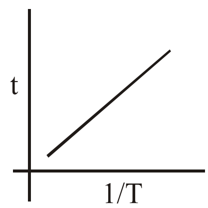
(1) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$ (2) 2

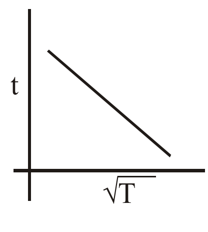
(3) $\frac{1}{2}$ (4) $\left(\frac{1}{2}\right)^{\gamma}$

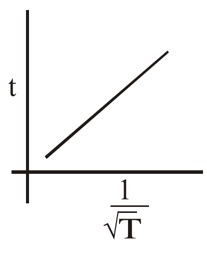
7. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/ g and heat of fusion of ice is 80 cal/g], the value of M is______.

8. The plot that depicts the behavior of the mean free time t (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale)

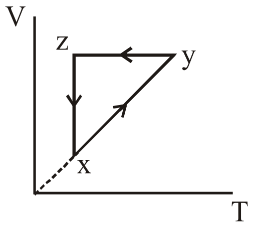
(1)


(2)


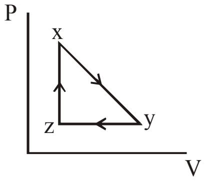
(3)


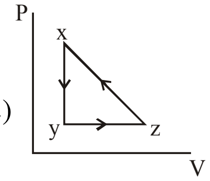
(4)


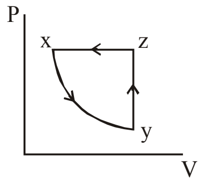
9. A thermodynamic cycle xyzx is shown on a V-T diagram.

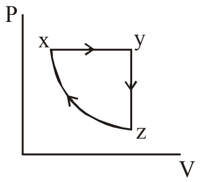


The P-V diagram that best describes this cycle is : (Diagrams are schematic and not to scale)

(1) 

(2) 

(3) 

(4) 

10. A leak proof cylinder of length 1m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to :

(1) 1.01 (2) 1.04 (3) 1.03 (4) 1.26

11. A carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is :

(1) 99 J (2) 100 J (3) 90 J (4) 1 J

12. Consider a mixture of n moles of helium gas and 2 n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_P/C_V value will be :

(1) 67/45 (2) 19/13 (3) 23/15 (4) 40/27

13. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in litres) are taken from each containers and mixed (assume no loss of heat during the process)

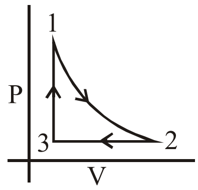
C_1	C_2	C_3	T
$1l$	$2l$	–	60°C
–	$1l$	$2l$	30°C
$2l$	–	$1l$	60°C
$1l$	$1l$	$1l$	θ

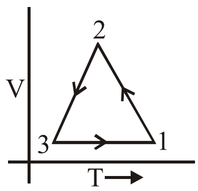
The value of θ (in °C to the nearest integer) is ....

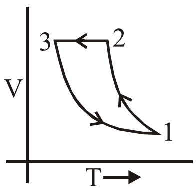
14. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats (C_V^A and C_V^B) of gas A and B, respectively is :

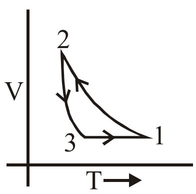
(1) 7 : 9 (2) 5 : 7 (3) 3 : 5 (4) 5 : 9

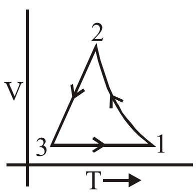
15. Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure ? where, 1 → 2 is adiabatic. (Graphs are schematic and are not to scale)



(1) 

(2) 

(3) 

(4) 

16. Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The raito of their respective mean free times is closest to :

(1) 3.67 (2) 4.67 (3) 1.83 (4) 2.3

17. Starling at temperature 300 K, one mole of an ideal diatomic gas ($\gamma = 1.4$) is first compressed adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) ________.

18. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is :

(1) 11 (2) 15 (3) 20 (4) 13

19. An engine takes in 5 moles of air at 20°C and 1 atm, and compresses it adiabaticaly to $1/10^{th}$ of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is ________.

20. A heat engine is involved with exchange of heat of 1915 J, –40 J, +125 J and QJ, during one cycle achieving an efficiency of 50.0%. The value of Q is:

(1) 640 J (2) 400 J (3) 980 J (4) 40 J

21. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true ?

(A) the mean free path of the molecules decreases.

(B) the mean collision time between the molecules decreases.

(C) the mean free path remains unchanged.

(D) the mean collision time remains unchanged.

(1) (C) and (D) (2) (A) and (B)

(3) (A) and (D) (4) (B) and (C)

22. When the temperature of a metal wire is increased from 0°C to 10°C, its length increases by 0.02%. The percentage change in its mass density will be closest to :

(1) 0.008 (2) 0.06

(3) 0.8 (4) 2.3

23. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as :

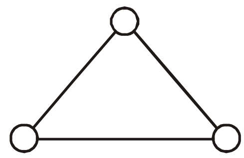
(1) Irreversible isothermal

(2) Irreversible adiabatic

(3) Reversible adiabatic

(4) Reversible isothermal

24.



Consider a gas of triatomic molecules. The molecules are assumed to the triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is :

(1) $\frac{9}{2}RT$ (2) $\frac{3}{2}RT$

(3) $\frac{5}{2}RT$ (4) 3RT

25. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}$ °C^{-1} and $\gamma_{(mercury)} = 1.5 \times 10^{-4}$ °C^{-1}, where γ is the coefficient of volume expansion, then V_m (in cc) is close to__________.

26. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal) ?

(1) 5 (2) 3 (3) 6 (4) 7

27. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to :

(1) 33°C (2) 35°C (3) 31°C (4) 28°C

28. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixure is 31°C. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal g^{-1} $°C^{-1}$)

(1) 2.6 (2) 2 (3) 4 (4) 3.2

29. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surrounding at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer?

30. Match the C_P/C_V ratio for ideal gases with different type of molecules :

Molecular type	C_P/C_V
(A) Monoatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non-rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

(1) A-IV, B-I, C-II, D-III
(2) A-IV, B-II, C-I, D-III
(3) A-III, B-IV, C-II, D-I
(4) A-II, B-III, C-I, D-IV

31. Dimensional formula for thermal conductivity is (here K denotes the temperature)

(1) $MLT^{-3}K$ (2) $MLT^{-2}K$
(3) $MLT^{-2}K^{-2}$ (4) $MLT^{-3}K^{-1}$

32. The specific heat of water = 4200 J kg^{-1} K^{-1} and the latent heat of ice = 3.4×10^5 J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams) :

(1) 61.7 (2) 63.8 (3) 69.3 (4) 64.6

33. A closed vessel contains 0.1 mole of a monoatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be closed to ______.

34. Match the thermodynamic processes taking place in a system with the correct conditions. In the table : ΔQ is the heat supplied, ΔW is the work done and ΔU is change in internal energy of the system :

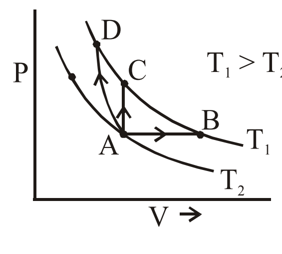
Process	Condition
(I) Adiabatic	(A) $\Delta W = 0$
(II) Isothermal	(B) $\Delta Q = 0$
(III) Isochoric	(C) $\Delta U \neq 0, \Delta W \neq 0, \Delta Q \neq 0$
(IV) Isobaric	(D) $\Delta U = 0$

(1) I-B, II-D, III-A, IV-C
(2) I-B, II-A, III-D, IV-C
(3) I-A, II-A, III-B, IV-C
(4) I-A, II-B, III-D, IV-D

35. The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm respectively. If $|\Delta T| = C|\Delta P|$ then value of C in (K/atm) is ______:

36. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as $A \to B$, $A \to C$ and $A \to D$. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the workdone as W_{AB}, W_{AC} and W_{AD}.

The correct relation between these parameters are :



(1) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$
(2) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(3) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$
(4) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$

37. A bullet of mass 5g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal/(g–°C) (1 cal = 4.2×10^7 ergs) close to :

(1) 83.3°C (2) 87.5°C

(3) 119.2°C (4) 38.4°C

38. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to ? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, g = 980 cm/s^2, density of mercury = 13.6 g/cm^3)

(1) 5.8×10^{18} (2) 5.8×10^{16}

(3) 4.0×10^{18} (4) 4.0×10^{16}

39. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is:

(1) 326 (2) $\frac{1}{32}$

(3) 32 (4) 128

40. Two different wires having lengths L_1 and L_2, and respective temperature coefficient of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is :

(1) $4\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\frac{L_2L_1}{(L_2+L_1)^2}$ (2) $2\sqrt{\alpha_1\alpha_2}$

(3) $\frac{\alpha_1+\alpha_2}{2}$ (4) $\frac{\alpha_1L_1+\alpha_2L_2}{L_1+L_2}$

41. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is _____.

(Molar mass of N_2 gas 28 g)

42. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_P}{C_v}\right)$ given, respectively, by:

(1) $U=\frac{5}{2}RT$ and $\gamma=\frac{6}{5}$

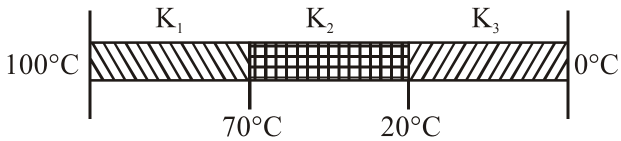
(2) U = 5RT and $\gamma=\frac{7}{5}$

(3) U = 5RT and $\gamma=\frac{6}{5}$

(4) $U=\frac{5}{2}RT$ and $\gamma=\frac{7}{5}$

43. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is.

44. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2, and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is :

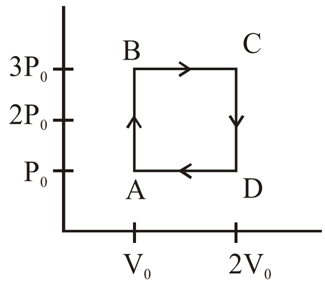


(1) $K_1 : K_3 = 2 : 3$; $K_2 : K_3 = 2 : 5$
(2) $K_1 < K_2 < K_3$
(3) $K_1 : K_2 = 5 : 2$; $K_1 : K_3 = 3 : 5$
(4) $K_1 > K_2 > K_3$

45. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T as :

(1) $\sqrt{T}$ (2) $\frac{1}{T}$ (3) $\frac{1}{\sqrt{T}}$ (4) T

46. An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to _____



KINEMATICS

1. A particle is moving along the x-axis with its coordinate with the time 't' given be $x(t) = 10 + 8t - 3t^2$. Another particle is moving the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is __________.

2. A particle moves such that its position vector $\vec{r}(t) = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle :

(1) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

(2) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$

(3) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

(4) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

3. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is _____

4. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is __________ .

5. A particle starts from the origin at t = 0 with an initial velocity of $3.0\,\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})\,m/s^2$. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is :-

(1) 50 (2) 32 (3) 60 (4) 40

6. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hr. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(1) 30.5 ms^{-1} (2) 29.5 ms^{-1}

(3) 31.5 ms^{-1} (4) 28.5 ms^{-1}

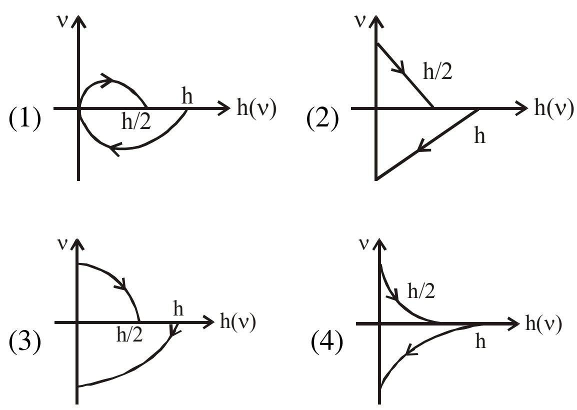
7. Starting from the origin at time t = 0, with initial velocity $5\hat{j}\,ms^{-1}$, a particle moves in the x-y plane with a constant acceleration of $(10\hat{i} + 4\hat{j})\,ms^{-2}$. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0, are respectively :

(1) 4s and 52 m (2) 2s and 24 m

(3) 2s and 18 m (4) 5s and 25 m

8. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by :

(graph are drawn schematically and on not to scale)

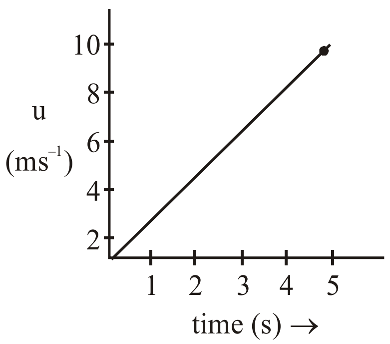


9. A small ball of mass is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2 where v is its speed. The maximum height attained by the ball is :

(1) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{g}$ (2) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$

(3) $\frac{1}{k}\tan^{-1}\frac{ku^2}{2g}$ (4) $\frac{1}{k}\ln\left(1+\frac{ku^2}{2g}\right)$

10. The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval t = 0 to t = 5s will be ______ :



11. A helicopter reises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] :

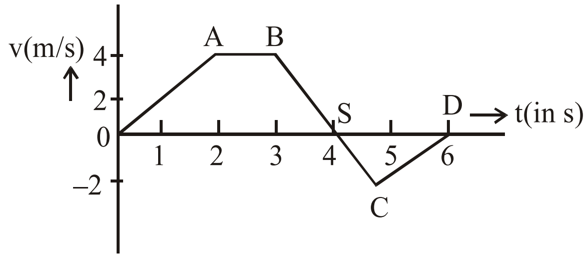
(1) $t=\sqrt{\frac{2h}{3g}}$

(2) $t=1.8\sqrt{\frac{h}{g}}$

(3) $t=3.4\sqrt{\left(\frac{h}{g}\right)}$

(4) $t=\frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

12. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6s is :



(1) 12m (2) $\frac{49}{4}$ m

(3) 11 m (4) $\frac{37}{3}$ m

13. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45°. The value of β is close to:

(1) 0.41 (2) 0.50

(3) 0.37 (4) 0.73

MAGNETISM

1. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct ?

(1) $\phi_i = -\phi_0$ (2) $\phi_i = \phi_0$

(3) $\phi_i < \phi_0$ (4) $\phi_i > \phi_0$

2. A loop ABCDEFA of straight edges has six corner points A(0,0,0), B(5,0,0), C(5,5,0), D(0, 5, 0), E(0, 5, 5) and F(0, 0, 5). The magnetic field in this region is $\vec{B}=\left(3\hat{i}+4\hat{k}\right)T$.

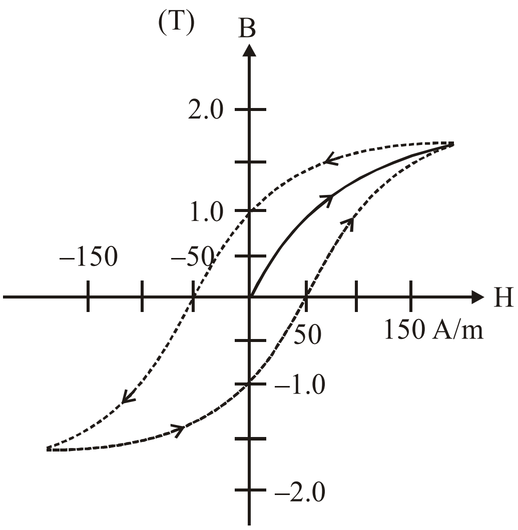
The quantity of flux through the loop ABCDEFA (in Wb) is ______ .

3. A particle of mass m and charge q has an initial velocity $\vec{\upsilon}=\upsilon_0\hat{j}$. If an electric field $\vec{E}=E_0\hat{i}$ and magnetic field $\vec{B}=B_0\hat{i}$ act on the particle, its speed will double after a time:

(1) $\frac{2m\upsilon_0}{qE_0}$ (2) $\frac{3m\upsilon_0}{qE_0}$

(3) $\frac{\sqrt{3}m\upsilon_0}{qE_0}$ (4) $\frac{\sqrt{2}m\upsilon_0}{qE_0}$

4. The figure gives experimentally measured B vs. H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are:



(1) 150 A/m, 1.0 T and 1.5 T

(2) 1.0 T, 50 A/m and 1.5 T

(3) 1.5 T, 50 A/m and 1.0 T

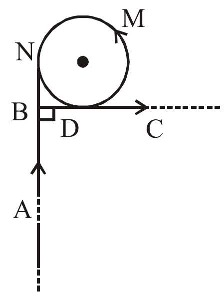
(4) 1.5 T, 50 A/m and 1.0 T

5. Photon with kinetic energy of 1MeV moves from south to north. It gets an acceleration of 10^{12} m/s^2 by an applied magnetic field (west to east). The value of magnetic field : (Rest mass of proton is 1.6×10^{-27} kg) :

(1) 71mT (2) 7.1mT

(3) 0.071mT (4) 0.71mT

6. A very long wire ABDMNDC is shown in figure carrying current I. AB and BC parts are straight, long and at right angle. At D wire forms a circular turn DMND of radius R. AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is :



(1) $\frac{\mu_0 I}{2R}$

(2) $\frac{\mu_0 I}{2\pi R}(\pi+1)$

(3) $\frac{\mu_0 I}{2\pi R}\left(\pi+\frac{1}{\sqrt{2}}\right)$

(4) $\frac{\mu_0 I}{2\pi R}\left(\pi-\frac{1}{\sqrt{2}}\right)$

7. A long, straight wire of radius a carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to the wire at distance $\frac{a}{3}$ and 2a, respectively from the axis of the wire is :

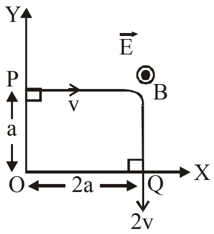
(1) $\frac{2}{3}$

(2) $\frac{3}{2}$

(3) $\frac{1}{2}$

(4) 2

8. A charged particle of mass 'm' and charge 'q' moving under the influence of uniform electric field $E\vec{i}$ and a uniform magnetic field $B\vec{k}$ follows a trajectory from point P to Q as shown in figure. The velocities at P and Q are respectively, $v\vec{i}$ and $-2v\vec{j}$. Then which of the following statements (A, B, C, D) are the correct ? (Trajectory shown is schematic and not to scale) :



(A) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

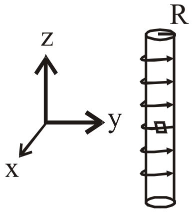
(B) Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

(C) Rate of work done by both the fields at Q is zero

(D) The difference between the magnitude of angular momentum of the particle at P and Q is 2 mav.

(1) (A), (B), (C), (D) (2) (A), (B), (C)
(3) (B), (C), (D) (4) (A), (C), (D)

9. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/length and carries a current I. The electron gun shoots an electron along the radius of the solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning) :



(1) $\frac{e\mu_0 nIR}{m}$ (2) $\frac{e\mu_0 nIR}{2m}$

(3) $\frac{2e\mu_0 nIR}{m}$ (4) $\frac{e\mu_0 nIR}{4m}$

10. A small circular loop of conducting wire has radius a and carries current I. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m then :

(1) $T = \sqrt{\frac{\pi m}{2IB}}$ (2) $T = \sqrt{\frac{2\pi m}{IB}}$

(3) $T = \sqrt{\frac{\pi m}{IB}}$ (4) $T = \sqrt{\frac{2m}{IB}}$

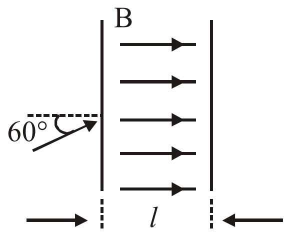
11. A beam of protons with speed 4×10^5 ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to: (Mass of the proton = 1.67×10^{-27} kg, charge of the proton = 1.69×10^{-19} C

(1) 12 cm (2) 4 cm (3) 5 cm (4) 2 cm

12. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required ?

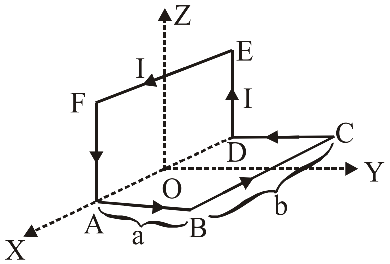
(1) T : Large retentivity, small coercivity
(2) P : Small retentivity, large coercivity
(3) T : Large retentivity, large coercivity
(4) P : Large retentivity, large coercivity

13. The figure shows a region of length 'l' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity 4×10^5 ms^{-1} making an angle 60° with the field. If the proton completes 10 revolution by the time it cross the region shown, 'l' is close to (mass of proton = 1.67×10^{-27} kg, charge of the proton = 1.6×10^{-19} C)



(1) 0.11 m (2) 0.22 m
(3) 0.44 m (4) 0.88 m

14. A wire carrying current I is bent in the shape ABCDEFA as shown, where rectangle ABCDA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop ABCDEFA is :



(1) $\sqrt{2}abI$, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

(2) $\sqrt{2}abI$, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$

(3) abI, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

(4) abI, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$

15. A charged particle carrying charge 1 μC is moving with velocity $\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ ms^{-1}. If an external magnetic field of $\left(5\hat{i}+3\hat{j}-6\hat{k}\right)\times 10^{-3}$ T exists in the region where the particle is moving then the force on the particle is $\vec{F}\times 10^{-9}$ N. The vector $\vec{F}$ is :

(1) $-0.30\hat{i}+0.32\hat{j}-0.09\hat{k}$

(2) $-300\hat{i}+320\hat{j}-90\hat{k}$

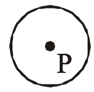
(3) $-30\hat{i}+32\hat{j}-9\hat{k}$

(4) $-3.0\hat{i}+3.2\hat{j}-0.9\hat{k}$

16. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is :

(1) $250\sqrt{3}$ (2) $5\sqrt{3}$ (3) $500\sqrt{3}$ (4) $50\sqrt{3}$

17. A perfectly dimagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is:



(1) Zero

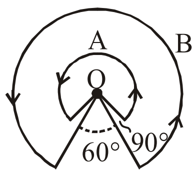
(2) $\vec{B}$

(3) much large than $|\vec{B}|$ but opposite to $\vec{B}$

(4) much large than $|\vec{B}|$ and parallel to $\vec{B}$

18. A galvanometer coil has 500 turns and each turn has an average area of 3×10^{-4} m^2. If a torque of 1.5 Nm is required to keep this coil parallel to magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is _______ .

19. A wire A, bent in the shape of an arc of a circle, carrying a current of 2A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is :



(1) 4 : 6 (2) 6 : 4 (3) 6 :5 (4) 2 : 5

20. A small bar magnet placed with its axis at 30° with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is :

(1) 9.2×10^{-3} J (2) 6.4×10^{-2} J
(3) 11.7×10^{-3} J (4) 7.2×10^{-2} J

21. A paramagnetic sample shows a net magnetisation of 6 A/m when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be :

(1) 4 A/m (2) 0.75 A/m
(3) 2.25 A/m (4) 1 A/m

22. A square loop of side 2a, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z-axis and passing through the point (0, b, 0), (b > > a). The magnitude of the torque on the loop about z-axis is given by:

(1) $\frac{2\mu_0 I^2 a^2}{\pi b}$ (2) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$

(3) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (4) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$

23. An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be :

(1) 0.5×10^2 Am^2 (2) 50×10^2 Am^2

(3) 500×10^2 Am^2 (4) 5×10^2 Am^2

24. A particle of charge q and mass m is moving with a velocity $-\upsilon\hat{i}(\upsilon \neq 0)$ towards a large screen placed in the Y-Z plane at a distance d. If there is a magnetic field $\vec{B} = B_0\hat{k}$, the minimum value of υ for which the particle will not hit the screen is:

(1) $\frac{qdB_0}{2m}$ (2) $\frac{qdB_0}{m}$

(3) $\frac{2qdB_0}{m}$ (4) $\frac{qdB_0}{3m}$

25. An electron is moving along + x direction with a velocity of 6×10^6 ms^{-1}. It enters a region of uniform electric field of 300 V/cm pointing along + y direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be:

(1) 5×10^{-3} T, along +z direction

(2) 3×10^{-4} T, along –z direction

(3) 3×10^{-4} T, along +z direction

(4) 5×10^{-3} T, along –z direction

26. A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane with speed v under the influence of magnetic field $\vec{B}$. The magnetic moment of this moving particle :

(1) $-\frac{mv^2\vec{B}}{B^2}$ (2) $-\frac{mv^2\vec{B}}{2\pi B^2}$

(3) $\frac{mv^2\vec{B}}{2B^2}$ (4) $-\frac{mv^2\vec{B}}{2B^2}$

27. A square loop of side 2a and carrying current I is kept in xz plane with its centre at origin. A long wire carrying the same current I is placed parallel to z-axis and passing through point (0, b, 0), (b >> a). The magnitude of torque on the loop about z-ax is will be :

(1) $\frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$ (2) $\frac{\mu_0 I^2 a^2 b}{2\pi(a^2+b^2)}$

(3) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (4) $\frac{2\mu_0 I^2 a^2}{\pi b}$

MODERN PHYSICS

1. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16} s. The frequency of revolution of the electron in its first excited state (in s^{-1}) is:

(1) 6.2×10^{15} (2) 5.6×10^{12}

(3) 7.8×10^{14} (4) 1.6×10^{14}

2. A beam of electromagnetic radiation of intensity 6.4×10^{-5} W/cm^2 is comprised of wavelength, λ = 310 nm. It falls normally on a metal (work function φ = 2eV) of surface area of 1 cm^2. If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x. (hc=1240 eVnm, 1eV=1.6×10^{-19} J), then x is____.

3. The activity of a radioactive sample falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its half life is close to :

(1) 66 min (2) 52 min

(3) 72 min (4) 62 min

4. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum)

(1) $\left(\frac{E}{2m}\right)^{1/2}$ (2) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

(3) $c(2mE)^{1/2}$ (4) $\frac{1}{c}\left(\frac{2E}{m}\right)^{1/2}$

5. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV end de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is :

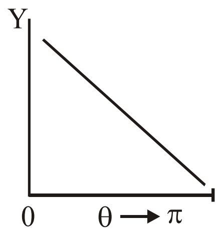
(1) 3eV (2) 2eV

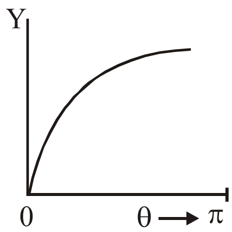
(3) 4eV (4) 1.5eV

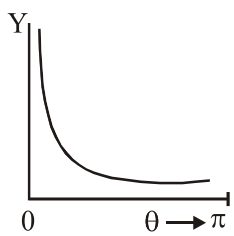
6. The graph which depicts the results of Rutherform gold foil experiment with α-particales is :

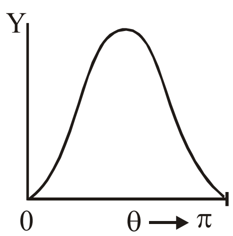
θ : Scattering angle

Y : Number of scattered α-particles detected (Plots are schematic and not to scale)

(1) 

(2) 

(3) 

(4) 

7. An electron (mass m) with initial velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$ is in an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wave length at time t is given by :

(1) $\frac{\lambda_0\sqrt{2}}{\sqrt{1+\frac{e^2E^2t^2}{m^2v_0^2}}}$ (2) $\frac{\lambda_0}{\sqrt{2+\frac{e^2E^2t^2}{m^2v_0^2}}}$

(3) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E^2t^2}{2m^2v_0^2}}}$ (4) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$

8. The first member of the Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is:

9. A particle moving with kinetic energy E has de Broglie wavelength λ. If energy ΔE is added to its energy, the wavelength become λ/2. Value of ΔE, is :

(1) 2E (2) E (3) 3E (4) 4E

10. Radiation, with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to :

(1) 1.8eV (2) 1.1eV (3) 0.8eV (4) 1.6eV

11. The energy required to ionise a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state ?

(1) 35.8 nm (2) 24.2 nm

(3) 8.6 nm (4) 11.4 nm

12. An electron of mass m and magnitude of charge |e| initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t ignoring relativistic effects is :

(1) $\frac{-h}{|e|Et^2}$ (2) $\frac{|e|Et}{h}$

(3) $-\frac{h}{|e|E\sqrt{t}}$ (4) $-\frac{h}{|e|Et}$

13. In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, N = 6.023×10^{26} per kilo mole and 1 eV = 1.6×10^{-19} J. The power output of the reactor is close to :

(1) 125 MW (2) 60 MW
(3) 35 MW (4) 54 MW

14. When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation of wavelength 3λ, the stopping potential is $\frac{V}{4}$. If the threshold wavelength for the metallic surface is $n\lambda$ then value of n will be _______.

15. In a hydrogen atom the electron makes a transition from $(n + 1)^{th}$ level to the n^{th} level. If n>>l, the frequency of radiation emitted is proportional to :

(1) $\frac{1}{n^4}$ (2) $\frac{1}{n^3}$ (3) $\frac{1}{n^2}$ (4) $\frac{1}{n}$

16. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to :

(1) 4.8×10^{-27} kg
(2) 1.2×10^{-28} kg
(3) 9.1×10^{-31} kg
(4) 9.7×10^{-28} kg

17. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to :

(1) 0.61 eV (2) 0.52 eV
(3) 0.81 eV (4) 1.02 eV

18. In a radioactive material, fraction of active material remaining after time t is 9/16. The fraction that was remaining after t/2 is :

(1) $\frac{3}{4}$ (2) $\frac{7}{8}$ (3) $\frac{4}{5}$ (4) $\frac{3}{5}$

19. The radius of R of a nucleus of mass number A can be estimated by the formula $R = (1.3 \times 10^{-15})A^{1/3}$ m. It follows that the mass density of a nucleus is of the order of:

$\left(M_{prot.} \cong M_{neut.} \simeq 1.67 \times 10^{-27} kg\right)$

(1) 10^{24} kg m^{-3}
(2) 10^{3} kg m^{-3}
(3) 10^{17} kg m^{-3}
(4) 10^{10} kg m^{-3}

20. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to:

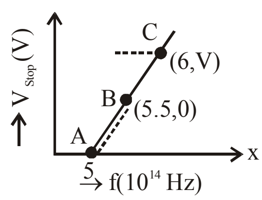
(1) 5 : 7 (2) 1 : 2
(3) 10 : 7 (4) 2 : 1

21. Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X-rays to the number densitty of photons of the visible light of the given wavelengths is :

(1) $\frac{1}{500}$ (2) 500

(3) 250 (4) $\frac{1}{250}$

22. Given figure shows few data points in a photo electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surfface is : (Plancks constant h = 6.62×10^{-34} J.s)



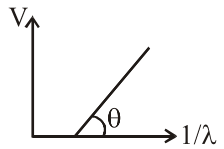
(1) 2.27 eV
(2) 2.59 eV
(3) 1.93 eV
(4) 2.10 eV

23. Particle A of mass $m_A = \frac{m}{2}$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \frac{m}{3}$. If both particles move along the x-axis after the collision, the change $\Delta\lambda$ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is :

(1) $\Delta\lambda = 4\lambda_0$
(2) $\Delta\lambda = \frac{5}{2}\lambda_0$
(3) $\Delta\lambda = 2\lambda_0$
(4) $\Delta\lambda = \frac{3}{2}\lambda_0$

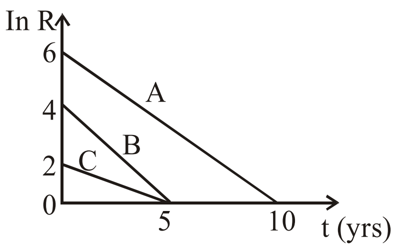
24. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is 304 Å. The corresponding difference for the Paschan series in Å is : ______.

25. In a photoelectric effect experiment, the graph of stopping potential V versus reciprocal of wavelength obtained is shown in the figure. As the intensity of incident radiation is increased :



(1) Slope of the straight line get more steep
(2) Straight line shifts to left
(3) Graph does not change
(4) Straight line shifts to right

26. Activities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives $T_{\frac{1}{2}}(A) : T_{\frac{1}{2}}(B) : T_{\frac{1}{2}}(C)$ are in the ratio :



(1) 3 : 2 : 1
(2) 4 : 3 : 1
(3) 2 : 1 : 3
(4) 2 : 1 : 1

27. A particle of mass 200 MeV/c^2 collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is :
(Given the mass of the hydrogen atom to be 1 GeV/c^2) ________.

28. A radioactive nucleus decays by two different processes. The half life for the first process is 10 s and that for the second is 100s. the effective half life of the nucleus is close to:

(1) 9 sec
(2) 55 sec
(3) 6 sec
(4) 12 sec

29. The surface of a metal is illuminated alternately with photons of energies E_1 = 4eV and E_2 = 2.5 eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal in (eV) is ______.

30. An electron, a doubly ionized helium ion (He^{++}) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths λ_e, $\lambda_{He^{++}}$ and λ_P is:

(1) $\lambda_e < \lambda_P < \lambda_{He^{++}}$
(2) $\lambda_e < \lambda_{He^{++}} = \lambda_P$
(3) $\lambda_e > \lambda_{He^{++}} > \lambda_P$
(4) $\lambda_e > \lambda_P > \lambda_{He^{++}}$

31. You are given that Mass of ^7_3Li = 7.0160 u,
Mass of ^4_2He = 4.0026 u
and Mass of ^1_1H = 1.0079 u.
When 20 g of ^7_3Li is converted into ^4_2He by proton capture, the energy liberated, (in kWh), is: [Mass of nudeon = 1 GeV/c^2]

(1) 8×10^6
(2) 1.33×10^6
(3) 6.82×10^5
(4) 4.5×10^5

32. Given the masses of various atomic particles $m_p = 1.0072u$, $m_n = 1.0087u$, $m_e = 0.000548u$, $m_{\bar{\nu}} = 0$, $m_d = 2.0141u$, where p ≡ proton, n ≡ neutron, e ≡ electron, $\bar{\nu}$ ≡ antineutrino and d ≡ deuteron. Which of the following process is allowed by momentum and energy conservation ?

(1) $n + p \rightarrow d + \gamma$

(2) $e^+ + e^- \rightarrow \gamma$

(3) n + n → deuterium atom (electron bound to the nucleus)

(4) $p \rightarrow n + e^+ + \bar{\nu}$

33. Assuming the nitrogen molecule is moving with r.m.s. velocity at 400 K, the de-Broglie wavelength of nitrogen molecule is close to :
(Given : nitrogen molecule weight : 4.64×10^{-26}kg, Boltzman constant : 1.38×10^{-23} J/K, Planck constant : 6.63×10^{-34} J.s)

(1) 0.34 Å (2) 0.24 Å (3) 0.20 Å (4) 0.44 Å

34. Find the bindng energy per nucleon for ${}^{120}_{50}\text{Sn}$. Mass of proton $m_P = 1.00783$ U, mass of neutron $m_n = 1.00867$ U and mass of tin nucleus $m_{Sn} = 119.902199$ U. (take 1U = 931 MeV)

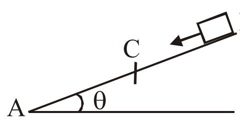
(1) 8.5 MeV (2) 7.5 MeV
(3) 8.0 MeV (4) 9.0 MeV

NLM & FRICTION

1. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take g= 10 ms^{-2} and the rope to be massless)

(1) 100 N (2) 90 N (3) 75 N (4) 70 N

2.



A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If BC = 2AC, the coefficient of friction is given by $\mu = k \tan\theta$). The value of k is ________.

3. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$, The nearest integer to I is __________ .

4. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is : ($g = 10ms^{-2}$)

(1) 0.80 m (2) 0.60 m
(3) 0.45 m (4) 0.20 m

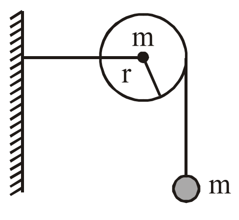
PRINCIPAL OF COMMUNICATION

1. An amplitude modulated wave is represented by the expression $v_m = 5(1+ 0.6 \cos 6280t) \sin(211 \times 10^4 t)$ volts. The minimum and maximum amplitudes of the amplitude modulated wave are, respectively :

(1) 5V, 8V (2) $\frac{3}{2}$V, 5V
(3) $\frac{5}{2}$V, 8V (4) 3V, 5V

ROTATIONAL MECHANICS

1. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be :



(1) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (2) $r\sqrt{\frac{3}{4gh}}$ (3) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (4) $r\sqrt{\frac{3}{2gh}}$

2. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is :

(1) $\frac{1}{8}l$

(2) $\sqrt{\frac{7}{48}}l$

(3) $\sqrt{\frac{3}{8}}l$

(4) $\frac{1}{4}l$

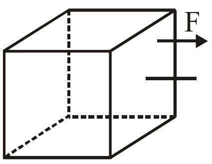
3. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the the axis, perpendicular to the plane and passing through its centre is :

(1) $2\pi a^4 \left(\frac{A}{4}+\frac{aB}{5}\right)$

(2) $\pi a^4 \left(\frac{A}{4}+\frac{aB}{5}\right)$

(3) $2\pi a^4 \left(\frac{aA}{4}+\frac{B}{5}\right)$

(4) $2\pi a^4 \left(\frac{A}{4}+\frac{B}{5}\right)$

4. 

Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for a box not to topple before moving is ______ .

5. Consider a uniform rod of mass M = 4m and length ℓ pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is :

(1) $\frac{3}{7\sqrt{2}}\frac{v}{\ell}$ (2) $\frac{3\sqrt{2}}{7}\frac{v}{\ell}$

(3) $\frac{4}{7}\frac{v}{\ell}$ (4) $\frac{3}{7}\frac{v}{\ell}$

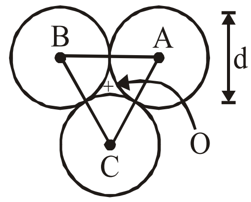
6. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is :

(1) 8.75×10^{-4} J

(2) 8.75×10^{-3} J

(3) 6.25×10^{-4} J

(4) 1.13×10^{-3} J

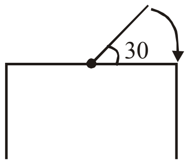
7. 

Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is :

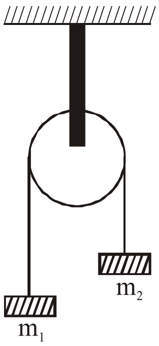
(1) $\frac{13}{23}$ (2) $\frac{15}{13}$

(3) $\frac{23}{13}$ (4) $\frac{13}{15}$

8. One end of a straight uniform 1m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\,s^{-1}$, where n is an integer. The value of n is __________ .



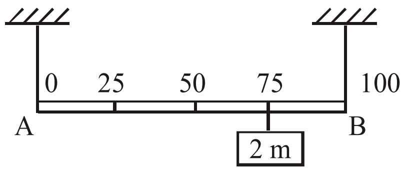
9. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is :



(1) $\left[\frac{m_1+m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}gh$ (2) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

(3) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$ (4) $\left[\frac{(m_1-m_2)}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}gh$

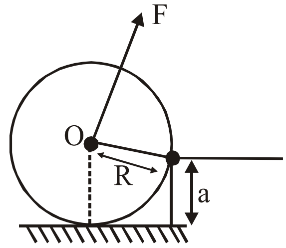
10.



Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is :

(1) 2 mg (2) 0.5 mg (3) 0.75 mg (4) 1 mg

11. A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is :



(1) $Mg\sqrt{1-\frac{a^2}{R^2}}$

(2) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(3) $Mg\frac{a}{R}$

(4) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

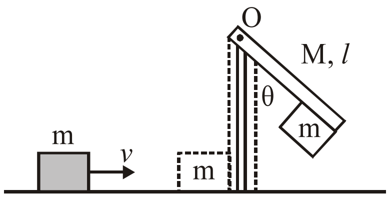
12. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The Kinetic energy of the combined system is :

(1) $\frac{10}{3}J$ (2) $\frac{2}{3}J$ (3) $\frac{5}{3}J$ (4) $\frac{20}{3}J$

13. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of material, the ratio L/R for it to have minimum possible I is :-

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{3}{2}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{2}{3}$

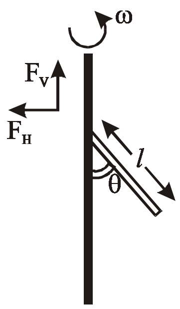
14. A block of mass m = 1 kg slides with velocity v = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg, and length l = 1 m, the value of θ is approximately :
(Take g = 10 m/s²)



(1) 69° (2) 63° (3) 55° (4) 49°

15. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis as 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre__________.

16.

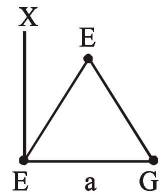


A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{m\ell^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that:

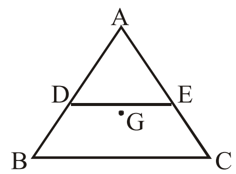
(1) $\cos\theta = \frac{g}{2\ell\omega^2}$ (2) $\cos\theta = \frac{3g}{2\ell\omega^2}$

(3) $\cos\theta = \frac{2g}{3\ell\omega^2}$ (4) $\cos\theta = \frac{g}{\ell\omega^2}$

17. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of intertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}ma^2$ where N is an integer. The value of N is ________.



18. ABC is a plane lamina of the shape of an equilateral triagnle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is _____.

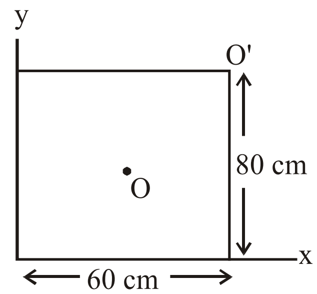


19. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is ______.

20. Consider two uniform discs of the same thickness and different radii R_1 = R and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2$ = 1 : 16 then the value of α is :

(1) $\sqrt{2}$ (2) 2 (3) 4 (4) $2\sqrt{2}$

21.



For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is :

(1) 1/2 (2) 2/3

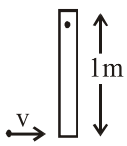
(3) 1/8 (4) 1/4

22. A wheel is rotaing freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of momet of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is :

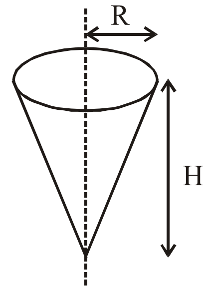
(1) 0 (2) $\frac{1}{4}$ (3) $\frac{3}{4}$ (4) $\frac{5}{6}$

23. A force $\vec{F} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)N$ acts at a point $\left(4\hat{i} + 3\hat{j} - \hat{k}\right)m$. Then the magnitude of torque about the point $\left(\hat{i} + 2\hat{j} + \hat{k}\right)m$ will be $\sqrt{x}$ N-m. The value of x is _____.

24. A thin rod of mass 0.9 kg and length 1m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be ______.



25. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is:



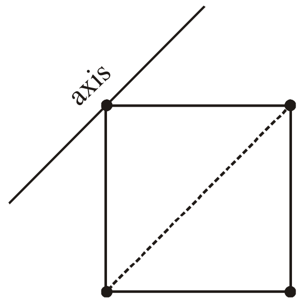
(1) $\frac{MR^2}{2}$

(2) $\frac{MH^2}{3}$

(3) $\frac{MR^2}{3}$

(4) $\frac{M\left(R^2 + H^2\right)}{4}$

26. Four point masses, each of mass m, are fixed at the corners of a square of side ℓ. The square is rotating with angular frequency ω, about an axis passing through oneof the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is:



(1) $2m\ell^2\omega$ (2) $3m\ell^2\omega$ (3) $m\ell^2\omega$ (4) $4m\ell^2\omega$

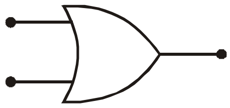
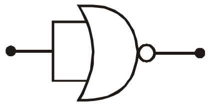
27. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1 + \frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is:

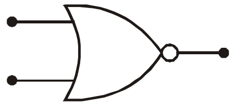
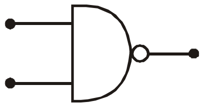
(1) $\frac{5}{12}ML^2$ (2) $\frac{3}{7}ML^2$

(3) $\frac{2}{5}ML^2$ (4) $\frac{7}{18}ML^2$

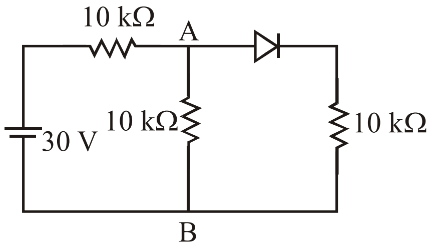
SEMICONDUCTOR

1. Which of the following gives a reversible operation?

(1)  (2) 

(3)  (4) 

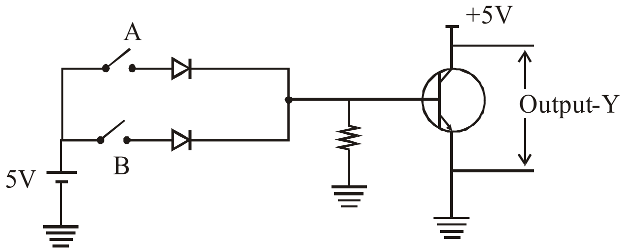
2. In the figure, potential difference between A and B is :



(1) 5V (2) 10 V

(3) zero (4) 15 V

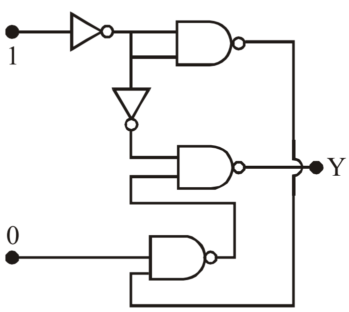
3. Boolean relation at the output stage-Y for the following circuit is :



(1) A + B (2) $\overline{A}+\overline{B}$

(3) $\overline{A}\cdot\overline{B}$ (4) $A\cdot B$

4. In the given circuit, value of Y is :



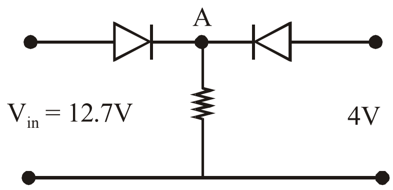
(1) will not execute

(2) 0

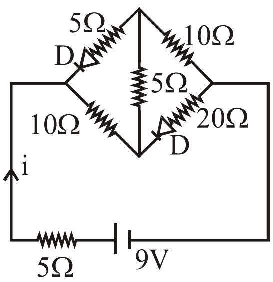
(3) toggles between 0 and 1

(4) 1

5. Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7 V. For the input voltages shown in the figure, the voltage (in Volts) at point A is __________ .



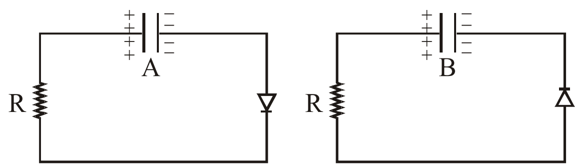
6. The current i in the network is :



(1) 0 A (2) 0.6 A

(3) 0.3 A (4) 0.2 A

7. Two identical capacitors A and B, charged to the same potential 5V are connected in two different circuits as shown below at time t = 0. If the charge on capacitors A and B at time t = CR is Q_A and Q_B respectively, then (Here e is the base of natural logarithm)



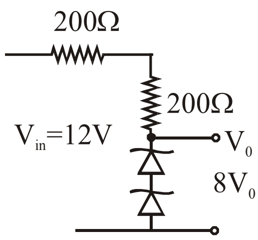
(1) $Q_A = VC, Q_B = \frac{VC}{e}$

(2) $Q_A = \frac{CV}{2}, Q_B = \frac{VC}{e}$

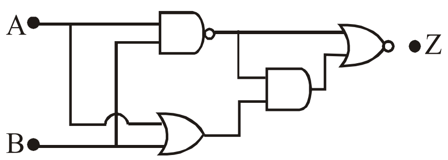
(3) $Q_A = VC,\ Q_B = CV$

(4) $Q_A = \frac{VC}{e}, Q_B = \frac{CV}{2}$

8. The circuit shown below is working as a 8 V dc regulated voltage source. When 12 V is used as input, the power dissipated (in mW) in each diode is; (considering both zener diodes are identical) _________.



9. In the following digital circuit, what will be the output at 'Z', when the input (A, B) are (1,0), (0,0), (1,1), (0,1):

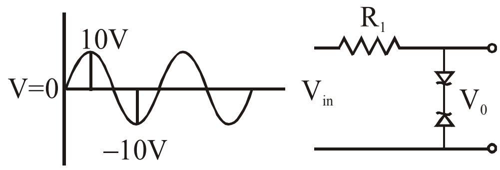


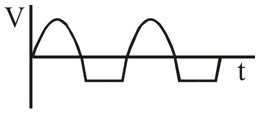
(1) 1, 0, 1, 1
(2) 0, 1, 0, 0
(3) 0, 0, 1, 0
(4) 1, 1, 0, 1

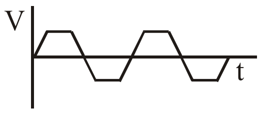
10. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is :
(1) 100 Ω
(2) 50 Ω
(3) 300 Ω
(4) 200 Ω

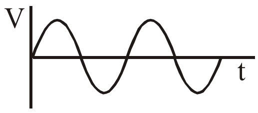
11. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is:
Planck's constant h = 6.63×10^{-34} J.s.
Speed of ligh c = 3×10^{8} m/s
(1) 2.0 eV
(2) 1.5 eV
(3) 3.1 eV
(4) 1.1 eV

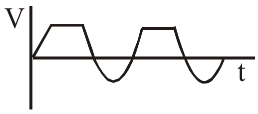
12. Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is : (Graphs drawn are schematic and not to scale)



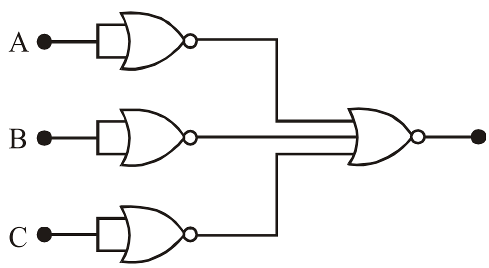
(1) 

(2) 

(3) 

(4) 

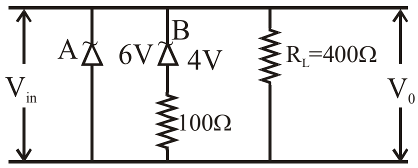
13. Identify the operation performed by the circuit given below :

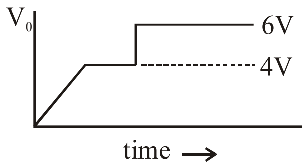


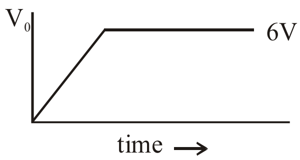
(1) AND (2) NAND
(3) OR (4) NOT

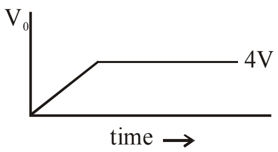
14. With increasing biasing voltage of a photodiode, the photocurrent magnitude :
(1) increases initially and saturates finally
(2) increases initially and after attaining certain value, it decreases
(3) increases linearly
(4) remains constant

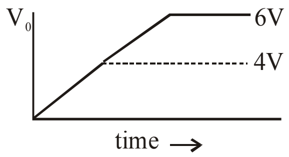
15. Two Zener diodes (A and B) having breakdown voltages of 6V and 4V respectively, are connected as shown in athe circuit below. The output voltage V_0 variation with input voltage linearly increasing with time, is given by : (V_{input} = 0V at t = 0) (figures are qualitative)



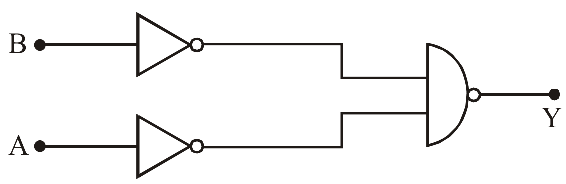
(1)


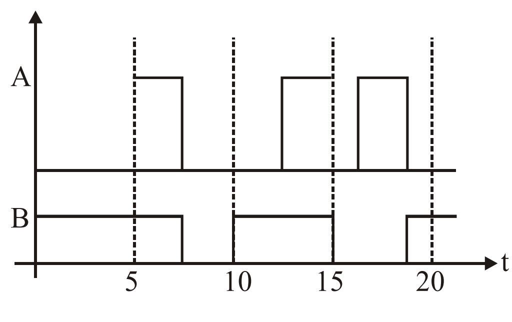
(2)


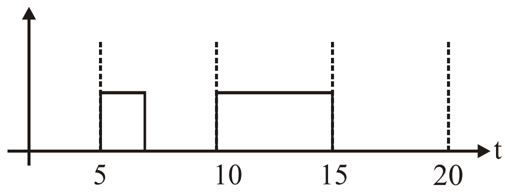
(3)


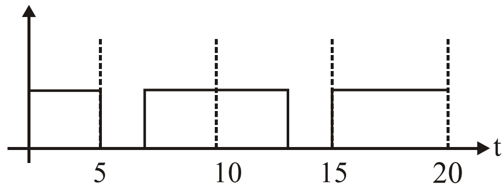
(4)


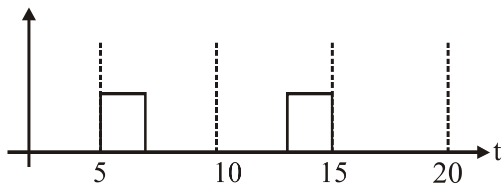
16. Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B.

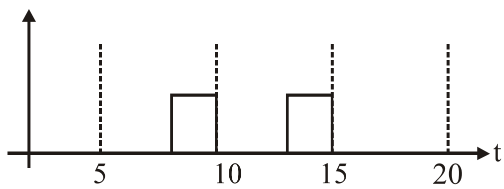




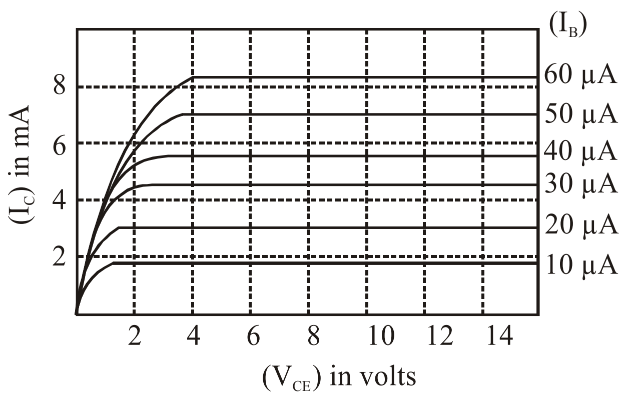
(1)


(2)


(3)


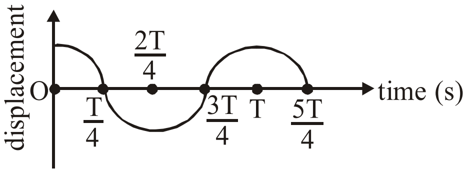
(4)


17. The output characteristics of a transistor is shown in the figure. When V_{CE} is 10 V and 1_C = 4.0 mA, then value of β_{ac} is ________ .



SIMPLE HARMONIC MOTION

1. The displacement time graph of a particle executing S.H.M. is given in figure :
(sketch is schematic and not to scale)



Which of the following statements is/are true for this motion ?

(A) The force is zero $t = \frac{3T}{4}$

(B) The acceleration is maximum at t = T

(C) The speed is maximum at $t = \frac{T}{4}$

(D) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(1) (A), (B) and (D) (2) (B), (C) and (D)
(3) (A) and (D) (4) (A), (B) and (C)

2. A block of mass m attached to massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become *f*A. The value of *f* is:

(1) $\frac{1}{2}$ (2) $\sqrt{2}$ (3) 1 (4) $\frac{1}{\sqrt{2}}$

3. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. the ratio $\frac{T_1}{T_2}$ will be :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{3}{\sqrt{2}}$

4. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0 \sin^2 \omega t$, where 'y' is measured from the lower end of unstretched spring. Then ω is :

(1) $\sqrt{\frac{g}{y_0}}$ (2) $\sqrt{\frac{g}{2y_0}}$

(3) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (4) $\sqrt{\frac{2g}{y_0}}$

UNIT & DIMENSION

1. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is :

(1) $ML^{-1} T^{-2}$ (2) $ML^2 T^{-1}$
(3) MLT^{-2} (4) $ML^2 T^{-2}$

2. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant 'h', speed of light 'c' and Gravitational constant 'G' and ampere A is :

(1) $h^2 G^{3/2} c^{1/3} A^{-1}$ (2) $h^{-2/3} c^{-1/3} G^{4/3} A^{-1}$
(3) $h^{1/3} G^{2/3} c^{1/3} A^{-1}$ (4) $h^{2/3} c^{5/3} G^{1/3} A^{-1}$

3. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of :

(1) Momentum (2) Area
(3) Energy (4) Volume

4. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be :

(1) $FA^{-1}V^0$ (2) FA^2V^{-1}
(3) FA^2V^{-3} (4) FA^2V^{-2}

5. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is :

(1) $[PA^{-1} T^{-2}]$ (2) $[PA^{1/2}T^{-1}]$
(3) $[P^2AT^{-2}]$ (4) $[P^{1/2}AT^{-1}]$

6. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is:

(1) ML^2T^{-2} (2) MLT^{-2}

(3) $M^2L^0T^{-1}$ (4) ML^0T^{-3}

7. A quantity x is given by (IFv^2/WL^4) in terms of moment of inertia I, force F, velocity v, work W and Length L. The dimensional formula for x is same as that of :

(1) Planck's constant

(2) Force constant

(3) Energy density

(4) Coefficient of viscosity

8. The quantities $x = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, $y = \frac{E}{B}$ and $z = \frac{1}{CR}$ are defined where C-capacitance, R-Resistance, l-length, E-Electric field, B-magnetic field and ϵ_0, μ_0,-free space permittivity and permeability respectively. Then :

(1) Only x and y have the same dimension

(2) x, y and z have the same dimension

(3) Only x and z have the same dimension

(4) Only y and z have the same dimension

WAVE MOTION

1. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is 16×10^{11} Nm^{-2}, the extension of wire over its natural length is :

(1) 0.02 mm (2) 0.04 mm

(3) 0.03 mm (4) 0.01 mm

2. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is $\nu_0 = 1400$ Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to :

(1) $\frac{1}{8}$ m/s (2) $\frac{1}{2}$ m/s (3) 1 m/s (4) $\frac{1}{4}$ m/s

3. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ________ Hz.

4. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T, the velocity changed to v/2. The value of T is close to :

(1) 10.2×10^2 N

(2) 5.15×10^3 N

(3) 2.50×10^4 N

(4) 30.5×10^4 N

5. Three harmonic waves having equal frequency ν and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to :

(1) 5.8 I_0 (2) 0.2 I_0

(3) I_0 (4) 3 I_0

6. A wire of length L and mass per unit length 6.0×10^{-3} kgm^{-1} is put under tension of 540 N. Two consecutive frequencies that it resonates at are : 420 Hz and 490 Hz. Then L in meters is :

(1) 8.1 m (2) 5.1 m

(3) 1.1 m (4) 2.1 m

7. Two identical strings X and Z made of same material have tension T_X and T_Z in them. It their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_X/T_Z is :

(1) 0.44 (2) 1.5

(3) 2.25 (4) 1.25

8. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire $Y = 9 \times 10^{10}$ Nm^{-2}), (to the nearest integer),_________.

9. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ?

(1) 9 (2) 12 (3) 6 (4) 3

10. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are :

(1) 1, 2, 3, ...

(2) $\frac{1}{2},\frac{1}{4},\frac{1}{6},...$

(3) 1, 3, 5, ...

(4) $\frac{1}{1},\frac{1}{3},\frac{1}{5},...$

11. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz, when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1}.

(1) 91 kmh^{-1}

(2) 71 kmh^{-1}

(3) 81 kmh^{-1}

(4) 61 kmh^{-1}

12. In a resonance tube experiment when the tube is filled with water up to height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is:

(1) 1100 Hz

(2) 3300 Hz

(3) 2200 Hz

(4) 550 Hz

13. Assume that the displacement(s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement(s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If Δp~10Pa, v~300 m/s, p~1 kg/m^3 and f~1000Hz, then s will be the order of (take multiplicative constant to be 1)

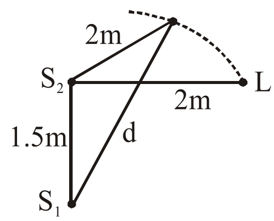
(1) 10 mm

(2) $\frac{3}{100}$ mm

(3) 1 mm

(4) $\frac{1}{10}$ mm

14. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, λ = 1 m, in phase. S_1 and S_2 are placed 1.5 m apart (see fig.) A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 2m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is :



(1) 12m

(2) 3m

(3) 5m

(4) 2m

15. A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is

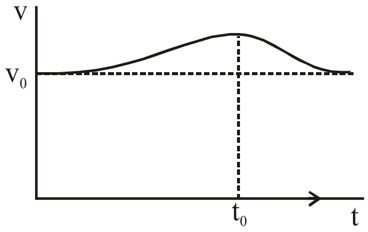
(1) 36 km/hr

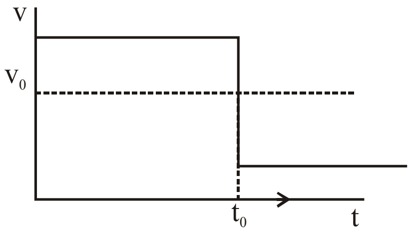
(2) 24 km/hr

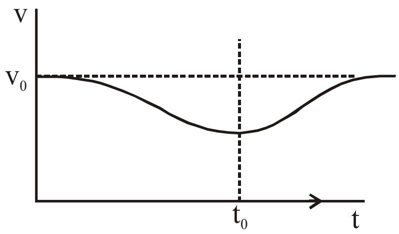
(3) 18 km/hr

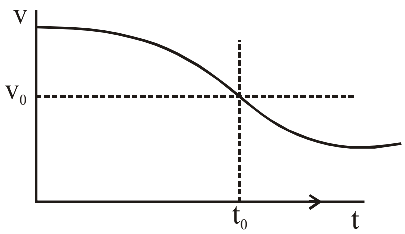
(4) 54 km/hr

16. A sound source S is moving along a straight track with speed v, and is emitting sound of frequency v_0 (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by :
(t_0 represents the instant when the distance between the source and observer is minimum)

(1)


(2)


(3)


(4)


WAVE OPTICS

1. Visible light of wavelength 6000×10^{-8} cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1, then θ_1 is close to :
(1) 20° (2) 45° (3) 30° (4) 25°

2. A polarizer - analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer - analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is :
(1) 18.4° (2) 71.6° (3) 90° (4) 45°

3. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is :
(1) 6.9 mm (2) 5.9 mm (3) 4.9 mm (4) 3.9 mm

4. In a double slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is :
(1) 0.568 (2) 0.672 (3) 0.760 (4) 0.853

5. The aperture diameter of a telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of 5500 Å, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to :
(1) 20 m (2) 600 m (3) 60 m (4) 200 m

6. A plane electromagnetic wave is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$, with its polarization along the direction $\hat{k}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant) :

(1) $B_0\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(2) $B_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(3) $B_0\hat{k}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(4) $B_0\frac{\hat{j}-\hat{i}}{\sqrt{2}}\cos\left(\omega t + k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

7. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source (λ = 632.8 nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to :

(1) 1.27 μm (2) 2 nm
(3) 2.87 nm (4) 2.05 μm

8. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be :

(1) 28 (2) 24 (3) 18 (4) 30

9. In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to :

(1) 0.07° (2) 0.17° (3) 1.7° (4) 0.57°

10. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is:

(1) $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$ (2) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$

(3) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_2}{n_2}\right)$ (4) $\frac{2\pi}{\lambda}(n_2L_1 - n_1L_2)$

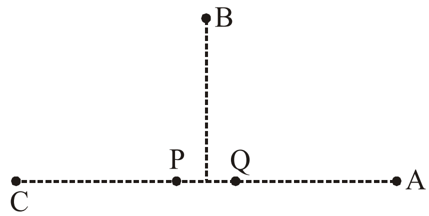
11. A beam of plane polarised light of large cross sectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polariser (cross sectional area 3×10^{-4} m^2) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to :

(1) 1.0×10^{-5} J (2) 5.0×10^{-4} J
(3) 1.0×10^{-4} J (4) 1.5×10^{-4} J

12. Orange light of wavelength 6000×10^{-10} m in illuminates a single slit of width 0.6×10^{-4} m. The maximum possible number of diffraction minima produced on both sides of the central maximum is ______.

13. A beam of electrons of energy E scatters from a target having atomic spacing of 1Å. The first maximum intensity occurs at θ = 60°. Then E (in eV) is ______.
(Planck constant h = 6.64×10^{-34} Js, 1eV = 1.6×10^{-19} J, electron mass m = 9.1×10^{-31} kg)

14. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio:



(1) 0 : 1 : 2 (2) 4 : 1 : 0
(3) 0 : 1 : 4 (4) 2 : 1 : 0

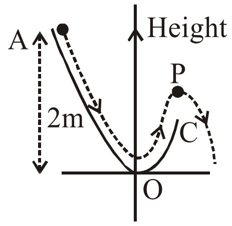
15. A Young's doublc-slit experiment is performed using monochromatic light of wavelength λ. The intensity of light at a point on the screen, where the path difference is λ, is K units. The intensity of light at a point where the path difference is A $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of n is ________ .

WORK POWER ENERGY

1. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms^{-2})

(1) 1.7 ms^{-1} (2) 2.0 ms^{-1}
(3) 1.9 ms^{-1} (4) 1.5 ms^{-1}

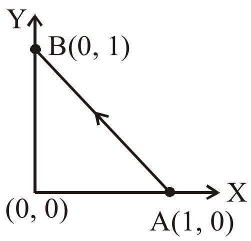
2. A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale; take g=10 ms^{-2})_____.



3. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator (g = 10 m/s^2) must be at least :

(1) 56300 W (2) 48000 W
(3) 66000 W (4) 62360 W

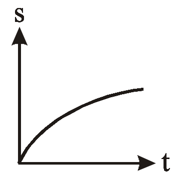
4. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is : (all quantities are in SI units)

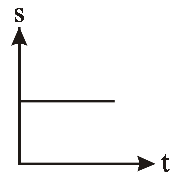


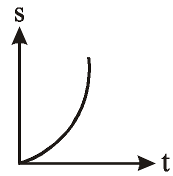
(1) $\frac{3}{2}$ (2) 1 (3) 2 (4) $\frac{1}{2}$

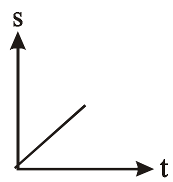
5. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F(in N) is (g = 10 ms^{-2})_______.

6. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) :

(1)


(2)


(3)


(4)


7. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance of 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box ?

(1) 5690 J
(2) 5250 J
(3) 3280 J
(4) 2780 J

8. A body of mass 2kg is driven by an engine delivering a constant power 1J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) _______.

9. If the potential energy between two molecules is given by $U = \frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are :

(1) $\left(\frac{B}{A}\right)^{1/6}, 0$ (2) $\left(\frac{B}{2A}\right)^{1/6}, -\frac{A^2}{2B}$

(3) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{4B}$ (4) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{2B}$

10. A particle moving in the xy plane experiences a velocity dependent force $\vec{F} = k(v_y\hat{i} + v_x\hat{j})$, where v_x and v_y are the x and y components of its velocity $\vec{v}$. If $\vec{a}$ Ls the acceleration of the particle, then which of the following statements is true for the particle ?

(1) quantity $\vec{v}\cdot\vec{a}$ is constant in time.

(2) kinetic energy of particle is constant in time.

(3) quantity $\vec{v}\times\vec{a}$ is constant in lime.

(4) $\vec{F}$ arises due to a magnetic field.

ANSWER KEY

BASIC MATHS & VECTOR

Que.	1	2
Ans.	90.00	1

CAPACITOR

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	6.00	3	4	NTA : 36.00 Allen : 4.033	2	8.00	1	NTA : (4) Allen : (1)	4
Que.	11	12	13							
Ans.	3	2	1							

CIRCULAR MOTION

Que.	1	2	3	4	5
Ans.	4	2	4	2	1

CENTRE OF MASS & COLLISION

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	4	1.00	3	4	2	4	3	4	10.00
Que.	11	12	13	14	15	16	17			
Ans.	23.00	1	4	4	120.00	4	3.00			

CURRENT ELECTRICITY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	4	12.00	4	10.00	1	30.00	2	40.00	4
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	4	4	4	2	2	2	1	1	1	3
Que.	21									
Ans.	1									

ELASTICITY					
Que.	1	2	3	4	5
Ans.	4.00	4	750.00	2	1

ELECTROSTATICS										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	4	4	3	4	4	3	–48.00	1	3
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	4	1	1	1	2	3	1	4	1	3
Que.	21	22	23							
Ans.	3	1	3							

EM WAVE										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	3	2	3	2	1	2	3	3	2
Que.	11	12	13	14						
Ans.	2	4	NTA : (275.00) Allen : (194.00)	2						

EMI & AC										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	1	NTA : (1) Allen : (2)	4	3	4	10.00	1	15.00	1
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	3	1	1	3	3	NTA : (1) Allen : (Bonus)	5.00	2	2	33.00
Que.	21									
Ans.	400.00									

ERROR & MEASUREMENT									
Que.	1	2	3	4	5	6	7	8	9
Ans.	3	2	Bonus	4	4	2	2	1050.00	3

FLUID										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	4	1	3	2	1	3	1	101.00	3
Que.	11	12	13	14						
Ans.	3	2	2	2						

GEOMETRICAL OPTICS										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	3	4	4	60.00	2	2	1	NTA : (1) Allen : (4)	90.00
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	
Ans.	158.00	1	NTA : (5.00) Allen : (4.48)	NTA : (5.00) Allen : (476.00)	4	50.00	5.00	NTA : (1,4) Allen : (3)	4	

GRAVITATION										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	2	4	16.00	1	1	3	1	2	2	1
Que.	**11**	**12**	**13**	**14**	**15**					
Ans.	3	4	4	1	2					

HEAT & THERMODYNAMICS										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	2	600.00	60.00	3	1	40.00	4	4	1
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	3	2	50.00	2	4	NTA : (1) Allen : (3)	1816.00 to 1820.00	2	NTA : (46.00) Allen : (46, 45.78)	3
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	4	2	2	4	20.00	3	1	2	8791.00	1
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	4	1	NTA : (266.00) Allen : (266.67)	1	150.00	1	2	3	4	4
Que.	**41**	**42**	**43**	**44**	**45**	**46**				
Ans.	NTA : (41.00) Allen : (40.93)	4	5.00	1	3	19.00				

KINEMATICS										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	580.00	1	8 or 2888	3.00	3	2	3	3	2	20.00
Que.	**11**	**12**	**13**							
Ans.	3	4	4							

MAGNETISM										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	175.00	3	2	4	3	1	2	2	2
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	2	4	3	1	3	3	1	20.00	3	4
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**			
Ans.	2	1	4	2	1	4	1			

MODERN PHYSICS

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	11.00	4	2	3	3	3	486.00	3	2
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	4	1	2	9.00	2	4	1	1	3	4
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	1	1	1	NTA : (10553) Allen : (10553.14)	3	3	51.00	1	2	4
Que.	**31**	**32**	**33**	**34**						
Ans.	2	1	2	1						

NLM & FRICTION

Que.	1	2	3	4
Ans.	1	3	346.00	4

PRINCIPAL OF COMMUNICATION

Que.	1
Ans.	3

ROTATIONAL MECHANICS

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	2	1	75.00	2	1	1	15.00	2	4
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	4	4	3	2	9.00	2	25.00	11.00	20.00	3
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**			
Ans.	4	3	195.00	20.00	1	2	4			

SEMICONDUCTOR

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	2	3	2	12.00	3	1	NTA : (12.00) Allen : (40.00)	3	1
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**			
Ans.	3	2	1	1	NTA : (4) Allen : (2)	NTA : (3) Allen : (2)	150.00			

SIMPLE HARMONIC MOTION

Que.	1	2	3	4
Ans.	4	4	1	2

UNIT & DIMENSION								
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Ans.	1	Bonus	3	1	2	4	3	2

WAVE MOTION										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	4	106.00 to 107.20	2	1	4	3	35.00	2	4
Que.	**11**	**12**	**13**	**14**	**15**	**16**				
Ans.	1	3	2	2	4	4				

WAVE OPTICS										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	4	1	2	4	3	1	1	1	4	1
Que.	**11**	**12**	**13**	**14**	**15**					
Ans.	3	NTA : (200.00) Allen : (198.00)	50.00	4	9.00					

WORK POWER ENERGY										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	10.00	3	2	150.00	3	2	18.00	3	3

IMPORTANT NOTES

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

path to success

Chapter Contents 02

JEE (MAIN) TOPICWISE TEST PAPERS JANUARY & SEPTEMBER 2020

CHEMISTRY

PHYSICAL CHEMISTRY

ORGANIC CHEMISTRY

INORGANIC CHEMISTRY

JANUARY & SEPTEMBER 2020 ATTEMPT (PC)

MOLE CONCEPT

1. Amongst the following statements, that which was not proposed by Dalton was :
(1) all the atoms of a given element have identical properties including identical mass. Atoms of different elements differ in mass.
(2) chemical reactions involve reorganisation of atoms. These are neither created nor destroyed in a chemical reaction.
(3) when gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are at the same T & P.
(4) matter consists of indivisible atoms.

2. The ammonia (NH_3) released on quantitative reaction of 0.6 g urea (NH_2CONH_2) with sodium hydroxide (NaOH) can be neutralized by :
(1) 100 ml of 0.1 N HCl
(2) 200 ml of 0.4 N HCl
(3) 100 ml of 0.2 N HCl
(4) 200 ml of 0.2 N HCl

3. Ferrous sulphate heptahydrate is used to fortify foods with iron. The amount (in grams) of the salt required to achieve 10 ppm of iron in 100 kg of wheat is _____.
Atomic weight : Fe = 55.85 ; S = 32.0 ; O = 16.00

4. $NaClO_3$ is used, even in spacecrafts, to produce O_2. The daily consumption of pure O_2 by a person is 492L at 1 atm, 300K. How much amount of $NaClO_3$, in grams, is required to produce O_2 for the daily consumption of a person at 1 atm, 300 K ?
$NaClO_3(s) + Fe(s) \rightarrow O_2(g) + NaCl(s) + FeO(s)$
$R = 0.082\ L\ atm\ mol^{-1}\ K^{-1}$

5. The first and second ionisation enthalpies of a metal are 496 and 4560 kJ mol^{-1}, respectively. How many moles of HCl and H_2SO_4, respectively, will be needed to react completely with 1 mole of the metal hydroxide ?
(1) 1 and 0.5 (2) 2 and 0.5
(3) 1 and 1 (4) 1 and 2

6. 5 g of zinc is treated separately with an excess of
(a) dilute hydrochloric acid and
(b) aqueous sodium hydroxide.
The ratio of the volumes of H_2 evolved in these two reactions is :
(1) 1 : 4 (2) 1 : 2 (3) 2 : 1 (4) 1 : 1

7. The minimum number of moles of O_2 required for complete combustion of 1 mole of propane and 2 moles of butane is ______.

8. The ratio of the mass percentages of 'C & H' and 'C & O' of a saturated acyclic organic compound 'X' are 4 : 1 and 3 : 4 respectively. Then, the moles of oxygen gas required for complete combustion of two moles of organic compound 'X' is _______ .

9. In an estimation of bromine by Carius method, 1.6 g of an organic compound gave 1.88 g of AgBr. The mass percentage of bromine in the compound is _______
(Atomic mass, Ag=108, Br = 80 g mol^{-1})

CONCENTRATION TERMS

1. The molarity of HNO_3 in a sample which has density 1.4 g/mL and mass percentage of 63% is _____. (Molecular Weight of HNO_3 = 63)

2. 10.30 mg of O_2 is dissolved into a liter of sea water of density 1.03 g/mL. The concentration of O_2 in ppm is__________.

3. The volume strength of 8.9 M H_2O_2 solution calculated at 273 K and 1 atm is _____. (R=0.0821 L atm K^{-1} mol^{-1}) (rounded off to the nearest integer)

4. The mole fraction of glucose ($C_6H_{12}O_6$) in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is ______.

5. 6.023×10^{22} molecules are present in 10 g of a substance 'x'. The molarity of a solution containing 5 g of substance 'x' in 2 L solution is ______ $\times 10^{-3}$.

6. The strengths of 5.6 volume hydrogen peroxide (of density 1 g/mL) in terms of mass percentage and molarity (M), respectively, are:
(Take molar mass of hydrogen peroxide as 34 g/mol)
(1) 1.7 and 0.25 (2) 1.7 and 0.5
(3) 0.85 and 0.5 (4) 0.85 and 0.25

7. A solution of two components containing n_1 moles of the 1st component and n_2 moles of the 2nd component is prepared. M_1 and M_2 are the molecular weights of component 1 and 2 respectively. If d is the density of the solution in g mL^{-1}, C_2 is the molarity and x_2 is the mole fraction of the 2nd component, then C_2 can be expressed as :

(1) $C_2 = \frac{1000x_2}{M_1 + x_2(M_2 - M_1)}$

(2) $C_2 = \frac{dx_2}{M_2 + x_2(M_2 - M_1)}$

(3) $C_2 = \frac{dx_1}{M_2 + x_2(M_2 - M_1)}$

(4) $C_2 = \frac{1000dx_2}{M_1 + x_2(M_2 - M_1)}$

REDOX REACTIONS

1. Oxidation number of potassium in K_2O, K_2O_2 and KO_2 , respectively, is :
(1) +1, +4 and +2 (2) +1, +2 and +4
(3) +1, +1 and +1 (4) +2 , +1 and $+\frac{1}{2}$

2. The strength of an aqueous NaOH solution is most accurately determined by titrating :
(Note : consider that an appropriate indicator is used)
(1) Aq. NaOH in a volumetric flask and concentrated H_2SO_4 in a conical flask
(2) Aq. NaOH in a pipette and aqueous oxalic acid in a burette
(3) Aq. NaOH in a burette and concentrated H_2SO_4 in a conical flask
(4) Aq. NaOH in a burette and aqueous oxalic acid in a conical flask

3. The compound that cannot act both as oxidising and reducing agent is :
(1) H_2O_2
(2) H_2SO_3
(3) HNO_2
(4) H_3PO_4

4. The hardness of a water sample containing 10^{-3} M $MgSO_4$ expressed as $CaCO_3$ equivalents (in ppm) is ______.
(molar mass of $MgSO_4$ is 120.37 g/mol)

5. Consider the following equations :
$2\,Fe^{2+} + H_2O_2 \rightarrow x\,A + y\,B$
(in basic medium)
$2MnO_4^- + 6H^+ + 5H_2O_2 \rightarrow x'\,C + y'\,D + z'\,E$
(in acidic medium)
The sum of the stoichiometric coefficients x, y, x', y' and z' for products A, B, C, D and E, respectively, is ___________.

6. A 100 mL solution was made by adding 1.43 g of $Na_2CO_3 \cdot xH_2O$. The normality of the solution is 0.1 N. The value of x is _______.
(The atomic mass of Na is 23g/mol) :-

7. A 20.0 mL solution containing 0.2 g impure H_2O_2 reacts completely with 0.316 g of $KMnO_4$ in acid solution. The purity of H_2O_2 (in %) is ______ (mol. wt. of H_2O_2 = 34; mol. wt. of $KMnO_4$ = 158)

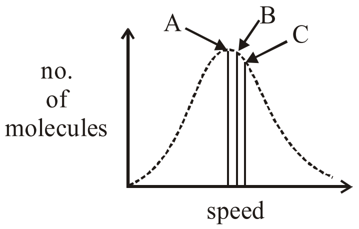
8. The volume (in mL) of 0.1 N NaOH required to neutralise 10 mL of 0.1 N phosphinic acid is ________ .

9. The volume, in mL, of 0.02 M $K_2Cr_2O_7$ solution required to react with 0.288 g of ferrous oxalate in acidic medium is_______.
(Molar mass of Fe = 56 g mol^{-1})

10. The oxidation states of transition metal atoms in $K_2Cr_2O_7$, $KMnO_4$ and K_2FeO_4, respectively, are x, y and z. The sum of x, y and z is ____.

IDEAL GAS

1. Identify the correct labels of A, B and C in the following graph from the options given below:



Root mean square speed (V_{rms}) ; most probable speed (V_{mp}) ; Average speed ($V_{av.}$)

(1) A – V_{rms} ; B –V_{mp} ; C – V_{av}

(2) A – V_{av} ; B –V_{rms} ; C – V_{mp}

(3) A – V_{mp} ; B –V_{rms} ; C – V_{av}

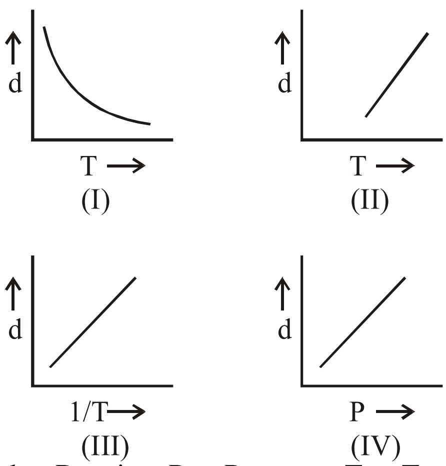
(4) A – V_{mp} ; B –V_{av} ; C – V_{rms}

2. A mixture of one mole each of H_2, He and O_2 each are enclosed in a cylinder of volume V at temperature T. If the partial pressure of H_2 is 2 atm, the total pressure of the gases in the cylinder is :

(1) 14 atm (2) 22 atm

(3) 6 atm (4) 38 atm

3. Which one of the following graphs is **not correct** for ideal gas ?



d = Density, P = Pressure, T = Temperature

(1) II (2) III

(3) I (4) IV

4. A spherical balloon of radius 3 cm containing helium gas has a pressure of 48×10^{-3} bar. At the same temperature, the pressure, of a spherical balloon of radius 12 cm containing the same amount of gas will be_____$\times 10^{-6}$ bar.

ATOMIC STRUCTURE

1. The number of orbitals associated with quantum numbers n = 5, $m_s = +\frac{1}{2}$ is :

(1) 11 (2) 25 (3) 15 (4) 50

2. For the Balmer series in the spectrum of H atom, $\bar{\nu} = R_H\left\{\frac{1}{n_1^2} - \frac{1}{n_2^2}\right\}$, the correct statements among (I) and (IV) are :

(I) As wavelength decreases, the lines in the series converge

(II) The integer n_1 is equal to 2

(III) The lines of longest wavelength corresponds to $n_2 = 3$

(IV) The ionization energy of hydrogen can be calculated from wave number of these lines

(1) (II), (III), (IV)

(2) (I), (II), (III)

(3) (I), (III), (IV)

(4) (I), (II), (IV)

3. The radius of the second Bohr orbit, in terms of the Bohr radius, a_0, in Li^{2+} is :

(1) $\frac{4a_0}{9}$ (2) $\frac{2a_0}{9}$

(3) $\frac{2a_0}{3}$ (4) $\frac{4a_0}{3}$

4. The de Broglie wavelength of an electron in the 4th Bohr orbit is :

(1) $8\pi a_0$ (2) $2\pi a_0$

(3) $4\pi a_0$ (4) $6\pi a_0$

5. The shortest wavelength of H atom in the Lyman series is λ_1. The longest wavelength in the Balmer series of He^+ is :-

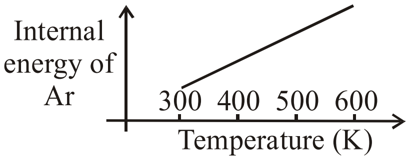
(1) $\frac{5\lambda_1}{9}$ (2) $\frac{27\lambda_1}{5}$ (3) $\frac{9\lambda_1}{5}$ (4) $\frac{36\lambda_1}{5}$

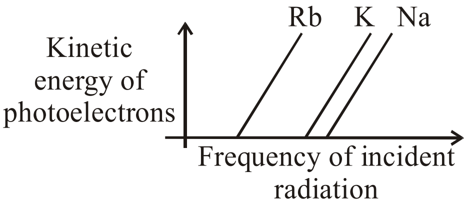
6. The difference between the radii of 3rd and 4th orbits of Li^{2+} is ΔR_1. The difference between the radii of 3rd and 4th orbits of He^+ is ΔR_2. Ratio $\Delta R_1 : \Delta R_2$ is :

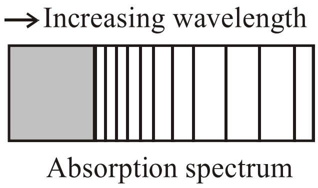
(1) 8 : 3 (2) 3 : 2

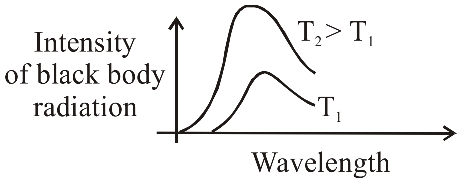
(3) 3 : 8 (4) 2 : 3

7. The figure that is not a direct manifestation of the quantum nature of atoms is :

(1) 

(2) 

(3) 

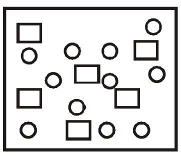
(4) 

8. The work function of sodium metal is 4.41×10^{-19} J. If the photons of wavelength 300 nm are incident on the metal, the kinetic energy of the ejected electrons will be ________ $\times 10^{-21}$ J.

($h = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^{8}$ m/s)

CHEMICAL EQUILIBRIUM

1. In the figure shown below reactant A (represented by square) is in equilibrium with product B (represented by circle). The equilibrium constant is :



(1) 2 (2) 1

(3) 8 (4) 4

2. Consider the following reaction

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$; $\Delta H^0 = +58$ kJ

For each of the following cases (a, b), the direction in which the equilibrium shifts is:

(a) Temperature is decreased

(b) Pressure is increased by adding N_2 at constant T

(1) (a) towards reactant, (b) no change

(2) (a) towards product, (b) towards reactant

(3) (a) towards product, (b) no change

(4) (a) towards reactant, (b) towards product

3. The value of K_C is 64 at 800 K for the reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The value of K_C for the following reaction is :

$NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{3}{2}H_2(g)$

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) 8 (4) $\frac{1}{64}$

IONIC EQUILIBRIUM

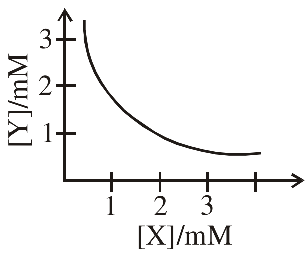
1. Two solutions A and B, each of 100 L was made by dissolving 4g of NaOH and 9.8 g of H_2SO_4 in water, respectively. The pH of the resultant solution obtained from mixing 40 L of solution A and 10 L of solution B is_______.

2. 3g of acetic acid is added to 250 mL of 0.1 M HCl and the solution made up to 500 mL. To 20 mL of this solution $\frac{1}{2}$ mL of 5 M NaOH is added. The pH of the solution is _____.

[Given : pK_a of acetic acid = 4.75, molar mass of acetic acid = 60 g/mol, log3 = 0.4771]

Neglect any changes in volume

3. The stoichiometry and solubility product of a salt with the solubility curve given below is, respectively :



(1) X_2Y, $2\times10^{-9}M^3$ (2) XY_2, $1\times10^{-9}M^3$

(3) XY_2, $4\times10^{-9}M^3$ (4) XY, $2\times10^{-6}M^3$

4. For the following Assertion and Reason, the correct option is :

Assertion : The pH of water increases with increase in temperature.

Reason : The dissociation of water into H^+ and OH^- is an exothermic reaction.

(1) Both assertion and reason are true, but the reason is not the correct explanation for the assertion.

(2) Both assertion and reason are false.

(3) Assertion is not true, but reason is true.

(4) Both assertion and reason are true, and the reason is the correct explanation for the assertion.

5. The K_{sp} for the following dissociation is 1.6×10^{-5}

$$PbCl_{2(s)} \rightleftharpoons Pb^{2+}_{(aq)} + 2Cl^-_{(aq)}$$

Which of the following choices is correct for a mixture of 300 mL 0.134 M $Pb(NO_3)_2$ and 100 mL 0.4 M NaCl ?

(1) $Q < K_{sp}$

(2) $Q > K_{sp}$

(3) $Q = K_{sp}$

(4) Not enough data provided

6. The solubility product of $Cr(OH)_3$ at 298 K is 6.0×10^{-31}. The concentration of hydroxide ions in a saturated solution of $Cr(OH)_3$ will be :

(1) $(18 \times 10^{-31})^{1/4}$

(2) $(2.22 \times 10^{-31})^{1/4}$

(3) $(4.86 \times 10^{-29})^{1/4}$

(4) $(18 \times 10^{-31})^{1/2}$

7. An acidic buffer is obtained on mixing :

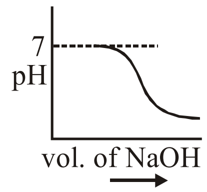
(1) 100 mL of 0.1 M CH_3COOH and 200 mL of 0.1 M NaOH

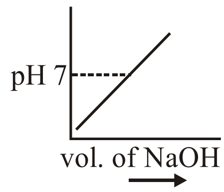
(2) 100 mL of 0.1 M CH_3COOH and 100 mL of 0.1 M NaOH

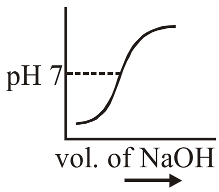
(3) 100 mL of 0.1 M HCl and 200 mL of 0.1 M CH_3COONa

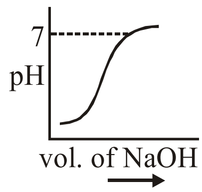
(4) 100 mL of 0.1 M HCl and 200 mL of 0.1 M NaCl

8. 100 mL of 0.1 M HCl is taken in a beaker and to it 100 mL of 0.1 M NaOH is added in steps of 2 mL and the pH is continuously measured. Which of the following graphs correctly depicts the change in pH?

(1)


(2)


(3)


(4)


9. A soft drink was bottled with a partial pressure of CO_2 of 3 bar over the liquid at room temperature. The partial pressure of CO_2 over the solution approaches a value of 30 bar when 44 g of CO_2 is dissolved in 1 kg of water at room temperature. The approximate pH of the soft drink is __________ $\times 10^{-1}$.

(First dissociation constant of $H_2CO_3 = 4.0 \times 10^{-7}$; log 2 = 0.3; density of the soft drink = 1 g mL^{-1})

10. If the solubility product of AB_2 is 3.20×10^{-11} M^3, then the solubility of AB_2 in pure water is_ _ _ _ _ $\times 10^{-4}$ mol L^{-1}. [Assuming that neither kind of ion reacts with water]

11. Arrange the following solutions is the decreasing order of pOH :

(A) 0.01 M HC1

(B) 0.01 M NaOH

(C) 0.01 M CH_3COONa

(D) 0.01 M NaCl

(1) (B) > (C) > (D) > (A)

(2) (A) > (C) > (D) > (B)

(3) (B) > (D) > (C) > (A)

(4) (A) > (D) > (C) > (B)

THERMODYNAMICS

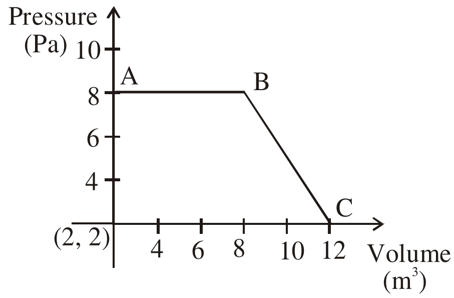
1. For the reaction ;

$A(l) \rightarrow 2B(g)$

$\Delta U = 2.1$ kcal , $\Delta S = 20$ cal K^{-1} at 300 K

Hence ΔG in kcal is_________ .

2. The magnitude of work done by a gas that undergoes a reversible expansion along the path ABC shown in the figure is ______



3. At constant volume, 4 mol of an ideal gas when heated from 300 K to 500K changes its internal energy by 5000 J. The molar heat capacity at constant volume is _______.

4. The true statement amongst the following is:

(1) Both ΔS and S are functions of temperature.

(2) S is not a function of temperature but ΔS is a function of temperature.

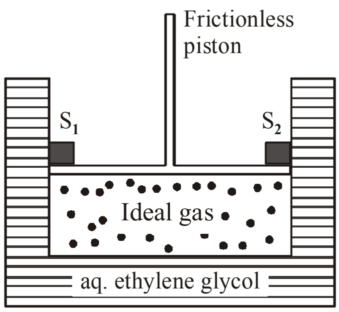
(3) Both S and ΔS are not functions of temperature.

(4) S is a function of temperature but ΔS is not a function of temperature.

5. A cylinder containing an ideal gas (0.1 mol of 1.0 dm^3) is in thermal equilibrium with a large volume of 0.5 molal aqueous solution of ethylene glycol at its freezing point. If the stoppers S_1 and S_2 (as shown in the figure) are suddenly withdrawn, the volume of the gas in litres after equilibrium is achieved will be____.

(Given, K_f (water) = 2.0 K kg mol^{-1},

R = 0.08 dm^3 atm K^{-1} mol^{-1})



6. Five moles of an ideal gas at 1 bar and 298 K is expanded into vacuum to double the volume. The work done is :-

(1) $C_V(T_2 - T_1)$ (2) $-RT \ln V_2/V_1$

(3) $-RT(V_2 - V_1)$ (4) zero

7. For one mole of an ideal gas, which of these statements must be true ?

(a) U and H each depends only on temperature

(b) Compressibility factor z is not equal to 1

(c) $C_{P,m} - C_{V,m} = R$

(d) $dU = C_V dT$ for any process

(1) (a), (c) and (d) (2) (b), (c) and (d)

(3) (c) and (d) (4) (a) and (c)

8. For a dimerization reaction,

$2\,A(g) \rightarrow A_2(g)$

at 298 K, $\Delta U^\ominus$, = – 20kJ mol^{-1}, $\Delta S^\ominus = -30$ J K^{-1} mol^{-1}, then the $\Delta G^\ominus$ will be _________J.

9. The internal energy change (in J) when 90g of water undergoes complete evaporation at 100°C is ____ .

(Given : ΔH_{vap} for water at 373 K = 41 kJ/mol, R = 8.314 JK^{-1} mol^{-1})

10. The Gibbs energy change (in J) for the given reaction at $[Cu^{2+}] = [Sn^{2+}] = 1$ M and 298K is:

$Cu(s) + Sn^{2+}(aq.) \rightarrow Cu^{2+}(aq.) + Sn(s)$;

$(E^0_{Sn^{2+}|Sn} = -0.16V,\ E^0_{Cu^{2+}|Cu} = 0.34V,$

Take F = 96500 C mol^{-1})

11. The variation of equilibrium constant with temperature is given below :

Temperature	Equilibrium constant
$T_1 = 25°C$	$K_1 = 100$
$T_2 = 100°C$	$K_2 = 100$

The values of $\Delta H°$, $\Delta G°$ at T_1 and $\Delta G°$ at T_2 (in kJ mol^{-1}) respectively, are close to

[Use R = 8.314 $JK^{-1}mol^{-1}$]

(1) 0.64, –5.71 and –14.29

(2) 28.4, –7.14 and –5.71

(3) 28.4, –5.71 and –14.29

(4) 0.64, –7.14 and –5.71

THERMOCHEMISTRY

1. The standard heat of formation $\left(\Delta_f H^0_{298}\right)$ of ethane in (kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are –1560, –393.5 and –286 kJ/mol, respectively is _______

2. If enthalpy of atomisation for $Br_{2(l)}$ is x kJ/mol and bond enthalpy for Br_2 is y kJ/mol, the relation between them :

(1) is x = y (2) is x < y

(3) does not exist (4) is x > y

3. Lattice enthalpy and enthalpy of solution of NaCl are 788 kJ mol^{-1} and 4 kJ mol^{-1}, respectively. The hydration enthalpy of NaCl is :

(1) –780 kJ mol^{-1} (2) –784 kJ mol^{-1}

(3) 780 kJ mol^{-1} (4) 784 kJ mol^{-1}

4. The heat of combustion of ethanol into carbon dioxide and water is –327 kcal at constant pressure. The heat evolved (in cal) at constant volume and 27°C (if all gases behave ideally) is (R = 2 cal mol^{-1} K^{-1})

SOLID STATE

1. Which of the following compounds is likely to show both Frenkel and Schottky defects in its crystalline form?

(1) AgBr (2) ZnS

(3) KBr (4) CsCl

2. An element with molar mass 2.7×10^{-2} $kgmol^{-1}$ forms a cubic unit cell with edge length 405 pm. If its density is 2.7×10^3 kgm^{-3}, the radius of the element is approximately ____ $\times 10^{-12}$ m (to the nearest integer).

3. An element crystallises in a face-centred cubic (fcc) unit cell with cell edge a. The distance between the centres of two nearest octahedral voids in the crystal lattice is

(1) a (2) $\sqrt{2}a$

(3) $\frac{a}{\sqrt{2}}$ (4) $\frac{a}{2}$

4. A crystal is made up of metal ions 'M_1' ana 'M_2' and oxide ions. Oxide ions form a ccp lattice structure. The cation 'M_1' occupies 50% of octahedral voids and the cation 'M_2' occupies 12.5% of tetrahedral voids of oxide lattice. The oxidation numbers of 'M_1' and 'M_2' are, respectively :

(1) +2, +4 (2) +3, +1

(3) +1, +3 (4) +4, +2

CHEMICAL KINETICS

1. For the reaction

$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$

the observed rate expression is, rate = $k_f[NO]^2[H_2]$. The rate expression of the reverse reaction is :

(1) $k_b[N_2][H_2O]^2/[NO]$

(2) $k_b[N_2][H_2O]$

(3) $k_b[N_2][H_2O]^2$

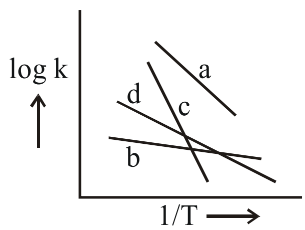
(4) $k_b[N_2][H_2O]^2/[H_2]$

2. The rate of a certain biochemical reaction at physiological temperature (T) occurs 10^6 times faster with enzyme than without. The change in the activation energy upon adding enzyme is :

(1) –6RT (2) +6RT

(3) +6(2.303)RT (4) –6(2.303)RT

3. Consider the following plots of rate constant versus $\frac{1}{T}$ for four different reactions. Which of the following orders is correct for the activation energies of these reactions?



(1) $E_b > E_d > E_c > E_a$

(2) $E_a > E_c > E_d > E_b$

(3) $E_c > E_a > E_d > E_b$

(4) $E_b > E_a > E_d > E_c$

4. For the following reactions

$A \xrightarrow{700\,K} Product$

$A \xrightarrow[catalyst]{500\,K} Product$

it was found that E_a is decreased by 30 kJ/mol in the presence of catalyst.
If the rate remains unchanged, the activation energy for catalysed reaction is (Assume pre exponential factor is same):

(1) 135 kJ/mol (2) 105 kJ/mol
(3) 198 kJ/mol (4) 75 kJ/mol

5. A sample of milk splits after 60 min. at 300 K and after 40 min. at 400 K when the population of *lactobacillus acidophilus* in it doubles. The activa tion energy (in kJ/ mol) for this process is closest to__________.

(Given, R = 8.3 J mol^{-1} K^{-1}, $ln\left(\frac{2}{3}\right) = 0.4$, $e^{-3} = 4.0$)

6. The number of molecules with energy greater than the threshold energy for a reaction increases five fold by a rise of temperature from 27 °C to 42 °C. Its energy of activation in J/mol is _______. (Take ln 5 = 1.6094; R = 8.314 J $mol^{-1}K^{-1}$)

7. If 75% of a first order reaction was completed in 90 minutes, 60% of the same reaction would be completed in approximately (in minutes) ________. (Take : log 2 = 0.30; log 2.5 = 0.40)

8. It is true that :

(1) A zero order reaction is a single step reaction
(2) A second order reaction is always a multistep reaction
(3) A first order reaction is always a single step reaction
(4) A zero order reaction is a multistep reaction

9. For the reaction $2A + 3B + \frac{3}{2}C \rightarrow 3P$, which statement is correct ?

(1) $\frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{dn_C}{dt}$

(2) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

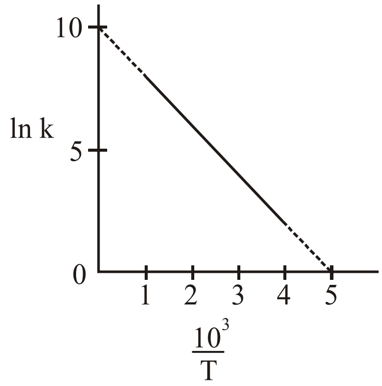
(3) $\frac{dn_A}{dt} = \frac{3}{2}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

(4) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$

10. A flask contains a mixture of compounds A and B. Both compounds decompose by first-order kinetics. The half-lives for A and B are 300 s and 180 s, respectively. If the concentrations of A and B are equal initially, the time required for the concentration of A to be four times that of B(in s) : (Use ln 2 = 0.693)

(1) 180 (2) 120
(3) 300 (4) 900

11. The rate constant (k) of a reaction is measured at different temperatures (T), and the data are plotted in the given figure. The activation energy of the reaction in kJ mol^{-1} is :
(R is gas constant)



(1) 2R (2) R
(3) 1/R (4) 2/R

12. The results given in the below table were obtained during kinetic studies of the following reaction:

$2A + B \longrightarrow C + D$

Experiment	[A]/molL^{-1}	[B]/molL^{-1}	Initial rate/molL^{-1} min^{-1}
I	0.1	0.1	6.00×10^{-3}
II	0.1	0.2	2.40×10^{-2}
III	0.2	0.1	1.20×10^{-2}
IV	X	0.2	7.20×10^{-2}
V	0.3	Y	2.88×10^{-1}

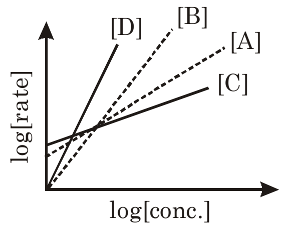
X and Y in the given table are respectively :

(1) 0.3, 0.4
(2) 0.4, 0.3
(3) 0.4, 0.4
(4) 0.3, 0.3

13. Consider the following reactions :

A → P1 ; B → P2 ; C → P3 ; D → P4

The order of the above reactions are a, b, c, and d, respectively. The following graph is obtained when log [rate] vs. log[conc] are plotted:



Among the following , the correct sequence for the order of the reactions is:

(1) a > b > c > d
(2) c > a > b > d
(3) d > b > a > c
(4) d > a > b > c

14. The rate of a reaction decreased by 3.555 times when the temperature was changed from 40°C to 30°C. The activation energy (in kJ mol^{-1}) of the reaction is_ _ _ _ _.

Take; R=8.314 J mol^{-1} K^{-1} In 3.555 = 1.268

RADIOACTIVITY

1. During the nuclear explosion, one of the products is ^{90}Sr with half life of 6.93 years. if 1 μg of ^{90}Sr was absorbed in the bones of a newly born baby in place of Ca, how much time, in years , is required to reduce it by 90% if it is not lost metabolically______ .

SURFACE CHEMISTRY

1. The flucculation value of HCl for arsenic sulphide sol. is 30 m mole L^{-1}. If H_2SO_4 is used for the flocculation of arsenic sulphide, the amount, in grams, of H_2SO_4 in 250 ml required for the above purpose is ______.

(molecular mass of H_2SO_4 = 98 g/mol)

2. As per Hardy-Schulze formulation, the flocculation values of the following for ferric hydroxide sol are in the order :

(1) $AlCl_3 > K_3[Fe(CN)_6] > K_2CrO_4 > KBr=KNO_3$
(2) $K_3[Fe(CN)_6] < K_2CrO_4 < AlCl_3 < KBr < KNO_3$
(3) $K_3[Fe(CN)_6] > AlCl_3 > K_2CrO_4 > KBr > KNO_3$
(4) $K_3[Fe(CN)_6] < K_2CrO_4 < KBr=KNO_3=AlCl_3$

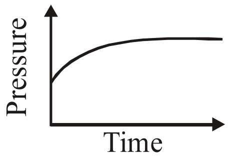
3. For the following Assertion and Reason, the correct option is

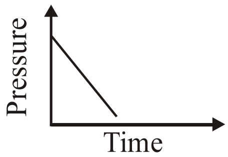
Assertion : For hydrogenation reactions, the catalytic activity increases from Group 5 to Group 11 metals with maximum activity shown by Group 7-9 elements.

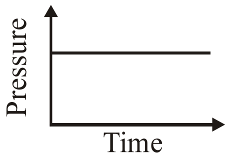
Reason : The reactants are most strongly adsorbed on group 7-9 elements.

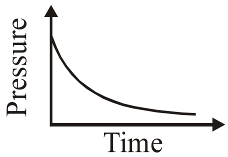
(1) Both assertion and reason are true but the reason is not the correct explanation for the assertion.
(2) Both assertion and reason are false.
(3) Both assertion and reason are true and the reason is the correct explanation for the assertion.
(4) The assertion is true, but the reason is false.

4. A mixture of gases O_2, H_2 and CO are taken in a closed vessel containing charcoal. The graph that represents the correct behaviour of pressure with time is :

(1)


(2)


(3)


(4)


5. Match the following :

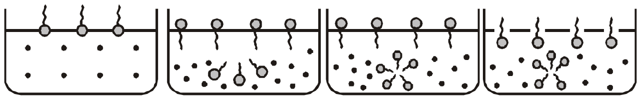
(i) Foam	(a) smoke
(ii) Gel	(b) cell fluid
(iii) Aerosol	(c) jellies
(iv) Emulsion	(d) rubber
	(e) froth
	(f) milk

(1) (i)-(b), (ii)-(c), (iii)-(e), (iv)-(d)
(2) (i)-(d), (ii)-(b), (iii)-(e), (iv)-(f)
(3) (i)-(e), (ii)-(c), (iii)-(a), (iv)-(f)
(4) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(e)

6. Tyndall effect of observed when :

(1) The diameter of dispersed particles is much smaller than the wavelength of light used
(2) The diameter of dispersed particles is much larger than the wavelength of light used
(3) The diameter of dispersed particles is similar to the wavelength of light used
(4) The refractive index of dispersed phase is greater than that of the dispersion medium

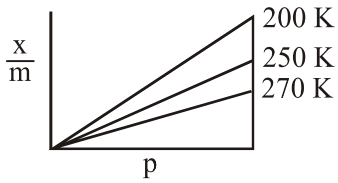
7. Identify the correct molecular picture showing that happens at the critical micellar concentration (CMC) of an aqueous solution of a surfactant (⚬ polar head; ∽ non-polar tail; • water).

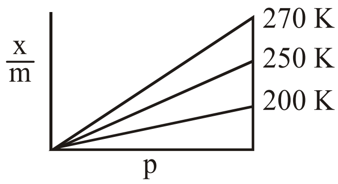


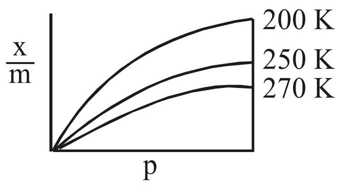
(A) (B) (C) (D)

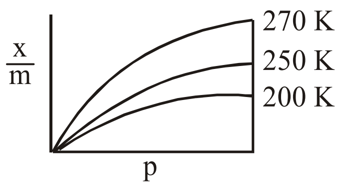
(1) (B) (2) (A) (3) (D) (4) (C)

8. Adsorption of a gas follows Freundlich adsorption isotherm. If x is the mass of the gas adsorbed on mass m of the adsorbent, the correct plot of $\frac{x}{m}$ versus p is :

(1)


(2)


(3)


(4)


9. The mass of gas adsorbed, x, per unit mass of adsorbate, m, was measured at various pressures, p. A graph between log $\frac{x}{m}$ and log p gives a straight line with slope equal to 2 and the intercept equal to 0.4771. The value of $\frac{x}{m}$ at a pressure of 4 atm is : (Given log 3 = 0.4771)

10. Amongst the following statements regarding adsorption, those that are valid are :

(a) ΔH becomes less negative as adsorption proceeds.

(b) On a given adsorbent, ammonia is adsorbed more than nitrogen gas.

(c) On adsorption, the residual force acting along the surface of the adsorbent increases.

(d) With increase in temperature, the equilibrium concentration of adsorbate increases.

(1) (b) and (c) (2) (a) and (b)

(3) (d) and (a) (4) (c) and (d)

11. For Freundlich adsorption isotherm, a plot of log (x/m) (y-axis) and log p (x-axis) gives a straight line. The intercept and slope for the line is 0.4771 and 2, respectively. The mass of gas, adsorbed per gram of adsorbent if the initial pressure is 0.04 atm, is_ _ _ _ _ _ × 10^{-4}g. (log 3 = 0.4771)

ELECTROCHEMISTRY

1. Given that the standard potentials (E°) of Cu^{2+}/Cu and Cu^{+}/Cu are 0.34 V and 0.522 V respectively, the E° of Cu^{2+}/Cu^{+} is :

(1) +0.158 V (2) 0.182 V

(3) –0.182 V (4) –0.158 V

2. The equation that is incorrect is -

(1) $\left(\Lambda_m^0\right)_{NaBr}-\left(\Lambda_m^0\right)_{NaI}=\left(\Lambda_m^0\right)_{KBr}-\left(\Lambda_m^0\right)_{NaBr}$

(2) $\left(\Lambda_m^0\right)_{H_2O}=\left(\Lambda_m^0\right)_{HCl}+\left(\Lambda_m^0\right)_{NaOH}-\left(\Lambda_m^0\right)_{NaCl}$

(3) $\left(\Lambda_m^0\right)_{KCl}-\left(\Lambda_m^0\right)_{NaCl}=\left(\Lambda_m^0\right)_{KBr}-\left(\Lambda_m^0\right)_{NaBr}$

(4) $\left(\Lambda_m^0\right)_{NaBr}-\left(\Lambda_m^0\right)_{NaCl}=\left(\Lambda_m^0\right)_{KBr}-\left(\Lambda_m^0\right)_{KCl}$

3. What would be the electrode potential for the given half cell reaction at pH = 5 ?______

$2H_2O \rightarrow O_2 + 4H^{\oplus} + 4e^-$; E^0_{red} = 1.23 V

(R = 8.314 J mol^{-1} K^{-1} ; Temp = 298 K; oxygen under std. atm. pressure of 1 bar)

4. For an electrochemical cell

$Sn(s)\,|\,Sn^{2+}\,(aq,1M)\,||\,Pb^{2+}\,(aq,1M)\,|\,Pb(s)$

the ratio $\frac{[Sn^{2+}]}{[Pb^{2+}]}$ when this cell attains equilibrium is _________.

(Given $E^0_{Sn^{2+}|Sn}=-0.14V$,

$E^0_{Pb^{2+}|Pb}=-0.13V, \frac{2.303RT}{F}=0.06$)

5. 108 g of silver (molar mass 108 g mol^{-1}) is deposited at cathode from $AgNO_3(aq)$ solution by a certain quantity of electricity. The volume (in L) of oxygen gas produced at 273 K and 1 bar pressure from water by the same quantity of electricity is _______.

6. Amongst the following, the form of water with the lowest ionic conductance at 298 K is:

(1) distilled water

(2) water from a well

(3) saline water used for intravenous injection

(4) sea water

7. 250 mL of a waste solution obtained from the workshop of a goldsmith contains 0.1 M $AgNO_3$ and 0.1 M AuCl. The solution was electrolyzed at 2 V by passing a current of 1 A for 15 minutes. The metal/metals electrodeposited will be :-

$\left(E^0_{Ag^+/Ag} = 0.80V,\ E^0_{Au^+/Au} = 1.69V\right)$

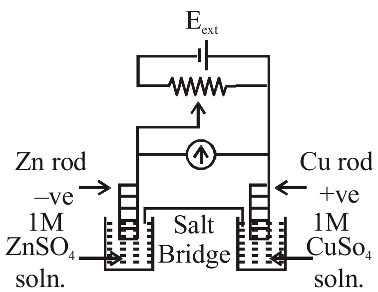
(1) only silver

(2) only gold

(3) silver and gold in equal mass proportion

(4) silver and gold in proportion to their atomic weights

8.



$E^o_{Cu^{2+}|Cu} = +0.34V$

$E^o_{Zn^{2+}|Zn} = -0.76V$

Identify the incorrect statement from the options below for the above cell :

(1) If $E_{ext} > 1.1$ V, Zn dissolves at Zn electrode and Cu deposits at Cu electrode

(2) If $E_{ext} > 1.1$ V, e^- flows from Cu to Zn

(3) If $E_{ext} = 1.1$ V, no flow of e^- or current occurs

(4) If $E_{ext} < 1.1$ V, Zn dissolves at anode and Cu deposits at cathode

9. The photoelectric current from Na (work function, $w_0 = 2.3$ eV) is stopped by the output voltage of the cell

$Pt(s)|H_2(g, 1bar)|HCl(aq., pH = 1)|AgCl(s)|Ag(s)$

The pH of aq. HCl required to stop the photoelectric current from K(w_0 = 2.25eV), all other conditions remaining the same, is ___× 10^{-2} (to the nearest integer).

Given, $2.303\frac{RT}{F} = 0.06V; E^0_{AgCl|Ag|Cl^-} = 0.22V$

10. An acidic solution of dichromate is electrolyzed for 8 minutes using 2A current. As per the following equation

$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

The amount of Cr^{3+} obtained was 0.104 g. The efficiency of the process(in%) is

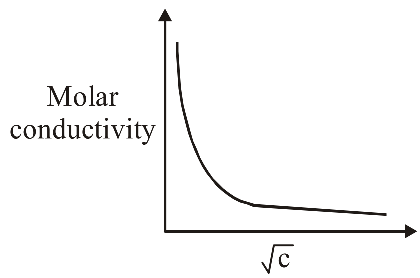
(Take : F = 96000 C, At. mass of chromium = 52)

______.

11. an oxidation-reduction reaction in which 3 electrons are transferred has a ΔG^o of 17.37 kJ mol^{-1} at 25°C. The value of E^o_{cell} (in V) is _____ × 10^{-2}

(1 F = 96,500 C mol^{-1})

12. The variation of molar conductivity with concentration of an electrolyte (X) in aqueous solution is shown in the given figure.



The electrolyte X is :

(1) CH_3COOH (2) KNO_3

(3) HCl (4) NaCl

13. For the disproportionation reaction $2Cu^+ (aq) \rightleftharpoons Cu(s) + Cu^{2+}(aq)$ at 298 K, ln K (where K is the equilibrium constant) is _____ × 10^{-1} .

Given

$(E^0_{Cu^{2+}/Cu^+} = 0.16V\ \ E^0_{Cu^+/Cu} = 0.52V\ \ \frac{RT}{F} = 0.025)$

14. Potassium chlorate is prepared by the electrolysis of KCl in basic solution

$6OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$

If only 60% of the current is utilized in the reaction, the time (rounded to the nearest hour) required to produce 10 g of $KClO_3$ using a current of 2 A is_____.

(Given : F = 96,500 C mol^{-1} molar mass of $KClO_3$=122 $gmol^{-1}$)

15. For the given cell ;

$Cu(s)|Cu^{2+}(C_1M)||Cu^{2+}(C_2M)|Cu(s)$ change in Gibbs energy (ΔG) is negative, if :

(1) $C_1 = 2C_2$ (2) $C_2 = C1/\sqrt{2}$

(3) $C_1 = C_2$ (4) $C_2 = \sqrt{2}C_1$

LIQUID SOLUTION

1. At 35°C, the vapour pressure of CS_2 is 512 mm Hg and that of acetone is 344 mm Hg. A solution of CS_2 in acetone has a total vapour pressure of 600 mm Hg. The false statement amongst the following is :

(1) heat must be absorbed in order to produce the solution at 35°C

(2) Raoult's law is not obeyed by this system

(3) a mixture of 100 mL CS_2 and 100 mL acetone has a volume < 200 mL

(4) CS_2 and acetone are less attracted to each other than to themselves

2. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non volatile solute are together sealed in a container. Over time -

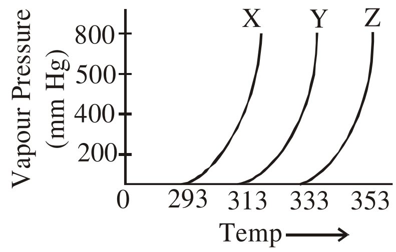
(1) The volume of the solution does not change and the volume of the solvent decreases

(2) The volume of the solution decrease and the volume of the solvent increases

(3) The volume of the solution increase and the volume of the solvent decreases

(4) The volume of the solution and the solvent does not change

3. A graph of vapour pressure and temperature for three different liquids X, Y and Z is shown below:



The following inferences are made :

(A) X has higher intermolecular interactions compared to Y.

(B) X has lower intermolecular interactions compared to Y.

(C) Z has lower intermolecular interactions compared to Y.

The correct inference(s) is/are :

(1) A

(2) (C)

(3) (B)

(4) (A) and (C)

4. How much amount of NaCl should be added to 600 g of water (ρ = 1.00 g/mL) to decrease the freezing point of water to – 0.2 °C ? _____. (The freezing point depression constant for water = 2K kg mol^{-1})

5. The osmotic pressure of a solution of NaCl is 0.10 atm and that of a glucose solution is 0.20 atm. The osmotic pressure of a solution formed by mixing 1 L of the sodium chloride solution with 2 L of the glucose solution is x × 10^{-3} atm. x is _____. (nearest integer) :-

6. At 300 K, the vapour pressure of a solution containing 1 mole of n-hexane and 3 moles of n-heptane is 550 mm of Hg. At the same temperature, if one more mole of n-heptane is added to this solution, the vapour pressure of the solution increases by 10 mm of Hg. What is the vapour pressure in mm Hg of n-heptane in its pure state ____ ?

7. Henry's constant (in kbar) for four gases α, β, γ and δ in water at 298 K is given below :

	α	β	γ	δ
K_H	50	2	2×10^{-5}	0.5

(density of water = 10^3 kg m^{-3} at 298 K)

This table implies that :

(1) The pressure of a 55.5 molal solution of γ is 1 bar

(2) The pressure of a 55.5 molal solution of δ is 250 bar

(3) Solubility of γ at 308 K is lower than at 298 K

(4) α has the highest solubility in water at a given pressure

8. If 250 cm^3 of an aqueous solution containing 0.73 g of a protein A is isotonic with one litre of another aqueous solution containing 1.65 g of a protein B, at 298 K, the ratio of the molecular mases of A and B is _______ $\times 10^{-2}$ (to the nearest integer).

9. A set of solutions is prepared using 180 g of water as a solvent and 10 g of different non-volatile solutes A, B and C. The relative lowering of vapour pressure in the presence of these solutes are in the order [Given, molar mass of A = 100 g mol^{-1}; B = 200 g mol^{-1}; C = 10,000 g mol^{-1}]

(1) A > B > C

(2) A > C > B

(3) C > B > A

(4) B > C > A

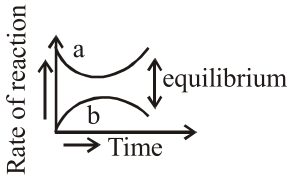
CHEMICAL EQUILIBRIUM

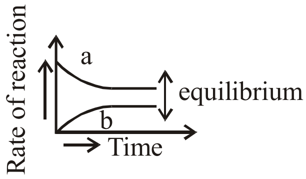
1. If the equilibrium constant for $A \rightleftharpoons B+C$ is $K_{eq}^{(1)}$ and that of $B+C \rightleftharpoons P$ is $K_{eq}^{(2)}$, the equilibrium constant for $A \rightleftharpoons P$ is :-

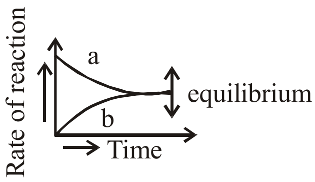
(1) $K_{eq}^{(2)} - K_{eq}^{(1)}$ (2) $K_{eq}^{(1)}K_{eq}^{(2)}$

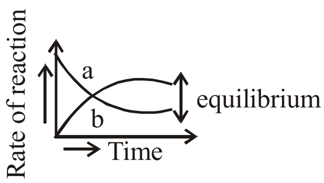
(3) $K_{eq}^{(1)} / K_{eq}^{(2)}$ (4) $K_{eq}^{(1)} + K_{eq}^{(2)}$

2. For the equilibrium $A \rightleftharpoons B$, the variation of the rate of the forward (a) and reverse (b) reaction with time is given by

(1) 

(2) 

(3) 

(4) 

3. For a reaction $X + Y \rightleftharpoons 2Z$, 1.0 mol of X, 1.5 mol of Y and 0.5 mol of Z were taken in a 1 L vessel and allowed to react. At equilibrium, the concentration of Z was 1.0 mol L^{-1}. The equilibrium constant of the reaction is ______ $\frac{x}{15}$. The value of x is ________ .

4. The value of K_C is 64 at 800 K for the reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The value of K_C for the following reaction is :

$$NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{3}{2}H_2(g)$$

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) 8 (4) $\frac{1}{64}$

5. For the reaction :

$$Fe_2N(s) + \frac{3}{2}H_2(g) \rightleftharpoons 2Fe(s) + NH_3(g)$$

(1) $K_C = K_P(RT)$ (2) $K_C = K_P(RT)^{-1/2}$

(3) $K_C = K_P(RT)^{-3/2}$ (4) $K_C = K_P(RT)^{1/2}$

ANSWER KEY

MOLE CONCEPT

Que.	1	2	3	4	5	6	7	8	9
Ans.	3	3	4.95 to 4.97	2120 to 2140	1	4	18	5.00	50.00

CONCENTRATION TERMS

Que.	1	2	3	4	5	6	7
Ans.	14.00	10	100	47	25	2	4

REDOX REACTIONS

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3.00	4	4	100	19	10	85	10.00	50.00	19.00

IDEAL GAS

Que.	1	2	3	4
Ans.	4	3	1	750.00

ATOMIC STRUCTURE

Que.	1	2	3	4	5	6	7	8
Ans.	2	2	4	1	3	4	1	222.00

CHEMICAL EQUILIBRIUM

Que.	1	2	3
Ans.	NTA-1, ALLEN 1 or Bonus	1	2

IONIC EQUILIBRIUM

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	10.60	5.22 to 5.24	3	2	2	1	3	3	37	2.00
Que.	**11**									
Ans.	4									

THERMODYNAMICS

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	–2.70 to –2.71	48.00	6.25	1	2.17 to 2.23	4	1	NTA:–13538.00 Allen 13537.57	NTA:–189494.00 Allen 189494.39	96500.00
Que.	**11**									
Ans.	3									

THERMODYNAMICS

Que.	1	2	3	4
Ans.	–192.50 or –85.00	4	2	NTA –326400.00 Allne 326400.00

SOLID STATE

Que.	1	2	3	4
Ans.	1	143	3	1

CHEMICAL KINETICS

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	4	3	4	3.98 to 4.00 or –3.98 to –4.00	NTA 84297 Allen 84297.47 or 84297.48	60	4	4	4
Que.	**11**	**12**	**13**	**14**						
Ans.	1	1	3	NTA 100.00 ALLEN 99.98						

RADIOACTIVITY

Que.	1
Ans.	23 to 23.03

SURFACE CHEMISTRY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	0.36 to 0.38	4	4	4	3	3	3	3	NTA 6.00 ALLEN 48.00	2
Que.	**11**									
Ans.	48.00									

ELECTROCHEMISTRY										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	1	–0.93 to –0.94	2.13 to 2.17	5.66 to 5.68	1	4	1	NTA 58 ALLEN 142	60
Que.	**11**	**12**	**13**	**14**	**15**					
Ans.	6	1	144.00	11.00	4					

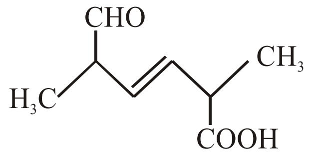
LIQUID SOLUTION										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
Ans.	3	3	3	1.74 to 1.76 or 0.03	167	600	2	177	1	

CHEMICAL EQUILIBRIUM					
Que.	**1**	**2**	**3**	**4**	**5**
Ans.	2	3	16	2	4

JANUARY & SEPTEMBER 2020 ATTEMPT (OC)

NOMENCLATURE

1. The IUPAC name for the following compound is:



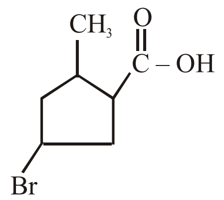
(1) 2, 5-dimethyl-6-carboxy-hex-3-enal

(2) 6-formyl-2-methyl-hex-3-enoic acid

(3) 2, 5-dimethyl-5-carboxy-hex-3-enal

(4) 2, 5-dimethyl-6-oxo-hex-3-enoic acid

2. The IUPAC name of the following compound is :



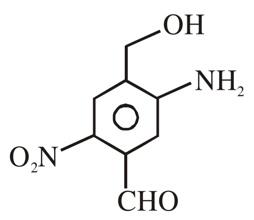
(1) 4-Bromo-2-methylcyclopentane carboxylic acid

(2) 5-Bromo-3-methylcyclopentanoic acid

(3) 3-Bromo-5-methylcyclopentane carboxylic acid

(4) 3-Bromo-5-methylcyclopentanoic acid

3. The IUPAC name of the following compound is :



(1) 3-amino-4-hydroxymethyl-5-nitrobenzaldehyde

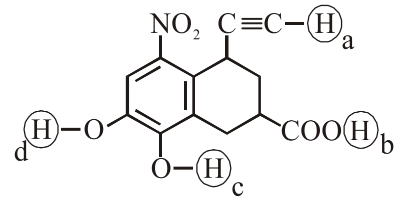
(2) 2-nitro-4-hydroxymethyl-5-aminobenzaldehyde

(3) 4-amino-2-formyl-5-hydroxymethylnitrobenzene

(4) 5-amino-4-hydroxymethyl-2-nitrobenzaldehyde

ACIDITY & BASICITY

1. Arrange the following labelled hydrogens in decreasing order of acidity :



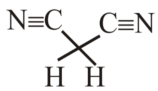
(1) b > c > d > a

(2) c > b > a > d

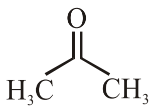
(3) b > a > c > d

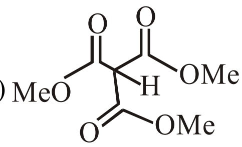
(4) c > b > d > a

2. Which one of the following compounds possesses the most acidic hydrogen ?

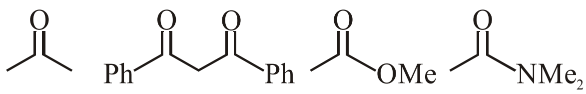
(1) 

(2) $H_3C–C≡C–H$

(3) 

(4) 

3. The increasing order of the acidity of the α-hydrogen of the following compounds is :



(A) (B) (C) (D)

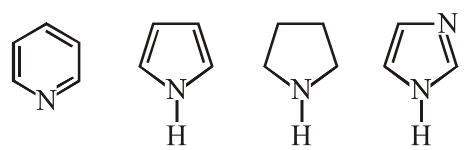
(1) (C) < (A) < (B) < (D)

(2) (B) < (C) < (A) < (D)

(3) (A) < (C) < (D) < (B)

(4) (D) < (C) < (A) < (B)

4. The increasing order of basicity of the following compounds is



(A) (B) (C) (D)

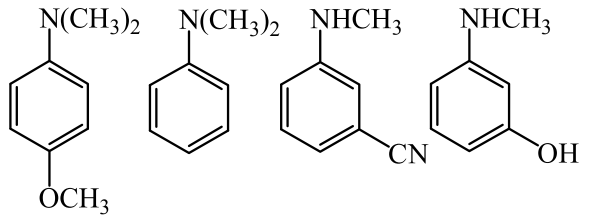
(1) (A) < (B) < (C) < (D)

(2) (B) < (A) < (C) < (D)

(3) (D) < (A) < (B) < (C)

(4) (B) < (A) < (D) < (C)

5. The increasing order of pK_b values of the following compounds is -



I II III IV

(1) I < II < IV < III

(2) II < IV < III < I

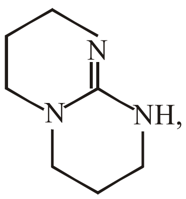
(3) II < I < III < IV

(4) I < II < III < IV

ELECTRONIC DISPLACEMENT EFFECT

1. The increasing order of pK_b for the following compounds will be :

$NH_2 - CH = NH$,
(A)

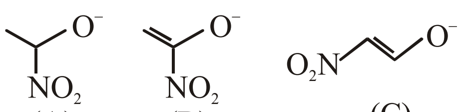


(B)

CH_3NHCH_3
(C)

(1) (A) < (B) < (C) (2) (C) < (A) < (B)
(3) (B) < (A) < (C) (4) (B) < (C) < (A)

2. The correct order of stability for the following alkoxides is :



(A) (B) (C)

(1) (C) > (B) > (A) (2) (C) > (A) > (B)
(3) (B) > (C) > (A) (4) (B) > (A) > (C)

3. Arrange the following compounds in increasing order of C–OH bond length : methanol, phenol, p-ethoxyphenol

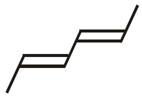
(1) phenol < methanol < p-ethoxyphenol

(2) phenol < p-ethoxyphenol < methanol

(3) methanol < p-ethoxyphenol < phenol

(4) methanol < phenol < p-ethoxyphenol

4. The correct order of heat of combustion for following alkadienes is :

(a)  (b)  (c) 

(1) (a) < (b) < (c) (2) (b) < (c) < (a)
(3) (c) < (b) < (a) (4) (a) < (c) < (b)

5. The increasing order of basicity for the following intermediates is (from weak to strong)

(i) $H_3C - \overset{\ominus}{C}(CH_3) - CH_3$

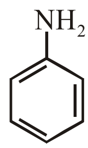
(ii) $H_2C = CH - \overset{\ominus}{C}H_2$

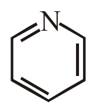
(iii) $HC \equiv \overset{\ominus}{C}$ (iv) $\overset{\ominus}{C}H_3$ (v) $\overset{\ominus}{C}N$

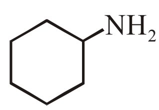
(1) (v) < (i) < (iv) < (ii) < (iii)
(2) (iii) < (i) < (ii) < (iv) < (v)
(3) (v) < (iii) < (ii) < (iv) < (i)
(4) (iii) < (iv) < (ii) < (i) < (v)

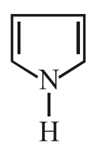
6. Which of the following has the shortest C-Cl bond?
(1) Cl–CH=CH–OCH_3
(2) C1–CH=CH–CH_3
(3) C1–CH=CH_2
(4) C1–CH=CH–NO_2

7. The decreasing order of basicity of the following amines is :


(I)

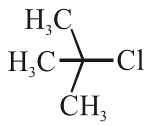

(II)

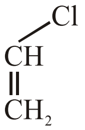
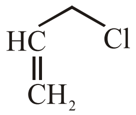

(III)


(IV)

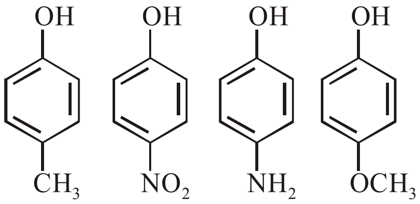
(1) (I) > (III) > (IV) > (II)
(2) (III) > (I) > (II) > (IV)
(3) (III) > (II) > (I) > (IV)
(4) (II) > (III) > (IV) > (I)

8. Among the following compounds, which one has the shortest C—Cl bond ?

(1) $H_3C–Cl$ (2) 

(3)  (4) 

9. The increasing order of boiling points of the following compounds is :



I II III IV

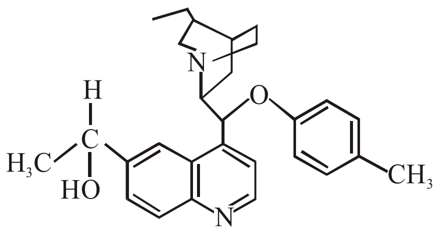
(1) I < IV < III < II (2) IV < I < II < III

(3) I < III < IV < II (4) III < I < II < IV

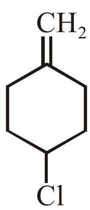
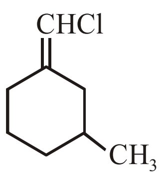
ISOMERISM

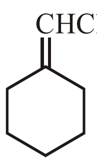
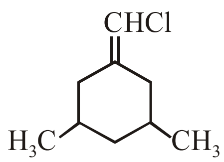
1. The number of chiral carbons in chloramphenicol is _______ .

2. The number of chiral carbons present in the molecule given below is _____ .



3. Among the following compounds, geometrical isomerism is exhibited by :

(1)  (2) 

(3)  (4) 

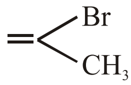
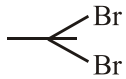
4. Which of the following compound shows geometrical isomerism

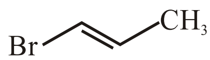
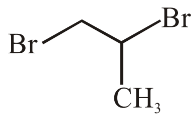
(1) 2-methylpent-2-ene (2) 4-methylpent-l-ene

(3) 4-methylpent-2-ene (4) 2-methylpent-l-ene

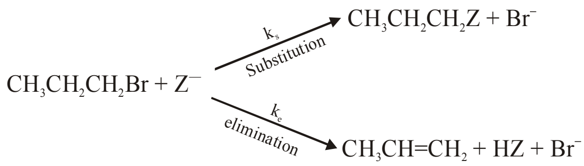
HALOGEN DERIVATIVE

1. 1-methyl ethylene oxide when treated with an excess of HBr produces :

(1)  (2) 

(3)  (4) 

2. For the following reactions :



where

$Z^- = CH_3CH_2O^-$ (A) or $H_3C–C(CH_3)_2–O^-$ (B),

k_s and k_e , are , respectively, the rate constants for the substitution and elimination, and $\mu = \frac{k_s}{k_e}$, the correct options is -

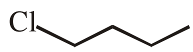
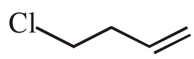
(1) $\mu_B > \mu_A$ and $k_e(B) > k_e(A)$

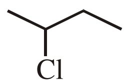
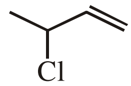
(2) $\mu_B > \mu_A$ and $k_e(A) > k_e(B)$

(3) $\mu_A > \mu_B$ and $k_e(B) > k_e(A)$

(4) $\mu_A > \mu_B$ and $k_e(A) > k_e(B)$

3. The decreasing order of reactivity towards dehydrohalogenation (E_1) reaction of the following compounds is :

(A)  (B) 

(C)  (D) 

(1) B > D > A > C (2) B > D > C > A

(3) D > B > C > A (4) B > A > D > C

4. Which of the following compounds will show retention in configuration on nucleophic substitution by OH^- ion ?

(1) $CH_3–CH(C_2H_5)–CH_2Br$ (2) $CH_3–CH(C_6H_5)–Br$

(3) $CH_3–CH(CH_3)–Br$ (4) $CH_3–C(Br)(H)–C_6H_{13}$

5. The major product obtained from E_2-elimination of 3-bromo-2-fluoropentane is:

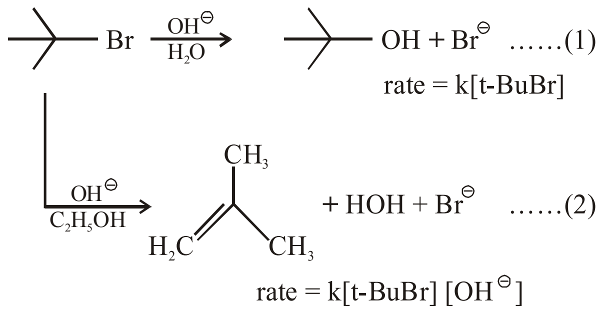
(1) $CH_3CH_2-\underset{}{\overset{Br}{\overset{|}{C}}}H-CH=CH_2$

(2) $CH_3-CH_2-\overset{Br}{\overset{|}{C}}=CH-CH_3$

(3) $CH_3-CH=CH-\overset{F}{\overset{|}{C}}H-CH_3$

(4) $CH_3CH_2CH=\underset{CH_3}{\underset{|}{C}}-F$

6. Consider the reaction sequence given below :



Which of the following statements is true :

(1) Changing the concentration of base will have no effect on reaction (1)

(2) Changing the concentration of base will have no effect on reaction (2)

(3) Changing the base from $OH^{\ominus}$ to $^{\ominus}OR$ will have no effect on reaction (2)

(4) Doubling the concentration of base will double the rate of both the reactions.

7. The mechanism of S_N1 reaction is given as :

$$R-X \rightarrow \underset{\text{Ion pair}}{R^{\oplus}X^{\ominus}} \rightarrow \underset{\text{Solvent separated ion pair}}{R^{\oplus}||X^{\ominus}} \xrightarrow{Y^{\ominus}} R-Y+X^{\ominus}$$

A student writes general characteristics based on the given mechanism as :

(a) The reaction is favoured by weak nucleophiles

(b) $R^{\oplus}$ would be easily formed if the substituents are bulky

(c) The reaction is accompained by recemization

(d) The reaction is favoured by non-polar solvents.

Which observations are correct ?

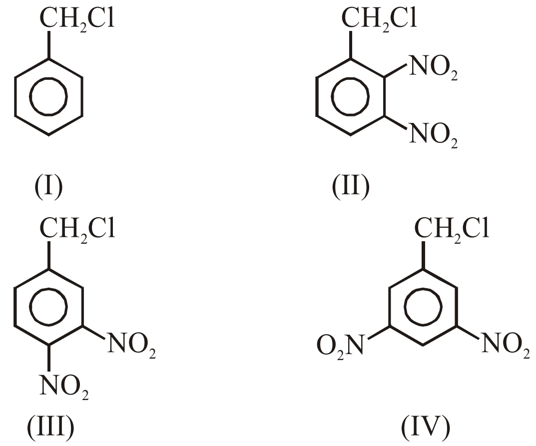
(1) b and d (2) a and c

(3) a, b and c (4) a and b

8. The total number of monohalogenated organic products in the following (including stereoisomers) reaction is ______.

$$\underset{\text{(simplest optically active alkene)}}{A} \xrightarrow[\text{(ii) } X_2/\Delta]{\text{(i) } H_2/Ni/\Delta}$$

9. The decreasing order of reactivity of the following compounds towards nucleophilic substitution (S_N2) is :



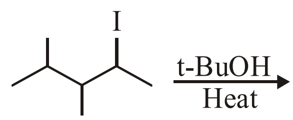
(1) (IV) > (II) > (III) > (I)

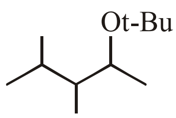
(2) (II) > (III) > (IV) > (I)

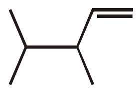
(3) (II) > (III) > (I) > (IV)

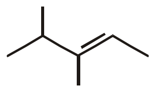
(4) (III) > (II) > (IV) > (I)

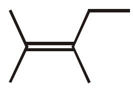
10. The major product in the following reaction is :



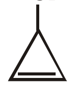
(1) 

(2) 

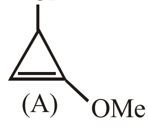
(3) 

(4) 

11. The decreasing order of reactivity of the following organic molecules towards $AgNO_3$ solution is :



(A)



(A)

$CH_3\underset{Cl}{\underset{|}{C}}HCH_3$ (C)

$CH_3\underset{Cl}{\underset{|}{C}}HCH_2NO_2$ (D)

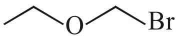
(1) (A) > (B) > (D) > (C)

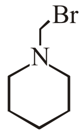
(2) (A) > (B) > (C) > (D)

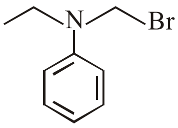
(3) (C) > (D) > (A) > (B)

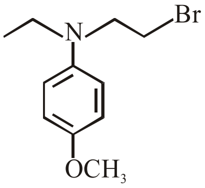
(4) (B) > (A) > (C) > (D)

12. Which of the following compounds will form the precipitate with aq. $AgNO_3$ solution most readily?

(1) 

(2) 

(3) 

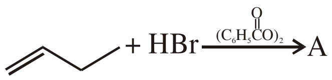
(4) 

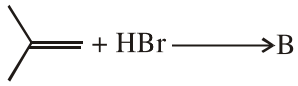
13. The major product formed in the following reaction is :

$CH_3CH = CHCH(CH_3)_2 \xrightarrow{HBr}$

(1) $CH_3\ CH_2\ CH_2\ C(Br)\ (CH_3)_2$
(2) $Br(CH_2)_3\ CH(CH_3)_2$
(3) $CH_3\ CH_2\ CH(Br)\ CH(CH_3)_2$
(4) $CH_3\ CH(Br)\ CH_2\ CH(CH_3)_2$

14. The increasing order of the boiling points of the major products A, B and C of the following reactions will be :

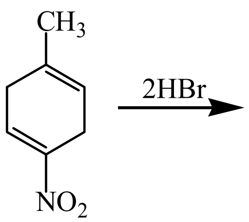
(a) 

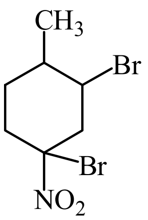
(b) 

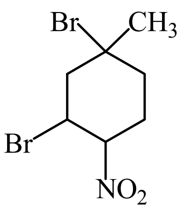
(c) 

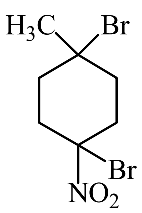
(1) C < A < B (2) B < C < A
(3) A < B < C (4) A < C < B

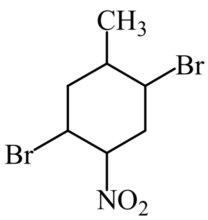
15. The major product of the following reaction is



(1) 

(2) 

(3) 

(4) 

ALCOHOL & ETHER

1. Consider the following reactions :

(a) $(CH_3)_3CCH(OH)CH_3 \xrightarrow{conc.H_2SO_4}$

(b) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{alc.KOH}$

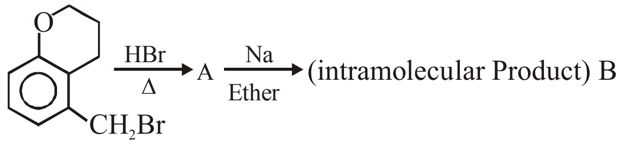
(c) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow[\text{It should be } (CH_3)_3CO^{\ominus}K^{\oplus}]{\text{given by NTA } (CH_3)_3O^{\ominus}K^{\oplus}}$

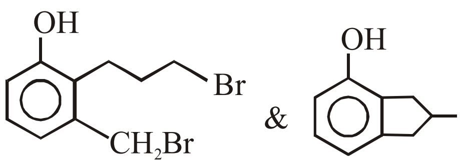
(d) $(CH_3)_2\underset{\displaystyle OH}{\underset{|}{C}}–CH_2–CHO \xrightarrow{\Delta}$

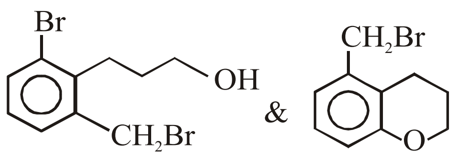
Which of these reaction(s) will not produce Saytzeff product ?

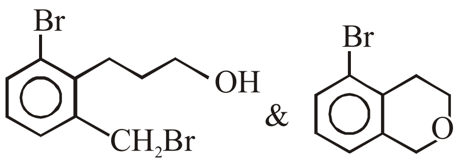
(1) (c) only
(2) (a), (c) and (d)
(3) (d) only
(4) (b) and (d)

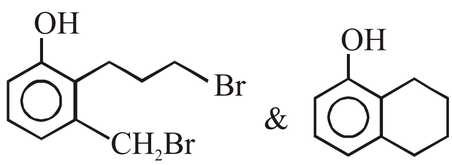
2. In the following reaction squence, structures of A and B, respectively will be :



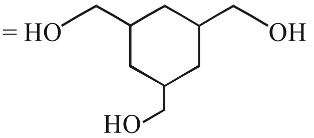
(1) 

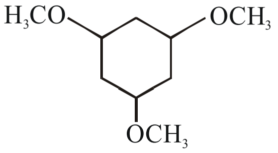
(2) 

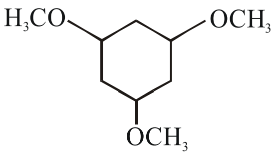
(3) 

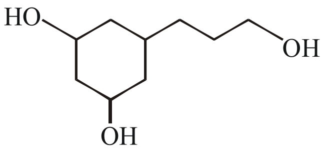
(4) 

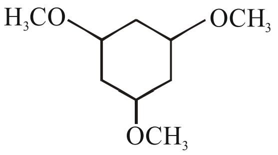
3. Among the compounds A and B with molecular formula $C_9H_{18}O_3$, A is having higher boiling point than B. The possible structures of A and B are :

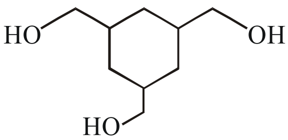
(1) A = 

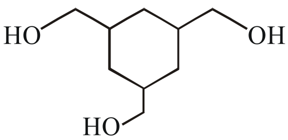
B = 

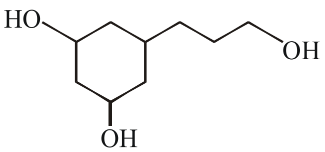
(2) A = 

B = 

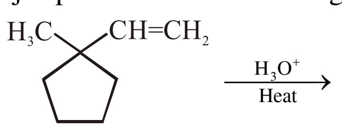
(3) A = 

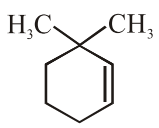
B = 

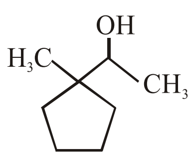
(4) A = 

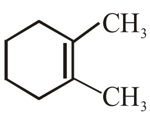
B = 

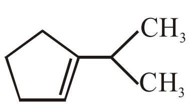
4. The major product in the following reaction is :



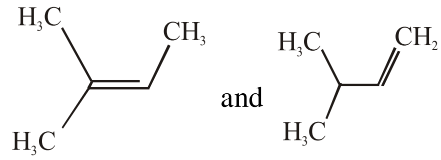
(1) 

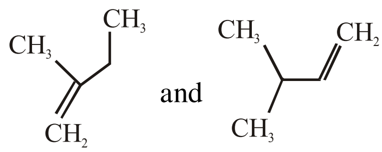
(2) 

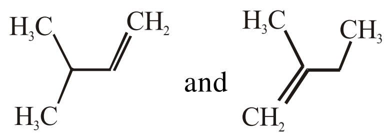
(3) 

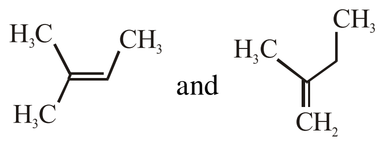
(4) 

5. When neopentyl alcohol is heated with an acid, it slowly converted into an 85 : 15 mixture of alkenes A and B, respectively. What are these alkenes ?

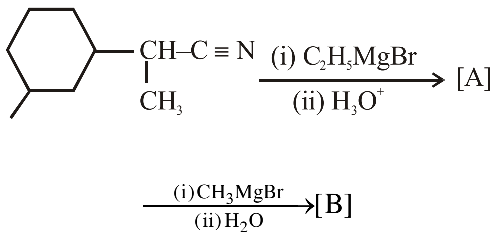
(1) 

(2) 

(3) 

(4) 

6. The number of chiral centres present in [B] is _______.



7. The major product [B] in the following reactions is :-

$$CH_3-CH_2-\underset{}{\overset{CH_3}{\overset{|}{C}H}}-CH_2-OCH_2-CH_3 \xrightarrow[\text{Heat}]{HI} [A]\ \text{alcohol} \xrightarrow[\Delta]{H_2SO_4} [B]$$

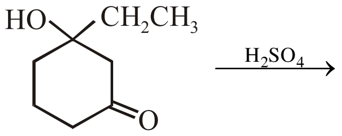
(1) $CH_3-CH_2-\overset{CH_3}{\overset{|}{C}}=CH_2$

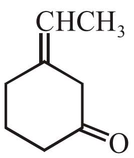
(2) $CH_3-CH_2-CH=CH-CH_3$

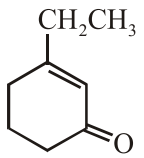
(3) $CH_2=CH_2$

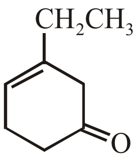
(4) $CH_3-CH=\overset{CH_3}{\overset{|}{C}}-CH_3$

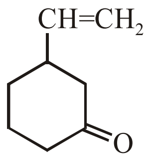
8. The major product of the following reaction is:



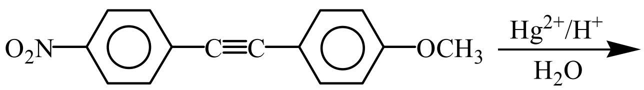
(1) 

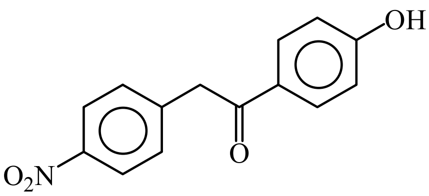
(2) 

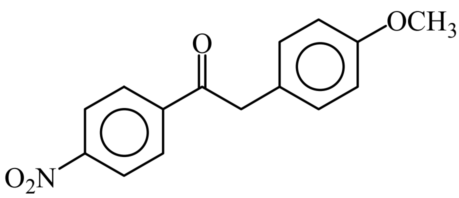
(3) 

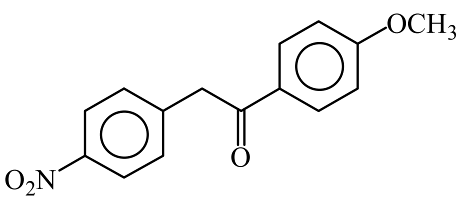
(4) 

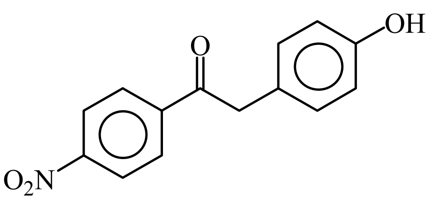
9. The major product obtained from the following reaction is -



(1) 

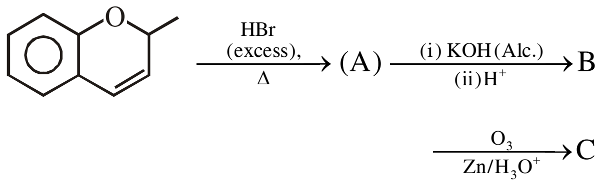
(2) 

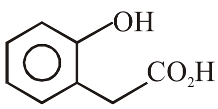
(3) 

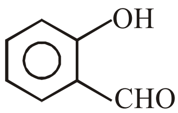
(4) 

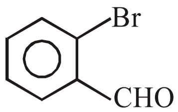
OXIDATION

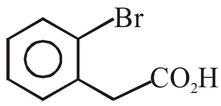
1. The major aromatic product C in the following reaction sequence will be :



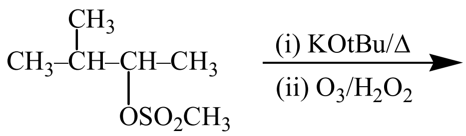
(1) 

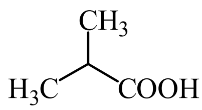
(2) 

(3) 

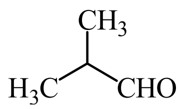
(4) 

2. The major products of the following reaction are :

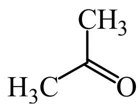


(1) 

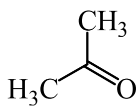
+ HCOOH

(2) 

+ HCHO

(3) 

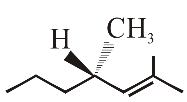
+ CH_3CHO

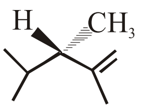
(4) 

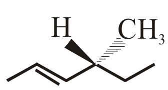
+ CH_3COOH

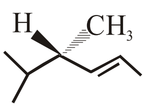
REDUCTION

1. Which of the following compounds produces an optically inactive compound on hydrogenation ?

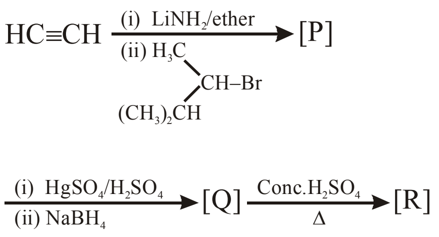
(1) 

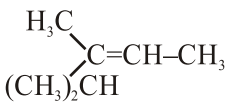
(2) 

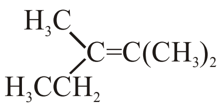
(3) 

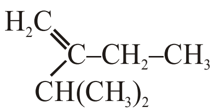
(4) 

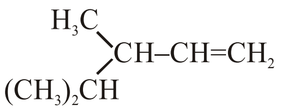
2. The major product [R] in the following sequence of reactions is :-



(1) 

(2) 

(3) 

(4) 

3. The most appropriate reagent for conversion of C_2H_5CN into $CH_3CH_2CH_2NH_2$ is :

(1) $Na(CN)BH_3$

(2) $LiAlH_4$

(3) $NaBH_4$

(4) CaH_2

4. The correct match between **Item-I** (starting material) and **Item-II** (reagent) for the preparation of benzaldehyde is :

Item-I	**Item-II**
(I) Benzene	(P) HCl and $SnCl_2$, H_3O^+
(II) Benzonitrile	(Q) H_2, Pd-$BaSO_4$, S and quinoline
(III) Benzoyl Chloride	(R) CO, HCl and $AlCl_3$

(1) (I)-(Q), (II)-(R) and (III)-(P)

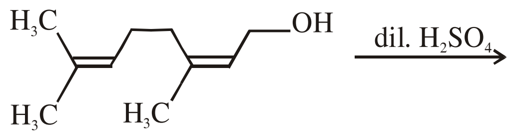
(2) (I)-(R), (II)-(Q) and (III)-(P)

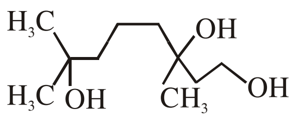
(3) (I)-(R), (II)-(P) and (III)-(Q)

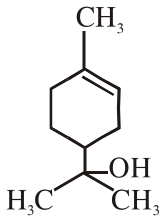
(4) (I)-(P), (II)-(Q) and (III)-(R)

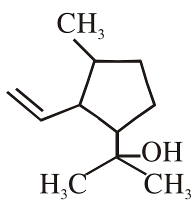
HYDROCARBON

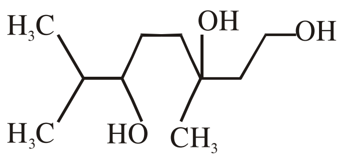
1. The major product of the following reaction is :



(1) 

(2) 

(3) 

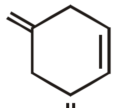
(4) 

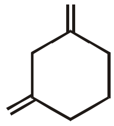
2. An unsaturated hydrocarbon X absorbs two hydrogen molecules on catalytic hydrogenattion, and also gives following reaction :

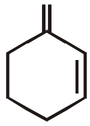
$$X \xrightarrow[Zn/H_2O]{O_3} A \xrightarrow{[Ag(NH_3)_2]^+}$$

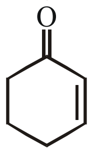
B(3 - oxo - hexanedicarboxylic acid)

X will be :-

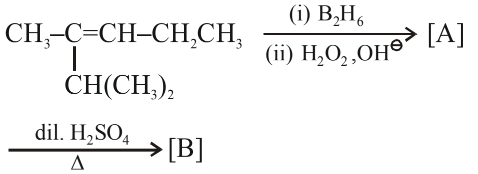
(1) 

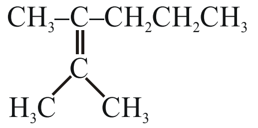
(2) 

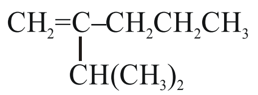
(3) 

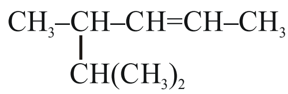
(4) 

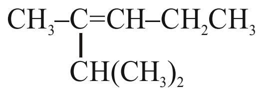
3. The major product [B] in the following sequence of reactions is :-



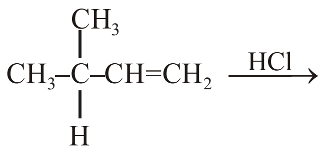
(1) 

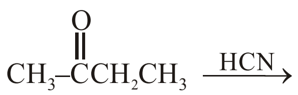
(2) 

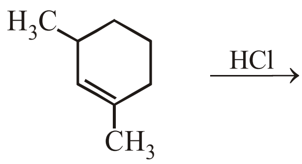
(3) 

(4) 

4. Which of the following reactions will not produce a racemic product ?

(1) 

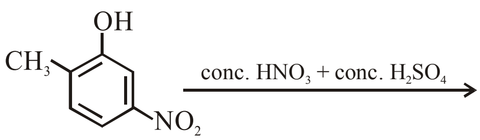
(2) 

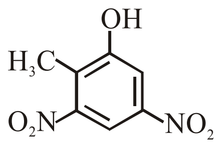
(3) 

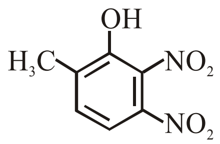
(4) $CH_3CH_2CH=CH_2 \xrightarrow{HBr}$

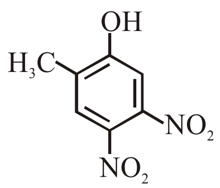
AROMATIC COMPOUND

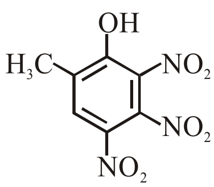
1. The major product of the following reaction is:



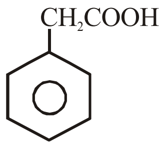
(1) 

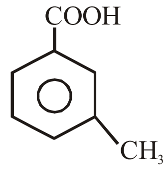
(2) 

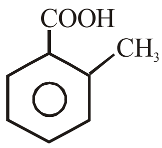
(3) 

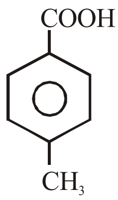
(4) 

2. [P] on treatment with $Br_2/FeBr_3$ in CCl_4 produced a single isomer $C_8H_7O_2$ Br while heating [P] with sodalime gave toluene. The compound [P] is :

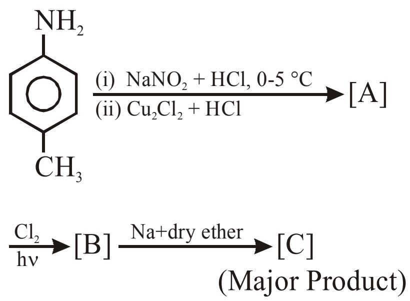
(1) 

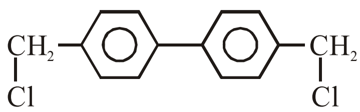
(2) 

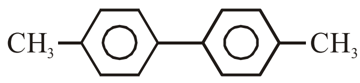
(3) 

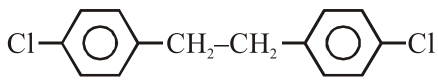
(4) 

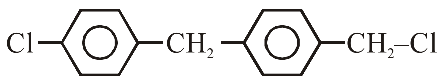
3. In the following reaction sequence, [C] is :-



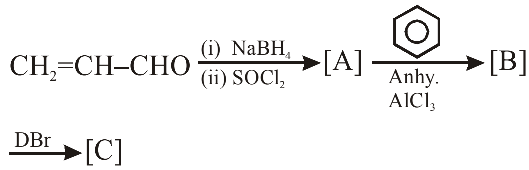
(1) 

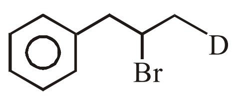
(2) 

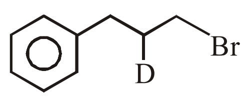
(3) 

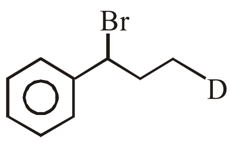
(4) 

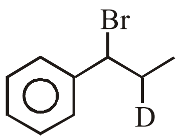
4. The major product [C] of the following reaction sequence will be :-



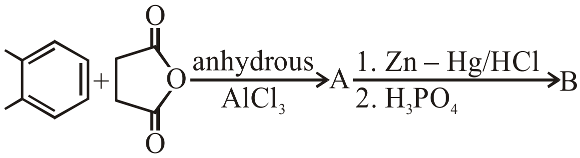
(1) 

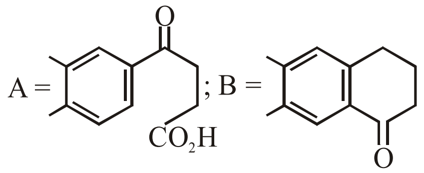
(2) 

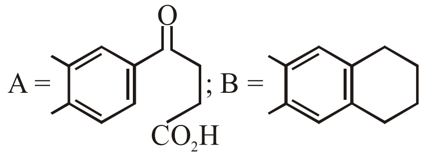
(3) 

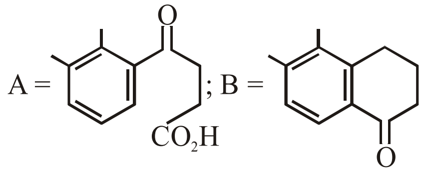
(4) 

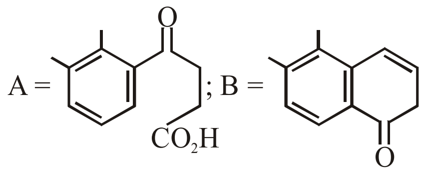
5. In the following reaction sequence the major products A and B are :



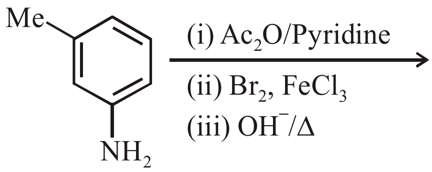
(1) 

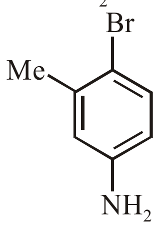
(2) 

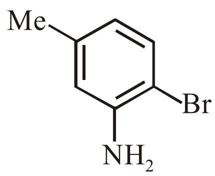
(3) 

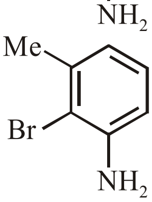
(4) 

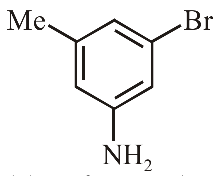
6. The final major product of the following reaction is :



(1) 

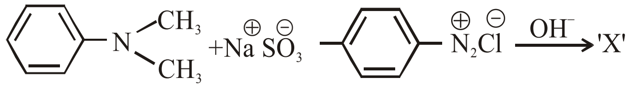
(2) 

(3) 

(4) 

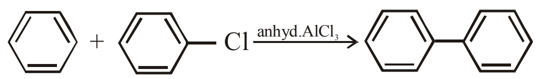
7. A solution of phenol in chloroform when treated with aqueous NaOH gives compound P as a major product. The mass percentage of carbon in P is________. (to the nearest integer)
(Atomic mass : C = 12; H = 1; O = 16)

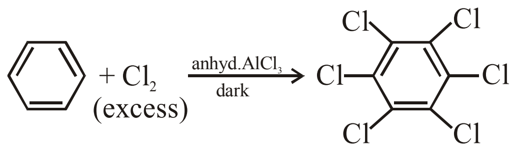
8. Consider the following reaction :



The product 'X' is used :
(1) in acid base titration as an indicator
(2) in protein estimation as an alternative to ninhydrin
(3) in laboratory test for phenols
(4) as food grade colourant

9. Consider the following reactions :

(a) 

(b) 

(c) 

(d) 

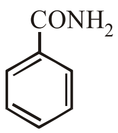
Which of these reactions are possible ?
(1) (a) and (d)
(2) (b) and (d)
(3) (a) and (b)
(4) (b) , (c) and (d)

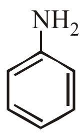
10. In the following sequence of reactions the maximum number of atoms present in molecule 'C' in one plane is _________.

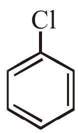
$$A \xrightarrow[\text{Cu tube}]{\text{Red hot}} B \xrightarrow[\text{Anhydrous AlCl}_3]{\text{CH}_3\text{Cl(1 eq.)}} C$$

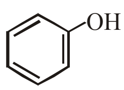
(A is a lowest molecular weight alkyne)

11. Which of these will produce the highest yield in Friedel Crafts reaction?

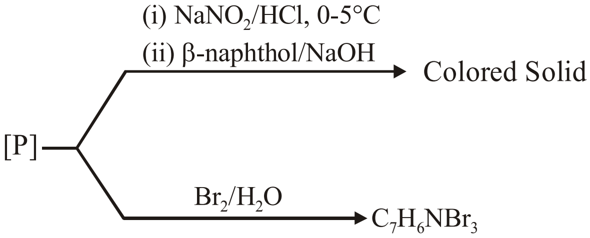
(1) 

(2) 

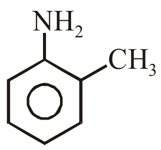
(3) 

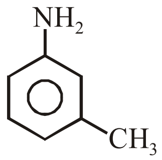
(4) 

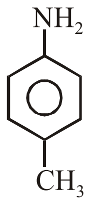
12. Consider the following reactions,

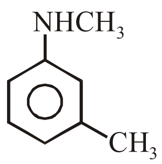


The compound [P] is :

(1) 

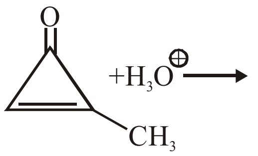
(2) 

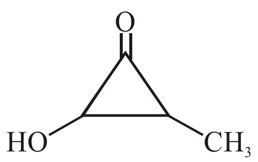
(3) 

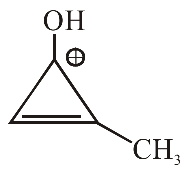
(4) 

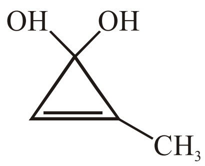
CARBONYL COMPOUNDS

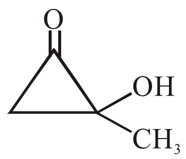
1. The major product in the following reaction is:



(1) 

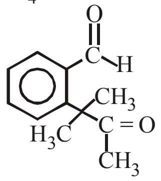
(2) 

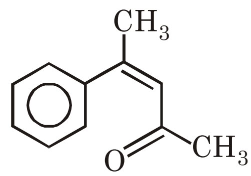
(3) 

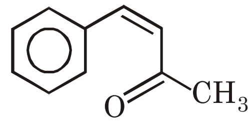
(4) 

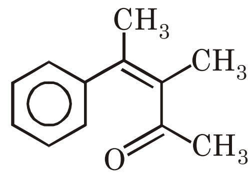
2. Identify (A) in the following reaction sequence :

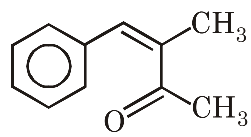
(A) Gives positive iodoform test $\xrightarrow[\text{(ii) } H^+, H_2O \text{ (iii) Conc. } H_2SO_4/\Delta]{\text{(i) } CH_3MgBr}$ (B) $\xrightarrow{O_3/Zn,\ H_2O}$



(1) 

(2) 

(3) 

(4) 

3. The major product (Y) in the following reactions is :

$CH_3-\underset{CH_3}{CH}-C\equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} X \xrightarrow[\text{(ii) Conc. } H_2SO_4/\Delta]{\text{(i) } C_2H_5MgBr, H_2O} Y$

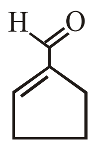
(1) $H_3C-\overset{CH_2}{\overset{\|}{C}}-\underset{C_2H_5}{CH}-CH_3$

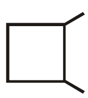
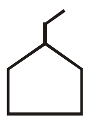
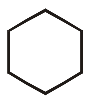
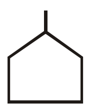
(2) $CH_3-\overset{CH_3}{CH}-\underset{CH_3}{C}=CH-CH_3$

(3) $CH_3-\overset{CH_3}{C}=\underset{CH_2CH_3}{C}-CH_3$

(4) $CH_3-\overset{CH_3}{CH}-\underset{CH_2CH_3}{C}=CH_2$

4. In the following reaction A is :

A $\xrightarrow{\text{(i) } Br_2, h\nu \text{ (ii) KOH (alc.) (iii) } O_3 \text{ (iv) } (CH_3)_2S \text{ (v) NaOH (aq)} + \Delta}$ 

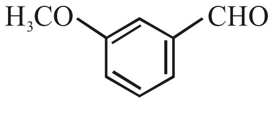
(1)  (2)  (3)  (4) 

5. Consider the following reactions

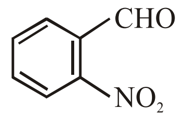
$$A \xrightarrow[\text{(ii)}H_3O^+]{\text{(i)}CH_3MgBr} B \xrightarrow[573K]{Cu} \text{2-methyl, 2-butene}$$

The mass percentage of carbon in A is ______.

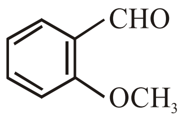
6. The increasing order of the following compounds towards HCN addition is :



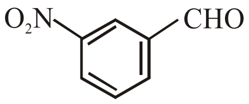
(i)



(ii)



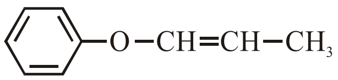
(iii)

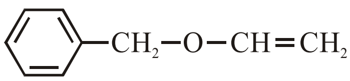


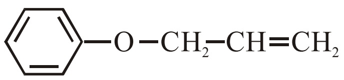
(iv)

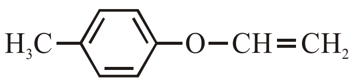
(1) (iii) < (iv) < (ii) < (i) (2) (iii) < (iv) < (i) < (ii)
(3) (iii) < (i) < (iv) < (ii) (4) (i) < (iii) < (iv) < (ii)

7. An organic compound 'A' ($C_9H_{10}O$) when treated with conc. HI undergoes cleavage to yield compounds 'B' and 'C'. 'B' gives yellow precipitate with $AgNO_3$ where as 'C' tautomerizes to 'D'. 'D' gives positive idoform test. 'A' could be :

(1) 

(2) 

(3) 

(4) 

8. The increasing order of the reactivity of the following compound in nucleophilic addition reaction is :

Propanal, Benzaldehyde, Propanone, Butanone

(1) Butanone < Propanone < Benzaldehyde < Propanal

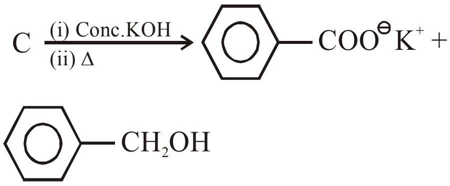
(2) Benzaldehyde < Butanone < Propanone < Propanal

(3) Propanal < Propanone < Butanone < Benzaldehyde

(4) Benzaldehyde < Propanal < Propanone < Butanone

9. The compound A in the following reaction is :

$$A \xrightarrow[\text{(ii) Conc. } H_2SO_4/\Delta]{\text{(i) } CH_3MgBr/H_2O} B \xrightarrow[\text{(ii) } Zn/H_2O]{\text{(i)}O_3} C + D$$



$$D \xrightarrow[\Delta]{Ba(OH)_2} H_3C-\overset{CH_3}{\overset{|}{C}}=CH-\overset{O}{\overset{\|}{C}}-CH_3$$

(1) $C_6H_5-\overset{O}{\overset{\|}{C}}-CH\begin{matrix}CH_3\\CH_3\end{matrix}$ (2) $C_6H_5-\overset{O}{\overset{\|}{C}}-CH_2CH_3$

(3) $C_6H_5-CH_2-\overset{O}{\overset{\|}{C}}-CH_3$ (4) $C_6H_5-\overset{O}{\overset{\|}{C}}-CH_3$

10. Consider the following reactions 'A' is -

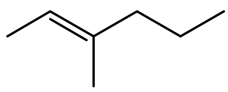
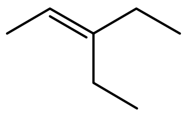
'A' (C_7H_{14}) $\xrightarrow{\text{ozonolysis}}$ 'B' + 'C'

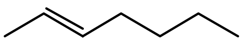
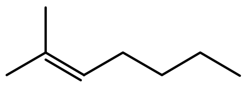
'B' $\xrightarrow[\Delta]{(I_2 + NaOH)}$ yellow ppt

'B' $\xrightarrow[\Delta]{Ag_2O}$ silver mirror

'C' $\xrightarrow[\Delta]{(I_2 + NaOH)}$ no yellow ppt

'C' $\xrightarrow{LiAlH_4}$ 'D' $\xrightarrow[\text{conc. HCl}]{\text{Anhydrous } ZnCl_2}$ gives white turbidity within 5 minutes

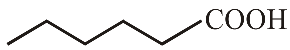
(1)  (2) 

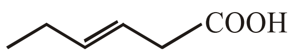
(3)  (4) 

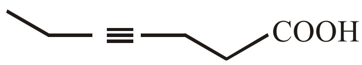
CARBOXYLIC ACID AND THEIR DERIVATIVES

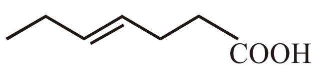
1. What is the product of following reaction ?

$$\text{Hex-3-ynal} \xrightarrow[\substack{\text{(ii) } PBr_3 \\ \text{(iii) Mg/ether} \\ \text{(iv) } CO_2/H_3O^+}]{\text{(i) } NaBH_4} ?$$

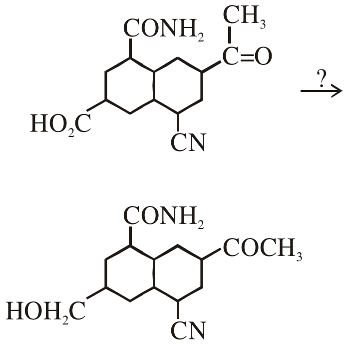
(1) 

(2) 

(3) 

(4) 

2. The most suitable reagent for the given conversion is :



(1) $LiAlH_4$ (2) $NaBH_4$ (3) H_2/Pd (4) B_2H_6

3. An organic compound [A], molecular formula $C_{10}H_{20}O_2$ was hydrolyzed with dilute sulphuric acid to give a carboxylic acid [B] and alcohol [C]. Oxidation of [C] with $CrO_3 – H_2SO_4$ produced [B]. Which of the following structures are not possible for [A] ?

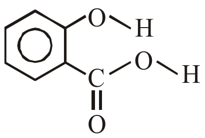
(1) $(CH_3)_3–C–COOCH_2C(CH_3)_3$

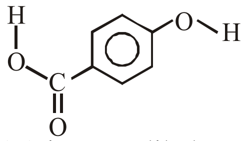
(2) $CH_3CH_2CH_2COOCH_2CH_2CH_2CH_3$

(3) $CH_3–CH_2–\underset{\displaystyle CH_3}{\underset{|}{CH}}–OCOCH_2\overset{\displaystyle CH_3}{\overset{|}{CH}}–CH_2CH_3$

(4) $CH_3–CH_2–\underset{\displaystyle CH_3}{\underset{|}{CH}}–COOCH_2–\overset{\displaystyle CH_3}{\overset{|}{CH}}–CH_2CH_3$

4. Consider the following molecules and statements related to them :

(A) 

(B) 

(a) (B) is more likely to be crystalline than (A)

(b) (B) has higher boiling point than (A)

(c) (B) dissolves more readily than (A) in water

Identify the correct option from below :

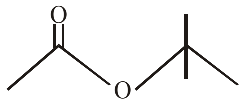
(1) only (a) is true

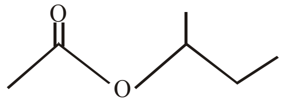
(2) (a) and (c) are true

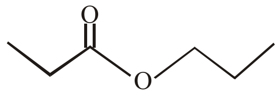
(3) (b) and (c) are true

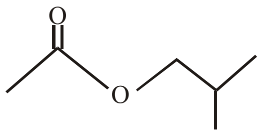
(4) (a) and (b) are true

5. An organic compound (A) (molecular formula $C_6H_{12}O_2$) was hydrolysed with dil. H_2SO_4 to give a carboxylic acid (B) and an alcohol (C). 'C' give white turbidity immediately when treated with anhydrous $ZnCl_2$ and conc. HCl. The organic compound (A) is :

(1) 

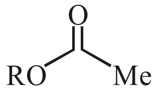
(2) 

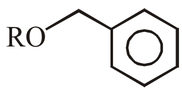
(3) 

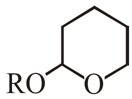
(4) 

6. Which of the following derivatives of alcohols is unstable in an aqueous base ?

(1) RO – CMe_3

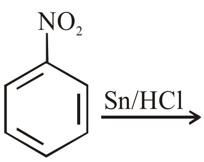
(2) 

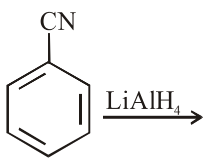
(3) 

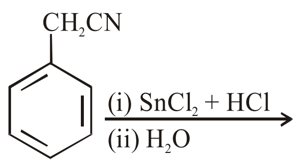
(4) 

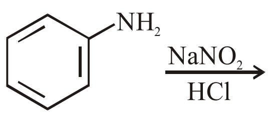
AMINES

1. The Kjeldahl method of Nitrogen estimation fails for which of the following reaction products ?

(a) 

(b) 

(c) 

(d) 

(1) a and d

(2) c and d

(3) a, c and d

(4) b and c

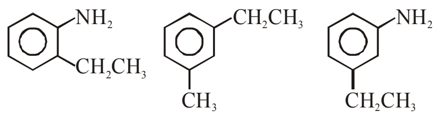
2. Three isomers A, B and C (mol. formula $C_8H_{11}N$) give the following results :

$$\text{A and C} \xrightarrow{\text{Diazotization}} P + Q \xrightarrow[\text{(ii) oxidation } (KMnO_4 + H^+)]{\text{(i) Hydrolysis}} \begin{matrix}\text{R(product of A)} \\ + \\ \text{S(product of C)}\end{matrix}$$

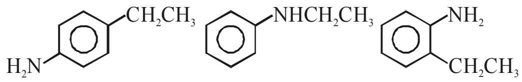
R has lower boiling point than S

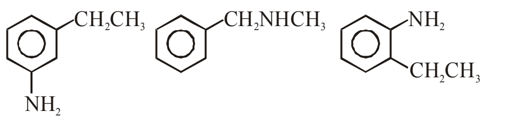
$\text{B} \xrightarrow{C_6H_5SO_2Cl}$ alkali-insoluble product

A, B and C, respectively are :

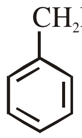
(1) 

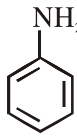
(2) 

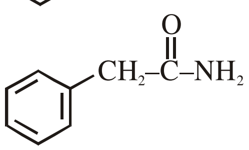
(3) 

(4) 

3. Which of the following compounds can be prepared in good yield by Gabriel phthalimide synthesis?

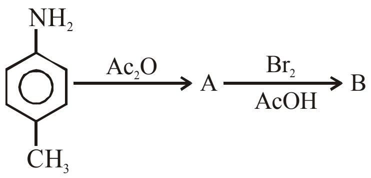
(1) 

(2) 

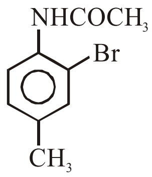
(3) 

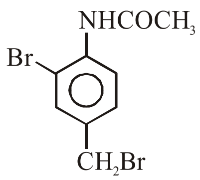
(4) $CH_3–CH_2–NHCH_3$

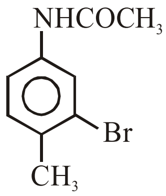
4. In the following reaction sequence

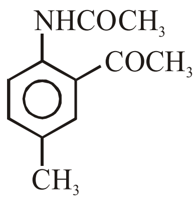


the major products B is -

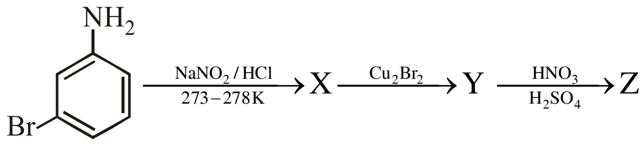
(1) 

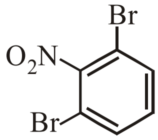
(2) 

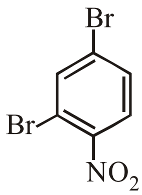
(3) 

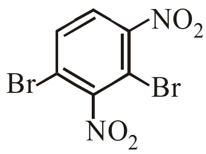
(4) 

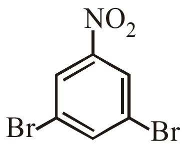
5. The major product Z obtained in the following reaction scheme is :



(1) 

(2) 

(3) 

(4) 

BIOMOLECULES

1. Which of the following statements is correct-

(1) Gluconic acid can form cyclic (acetal/ hemiacetal) structure

(2) Gluconic acid is a partial oxidation product of glucose

(3) Gluconic acid is obtained by oxidation of glucose with HNO_3

(4) Gluconic acid is a dicarboxylic acid

2. Which of the following statement is not true for glucose?

(1) The pentaacetate of glucose does not react with hydroxylamine to give oxime

(2) Glucose gives Schiff's test for aldehyde

(3) Glucose exists in two crystalline forms α and β

(4) Glucose reacts with hydroxylamine to form oxime

3. Two monomers in maltose are :

(1) α-D-glucose and β-D-glucose

(2) α-D-glucose and α-D-Fructose

(3) α-D-glucose and α-D-glucose

(4) α-D-glucose and α-D-galactose

4. A, B and C are three biomolecules. The results of the tests performed on them are given below:

	Molisch's Test	Barfoed Test	Biuret Test
A	Positive	Negative	Negative
B	Positive	Positive	Negative
C	Negative	Negative	Positive

A, B and C are respectively :
(1) A = Glucose, B = Fructose, C = Albumin
(2) A = Lactose, B = Fructose, C = Alanine
(3) A = Lactose, B = Glucose, C = Alanine
(4) A = Lactose, B = Glucose, C = Albumin

5. Consider the following reactions :

(i) Glucose + ROH $\xrightarrow{\text{dry HCl}}$ Acetal $\xrightarrow[(CH_3CO)_2O]{\text{x eq. of}}$ acetyl derivative

(ii) Glucose $\xrightarrow{Ni/H_2}$ A $\xrightarrow[(CH_3CO)_2O]{\text{y eq. of}}$ acetyl derivative

(iii) Glucose $\xrightarrow[(CH_3CO)_2O]{\text{z eq. of}}$ acetyl derivative

'x', 'y' and 'z' in these reactions are respectively.
(1) 5, 6, & 5 (2) 4, 5 & 5
(3) 5, 4 & 5 (4) 4, 6 & 5

6. The correct observation in the following reactions is :

$$\text{Sucrose} \xrightarrow[\text{Cleavage (Hydrolysis)}]{\text{Glycosidic bond}} A + B \xrightarrow[\text{reagent}]{\text{Seliwanoff's}} ?$$

(1) Formation of blue colour
(2) Formation of violet colour
(3) Formation of red colour
(4) Gives no colour

7. The number of $>C=O$ groups present in a tripeptide Asp – Glu – Lys is ________ .

8. Which of the following will react with $CHCl_3$ + alc. KOH ?
(1) Adenine and lysine
(2) Adenine and thymine
(3) Adenine and proline
(4) Thymine and proline

9. What are the functional groups present in the structure of maltose ?
(1) One ketal and one hemiketal
(2) One acetal and one hemiacetal
(3) Two acetals
(4) One acetal and one ketal

10. The number of chiral centres present in threonine is _________.

11. Which of the following is not an essential amino acid :
(1) Valine (2) Leucine
(3) Lysine (4) Tyrosine

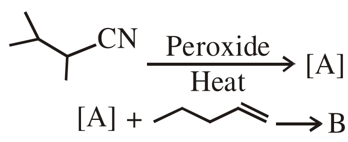
12. The number of chiral carbon(s) present in peptide, Ile-Arg-Pro, is _____.

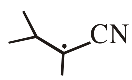
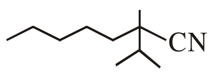
13. The number of chiral carbons present in sucrose is_______ .

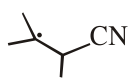
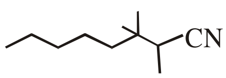
14. Which one of the following statements not true ?
(1) Lactose contains α-glycosidic linkage between C_1 of galactose and C_4 of glucose.
(2) Lactose ($C_{11}H_{22}O_{11}$) is a disaccharide and it contains 8 hydroxyl groups.
(3) On acid hydrolysis, lactose gives one molecule of D(+)-glucose and one molecule of D(+)-galactose.
(4) Lactose is a reducing sugar and it gives Fehling's test.

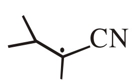
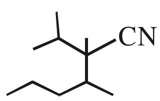
POLYMER

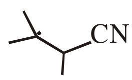
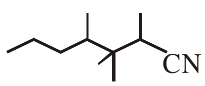
1. The major products A and B in the following reactions are :



(1) A =  and B = 

(2) A =  and B = 

(3) A =  and B = 

(4) A =  and B = 

2. Preparation of Bakelite proceeds via reactions.
(1) Condensation and elimination
(2) Electrophilic addition and dehydration
(3) Electrophilic substitution and dehydration
(4) Nucleophilic addition and dehydration

3. Which polymer has 'chiral' monomer(s) ?
(1) Buna-N
(2) Nylon 6,6
(3) Neoprene
(4) PHBV

4. Which one of the following polymers is not obtained by condensation polymerisation?
(1) Buna - N
(2) Bakelite
(3) Nylon 6
(4) Nylon 6, 6

5. The correct match between **Item-I** and **Item-II** :

Item-I		**Item-II**
(a) Natural rubber	(I)	1, 3-butadiene + styrene
(b) Neoprene	(II)	1, 3-butadiene + acrylonitrile
(c) Buna-N	(III)	Chloroprene
(d) Buna-S	(IV)	Isoprene

(1) (a) - (III), (b) - (IV), (c) - (I), (d) - (II)
(2) (a) - (IV), (b) - (III), (c) - (II), (d) - (I)
(3) (a) - (IV), (b) - (III), (c) - (I), (d) - (II)
(4) (a) - (III), (b) - (IV), (c) - (II), (d) - (I)

6. Consider the Assertion and Reason given below.
Assertion (A) : Ethene polymerized in the presence of Ziegler Natta Catalyst at high temperature and pressure is used to make buckets and dustbins.
Reason (R): High density polymers are closely packed and are chemically inert. Choose the correct answer from the following :
(1) (A) is correct but (R) is wrong.
(2) (A) and (R) both are wrong.
(3) Both (A) and (R) are correct and (R) is the correct explanation of (A).
(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

PRACTICAL ORGANIC CHEMISTRY (POC)

1. A solution of m-chloroaniline, m-chlorophenol and m-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of $NaHCO_3$ to give fraction A. The left over organic phase was extracted with dilute NaOH solution to give fraction B. The final organic layer was labelled as fraction C. Fractions A, B and C, contain respectively :
(1) m-chlorobenzoic acid, m-chloroaniline and m-chlorophenol
(2) m-chloroaniline, m-chlorobenzoic acid and m-chlorophenol
(3) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline
(4) m-chlorophenol, m-chlorobenzoic acid and m-chloroaniline

2. A chromatography column, packed with silica gel as stationary phase, was used to separate a mixture of compounds consisting of (A) benzanilide (B) aniline and (C) acetophenone. When the column is eluted with a mixture of solvents, hexane : ethyl acetate (20 : 80), the sequence of obtained compounds :
(1) (B), (C) and (A)
(2) (C), (A) and (B)
(3) (A), (B) and (C)
(4) (B), (A) and (C)

3. A flask contains a mixture of isohexane and 3-methylpentane. One of the liquids boils at 63°C while the other boils at 60°C. What is the best way to seprate the two liquids and which one will be distilled out first?
(1) simple distillation, 3-methylpentane
(2) simple distillation, isohexane
(3) fractional distillation, isohexane
(4) fractional distillation, 3-methylpentane

4. Kjeldahl's method cannot be used to estimate nitrogen for which of the following compounds?
(1) $C_6H_5NO_2$ (2) $C_6H_5NH_2$
(3) $CH_3CH_2-C\equiv N$ (4) $NH_2-\overset{\overset{\large O}{\|}}{C}-NH_2$

5. A chemist has 4 samples of artificial sweetener A, B, C and D. To identify these samples, he performed certain experiments and noted the following observations :

(i) A and D both form blue-violet colour with ninhydrin.

(ii) Lassaigne extract of C gives positive $AgNO_3$ test and negative $Fe_4[Fe(CN)_6]_3$ test.

(iii) Lassaigne extract of B and D gives positive sodium nitroprusside test

Based on these observations which option is correct?

(1) A : Aspartame ; B : Saccharin ; C : Sucralose ; D ; Alitame

(2) A : Alitame ; B : Saccharin ; C : Aspartame ; D ; Sucralose

(3) A : Saccharin ; B : Alitame ; C : Sucralose ; D ; Aspartame

(4) A : Aspartame ; B : Alitame ; C : Saccharin ; D ; Sucralose

6. Two compounds A and B with same molecular formula (C_3H_6O) undergo Grignard's reaction with methylmagnesium bromide to give products C and D. Products C and D show following chemical tests.

Test	C	D
Ceric ammonium nitrate Test	Positive	Positive
Lucas Test	Turbidity obtained after five minutes	Turbidity obtained immediately
Iodoform Test	Positive	Negative

C and D respectively are :

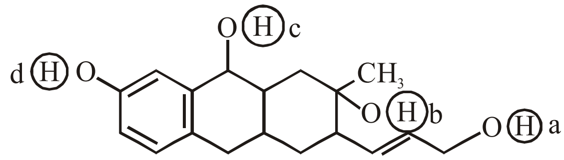
(1) C = $H_3C-C(CH_3)_2-OH$; D = $H_3C-CH_2-CH(OH)-CH_3$

(2) C = $H_3C-CH_2-CH_2-CH_2-OH$; D = $H_3C-C(CH_3)_2-OH$

(3) C = $H_3C-CH_2-CH(OH)-CH_3$; D = $H_3C-C(CH_3)_2-OH$

(4) C = $H_3C-CH_2-CH_2-CH_2-OH$; D = $H_3C-CH_2-CH(OH)-CH_3$

7. Consider the following reaction :



$\xrightarrow[\text{anhydride}]{\text{Chromic}}$ 'P'

The product 'P' gives positive ceric ammonium nitrate test. This is because of the presence of which of these –OH group(s) ?

(1) (c) and (d)

(2) (b) only

(3) (d) only

(4) (b) and (d)

8. Match the following :

	Test/Method		**Reagent**
(i)	Lucas Test	(a)	$C_6H_5SO_2Cl$/aq. KOH
(ii)	Dumas method	(b)	$HNO_3/AgNO_3$
(iii)	Kjeldahl's method	(c)	CuO/CO_2
(iv)	Hinsberg Test	(d)	Conc. HCl and $ZnCl_2$
		(e)	H_2SO_4

(1) (i)-(d), (ii)-(c), (iii)-(e), (iv)-(a)

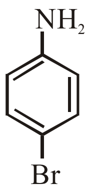
(2) (i)-(b), (ii)-(d), (iii)-(e), (iv)-(a)

(3) (i)-(d), (ii)-(c), (iii)-(b), (iv)-(e)

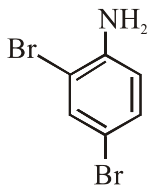
(4) (i)-(b), (ii)-(a), (iii)-(c), (iv)-(d)

PURIFICATION AND SEPRATION TECHNIQUE

1. In Carius method of estimation of halogen, 0.172g of an organic compound showed presence of 0.08g of bromine. Which of these is the **correct** structure of the compound :

(1) 

(2) $H_3C–CH_2–Br$

(3) 

(4) $H_3C–Br$

CHEMISTRY IN EVERYDAY LIFE

1. Match the following :

(i) Riboflavin	(a) Beriberi
(ii) Thiamine	(b) Scurvy
(iii)Pyridoxine	(c) Cheilosis
(iv)Ascorbic acid	(d) Convulsions

(1) (i)-(c), (ii)-(a), (iii)-(d), (iv)-(b)

(2) (i)-(c), (ii)-(d), (iii)-(a), (iv)-(b)

(3) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(c)

(4) (i)-(a), (ii)-(d), (iii)-(c), (iv)-(b)

2. The number of sp^2 hybridised carbons present in "Aspartame" is ________.

3. The number of chiral centres in penicillin is ______.

4. The mass percentage of nitrogen in histamine is _____.

5. Glycerol is separated in soap industries by :

(1) Steam distillation

(2) Differential extraction

(3) Distillation under reduced pressure

(4) Fractional distillation

6. The antifertility drug 'Novestrol" can react with :

(1) Br_2/water; $ZnCl_2$/HCl; $FeCl_3$

(2) Alcoholic HCN; NaOCl; $ZnCl_2$/HCl

(3) Br_2/water; $ZnCl_2$/HCl; NaOCl

(4) $ZnCl_2$/HCl; $FeCl_3$; Alcoholic HCN

7. Match the following drugs with their therapeutic actions :

(i) Ranitidine	(a) Antidepressant
(ii) Nardil (Phenelzine)	(b) Antibiotic
(iii)Chloramphenicol	(c) Antihistamine
(iv)Dimetane (Brompheniramine)	(d) Antacid
	(e) Analgesic

(1) (i)-(a); (ii)-(c); (iii)-(b); (iv)-(e)

(2) (i)-(e); (ii)-(a); (iii)-(c); (iv)-(d)

(3) (i)-(d); (ii)-(a); (iii)-(b); (iv)-(c)

(4) (i)-(d); (ii)-(c); (iii)-(a); (iv)-(e)

8. If a person is suffering from the deficiency of nor-adrenaline, what kind of drug can be suggested ?

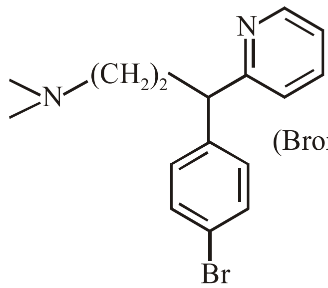
(1) Anti-inflammatory

(2) Analgesic

(3) Antihistamine

(4) Antidepressant

9. The following molecule acts as an :



(Brompheniramine)

(1) Antiseptic

(2) Anti-bacterial

(3) Anti-histamine

(4) Anti-depressant

ANSWER KEY

NOMENCLATURE

Que.	1	2	3
Ans.	4	1	4

ACIDITY & BASICITY

Que.	1	2	3	4	5
Ans.	1	4	4	4	1

ELECTRONIC DISPLACEMENT EFFECT

Que.	1	2	3	4	5	6	7	8	9
Ans.	3	1	2	1	3	4	3	3	1

ISOMERISM

Que.	1	2	3	4
Ans.	2	5.00	NTA-2 Allen-2 & 4	3

HALOGEN DERIVATIVE

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	3	3	1	4	1	2	8	2	4
Que.	11	12	13	14	15					
Ans.	4	2	NTA 4 ALLEN 4	2	2					

ALCOHOL & ETHER

Que.	1	2	3	4	5	6	7	8	9
Ans.	1	4	1	3	4	4	4	2	3

OXIDATION

Que.	1	2
Ans.	2	1

REDUCTION

Que.	1	2	3	4
Ans.	2	2	2	3

HYDROCARBON

Que.	1	2	3	4
Ans.	2	1	1	1

AROMATIC COMPOUND

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	4	3	3	1	1	NTA 69.00 ALLEN 68.85	1	2	13
Que.	11	12								
Ans.	3	2								

CARBONYL COMPOUNDS										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	2	4	3	3	66.65 to 66.70	3	2	1	3	2

CARBOXYLIC ACID AND THEIR DERIVATIVES						
Que.	**1**	**2**	**3**	**4**	**5**	**6**
Ans.	3	4	NTA (3) ALLEN (2 & 3)	NTA (3) ALLEN (2, 3 & 4)	1	2

AMINES					
Que.	**1**	**2**	**3**	**4**	**5**
Ans.	2	2	1	1	2

BIOMOLECULES										
Que.	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
Ans.	2	2	3	4	4	3	5	1	2	2
Que.	**11**	**12**	**13**	**14**						
Ans.	4	4	9	1						

POLYMER						
Que.	**1**	**2**	**3**	**4**	**5**	**1**
Ans.	1	3	4	1	2	3

PRACTICAL ORGANIC CHEMISTRY (POC)								
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Ans.	3	2	3	1	1	3	2	1

PURIFICATION AND SEPRATION TECHNIQUE	
Que.	**1**
Ans.	1

CHEMISTRY IN EVERYDAY LIFE									
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Ans.	1	9.00	3.00	37.80 to 38.20	3	1	3	4	3

JANUARY & SEPTEMBER 2020 ATTEMPT (IOC)

QUANTUM NUMBER

1. The correct electronic configuration and spin-only magnetic moment (BM) of Gd^{3+} (Z = 64), respectively, are

(1) $[Xe]5f^7$ and 8.9

(2) $[Xe]4f^7$ and 7.9

(3) $[Xe]5f^7$ and 7.9

(4) $[Xe]4f^7$ and 8.9

2. In the sixth period, the orbitals that are filled are

(1) 6s, 5f, 6d, 6p

(2) 6s, 6p, 6d, 6f

(3) 6s, 5d, 5f, 6p

(4) 6s, 4f, 5d, 6p

3. Consider the hypothetical situation where the azimuthal quantum number, l, takes values 0, 1, 2, ... n + 1, where n is the principal quantum number. Then, the element with atomic number :

(1) 13 has a half-filled valence subshell

(2) 9 is the first alkali metal

(3) 8 is the first noble gas

(4) 6 has a 2p-valence subshell

4. The number of subshells associated with n = 4 and m = –2 quantum numbers is :

(1) 4 (2) 8

(3) 16 (4) 2

PERIODIC TABLE

1. The electron gain enthalpy (in kJ/mol) of fluorine, chlorine, bromine and iodine, respectively are :

(1) – 333, – 349, – 325 and – 296

(2) –296, – 325, – 333 and – 349

(3) – 333, – 325, – 349 and – 296

(4) –349, – 333, – 325 and – 296

2. Within each pair of elements of F & Cl , S & Se, and Li & Na, respectively, the elements that release more energy upon an electron gain are-

(1) F, Se and Na (2) F, S and Li

(3) Cl, S and Li (4) Cl, Se and Na

3. The first ionization energy (in kJ/mol) of Na, Mg, Al and Si respectively, are :

(1) 496, 737, 577, 786

(2) 786, 737, 577, 496

(3) 496, 577, 737, 786

(4) 496, 577, 786, 737

4. The increasing order of the atomic radii of the following elements is :-

(a) C (b) O (c) F (d) Cl

(e) Br

(1) (b) < (c) < (d) < (a) < (e)

(2) (a) < (b) < (c) < (d) < (e)

(3) (d) < (c) < (b) < (a) < (e)

(4) (c) < (b) < (a) < (d) < (e)

5. B has a smaller first ionization enthalpy than Be. Consider the following statements :

(I) It is easier to remove 2p electron than 2s electron

(II) 2p electron of B is more shielded from the nucleus by the inner core of electrons than the 2s electrons of Be.

(III) 2s electron has more penetration power than 2p electron.

(IV) atomic radius of B is more than Be

(Atomic number B = 5, Be = 4)

The correct statements are :

(1) (I), (II) and (III)

(2) (II), (III) and (IV)

(3) (I), (III) and (IV)

(4) (I), (II) and (IV)

6. The correct order of the ionic radii of O^{2-}, N^{3-}, F^-, Mg^{2+}, Na^+ and Al^{3+} is :

(1) $Al^{3+} < Na^+ < Mg^{2+} < O^{2-} < F^- < N^{3-}$

(2) $N^{3-} < O^{2-} < F^- < Na^+ < Mg^{2+} < Al^{3+}$

(3) $Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-} < N^{3-}$

(4) $N^{3-} < F^- < O^{2-} < Mg^{2+} < Na^+ < Al^{3+}$

7. Lattice enthalpy and enthalpy of solution of NaCl are 788 kJ mol^{-1} and 4 kJ mol^{-1}, respectively. The hydration enthalpy of NaCl is :

(1) –780 kJ mol^{-1}

(2) –784 kJ mol^{-1}

(3) 780 kJ mol^{-1}

(4) 784 kJ mol^{-1}

8. The process that is NOT endothermic in nature is :-

(1) $Ar_{(g)} + e^- \rightarrow Ar^-_{(g)}$ (2) $H_{(g)} + e^- \rightarrow H^-_{(g)}$

(3) $Na_{(g)} \rightarrow Na^+_{(g)} + e^-$ (4) $O^-_{(g)} + e^- \rightarrow O^{2-}_{(g)}$

9. The ionic radii of O_2^-, F^-, Na^+ and Mg^{2+} are in the order :

(1) $F^- > O^{2-} > Na^+ > Mg^{2+}$

(2) $Mg^{2+} > Na^+ > F^- > O^{2-}$

(3) $O^{2-} > F^- > Mg^{2+} > Na^+$

(4) $O^{2-} > F^- > Na^+ > Mg^{2+}$

10. The elements with atomic numbers 101 and 104 belong to, respectively :

(1) Group 11 and Group 4

(2) Actinoids and Group 4

(3) Actinoids and Group 6

(4) Group 6 and Actinoids

11. The five successive ionization enthalpies of an element are 800, 2427, 3658, 25024 and 32824 kJ mol^{-1}. The number of valence electrons in the element is :

(1) 2 (2) 3 (3) 4 (4) 5

12. Among the statements (I – IV), the correct ones are:

(I) Be has smaller atomic radius compared to Mg.

(II) Be has higher ionization enthalpy than Al.

(III) Charge/radius ratio of Be is greater than that of Al.

(IV) Both Be and Al form mainly covalent compounds.

(1) (I), (II) and (IV)

(2) (II), (III) and (IV)

(3) (I), (II) and (III)

(4) (I), (III) and (IV

13. The atomic number of the element unnilennium is :

(1) 119 (2) 108 (3) 102 (4) 109

14. Three elements X, Y and Z are in the 3rd period of the periodic table. The oxides of X, Y and Z, respectively, are basic, amphoteric and acidic. The correct order of the atomic numbers of X, Y and Z is :

(1) Z < Y < X (2) X < Z < Y

(3) X < Y < Z (4) Y < X < Z

15. In general, the property (magnitudes only) that shows an opposite trend in comparison to other properties across a period is

(1) Electronegativity

(2) Electron gain enthalpy

(3) Ionization enthalpy

(4) Atomic radius

16. The atomic number of Unnilunium is _____.

CHEMICAL BONDING

1. The dipole moments of CCl_4, $CHCl_3$ and CH_4 are in the order :

(1) $CH_4 = CCl_4 < CHCl_3$

(2) $CH_4 < CCl_4 < CHCl_3$

(3) $CCl_4 < CH_4 < CHCl_3$

(4) $CHCl_3 < CH_4 = CCl_4$

2. The relative strength of interionic/ intermolecular forces in decreasing order is :

(1) ion-dipole > ion-ion > dipole-dipole

(2) dipole-dipole > ion-dipole > ion-ion

(3) ion-dipole > dipole-dipole > ion-ion

(4) ion-ion > ion-dipole > dipole-dipole

3. The bond order and the magnetic characteristics of CN^- are :

(1) 3, diamagnetic

(2) $2\frac{1}{2}$, paramagnetic

(3) 3, paramagnetic

(4) $2\frac{1}{2}$, diamagnetic

4. The predominant intermolecular forces present in ethyl acetate, a liquid, are :

(1) hydrogen bonding and London dispersion

(2) Dipole-dipole and hydrogen bonding

(3) London dispersion and dipole-dipole

(4) London dispersion, dipole-dipole and hydrogen bonding

5. Arrange the following bonds according to their average bond energies in descending order :
C–Cl, C–Br, C–F, C–I
(1) C–I > C–Br > C–Cl > C–F
(2) C–Br > C–I > C–Cl > C–F
(3) C–F > C–Cl > C–Br > C–I
(4) C–Cl > C–Br > C–I > C–F

6. 'X' melts at low temperature and is a bad conductor of electricity in both liquid and solid state. X is :
(1) Carbon tetrachloride (2) Mercury
(3) Silicon carbide (4) Zinc sulphide

7. If the magnetic moment of a dioxygen species is 1.73 B.M, it may be :
(1) O_2^- or O_2^+ (2) O_2 or O_2^+
(3) O_2 or O_2^- (4) O_2, O_2^- or O_2^+

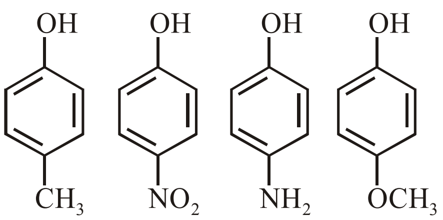
8. The acidic, basic and amphoteric oxides, respectively, are :
(1) MgO, Cl_2O, Al_2O_3
(2) Cl_2O, CaO, P_4O_{10}
(3) Na_2O, SO_3, Al_2O_3
(4) N_2O_3, Li_2O, Al_2O_3

9. The number of sp^2 hybrid orbitals in a molecule of benzene is :
(1) 24 (2) 6 (3) 12 (4) 18

10. Among the sulphates of alkaline earth metals, the solubilities of $BeSO_4$ and $MgSO_4$ in water, respectively, are:
(1) high and high (2) poor and poor
(3) high and poor (4) poor and high

11. The number of CI = O bonds in perchloric acid is, "_________"

12. The increasing order of boiling points of the following compounds is :


I II III IV

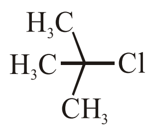
(1) I < IV < III < II
(2) IV < I < II < III
(3) I < III < IV < II
(4) III < I < II < IV

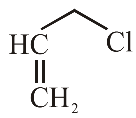
13. The compound that has the largest H–M–H bond angle (M=N, O, S, C), is :
(1) H_2O (2) CH_4
(3) NH_3 (4) H_2S

14. Hydrogen peroxide, in the pure state, is :
(1) non-planar and almost colorless
(2) linear and almost colorless
(3) planar and blue in color
(4) linear and blue in color

15. The structure of PCl_5 in the solid state is
(1) square pyramidal
(2) tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]^-$
(3) square planar $[PCl_4]^+$ and octahedral $[PCl_6]^-$
(4) trigonal bipyramidal

16. Among the following compounds, which one has the shortest C—Cl bond ?

(1) H_3C–Cl (2) 

(3)  (4) 

17. The reaction in which the hybridisation of the underlined atom is affected is :-
(1) $\underline{N}H_3 \xrightarrow{H^+}$
(2) $\underline{Xe}F_4 + SbF_5 \rightarrow$
(3) $H_2\underline{S}O_4 + NaCl \xrightarrow{420\ K}$
(4) $H_3\underline{P}O_2 \xrightarrow{\text{Disproportionation}}$

18. Of the species, NO, NO^+, NO^{2+}, NO^-, the one with minimum bond strength is :
(1) NO^{2+} (2) NO^+
(3) NO (4) NO^-

19. In a molecule of pyrophosphoric acid, the number of P–OH, P=O and P–O–P bonds/ moiety(ies) respectivey are :
(1) 3, 3 and 3
(2) 2, 4 and 1
(3) 4, 2 and 0
(4) 4, 2 and 1

20. Match the type of interaction in Column A with the distance dependence of their interaction energy in Column B :

A	B
(I) ion - ion	(a) $\frac{1}{r}$
(II) dipole - dipole	(b) $\frac{1}{r^2}$
(III) London dispersion	(c) $\frac{1}{r^3}$
	(d) $\frac{1}{r^6}$

(1) (I)-(a), (II)-(b), (III)-(c)
(2) (I)-(a), (II)-(c), (III)-(d)
(3) (I)-(a), (II)-(b), (III)-(d)
(4) (I)-(b), (II)-(d), (III)-(c)

21. The molecular geometry of SF_6 is octahedral. What is the geometry of SF_4 (including lone pair(s) of electrons, if any) ?
(1) Trigonal bipyramidal
(2) Square planar
(3) Tetrahedral
(4) Pyramidal

22. If AB_4 molecule is a polar molecule, a possible geometry of AB_4 is :
(1) Square pyramidal
(2) Tetrahedral
(3) Square planar
(4) Rectangular planar

23. The shape/structure of $[XeF_5]^-$ and XeO_3F_2, respectively, are :
(1) pentagonal planar and trigonal bipyramidal
(2) trigonal bipyramidal and pentagonal planar
(3) octahedral and square pyramidal
(4) trigonal bipyramidal and trigonal bipyramidal

COORDINATION CHEMISTRY

1. The theory that can completely/properly explain the nature of bonding in $[Ni(CO)_4]$ is:
(1) Werner's theory
(2) Crystal field theory
(3) Valence bond theory
(4) Molecular orbital theory

2. The IUPAC name of the complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is :
(1) Diammine (methanamine) chlorido platinum (II) chloride
(2) Bisammine (methanamine) chlorido platinum (II) chloride
(3) Diamminechlorido (aminomethane) platinum(II) chloride
(4) Diamminechlorido (methanamine) platinum (II) chloride

3. Among the statements(a)-(d), the incorrect ones are-
(a) Octahedral Co(III) complexes with strong field ligands have very high magnetic moments
(b) When $\Delta_0 < P$, the d-electron configuration of Co(III) in an octahedral complex is $t_{eg}^4\, e_g^2$
(c) Wavelength of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$
(d) If the Δ_0 for an octahedral complex of Co(III) is 18,000 cm^{-1}, the Δ_t for its tetrahedral complex with the same ligand will be 16,000 cm^{-1}
(1) (a) and (b) only (2) (c) and (d) only
(3) (b) and (c) only (4) (a) and (d) only

4. The number of possible optical isomers for the complexes MA_2B_2 with sp^3 and dsp^2 hybridised metal atom, respectively, is :
Note : A and B are unidentate neutral and unidentate monoanionic ligands, respectively
(1) 0 and 0 (2) 0 and 2
(3) 0 and 1 (4) 2 and 2

5. The complex that can show fac-and mer-isomers is :
(1) $[Pt(NH_3)_2Cl_2]$
(2) $[Co(NH_3)_4Cl_2]^+$
(3) $[Co(NH_3)_3(NO_2)_3]$
(4) $[CoCl_2(en)_2]$

6. The volume (in mL) of 0.125 M $AgNO_3$ required to quantitatively precipitate chloride ions in 0.3 g of $[Co(NH_3)_6]Cl_3$ is _______.
$^{M}[Co(NH_3)_6]Cl_3$ = 267.46 g/mol
$^{M}AgNO_3$ = 169.87 g/mol

7. Among (a) – (d) the complexes that can display geometrical isomerism are :
(a) $[Pt(NH_3)_3Cl]^+$ (b) $[Pt(NH_3)Cl_5]^-$
(c) $[Pt(NH_3)_2Cl(NO_2)]$ (d) $[Pt(NH_3)_4ClBr]^{2+}$
(1) (d) and (a) (2) (a) and (b)
(3) (b) and (c) (4) (c) and (d)

8. The correct order of the calculated spin-only magnetic moments of complexes (A) to (D) is:
(A) $Ni(CO)_4$
(B) $[Ni(H_2O)_6]Cl_2$
(C) $Na_2[Ni(CN)_4]$
(D) $PdCl_2(PPh_3)_2$
(1) (A) ≈ (C) ≈ (D) < (B)
(2) (A) ≈ (C) < (B) ≈ (D)
(3) (C) < (D) < (B) < (A)
(4) (C) ≈ (D) < (B) < (A)

9. Complexes (ML_5) of metals Ni and Fe have ideal square pyramidal and trigonal bipyramidal grometries, respectively. The sum of the 90°, 120° and 180° L-M L angles in the two complexes is ________.

10. $[Pd(F)(Cl)(Br)(I)]^{2-}$ has n number of geometrical isomers. Then, the spin-only magnetic moment and crystal field stabilisation energy [CFSE] of $[Fe(CN)_6]^{n-6}$, respectively, are:
[Note : Ignore the pairing energy]
(1) 2.84 BM and $-1.6\ \Delta_0$
(2) 1.73 BM and $-2.0\ \Delta_0$
(3) 0 BM and $-2.4\ \Delta_0$
(4) 5.92 BM and 0

11. Complex X of composition $Cr(H_2O)_6Cl_n$ has a spin only magnetic moment of 3.83 BM. It reacts with $AgNO_3$ and shows geometrical isomerism. The IUPAC nomenclature of X is:
(1) Tetraaquadichlorido chromium (III) chloride dihydrate
(2) Hexaaqua chromium (III) chloride
(3) Dichloridotetraaqua chromium (IV) chloride dihydrate
(4) Tetraaquadichlorido chromium(IV) chloride dihydrate

12. The correct order of the spin-only magnetic moments of the following complexes is :
(I) $[Cr(H_2O)_6]Br_2$
(II) $Na_4[Fe(CN)_6]$
(III) $Na_3[Fe(C_2O_4)_3]$ $(\Delta_0 > P)$
(IV) $(Et_4N)_2[CoCl_4]$
(1) (III) > (I) > (II) > (IV)
(2) (I) > (IV) > (III) > (II)
(3) (II) ≈ (I) > (IV) > (III)
(4) (III) > (I) > (IV) > (II)

13. The isomer(s) of $[Co(NH_3)_4Cl_2]$ that has/have a Cl–Co–Cl angle of 90°, is/are :
(1) meridional and trans
(2) cis and trans
(3) trans only
(4) cis only

14. The species that has a spin only magnetic moment of 5.9 BM, is -
(1) $Ni(CO)_4(T_d)$
(2) $[MnBr_4]^{2-}(T_d)$
(3) $[NiCl_4]^{2-}(T_d)$
(4) $[Ni(CN)_4]^{2-}$ (square planar)

15. For a d^4 metal ion in an octahedral field, the correct electronic configuration is :
(1) $t_{2g}^4 e_g^0$ when $\Delta_O < P$
(2) $e_g^2 t_{2g}^2$ when $\Delta_O < P$
(3) $t_{2g}^3 e_g^1$ when $\Delta_O < P$
(4) $t_{2g}^3 e_g^1$ when $\Delta_O > P$

16. Considering that $\Delta_0 > P$, the magnetic moment (in BM) of $[Ru(H_2O)_6]^{2+}$ would be________ .

17. Consider the complex ions, *trans*-$[Co(en)_2Cl_2]^+$ (A) and *cis*-$[Co(en)_2Cl_2]^+$ (B). The correct statement regarding them is :
(1) both (A) and (B) can be optically active
(2) both (A) and (B) cannot be optically active
(3) (A) can be optically active, but (B) cannot be optically active
(4) (A) cannot be optically active, but (B) can be optically active

18. The total number of coordination sites in ethylenediaminetetraacetate ($EDTA^{4-}$) is _______.

19. The values of the crystal field stabilization energies for a high spin d^6 metal ion in octahedral and tetrahedral fields, respectively, are :

(1) $-0.4\ \Delta_0$ and $-0.27\ \Delta_t$

(2) $-1.6\ \Delta_0$ and $-0.4\ \Delta_t$

(3) $-0.4\ \Delta_0$ and $-0.6\ \Delta_t$

(4) $-2.4\ \Delta_0$ and $-0.6\ \Delta_t$

20. The molecule in which hybrid MOs involve only one d-orbital of the central atom is :-

(1) $[Ni(CN)_4]^{2-}$ (2) $[CrF_6]^{3-}$

(3) BrF_5 (4) XeF_4

21. The one that can exhibit highest paramagnetic behaviour among the following is :-

gly = glycinato; bpy = 2, 2'-bipyridine

(1) $[Pd(gly)_2]$

(2) $[Ti(NH_3)_6]^{3+}$

(3) $[Co(OX)_2(OH)_2]^-$ ($\Delta_0 > P$)

(4) $[Fe(en)(bpy)(NH_3)_2]^{2+}$

22. The crystal Field stabilization Energy (CFSE) of $[CoF_3(H_2O)_3]$($\Delta_0 < P$) is :-

(1) $-0.8\ \Delta_0$

(2) $-0.4\ \Delta_0 + P$

(3) $-0.8\ \Delta_0 + 2P$

(4) $-0.4\ \Delta_0$

23. The pair in which both the species have the same magnetic moment (spin only) is :

(1) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)]^{2+}$

(2) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

(3) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

(4) $[Co(OH)_4]^{2-}$ and $[Fe(NH_3)_6]^{2+}$

24. The number of isomers possible for $[Pt(en)(NO_2)_2]$ is :

(1) 3 (2) 2

(3) 1 (4) 4

25. Complex A has a composition of $H_{12}O_6Cl_3Cr$. If the complex on treatment with conc. H_2SO_4 loses 13.5% of its original mass, the correct molecular formula of A is :

[Given : atomic mass of Cr = 52 amu and Cl = 35 amu]

(1) $[Cr(H_2O)_5Cl]Cl_2 \cdot H_2O$

(2) $[Cr(H_2O)_3Cl_3] \cdot 3H_2O$

(3) $[Cr(H_2O)_4Cl_2]Cl \cdot 2H_2O$

(4) $[Cr(H_2O)_6]Cl_3$

26. The d-electron configuration of $[Ru(en)_3]Cl_2$ and $[Fe(H_2O)_6]Cl_2$, respectively are :

(1) $t_{2g}^4 e_g^2$ and $t_{2g}^6 e_g^0$

(2) $t_{2g}^6 e_g^0$ and $t_{2g}^6 e_g^0$

(3) $t_{2g}^6 e_g^0$ and $t_{2g}^4 e_g^2$

(4) $t_{2g}^4 e_g^2$ and $t_{2g}^4 e_g^2$

27. The electronic spectrum of $[Ti(H_2O)_6]^{3+}$ shows a single broad peak with a maximum at 20,300 cm^{-1}. The crystal field stabilization energy (CFSE) of the complex ion, in kJ mol^{-1}, is :

(1) 242.5 (2) 83.7

(3) 145.5 (4) 97

28. The complex that can show optical activity is:

(1) trans-$[Fe(NH_3)_2(CN)_4]^-$

(2) cis-$[Fe(NH_3)_2(CN)_4]^-$

(3) cis-$[CrCl_2(ox)_2]^{3-}$ (ox = oxalate)

(4) trans-$[Cr(Cl_2)(ox)_2]^{3-}$

29. The one that is not expected to show isomerism is :

(1) $[Ni(NH_3)_4(H_2O)_2]^{2+}$

(2) $[Ni(NH_3)_2Cl_2]$

(3) $[Pt(NH_3)_2Cl_2]$

(4) $[Ni(en)_3]^{2+}$

30. For octahedral Mn(II) and tetrahedral Ni(II) complexes, consider the following statements:

(I) both the complexes can be high spin

(II) Ni(II) complex can very rarely be low spin.

(III) with strong field ligands, Mn(II) complexes can be low spin.

(IV) aqueous solution of Mn(II) ions is yellow in color.

The **correct** statements are :

(1) (I), (III) and (IV) only

(2) (II), (III) and (IV) only

(3) (I), (II) and (III) only

(4) (I) and (II) only

31. Consider that a d^6 metal ion (M^{2+}) forms a complex with aqua ligands, and the spin only magnetic moment of the complex is 4.90 BM. The geometry and the crystal field stabilization energy of the complex is :

(1) tetrahedral and $-1.6\ \Delta_t + 1P$

(2) tetrahedral and $-0.6\ \Delta_t$

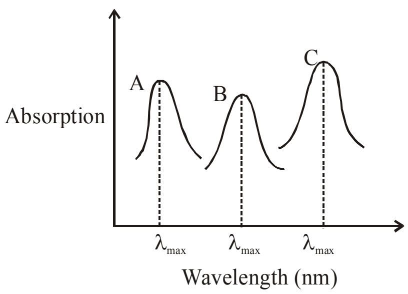
(3) octahedral and $-1.6\ \Delta_0$

(4) octahedral and $-2.4\ \Delta_0 + 2P$

32. The oxidation states of iron atoms in compounds (A), (B) and (C), respectively, are x, y and z. The sum of x,y and z is ____.

$Na_4[Fe(CN)_5NOS)]$	$Na_4[FeO_4]$	$[Fe_2(CO)_9]$
(A)	(B)	(C)

33. Simplified absorption spectra of three complexes ((i), (ii) and (iii)) of M^{n+} ion are provided below; their λ_{max} values are marked as A, B and C respectively. The correct match between the complexes and their λ_{max} values is :



(i) $[M(NCS)_6]^{(-6+n)}$ (ii) $[MF_6]^{(-6+n)}$

(iii) $[M(NH_3)_6]^{n+}$

(1) A-(ii), B-(i), C-(iii) (2) A-(iii), B-(i), C-(ii)

(3) A-(ii), B-(iii), C-(i) (4) A-(i), B-(ii), C-(iii)

METALLURGY

1. The purest form of commercial iron is

(1) scrap iron and pig iron

(2) wrought iron

(3) cast iron

(4) pig iron

2. The refining method used when the metal and the impurities have low and high melting temperatures, respectively, is -

(1) zone refining

(2) liquation

(3) vapour phase refining

(4) distillation

3. Among the reactions (a) - (d), the reaction(s) that does/do not occur in the blast furnace during the extraction of iron is/are :

(a) $CaO + SiO_2 \rightarrow CaSiO_3$

(b) $3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$

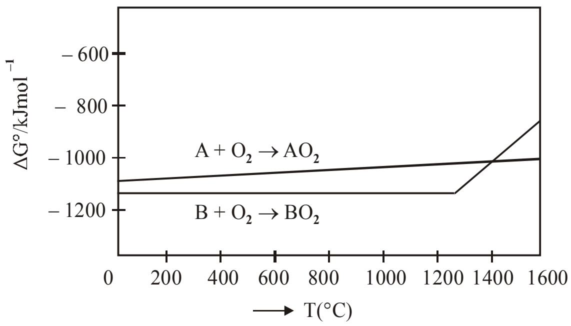
(c) $FeO + SiO_2 \rightarrow FeSiO_3$

(d) $FeO \rightarrow Fe + \frac{1}{2}O_2$

(1) (c) and (d) (2) (a) and (d)

(3) (d) (4) (a)

4. According to the following diagram, A reduces BO_2 when the temperature is :



(1) < 1400 °C

(2) > 1400 °C

(3) < 1200 °C

(4) > 1200 °C but < 1400 °C

5. The element that can be refined by distillation is :

(1) nickel (2) zinc (3) gallium (4) tin

6. Boron and silicon of very high purity can be obtained through :

(1) vapour phase refining

(2) electrolytic refining

(3) liquation

(4) zone refining

7. An Ellingham diagram provides information about :

(1) the pressure dependence of the standard electrode potentials of reduction reactions involved in the extraction of metals.

(2) the kinetics of the reduction process.

(3) the temperature dependence of the standard Gibbs energies of formation of some metal oxides.

(4) the conditions of pH and potential under which a species is thermodynamically stable.

8. The processes of calcination and roasting in metallurgical industries, respectively, can lead to :-

(1) Global warming and acid rain

(2) Photochemical smog and ozone layer depletion

(3) Global warming and photochemical smog

(4) Photochemical smog and global warming

9. Among statements (a) -(d), the correct ones are :

(a) Lime stone is decomposed to CaO during the extraction of iron from its oxides.

(b) In the extraction of silver, silver is extracted as an anionic complex.

(c) Nickel is purified by Mond's process.

(d) Zr and Ti are purified by Van Arkel method.

(1) (c) and (d) only

(2) (a), (c) and (d) only

(3) (b), (c) and (d) only

(4) (a), (b), (c) and (d)

10. Cast iron is used for the manufacture of :

(1) wrought iron and pig iron

(2) wrought iron and steel

(3) wrought iron, pig iron and steel

(4) pig iron, scrap iron and steel

HYDROGEN & IT'S COMPOUND

1. Dihydrogen of high purity (> 99.95%) is obtained through:

(1) the electrolysis of warm $Ba(OH)_2$ solution using Ni electrodes.

(2) the reaction of Zn with dilute HCl

(3) the electrolysis of brine solution.

(4) the electrolysis of acidified water using Pt electrodes.

2. The one that is NOT suitable for the removal of permanent hardness of water is :

(1) Treatment with sodium carbonate

(2) Calgon's method

(3) Clark's method

(4) Ion-exchange method

SALT ANALYSIS

1. Reaction of an inorganic sulphite X with dilute H_2SO_4 generates compound Y. Reaction of Y with NaOH gives X. Further, the reaction of X with Y and water affords compound Z. Y and Z, respectively, are:

(1) S and Na_2SO_3

(2) SO_2 and $NaHSO_3$

(3) SO_3 and $NaHSO_3$

(4) SO_2 and Na_2SO_3

COMPLETE S-BLOCK

1. In the following reactions products(A) and (B), respectively , are :

$NaOH + Cl_2 \rightarrow (A) +$ side products
(hot and conc.)

$Ca(OH)_2 + Cl_2 \rightarrow (B) +$ side products
(dry)

(1) $NaClO_3$ and $Ca(OCl)_2$

(2) $NaOCl$ and $Ca(ClO_3)_2$

(3) $NaClO_3$ and $Ca(ClO_3)_2$

(4) $NaOCl$ and $Ca(OCl)_2$

2. When gypsum is heated to 393 K, it forms :

(1) Dead burnt plaster

(2) Anhydrous $CaSO_4$

(3) $CaSO_4.5H_2O$

(4) $CaSO_4.0.5H_2O$

3. A metal (A) on heating in nitrogen gas gives compound B. B on treatment with H_2O gives a colourless gas which when passed through $CuSO_4$ solution gives a dark blue-violet coloured solution. A and B respectively, are :
(1) Mg and Mg_3N_2
(2) Na and $NaNO_3$
(3) Mg and $Mg(NO_3)_2$
(4) Na and Na_3N

4. Among the statements (a)-(d) the correct ones are:
(a) Lithium has the highest hydration enthalpy among the alkali metals.
(b) Lithium chloride is insoluble in pyridine.
(c) Lithium cannot form ethynide upon its reaction with ethyne.
(d) Both lithium and magnesium react slowly with H_2O.
(1) (a), (b) and (d) only
(2) (b) and (c) only
(3) (a), (c) and (d) only
(4) (a) and (d) only

5. Match the following compounds (Column-I) with their uses (Column-II) :

S.No.	Column – I	S.No.	Column – II
(I)	$Ca(OH)_2$	(A)	casts of statues
(II)	NaCl	(B)	white wash
(III)	$CaSO_4.\frac{1}{2}H_2O$	(C)	antacid
(IV)	$CaCO_3$	(D)	washing soda preparation

(1) (I)-(D), (II)-(A), (III)-(C), (IV)-(B)
(2) (I)-(B), (II)-(C), (III)-(D), (IV)-(A)
(3) (I)-(C), (II)-(D), (III)-(B), (IV)-(A)
(4) (I)-(B), (II)-(D), (III)-(A), (IV)-(C)

6. An alkaline earth metal 'M' readily forms water soluble sulphate and water insoluble hydroxide. Its oxide MO is very stable to heat and does not have rock-salt structure. M is :-
(1) Ca (2) Be (3) Mg (4) Sr

7. On combustion Li, Na and K in excess of air, the major oxides formed, respectively, are :
(1) Li_2O, Na_2O and K_2O_2
(2) Li_2O, Na_2O_2 and K_2O
(3) Li_2O, Na_2O_2 and KO_2
(4) Li_2O_2, Na_2O_2 and K_2O_2

8. If you spill a chemical toilet cleaning liquid on your hand, your first aid would be :
(1) aqueous NH_3 (2) vinegar
(3) aqueous $NaHCO_3$ (4) aqueous NaOH

9. The metal mainly used in devising photoelectric cells is:
(1) Na (2) Rb (3) Li (4) Cs

10. Two elements A and B have similar chemical properties. They don't form solid hydrogencarbonates, but react with nitrogen to form nitrides. A and B, respectively, are :
(1) Na and C (2) Li and Mg
(3) Cs and Ba (4) Na and Rb

COMPLETE D-BLOCK

1. The atomic radius of Ag is closest to :
(1) Cu (2) Hg (3) Au (4) Ni

2. Conside the following reactions :
$NaCl + K_2Cr_2O_7 + H_2SO_4(Conc.) \rightarrow (A) +$ Side products
$(A) + NaOH \rightarrow (B) +$ Side product
$(B) + H_2SO_4(dilute) + H_2O_2 \rightarrow (C) +$ Side product
The sum of the total number of atoms in one molecule each of (A), (B) and (C) is

3. The third ionization enthalpy is minimum for :
(1) Fe (2) Ni
(3) Co (4) Mn

4. The sum of the total number of bonds between chromium and oxygen atoms in chromate and dichromate ions is ____________.

5. The set that contains atomic number of only transition element is -
(1) 21, 32, 53, 64
(2) 21, 25, 42, 72
(3) 9, 17, 34, 38
(4) 37, 42, 50, 64

6. The incorrect statement(s) among (a) - (c) is (are) :-
(a) W(VI) is more stable than Cr(VI).
(b) in the presence of HCl, permanganate titrations provide satisfactory results.
(c) some lanthanoid oxides can be used as phosphors.
(1) (a) and (b) only (2) (a) only
(3) (b) and (c) only (4) (b) only

7. The INCORRECT statement is :
(1) bronze is an alloy of copper and tin.
(2) brass is an alloy of copper and nickel
(3) cast iron is used to manufacture wrought iron
(4) german silver is an alloy of zinc, copper and nickel

8. The incorrect statement is :
(1) In manganate and permanganate ions, the π-bonding takes place by overlap of p-orbitals of oxygen and d-orbitals of manganese
(2) Manganate ion is green in colour and permanganate ion in purple in colour
(3) Manganate and permanganate ions are paramagnetic
(4) Manganate and permanganate ions are tetrahedral

COMPLETE P-BLOCK

1. Chlorine reacts with hot and concentrated NaOH and produces compounds (X) and (Y). Compound (X) gives white precipitate with silver nitrate solution. The average bond order between Cl and O atoms in (Y) is ______.

2. The redox reaction among the following is :
(1) Combination of dinitrogen with dioxygen at 2000 K
(2) Formation of ozone from atmosphereic oxygen in the presence of sunlight
(3) Reaction of H_2SO_4 with NaOH
(4) Reaction of $[Co(H_2O)_6]Cl_3$ with $AgNO_3$

3. Among the statements (a) - (d), the correct ones are -
(a) Decomposition of hydrogen peroxide gives dioxygen
(b) Like hydrogen peroxide, compounds , such as $KClO_3$, $Pb(NO_3)_2$ and $NaNO_3$when heated liberated dioxygen
(c) 2-Ethylanthraquinone is useful for the industrial preparation of hydrogen peroxide.
(d) Hydrogen peroxide is used for the manufacture of sodium perborate
(1) (a), (b) and (c) only
(2) (a) and (c) only
(3) (a), (b) , (c) and (d)
(4) (a), (c) and (d) only

4. The number of bonds between sulphur and oxygen atoms in $S_2O_8^{2-}$ and the number of bonds between sulphur and sulphur atoms in rhombic sulphur, respectively, are :
(1) 4 and 8 (2) 4 and 6
(3) 8 and 8 (4) 8 and 6

5. White Phosphorus on reaction with concentrated NaOH solution in an inert atmosphere of CO_2 gives phosphine and compound (X). (X) on acidification with HCl gives compound (Y). The basicity of compound (Y) is :
(1) 4 (2) 1 (3) 2 (4) 3

6. The reaction of $H_3N_3B_3Cl_3$ (A) with $LiBH_4$ in tetrahydrofuran gives inorganic benzene (B). Further, the reaction of (A) with (C) leads to $H_3N_3B_3(Me)_3$. Compounds (B) and (C) respectively, are:
(1) Boron nitride and MeBr
(2) Borazine and MeMgBr
(3) Borazine and MeBr
(4) Diborane and MeMgBr

7. The reaction of NO with N_2O_4 at 250 K gives :
(1) N_2O_5 (2) NO_2
(3) N_2O (4) N_2O_3

8. Reaction of ammonia with excess Cl_2 gives :
(1) NH_4Cl and N_2
(2) NCl_3 and NH_4Cl
(3) NH_4Cl and HCl
(4) NCl_3 and HCl

9. The correct statement with respect to dinitrogen is :
(1) liquid dinitrogen is not used in cryosurgery.
(2) it can be used as an inert diluent for reactive chemicals.
(3) it can combine with dioxygen at 25°C
(4) N_2 is paramagnetic in nature.

10. The equation that represents the water-gas shift reaction is :
(1) $CO(g) + H_2O(g) \xrightarrow[\text{Catalyst}]{673K} CO_2(g) + H_2(g)$
(2) $CH_4(g) + H_2O(g) \xrightarrow[\text{Ni}]{1270K} CO(g) + 3H_2(g)$
(3) $C(s) + H_2O(g) \xrightarrow{1270K} CO(g) + H_2(g)$
(4) $2C(s) + O_2(g) + 4N_2(g) \xrightarrow{1273K} 2CO(g) + 4N_2(g)$

11. On heating, lead(II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with NO changes to a blue solid (C). The oxidation number of nitrogen in solid (C) is :

(1) +5 (2) +2

(3) +4 (4) +3

12. The statement that is not true about ozone is :

(1) in the stratosphere, it forms a protective shield against UV radiation.

(2) it is a toxic gas and its reaction with NO gives NO_2.

(3) in the atmosphere, it is depleted by CFCs.

(4) in the stratophere, CFCs release chlorine free radicals ($\dot{Cl}$) which reacts with O_3 to give chlorine dioxide radicals.

13. On heating compound (A) gives a gas (B) which is constituent of air. This gas when treated with H_2 in the presence of a catalyst gives another gas (C) which is basic in nature. (A) should not be:

(1) $(NH_4)_2Cr_2O_7$

(2) $Pb(NO_3)_2$

(3) NaN_3

(4) NH_4NO_2

HYDROGEN AND ITS COMPOUND

1. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resins method is :

(1) less efficient as it exchanges only anions

(2) more efficient as it can exchange only cations

(3) less efficient as the resins cannot be regenerated

(4) more efficient as it can exchange both cations as well as anions

2. Hydrogen has three isotopes (A), (B) and (C). If the number of neutron(s) in (A), (B) and (C) respectively, are (x), (y) and (z), the sum of (x), (y) an (z) is :

(1) 4 (2) 3 (3) 2 (4) 1

ENVIRONMENTAL CHEMISTRY

1. Among the gases (a) - (e), the gases that cause greenhouse effect are :

(a) CO_2 (b) H_2O (c) CFCs

(d) O_2 (e) O_3

(1) (a), (b), (c) and (d)

(2) (a), (c), (d) and (e)

(3) (a) and (d)

(4) (a), (b), (c) and (e)

2. Biochemical Oxygen Demand (BOD) is the amount of oxygen required (in ppm):

(1) by anaerobic bacteria to breakdown inorganic waste present in a water body.

(2) for the photochemical breakdown of waste present in 1 m^3 volume of a water body.

(3) by bacteria to break-down organic waste in a certain volume of a water sample.

(4) for sustaining life in a water body.

F-BLOCK

1. The electronic configurations of bivalent europium and trivalent cerium are

(atomic number : Xe = 54, Ce = 58, Eu = 63)

(1) [Xe] $4f^4$ and [Xe] $4f^9$

(2) [Xe] $4f^7$ and [Xe] $4f^1$

(3) [Xe] $4f^7$ $6s^2$ and [Xe] $4f^2$ $6s^2$

(4) [Xe] $4f^2$ and [Xe] $4f^7$

2. The lanthanoid that does NOT show +4 oxidation state is

(1) Dy (2) Eu (3) Ce (4) Tb

3. Mischmetal is an alloy consisting mainly of:

(1) lanthanoid metals

(2) actinoid metals

(3) actinoid and transition metals

(4) lanthanoid and actinoid metals

ANSWER KEY

QUANTUM NUMBER				
Que.	1	2	3	4
Ans.	2	4	NTA (1) ALLEN (2,3)	4

PERIODIC TABLE										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	3	1	4	1	3	2	2	4	2
Que.	11	12	13	14	15	16				
Ans.	2	3	4	3	4	101.00				

CHEMICAL BONDING										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	4	1	3	3	1	1	4	4	1
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	3.00	1	2	1	2	3	2	4	4	NTA (3) ALLEN (2)
Que.	21	22	23							
Ans.	1	1	1							

COORDINATION CHEMISTRY										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	4	4	1	3	26.60 to 27.00	4	1	20	2
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	1	2	4	2	3	0	4	6	3	1
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	2	NTA (4) ALLEN (2, 4)	3	1	3	3	4	3	2	3
Que.	31	32	33							
Ans.	3	6	2							

METALLURGY										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	2	1	2	2	4	3	1	4	2

HYDROGEN & IT'S COMPOUND		
Que.	**1**	**2**
Ans.	1	3

SALT ANALYSIS	
Que.	**1**
Ans.	2

COMPLETE S-BLOCK										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	4	1	3	4	2	3	3	4	2

COMPLETE D-BLOCK								
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Ans.	3	18	1	NTA (12.00) ALLEN (18.00)	2	4	2	3

COMPLETE P-BLOCK										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1.66 to 1.67	1	3	3	2	2	4	4	2	1
Que.	**11**	**12**	**13**							
Ans.	4	4	2							

HYDROGEN AND ITS COMPOUND		
Que.	**1**	**2**
Ans.	4	2

ENVIRONMENTAL CHEMISTRY		
Que.	**1**	**2**
Ans.	4	3

F-BLOCK			
Que.	**1**	**2**	**3**
Ans.	2	2	1

IMPORTANT NOTES

ALLEN® CAREER INSTITUTE KOTA (RAJASTHAN) path to success

Chapter Contents 03

JEE (MAIN) TOPICWISE TEST PAPERS JANUARY & SEPTEMBER 2020

MATHEMATICS

JANUARY AND SEPTEMBER 2020 ATTEMPT (MATHEMATICS)

LOGARITHM

1. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\text{to }\infty\right)}$ is equal to ________ .

COMPOUND ANGLE

1. Let α and β be two real roots of the equation $(k + 1)\tan^2x - \sqrt{2}\,.\,\lambda\tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is ;

(1) 5 (2) 10

(3) $5\sqrt{2}$ (4) $10\sqrt{2}$

2. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, α, $\beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to _____.

3. The value of

$$\cos^3\left(\frac{\pi}{8}\right)\cdot\cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right)\cdot\sin\left(\frac{3\pi}{8}\right) \text{ is :}$$

(1) $\frac{1}{4}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{2}$

4. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then :

(1) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(2) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

(3) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

(4) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

QUADRATIC EQUATION

1. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true ?

(1) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(2) $p_5 = 11$

(3) $p_3 = p_5 - p_4$

(4) $p_5 = p_2 \cdot p_3$

2. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

(1) is an empty set.

(2) contains at least four elements.

(3) contains exactly two elements.

(4) is a singleton.

3. The least positive value of 'a' for which the equation $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is

4. Let a, b $\in$ R, a $\neq$ 0 be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to :

(1) 26 (2) 25

(3) 28 (4) 24

5. If $A = \{x \in \mathbf{R} : |x| < 2\}$ and $B = \{x \in \mathbf{R} : |x - 2| \geq 3\}$; then :

(1) $A \cup B = \mathbf{R} - (2, 5)$ (2) $A \cap B = (-2, -1)$

(3) $B - A = \mathbf{R} - (-2, 5)$ (4) $A - B = [-1, 2)$

6. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is :

(1) 4 (2) 2

(3) 3 (4) 1

7. Let α and β be the roots of the equation $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1,2,3...., then :

(1) $5S_6 + 6S_5 = 2S_4$

(2) $5S_6 + 6S_5 + 2S_4 = 0$

(3) $6S_6 + 5S_5 + 2S_4 = 0$

(4) $6S_6 + 5S_5 = 2S_4$

8. Let f(x) be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in :

(1) (–3, –1) (2) (1, 3)

(3) (–1, 0) (4) (0, 1)

9. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ is equal to :

(1) $\frac{9}{4}(9 + p^2)$ (2) $\frac{9}{4}(9 - q^2)$

(3) $\frac{9}{4}(9 - p^2)$ (4) $\frac{9}{4}(9 + q^2)$

10. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

(1) (–3, –1) (2) (1, 3]

(3) (0, 2) (4) (2, 4]

11. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio (2q + p) : (2q – p) is :

(1) 3 : 1 (2) 33 : 31

(3) 9 : 7 (4) 5 : 3

12. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to :

(1) 36 (2) 27

(3) 9 (4) 18

13. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is

(1) $\frac{25}{9}$ (2) $\frac{25}{81}$

(3) $\frac{5}{27}$ (4) $\frac{5}{9}$

14. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to:

(1) $\frac{27}{16}$ (2) $\frac{1}{24}$

(3) $\frac{27}{32}$ (4) $\frac{3}{8}$

15. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is

(1) 1 (2) 3

(3) 4 (4) 2

16. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to :

(1) $2\alpha^2$ (2) $2\alpha(\alpha + 1)$

(3) $-2\alpha(\alpha + 1)$ (4) $2\alpha(\alpha - 1)$

SEQUENCE & PROGRESSION

1. If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ... is (102)m, then m is equal to :

(1) 20 (2) 5

(3) 10 (4) 25

2. Let $a_1, a_2, a_3, \ldots$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to :

(1) –171 (2) 171

(3) $\frac{511}{3}$ (4) –513

3. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is :

(1) $\frac{21}{2}$ (2) 27

(3) 16 (4) 7

4. The greatest positive integer k, fr which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \ldots\ 49^2 + 49 + 1$, is :

(1) 32 (2) 60

(3) 63 (4) 65

5. If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is

(1) $50\frac{1}{4}$ (2) $100\frac{1}{2}$

(3) 50 (4) 100

6. The sum, $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to ________.

7. The sum $\sum_{k=1}^{20} (1+2+3+\ldots+k)$ is ....

8. If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then :

(1) $y(1 + x) = 1$ (2) $x(1 + y) = 1$

(3) $y(1 - x) = 1$ (4) $x(1 - y) = 1$

9. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to :

(1) 225 (2) 175

(3) 300 (4) 150

10. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is ______.

11. The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \ldots$ to ∞ is equal to :

(1) $2^{\frac{1}{2}}$ (2) $2^{\frac{1}{4}}$

(3) 2 (4) 1

12. If $|x| < 1, |y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x+y) + (x^2+xy+y^2) + (x^3+x^2y + xy^2+y^3)+\ldots$

(1) $\frac{x+y-xy}{(1-x)(1-y)}$ (2) $\frac{x+y-xy}{(1+x)(1+y)}$

(3) $\frac{x+y+xy}{(1+x)(1+y)}$ (4) $\frac{x+y+xy}{(1-x)(1-y)}$

13. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

(1) $[-3, \infty)$ (2) $(-\infty, 9]$

(3) $(-\infty, -9] \cup [3, \infty)$ (4) $(-\infty, -3] \cup [9, \infty)$

14. If the sum of first 11 terms of an A.P., a_1 a_2, a_3,... is 0 ($a_1 \neq 0$), then the sum of the A.P., a_1, a_3, a_5,...,a_{23} is ka_1, where k is equal to :

(1) $\frac{121}{10}$ (2) $-\frac{72}{5}$

(3) $\frac{72}{5}$ (4) $-\frac{121}{10}$

15. Let S be the sum of the first 9 terms of the series: $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3+(k+4)a\} + \{x^4+(k + 6)a\}+\ldots$ where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to :

(1) –5 (2) 1

(3) –3 (4) 3

16. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is :

(1) $\frac{1}{4}$ (2) $\frac{1}{5}$

(3) $\frac{1}{7}$ (4) $\frac{1}{6}$

17. If the sum of the series $20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+....$ upto n^{th} term is 488 and the n^{th} term is negative, then :

(1) n^{th} term is $-4\frac{2}{5}$ (2) n = 41

(3) n^{th} term is –4 (4) n = 60

18. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to _______.

19. If $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+.....+(1-20^2.19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to:

(1) (10, 97) (2) (11, 103)

(3) (10, 103) (4) (11, 97)

20. Let $a_1, a_2..., a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + ..+ a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to :

(1) (2480, 249) (2) (2490, 249)

(3) (2490, 248) (4) (2480, 248)

21. The minimum value of $2^{\sin x} + 2^{\cos x}$ is :-

(1) $2^{1-\frac{1}{\sqrt{2}}}$ (2) $2^{-1+\sqrt{2}}$

(3) $2^{1-\sqrt{2}}$ (4) $2^{-1+\frac{1}{\sqrt{2}}}$

22. If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P. is :

(1) 66 (2) 65

(3) 81 (4) 78

23. If $2^{10}+2^9\cdot3^1+2^8\cdot3^2+....+2\cdot3^9+3^{10} = S-2^{11}$, then S is equal to :

(1) $\frac{3^{11}}{2}+2^{10}$ (2) $3^{11} - 2^{12}$

(3) 3^{11} (4) $2\cdot3^{11}$

24. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + ...$ is 460, then x is equal to:

(1) $7^{46/21}$ (2) $7^{1/2}$

(3) e^2 (4) 7^2

25. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is :

(1) $\frac{2}{13}(3^{50}-1)$ (2) $\frac{1}{26}(3^{50}-1)$

(3) $\frac{1}{13}(3^{50}-1)$ (4) $\frac{1}{26}(3^{49}-1)$

26. If $f(x+y) = f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2, x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is

(1) $\frac{1}{9}$ (2) $\frac{4}{9}$

(3) $\frac{1}{3}$ (4) $\frac{2}{3}$

27. Let a,b,c,d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then :

(1) a,c,p are in G.P.

(2) a,c,p are in A.P.

(3) a,b,c,d are in G.P.

(4) a,b,c,d are in A.P.

28. The common difference of the A.P. $b_1, b_2, ..., b_m$ is 2 more than the common difference of A . P . $a_1, a_2, ..., a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to :

(1) –127 (2) –81

(3) 81 (4) 127

TRIGONOMETRIC EQUATION

1. The number of distinct solutions of the equation $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is ———.

2. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval :

(1) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$ (2) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$

(3) $\left(-\frac{5}{4}, -1\right)$ (4) $\left[-1, -\frac{1}{2}\right]$

3. Let $a, b, c \in R$ be such that $a^2 + b^2 + c^2 = 1$. If $a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is :

(1) $\frac{\pi}{2}$ (2) 0

(3) $\frac{\pi}{9}$ (4) $\frac{2\pi}{3}$

SOLUTION OF TRIANGLE

1. If a ΔABC has vertices $A(-1, 7)$, $B(-7, 1)$ and $C(5, -5)$, then its orthocentre has coordinates:

(1) (3, –3) (2) $\left(-\frac{3}{5}, \frac{3}{5}\right)$

(3) (–3, 3) (4) $\left(\frac{3}{5}, -\frac{3}{5}\right)$

HEIGHT & DISTANCE

1. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :

(1) 20/3 (2) 5

(3) 10/3 (4) 6

2. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30º. If the angle of depression of the image of C in the lake from the point P is 60º, then PC (in m) is equal to :

(1) 400 (2) $400\sqrt{3}$

(3) 100 (4) $200\sqrt{3}$

3. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is_.

4. The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climding up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is :

(1) $\frac{1}{\sqrt{3}-1}$ (2) $\frac{1}{\sqrt{3}+1}$

(3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (4) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

DETERMINANT

1. If the system of linear equations,

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$3x + 2y + \lambda z = \mu$$

has more two solutions, then $\mu - \lambda^2$ is equal to

____

2. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0,$

where a, b, c $\in$ R are non-zero and distinct; has a non-zero solution, then :

(1) a, b, c are in A.P.

(2) a + b + c = 0

(3) a, b, c are in G.P.

(4) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

3. The system of linear equations

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has

(1) infinitely many solutions when $\lambda = 2$

(2) a unique solution when $\lambda = -8$

(3) no solution when $\lambda = 8$

(4) no solution when $\lambda = 2$

4. For which of the following ordered pairs (μ,δ), the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent ?

(1) (1,0) (2) (4,6)

(3) (3,4) (4) (4,3)

5. Let a – 2b + c = 1. If

$$f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$, then :

(1) $f(-50) = 501$ (2) $f(-50) = -1$

(3) $f(50) = 1$ (4) $f(50) = -501$

6. The following system of linear equations

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$, has

(1) infinitely many solutions, (x, y, z) satisfying x = 2z

(2) no solution

(3) only the trivial solution

(4) infinitely many solutions, (x, y, z) satisfying y = 2z

7. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

has no solution. Then the set S

(1) contains more than two elements.

(2) is a singleton.

(3) contains exactly two elements.

(4) is an empty set.

8. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to ________.

9. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$, then B + C is equal to :

(1) –1 (2) 1

(3) –3 (4) 9

10. If the system of equations

$x - 2y + 3z = 9$

$2x + y + z = b$

$x - 7y + az = 24,$

has infinitely many solutions, then a – b is equal to ________ .

11. If the system of equations

$x + y + z = 2$

$2x + 4y - z = 6$

$3x + 2y + \lambda z = \mu$

has infinitely many solutions, then :

(1) $\lambda - 2\mu = -5$ (2) $2\lambda - \mu = 5$

(3) $2\lambda + \mu = 14$ (4) $\lambda + 2\mu = 14$

12. If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \to R$, defined by :

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$ are m and M

respectively, then the ordered pair (m, M) is equal to:

(1) (0, 4) (2) (–4, 4)

(3) $(0, 2\sqrt{2})$ (4) (–4, 0)

13. Let $\lambda \in R$. The system of linear equations

$2x_1 - 4x_2 + \lambda x_3 = 1$

$x_1 - 6x_2 + x_3 = 2$

$\lambda x_1 - 10x_2 + 4x_3 = 3$

is inconsistent for :

(1) exactly one negative value of λ.

(2) exactly one positive value of λ.

(3) every value of λ.

(4) exactly two values of λ.

14. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to :

(1) 9 (2) –3

(3) –9 (4) 3

15. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$$ is equal to :

(1) 0 (2) y(a – b)

(3) y (b – a) (4) y(a – c)

16. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively

(1) 5 and 7 (2) 6 and 8

(3) 4 and 9 (4) 5 and 8

17. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$. Then the ordered pair (m,M) is equal to

(1) (–3,–1) (2) (–4,–1)

(3) (1,3) (4) (–3,3)

18. The sum of distinct values of λ for which the system of equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0,$

has non-zero solutions, is ___________.

STRAIGHT LINE

1. The locus of the mid-points of the perpendiculars drawn from points on the line, x = 2y to the line x = y is :

(1) $2x - 3y = 0$ (2) $7x - 5y = 0$

(3) $5x - 7y = 0$ (4) $3x - 2y = 0$

2. Let A(1, 0), B(6, 2) and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is __________.

3. Let two points be A(1,–1) and B(0,2). If a point P(x',y') be such that the area of $\Delta PAB = 5$ sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of λ is

(1) 1 (2) 4

(3) 3 (4) –3

4. Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point :

(1) (7, 6) (2) (–9, –6)

(3) (–9, –7) (4) (9, 7)

5. The set of all possible values of θ in the interval $(0, \pi)$ for which the points (1, 2) and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is :

(1) $\left(0, \frac{\pi}{4}\right)$ (2) $\left(0, \frac{3\pi}{4}\right)$

(3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (4) $\left(0, \frac{\pi}{2}\right)$

6. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90°$, and $ar(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

(1) $2+\sqrt{5}$ (2) $1+\sqrt{5}$

(3) $1+2\sqrt{5}$ (4) $2\sqrt{5}-1$

7. If the perpendicular bisector of the line segment joining the points P (1, 4) and Q (k, 3) has y-intercept equal to –4, then a value of k is :-

(1) $\sqrt{15}$ (2) –2

(3) $\sqrt{14}$ (4) –4

8. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is _____

9. A ray of light coming from the point $\left(2, 2\sqrt{3}\right)$ is incident at an angle 30° on the line x=l at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point:

(1) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (2) $\left(3, -\sqrt{3}\right)$

(3) $\left(4, -\frac{\sqrt{3}}{2}\right)$ (4) $\left(4, -\sqrt{3}\right)$

10. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point (–1, –4) in this line is :

(1) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (2) $\left(\frac{29}{5}, \frac{11}{5}\right)$

(3) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (4) $\left(\frac{29}{5}, \frac{8}{5}\right)$

CIRCLE

1. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to :

(1) $\frac{52}{5}$ (2) $\frac{32}{5}$

(3) $\frac{56}{5}$ (4) $\frac{64}{5}$

2. If a line, $y = mx + c$ is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then

(1) $c^2 - 6c + 7 = 0$ (2) $c^2 + 6c + 7 = 0$

(3) $c^2 + 7c + 6 = 0$ (4) $c^2 - 7c + 6 = 0$

3. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is______.

4. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is ______.

5. The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x = 3$ and $y = 2$, is _______ .

6. The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point :

(1) (1, –3) (2) (–1, 3)
(3) (–3, 1) (4) (–3, 6)

7. Let PQ be a diameter of the circle $x^2+y^2=9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is ____

8. If the length of the chord of the circle, $x^2 + y^2 = r^2$ $(r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to:

(1) $\frac{9}{5}$ (2) $\frac{12}{5}$
(3) 12 (4) $\frac{24}{5}$

PERMUTATION & COMBINATION

1. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is:

(1) $\frac{5}{2}(6!)$ (2) 5^6
(3) $\frac{1}{2}(6!)$ (4) 6!

2. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _______.

3. If a,b and c are the greatest value of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then

(1) $\frac{a}{11}=\frac{b}{22}=\frac{c}{21}$ (2) $\frac{a}{10}=\frac{b}{11}=\frac{c}{21}$
(3) $\frac{a}{10}=\frac{b}{11}=\frac{c}{42}$ (4) $\frac{a}{11}=\frac{b}{22}=\frac{c}{42}$

4. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is ....

5. If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is 336 k, then k is equal to :

(1) 8 (2) 6
(3) 4 (4) 7

6. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is ______.

7. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is :-

(1) 199 (2) 101
(3) 201 (4) 200

8. The value of $(2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2 - ....$ up to 51^{th} term$) + (1! - 2! + 3! - .....$ up to 51^{th} term$)$ is equal to :

(1) $1 + (51)!$ (2) $1 - 51(51)!$
(3) $1 + (52)!$ (4) 1

9. The total number of 3-digit numbers, whose sum of digits is 10, is _______.

10. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ____

11. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is ________.

12. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is :

(1) 1500 (2) 2255
(3) 3000 (4) 2250

13. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is ___________.

BINOMIAL THEOREM

1. The number of ordered pairs (r, k) for which $6\cdot{}^{35}C_r = (k^2-3)\cdot{}^{36}C_{r+1}$, where k is an integer, is:

(1) 3 (2) 2
(3) 4 (4) 6

2. The coefficient of x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \ldots + x^{10}$ is :

(1) 120 (2) 330
(3) 210 (4) 420

3. If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$ is 61, then n is equal to _________.

4. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6$, then

(1) $\alpha + \beta = 60$ (2) $\alpha + \beta = -30$
(3) $\alpha - \beta = -132$ (4) $\alpha - \beta = 60$

5. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if ℓ_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and ℓ_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $\ell_2 : \ell_1$ is equal to :

(1) 1 : 8 (2) 1 : 16
(3) 8 : 1 (4) 16 : 1

6. If $C_r \equiv {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + \ldots + (101).C_{25} = 2^{25}.k$, then k is equal to _____.

7. The coefficient of x^4 is the expansion of $(1 + x + x^2)^{10}$ is ______.

8. Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$ is 10k, then k is equal to :

(1) 176 (2) 336
(3) 352 (4) 84

9. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to________.

10. If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is :

(1) 264 (2) 256
(3) 128 (4) 248

11. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is k, then 18 k is equal to :

(1) 9 (2) 11
(3) 5 (4) 7

12. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to :

(1) ${}^{51}C_7 + {}^{30}C_7$ (2) ${}^{51}C_7 - {}^{30}C_7$
(3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

13. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to _________.

14. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is :-

(1) 792 (2) 252
(3) 462 (4) 330

15. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ________.

16. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is ______.

17. If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to

(1) $\frac{1}{8}$ (2) $\frac{5}{8}$

(3) $\frac{3}{8}$ (4) $\frac{7}{8}$

18. If the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then |k| equals :

(1) 2 (2) 1

(3) 3 (4) 9

SET

1. Let X = {n ∈ N : 1 ≤ n ≤ 50}. If A = {n ∈ X : n is a multiple of 2} and B = {n ∈ X : n is a multiple of 7}, then the number of elements in the smallest subset of X containing both A and B is_____

2. Consider the two sets :

A = {m ∈ R : both the roots of $x^2 - (m + 1)x + m + 4 = 0$ are real} and B = [–3, 5).

Which of the following is not true ?

(1) A – B = (–∞, –3) ∪ (5, ∞)

(2) A ∩ B = {–3}

(3) B – A = (–3, 5)

(4) A ∪ B = R

3. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to _______.

4. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

(1) 65 (2) 37

(3) 29 (4) 55

5. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to :

(1) 45 (2) 15

(3) 50 (4) 30

6. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

(1) 63 (2) 38

(3) 54 (4) 36

7. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _.

RELATION

1. If $R = \{(x,y) : x,y \in Z, x^2 + 3y^2 \le 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is:

(1) {–2, –1, 1,2} (2) {–1, 0, 1}
(3) {–2, –1, 0, 1, 2} (4) {0, 1}

2. Let R_1 and R_2 be two relations defined as follows :

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$,

where Q is the set of all rational numbers. Then:

(1) R_2 is transitive but R_1 is not transitive

(2) R_1 is transitive but R_2 is not transitive

(3) R_1 and R_2 are both transitive

(4) Neither R_1 nor R_2 is transitive

FUNCTION

1. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to

(1) $\frac{3}{2}$ (2) $-\frac{1}{2}$
(3) $-\frac{3}{2}$ (4) $\frac{1}{2}$

2. Let $f : (1,3) \to R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where [x] denotes the greatest integer ≤ x. Then the range of f is

(1) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (2) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$

(3) $\left(\frac{2}{5}, \frac{4}{5}\right]$ (4) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

3. Let $f : R \to R$ be such that for all $x \in R$ $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is

(1) 0 (2) 3
(3) 2 (4) 4

4. The inverse function of

$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1,1)$, is

(1) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

(2) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(3) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

(4) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

5. Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y) \forall x,y \in R$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which $g(n) = 20$, is :

(1) 5 (2) 9
(3) 20 (4) 4

6. Let [t] denote the greatest integer ≤ t. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has :

(1) no integral solution
(2) exactly four integral solutions
(3) exactly two solutions
(4) infinitely many solutions

7. Let A = {a, b, c} and B = {1, 2, 3, 4}. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one$\}$ is ________.

8. For a suitably chosen real constant a, let a function, $f : R - \{-a\} \to R$ be defined by $f(x) = \frac{a-x}{a+x}$. Further suppose that for any real number $x \ne -a$ and $f(x) \ne -a$, $(f o f)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) $\frac{1}{3}$ (2) 3
(3) –3 (4) $-\frac{1}{3}$

9. Suppose that a function $f : R \to R$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to __________.

INVERSE TRIGONOMETRY FUNCTION

1. The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a]\cup[a, \infty)$. Then a is equal to :

(1) $\frac{1+\sqrt{17}}{2}$ (2) $\frac{\sqrt{17}-1}{2}$

(3) $\frac{\sqrt{17}}{2}+1$ (4) $\frac{\sqrt{17}}{2}$

2. $2\pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}\right)$ is equal to:

(1) $\frac{7\pi}{4}$ (2) $\frac{5\pi}{4}$

(3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{2}$

3. If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\tan^{-1}\left(\frac{1}{21}\right)+\ldots$, then tan(S) is equal to :

(1) $\frac{5}{11}$ (2) $-\frac{6}{5}$

(3) $\frac{10}{11}$ (4) $\frac{5}{6}$

LIMIT

1. $\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}$ is equal to ____________.

2. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

(1) $\frac{1}{e}$ (2) e^2

(3) e (4) $\frac{1}{e^2}$

3. If $\lim_{x\to 1}\frac{x+x^2+x^3+\ldots+x^n-n}{x-1} = 820, (n \in N)$ then the value of n is equal to_______.

4. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to :

(1) 2 (2) e

(3) 1 (4) e^2

5. Let [t] denote the greatest integer $\leq$ t. If for some $\lambda \in R - \{0, 1\}$, $\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda - x+[x]}\right| = L$, then L is equal to :

(1) 1 (2) 2

(3) $\frac{1}{2}$ (4) 0

6. If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$, then the value of k is _______ .

7. $\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}} (a \neq 0)$ is equal to :

(1) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

(3) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

8. Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that f(1) = e and $\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x} = 0$. If f(x) = 1, then x is equal to :

(1) 2e (2) $\frac{1}{2e}$

(3) e (4) $\frac{1}{e}$

9. If α is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim_{x\to\alpha^+}\frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{3}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

10. $\lim_{x\to 0}\frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$

(1) does not exist. (2) is equal to $\sqrt{e}$.

(3) is equal to 0. (4) is equal to 1.

11. $\lim_{x\to 1}\left(\frac{\int_0^{(x-1)^2} t\cos\left(t^2\right)dt}{(x-1)\sin(x-1)}\right)$

(1) does not exist (2) is equal to $\frac{1}{2}$

(3) is equal to 1 (4) is equal to $-\frac{1}{2}$

CONTINUITY

1. If the function f defined on $\left(-\frac{1}{3},\frac{1}{3}\right)$ by

$$f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right) & ,\text{when } x\neq 0\\ k & ,\text{when } x=0\end{cases}$$ is

continuous, then k is equal to_______

2. Let [t] denote the greatest integer $\le$ t and $\lim_{x\to 0} x\left[\frac{4}{x}\right]=A$. Then the function, $f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to :

(1) $\sqrt{A+5}$ (2) $\sqrt{A+1}$

(3) $\sqrt{A}$ (4) $\sqrt{A+21}$

3. If $f(x)=\begin{cases}\frac{\sin(a+2)x+\sin x}{x} & ; x<0\\ b & ; x=0\\ \frac{(x+3x^2)^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{4}{3}}} & ; x>0\end{cases}$

is continuous at x = 0, then a + 2b is equal to :

(1) –1 (2) 1

(3) –2 (4) 0

4. Let $f(x) = x\cdot\left[\frac{x}{2}\right]$, for $-10 < x < 10$, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to ________.

DIFFERENTIABILITY

1. Let S be the set of points where the function, $f(x) = \left|2-|x-3|\right|$, $x\in R$, is not differentiable.

Then $\sum_{x\in S} f(f(x))$ is equal to __________.

2. If a function f(x) defined by

$$f(x)=\begin{cases}ae^x+be^{-x}, & -1\le x<1\\ cx^2 & , 1\le x\le 3\\ ax^2+2cx, & 3<x\le 4\end{cases}$$ be continuous

for some a, b, c $\in$ R and $f'(0) + f'(2) = e$, then the value of of a is :

(1) $\frac{e}{e^2-3e-13}$ (2) $\frac{e}{e^2+3e+13}$

(3) $\frac{1}{e^2-3e+13}$ (4) $\frac{e}{e^2-3e+13}$

3. Suppose a differentiable function f(x) satisfies the identity $f(x + y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim_{x\to 0}\frac{f(x)}{x}=1$, then f '(3) is equal to ________.

4. The function $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, |x| \le 1 \\ \frac{1}{2}(|x|-1), |x| > 1 \end{cases}$ is :

(1) continuous on R–{1} and differentiable on R – {–1, 1}.

(2) both continuous and differentiable on R – {–1}.

(3) continuous on R – {–1} and differentiable on R – {–1, 1}.

(4) both continuous and differentiable on R –{1}

5. If the function $f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to :

(1) $\left(\frac{1}{2}, 1\right)$ (2) (1, 1)

(3) $\left(\frac{1}{2}, -1\right)$ (4) (1, 0)

6. Let f : R → R be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2 & , \quad x < 0 \\ 0 & , \quad x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2 & , \quad x > 0 \end{cases}$$

. The value of λ for which $f''(0)$ exists, is _.

7. Let f : R → R be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then :

(1) {0, 1} (2) {0}

(3) ϕ(an empty set) (4) {1}

METHOD OF DIFFERENTIATION

1. Let y = y(x) be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to:

(1) $\frac{\sqrt{5}}{2}$ (2) $-\frac{\sqrt{5}}{2}$

(3) $\frac{2}{\sqrt{5}}$ (4) $-\frac{\sqrt{5}}{4}$

2. If $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1+\tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}, \alpha \in \left(\frac{3\pi}{4}, \pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is :

(1) 4 (2) $-\frac{1}{4}$

(3) $\frac{4}{3}$ (4) –4

3. Let $x^k + y^k = a^k$, (a, K > 0) and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is :

(1) $\frac{3}{2}$ (2) $\frac{1}{3}$

(3) $\frac{2}{3}$ (4) $\frac{4}{3}$

4. Let $f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1$, $|x| > 1$. If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}(f(x))\right)$ and $y(\sqrt{3}) = \frac{\pi}{6}$, then $y(-\sqrt{3})$ is equal to

(1) $\frac{5\pi}{6}$ (2) $-\frac{\pi}{6}$

(3) $\frac{\pi}{3}$ (4) $\frac{2\pi}{3}$

5. If x = 2sinθ – sin2θ and y = 2cosθ – cos2θ, θ ∈ [0, 2π], then $\frac{d^2y}{dx^2}$ at θ = π is :

(1) $\frac{3}{2}$ (2) $-\frac{3}{4}$

(3) $\frac{3}{4}$ (4) $-\frac{3}{8}$

6. Let f and g be differentiable functions on **R** such that fog is the identity function. If for some a, b $\in$ **R**, g'(a) = 5 and g(a) = b, then $f'(b)$ is equal to :

(1) $\frac{2}{5}$ (2) 1

(3) $\frac{1}{5}$ (4) 5

7. If $y=\sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx-\frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at x = 0 is________.

8. If $y^2 + \log_e(\cos^2 x) = y$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, then :

(1) $|y''(0)| = 2$

(2) $|y'(0)| + |y''(0)| = 3$

(3) $|y'(0)| + |y''(0)| = 1$

(4) $y''(0) = 0$

9. If $\left(a+\sqrt{2}\,b\cos x\right)\left(a-\sqrt{2}\,b\cos y\right)=a^2-b^2$, where a > b > 0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ is :

(1) $\frac{a-b}{a+b}$ (2) $\frac{a+b}{a-b}$

(3) $\frac{2a+b}{2a-b}$ (4) $\frac{a-2b}{a+2b}$

10. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=\frac{1}{2}$ is :

(1) $\frac{\sqrt{3}}{12}$ (2) $\frac{\sqrt{3}}{10}$

(3) $\frac{2\sqrt{3}}{5}$ (4) $\frac{2\sqrt{3}}{3}$

INDEFINITE INTEGRATION

1. If $\int\frac{\cos x\,dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}}=f(x)\left(1+\sin^6 x\right)^{1/\lambda}+c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to

(1) –2 (2) $-\frac{9}{8}$

(3) 2 (4) $\frac{9}{8}$

2. If $\int\frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)}=\lambda\tan\theta+2\log_e|f(\theta)|+C$ where C is a constant of integration, then the ordered pair (λ, $f(\theta)$) is equal to :

(1) (–1, 1 + tanθ) (2) (–1, 1 – tanθ)

(3) (1, 1 – tanθ) (4) (1, 1 + tanθ)

3. The integral $\int\frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to :

(where C is a constant of integration)

(1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+C$ (2) $-\left(\frac{x-3}{x+4}\right)^{-\frac{1}{7}}+C$

(3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}}+C$ (4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-\frac{13}{7}}+C$

4. If $\int\sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx=A(x)\tan^{-1}\left(\sqrt{x}\right)+B(x)+C$, where C is a constant of integration, then the ordered pair (A(x), B(x)) can be :

(1) $\left(x-1,\sqrt{x}\right)$ (2) $\left(x+1,\sqrt{x}\right)$

(3) $\left(x+1,-\sqrt{x}\right)$ (4) $\left(x-1,-\sqrt{x}\right)$

5. The integral $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to:

(where C is a constant of integration)

(1) $\sec x+\frac{x\tan x}{x\sin x+\cos x}+C$

(2) $\sec x-\frac{x\tan x}{x\sin x+\cos x}+C$

(3) $\tan x+\frac{x\sec x}{x\sin x+\cos x}+C$

(4) $\tan x-\frac{x\sec x}{x\sin x+\cos x}+C$

6. If

$\int(e^{2x}+2e^x-e^{-x}-1)e^{(e^x+e^{-x})}dx = g(x)e^{(e^x+e^{-x})}+c$,

where c is a constant of integration, then g(0) is equal to :

(1) 2 (2) e^2

(3) e (4) 1

7. If $\int\frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta = A\log_e|B(\theta)|+C$,

where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be:

(1) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$ (2) $\frac{2\sin\theta+1}{\sin\theta+3}$

(3) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$ (4) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$

DEFINITE INTEGRATION

1. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in (0, 2π) – {π} which satisfy the equation, $2\cot^2\theta-\frac{5}{\sin\theta}+4=0$, then $\int_{\theta_1}^{\theta_2}\cos^2 3\theta d\theta$ is equal to :

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}+\frac{1}{6}$

(3) $\frac{\pi}{9}$ (4) $\frac{\pi}{3}$

2. The value of α for which $4\alpha\int_{-1}^{2}e^{-\alpha|x|}dx=5$, is :

(1) $\log_e\left(\frac{3}{2}\right)$ (2) $\log_e\left(\frac{4}{3}\right)$

(3) $\log_e 2$ (4) $\log_e\sqrt{2}$

3. If $f(a+b+1-x)=f(x)$, for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x(f(x)+f(x+1))dx$ is equal to :

(1) $\int_{a+1}^{b+1}f(x)dx$ (2) $\int_{a+1}^{b+1}f(x+1)dx$

(3) $\int_{a-1}^{b-1}f(x+1)dx$ (4) $\int_{a-1}^{b-1}f(x)dx$

4. If $I=\int_1^2\frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then :

(1) $\frac{1}{9}<I^2<\frac{1}{8}$ (2) $\frac{1}{16}<I^2<\frac{1}{9}$

(3) $\frac{1}{6}<I^2<\frac{1}{2}$ (4) $\frac{1}{8}<I^2<\frac{1}{4}$

5. $\lim_{x\to 0}\frac{\int_0^x t\sin(10t)dt}{x}$ is equal to

(1) 0 (2) $-\frac{1}{5}$

(3) $-\frac{1}{10}$ (4) $\frac{1}{10}$

6. The value of $\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ is equal to :

(1) 2π (2) 4π

(3) $2\pi^2$ (4) π^2

7. If for all real triplets (a, b, c), $f(x)=a+bx+cx^2$; then $\int_0^1 f(x)dx$ is equal to :

(1) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$

(2) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$

(3) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

(4) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$

8. The integral $\int_0^2 ||x-1|-x|\,dx$ is equal to______.

9. Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is_______.

10. $\int_{-\pi}^{\pi} |\pi-|x||\,dx$ is equal to :

(1) π^2 (2) $2\pi^2$

(3) $\sqrt{2}\pi^2$ (4) $\frac{\pi^2}{2}$

11. If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to :

(1) $2\sqrt{3}-\pi$ (2) $3\sqrt{2}+\pi$

(3) $3\sqrt{2}-\pi$ (4) $2\sqrt{3}+\pi$

12. Let $f(x) = |x-2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$.

Then $\int_0^3 (g(x)-f(x))dx$ is equal to :

(1) $\frac{3}{2}$ (2) 0

(3) $\frac{1}{2}$ (4) 1

13. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}dx\ (x \ge 0)$. Then $f(3)-f(1)$ is equal to :

(1) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

14. $\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x(2\sec^2 x \cdot \sin^2 3x + 3\tan x \cdot \sin 6x)dx$ is equal to :

(1) $\frac{9}{2}$ (2) $-\frac{1}{9}$

(3) $-\frac{1}{18}$ (4) $\frac{7}{18}$

15. Let {x} and [x] denote the fractional part of x and the greatest integer $\le$ x respectively of a real number x. If $\int_0^n \{x\}dx, \int_0^n [x]dx$ and $10(n^2-n)$, $(n \in N, n > 1)$ are three consecutive terms of a G.P., then n is equal to____

16. The value of $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx$ is

(1) π (2) $\frac{3\pi}{2}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

17. If $I_1 = \int_0^1 (1-x^{50})^{100} dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to

(1) $\frac{5050}{5051}$ (2) $\frac{5050}{5049}$

(3) $\frac{5049}{5050}$ (4) $\frac{5051}{5050}$

18. The integral $\int_1^2 e^x.x^x(2+\log_e x)dx$ equal :

(1) e(4e + 1) (2) e(2e – 1)

(3) $4e^2 - 1$ (4) e(4e – 1)

TANGENT & NORMAL

1. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2,2) is

(1) $4\sqrt{2}$ (2) $2\sqrt{2}$

(3) 2 (4) $\sqrt{2}$

2. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then |m| is equal to

3. If the tangent to the curve y = x + sin y at a point (a, b) is parallel to the line joining $\left(0,\frac{3}{2}\right)$ and $\left(\frac{1}{2},2\right)$, then :

(1) b = a (2) $b=\frac{\pi}{2}+a$

(3) |b – a| = 1 (4) |a+b| = 1

4. The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at x = 0 is :

(1) y = 4x + 2 (2) x + 4y = 8

(3) y + 4x = 2 (4) 2y + x = 4

5. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is :

(1) 9 (2) 18

(3) 10 (4) 20

6. If the tangent of the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x-axis, then the value of c is ________.

7. If the lines x + y = a and x – y = b touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to________.

8. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, t > 0, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1,t_2]$ is attained at the point :

(1) $a(t_2 - t_1) + b$ (2) $(t_2 - t_1)/2$

(3) $2a(t_1 + t_2) + b$ (4) $(t_1 + t_2)/2$

MONOTONICITY

1. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when x ∈ [0, 1] is :

(1) $\frac{2}{3}$ (2) $\frac{\sqrt{7}-2}{3}$

(3) $\frac{4-\sqrt{5}}{3}$ (4) $\frac{4-\sqrt{7}}{3}$

2. Let the function, $f: [-7, 0] \to R$ be continuous on [–7, 0] and differentiable on (–7, 0). If $f(-7) = -3$ and $f'(x) \le 2$, for all x ∈ (–7, 0), then for all such functions f, $f(-1) + f(0)$ lies in the interval:

(1) [–6, 20] (2) (–∞, 20]

(3) (– ∞, 11] (4) [–3, 11]

3. Let S be the set of all functions $f : [0,1] \to R$, which are continuous on [0,1] and differentiable on (0,1). Then for every f in S, there exists a c ∈ (0,1), depending on f, such that

(1) $|f(c) - f(1)| < (1 - c)|f'(c)|$

(2) $|f(c) - f(1)| < |f'(c)|$

(3) $|f(c) + f(1)| < (1 + c)|f'(c)|$

(4) $\frac{f(1)-f(c)}{1-c} = f'(c)$

4. If c is a point at which Rolle's theorem holds for the function, $f(x)=\log_e\left(\frac{x^2+\alpha}{7x}\right)$ in the interval [3,4], where α ∈ R, then $f''(c)$ is equal to

(1) $\frac{\sqrt{3}}{7}$ (2) $\frac{1}{12}$

(3) $-\frac{1}{24}$ (4) $-\frac{1}{12}$

5. Let $f(x) = x\cos^{-1}(-\sin|x|)$, $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, then which of the following is true ?

(1) f' is decreasing in $\left(-\frac{\pi}{2},0\right)$ and increasing in $\left(0,\frac{\pi}{2}\right)$

(2) f is not differentiable at x = 0

(3) $f'(0) = -\frac{\pi}{2}$

(4) f' is increasing in $\left(-\frac{\pi}{2},0\right)$ and decreasing in $\left(0,\frac{\pi}{2}\right)$

6. Let f be any function continuous on [a, b] and twice differentiable on (a, b). If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than :

(1) $\frac{b+a}{b-a}$ (2) $\frac{b-c}{c-a}$

(3) $\frac{c-a}{b-c}$ (4) 1

7. Let $f : (-1, \infty) \to R$ be defined by $f(0) = 1$ and $f(x) = \frac{1}{x}\log_e(1+x), x \neq 0$. Then the function f:

(1) decreases in $(-1, \infty)$

(2) decreases in $(-1, 0)$ and increases in $(0, \infty)$

(3) increases in $(-1, \infty)$

(4) increases in $(-1, 0)$ and decreases in $(0, \infty)$

8. The function, $f(x) = (3x - 7)x^{2/3}$, $x \in R$, is increasing for all x lying in :

(1) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

(2) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(3) $\left(-\infty, \frac{14}{15}\right)$

(4) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

9. Let f be a twice differentiable function on (1, 6). If $f(2) = 8$, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1, 6)$, then :

(1) $f(5) \leq 10$ (2) $f'(5) + f''(5) \leq 20$

(3) $f(5) + f'(5) \geq 28$ (4) $f(5) + f'(5) \leq 26$

10. For all twice differentiable functions $f : R \to R$, with $f(0) = f(1) = f'(0) = 0$

(1) $f''(x) = 0$, for some $x \in (0, 1)$

(2) $f''(0) = 0$

(3) $f''(x) \neq 0$ at every point $x \in (0, 1)$

(4) $f''(x) = 0$ at every point $x \in (0, 1)$

11. If the tangent to the curve, $y = f(x) = x\log_e x$, $(x > 0)$ at a point $(c, f(c))$ is parallel to the line - segement joining the points (1, 0) and (e, e), then c is equal to :

(1) $\frac{1}{e-1}$ (2) $e^{\left(\frac{1}{1-e}\right)}$

(3) $e^{\left(\frac{1}{e-1}\right)}$ (4) $\frac{e-1}{e}$

MAXIMA & MINIMA

1. Let f(x) be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

(1) f is an odd function

(2) x = 1 is a point of minima and x = –1 is a point of maxima of f.

(3) x = 1 is a point of maxima and x = –1 is a point of minimum of f.

(4) $f(1) - 4f(-1) = 4$

2. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$, $f(x)$ has a critical point at $x = -1$ and $f'(x)$ has a critical point at $x = 1$. Then $f(x)$ has a local minima at x = ________.

3. Let a function $f : [0, 5] \to \mathbf{R}$ be continuous, $f(1) = 3$ and F be defined as : $F(x) = \int_1^x t^2 g(t)\, dt$, where $g(t) = \int_1^t f(u)\, du$. Then for the function F, the point x = 1 is :

(1) a point of local minima.

(2) not a critical point.

(3) a point of inflection.

(4) a point of local maxima.

4. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness the melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is :

(1) $\frac{1}{36\pi}$ (2) $\frac{5}{6\pi}$

(3) $\frac{1}{18\pi}$ (4) $\frac{1}{54\pi}$

5. Let P(h, k) be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at P is :

(1) $x + 3y - 62 = 0$ (2) $x - 3y - 11 = 0$

(3) $x - 3y + 22 = 0$ (4) $x + 3y + 26 = 0$

6. If p(x) be a polynomial of degree three that has a local maximum value 8 at x = 1 and a local minimum value 4 at x = 2; then p(0) is equal to:

(1) 12 (2) –24

(3) 6 (4) –12

7. Suppose f(x) is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in R | f(x) = f(0)\}$, then the sum of squares of all the elements of T is:

(1) 6 (2) 8

(3) 4 (4) 2

8. If x = 1 is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)\, e^x$, then :

(1) x = 1 is a local minima and $x = -\frac{2}{3}$ is a local maxima of f.

(2) x = 1 is a local maxima and $x = -\frac{2}{3}$ is a local minima of f.

(3) x = 1 and $x = -\frac{2}{3}$ are local minima of f.

(4) x = 1 and $x = -\frac{2}{3}$ are local maxima of f.

9. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is_.

10. The set of all real values of λ for which the function $f(x) = (1 - \cos^2 x).(\lambda + \sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is:

(1) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (4) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

DIFFERENTIAL EQUATION

1. Let y = y(x) be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying y(0) = 1. This curve intersects the x-axis at a point whose abscissa is :

(1) 2 + e (2) 2

(3) 2 – e (4) –e

2. If y = y(x) is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that y(0) = 0, then y(1) is equal to :

(1) $2 + \log_e 2$ (2) 2e

(3) $\log_e 2$ (4) $1 + \log_e 2$

3. The differential equation of the family of curves, $x^2 = 4b(y + b)$, $b \in R$, is

(1) $x(y')^2 = x + 2yy'$

(2) $x(y')^2 = 2yy' - x$

(3) $xy'' = y'$

(4) $x(y')^2 = x - 2yy'$

4. Let y = y(x) be a solution of the differential equation, $\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0, |x|<1$. If $y\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to

(1) $-\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{1}{\sqrt{2}}$

5. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$; y(1) = 1; then a value of x satisfying y(x) = e is :

(1) $\sqrt{2}e$ (2) $\frac{e}{\sqrt{2}}$

(3) $\frac{1}{2}\sqrt{3}e$ (4) $\sqrt{3}e$

6. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2}<x<\frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to :

(1) $\frac{\pi-1}{4}$ (2) $\frac{\pi+2}{4}$

(3) $\frac{\pi+1}{4}$ (4) $\frac{1}{4}$

7. If for x ≥ 0, y = y(x) is the solution of the differential equation $(x + 1)dy = ((x + 1)^2 + y - 3)dx$, y(2) = 0, then y(3) is equal to ——.

8. Let y = y(x) be the solution of the differential equation, $\frac{2+\sin x}{y+1}\cdot\frac{dy}{dx}=-\cos x, y>0, y(0)=1$.

If $y(\pi) = a$ and $\frac{dy}{dx}$ at $x = \pi$ is b, then the ordered pair (a, b) is equal to :

(1) (2, 1) (2) $\left(2,\frac{3}{2}\right)$

(3) (1, –1) (4) (1, 1)

9. If a curve y = f(x), passing through the point (1,2), is the solution of the differential equation, $2x^2dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to:

(1) $\frac{1}{1-\log_e 2}$ (2) $\frac{1}{1+\log_e 2}$

(3) $\frac{-1}{1+\log_e 2}$ (4) $1+\log_e 2$

10. The solution curve of the differential equation, $(1 + e^{-x})(1 + y^2)\frac{dy}{dx}=y^2$, which passes through the point (0, 1), is :

(1) $y^2 = 1 + y\log_e\left(\frac{1+e^x}{2}\right)$

(2) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^x}{2}\right)+2\right)$

(3) $y^2 = 1 + y\log_e\left(\frac{1+e^{-x}}{2}\right)$

(4) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{-x}}{2}\right)+2\right)$

11. If $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$; y(2) = e and x > 1, then y(4) is equal to :

(1) $\frac{3}{2}+\sqrt{e}$ (2) $\frac{3}{2}\sqrt{e}$

(3) $\frac{1}{2}+\sqrt{e}$ (4) $\frac{\sqrt{e}}{2}$

12. Let y = y(x) be the solution of the differential equation, $xy' - y = x^2(x\cos x + \sin x)$, x > 0. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right)+y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $2+\frac{\pi}{2}$ (2) $1+\frac{\pi}{2}$

(3) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$ (4) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

13. The solution of the differential equation $\frac{dy}{dx}-\frac{y+3x}{\log_e(y+3x)}+3=0$ is :-

(where C is a constant of integration.)

(1) $x-2\log_e(y+3x)=C$

(2) $x-\log_e(y+3x)=C$

(3) $x-\frac{1}{2}(\log_e(y+3x))^2=C$

(4) $y+3x-\frac{1}{2}(\log_e x)^2=C$

14. If y = y(x) is the solution of the differential equation $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx}+e^x=0$ satisfying y(0) = 1, then a value of $y(\log_e 13)$ is :

(1) 1 (2) –1

(3) 2 (4) 0

15. Let y = y(x) be the solution of the differential equation $\cos x\frac{dy}{dx}+2y\sin x=\sin 2x$, $x\in\left(0,\frac{\pi}{2}\right)$. If $y(\pi/3)=0$, then $y(\pi/4)$ is equal to :

(1) $\sqrt{2}-2$ (2) $\frac{1}{\sqrt{2}}-1$

(3) $2-\sqrt{2}$ (4) $2+\sqrt{2}$

16. Which of the following points lies on the tangent to the curve $x^4e^y+2\sqrt{y+1}=3$ at the point (1, 0)?

(1) (2, 2) (2) (–2, 6)

(3) (–2, 4) (4) (2, 6)

17. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$ is :

(where C is a constant of integration)

(1) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

(2) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

(3) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(4) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

18. If $y=\left(\frac{2}{\pi}x-1\right)\operatorname{cosec}x$ is the solution of the differential equation, $\frac{dy}{dx}+p(x)y=\frac{2}{\pi}\operatorname{cosec}x$, $0<x<\frac{\pi}{2}$, then the function p(x) is equal to

(1) cotx (2) tanx

(3) cosecx (4) secx

AREA UNDER THE CURVE

1. The area (in sq. units) of the region $\{(x, y)\in R^2|4x^2\le y\le 8x+12)$ is :

(1) $\frac{127}{3}$ (2) $\frac{125}{3}$

(3) $\frac{124}{3}$ (4) $\frac{128}{3}$

2. The area of the region, enclosed by the circle $x^2+y^2=2$ which is not common to the region bounded by the parabola $y^2=x$ and the straight line y = x, is :

(1) $\frac{1}{3}(12\pi-1)$ (2) $\frac{1}{6}(12\pi-1)$

(3) $\frac{1}{6}(24\pi-1)$ (4) $\frac{1}{3}(6\pi-1)$

3. The area (in sq. units) of the region $\{(x,y) \in R^2 : x^2 \le y \le 3 - 2x\}$, is

(1) $\frac{29}{3}$ (2) $\frac{31}{3}$

(3) $\frac{34}{3}$ (4) $\frac{32}{3}$

4. For a > 0, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b (0 < b < a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line x = b bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation

(1) $x^6 - 12x^3 + 4 = 0$
(2) $x^6 - 12x^3 - 4 = 0$
(3) $x^6 + 6x^3 - 4 = 0$
(4) $x^6 - 6x^3 + 4 = 0$

5. Given : $f(x) = \begin{cases} x & , 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , x = \frac{1}{2} \\ 1-x & , \frac{1}{2} < x \le 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is :

(1) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (2) $\frac{\sqrt{3}}{4} - \frac{1}{3}$

(3) $\frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{1}{2} - \frac{\sqrt{3}}{4}$

6. Area (in sq. units) of the region outside $\frac{|x|}{2} + \frac{|y|}{3} = 1$ and inside the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is :

(1) $3(4 - \pi)$ (2) $6(\pi - 2)$
(3) $3(\pi - 2)$ (4) $6(4 - \pi)$

7. Consider a region $R = \{(x, y) \in R^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true?

(1) $\alpha^3 - 6\alpha^2 + 16 = 0$
(2) $3\alpha^2 - 8\alpha + 8 = 0$
(3) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$
(4) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

8. The area (in sq. units) of the region $\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\}$ is:

(1) $\frac{79}{16}$ (2) $\frac{23}{6}$

(3) $\frac{79}{24}$ (4) $\frac{23}{16}$

9. The area (in sq. units) of the region $A = \{(x, y) : (x - 1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$, where [t] denotes the greatest integer function, is :

(1) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (2) $\frac{8}{3}\sqrt{2} - 1$

(3) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$ (4) $\frac{4}{3}\sqrt{2} + 1$

10. The area (in sq. units) of the region $A = \{(x,y) : |x| + |y| \le 1, 2y^2 \ge |x|\}$ is :

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$

(3) $\frac{7}{6}$ (4) $\frac{5}{6}$

11. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to :

(1) $\frac{4}{3}$ (2) $\frac{8}{3}$

(3) $\frac{16}{3}$ (4) $\frac{7}{2}$

MATRICES

1. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is :

(1) 3 (2) 1/3
(3) 1/81 (4) 1/9

2. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to:

(1) A^3 (2) A
(3) A^2 (4) I_3

3. If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to

(1) 4I – A (2) A – 6I
(3) 6I – A (4) A – 4I

4. The number of all 3×3 matrices A, with enteries from the set {–1,0,1} such that the sum of the diagonal elements of AA^T is 3, is

5. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, B = adjA and C = 3A, then $\frac{|adjB|}{|C|}$ is equal to :

(1) 72 (2) 2
(3) 8 (4) 16

6. Let A be a 2×2 real matrix with entries from {0, 1} and $|A| \neq 0$. Consider the following two statements:

(P) If $A \neq I_2$, then $|A| = -1$
(Q) If $|A| = 1$, then $tr(A) = 2$,

where I_2 denotes 2×2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then:

(1) (P) is true and (Q) is false
(2) Both (P) and (Q) are false
(3) Both (P) and (Q) are true
(4) (P) is false and (Q) is true

7. Let a, b, c∈R be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ satisfies $A^TA = I$, then a value of abc can be:

(1) $\frac{2}{3}$ (2) $-\frac{1}{3}$
(3) 3 (4) $\frac{1}{3}$

8. Let $A = \{X = (x, y, z)^T : PX = 0$ and $x^2 + y^2 + z^2 = 1\}$ where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$, then the set A :

(1) is a singleton
(2) contains exactly two elements
(3) contains more than two elements
(4) is an empty set

9. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to _______ .

10. Let A be a 3×3 matrix such that adj $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and B = adj (adj A). If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to :

(1) $\left(9, \frac{1}{9}\right)$ (2) $\left(9, \frac{1}{81}\right)$
(3) $\left(3, \frac{1}{81}\right)$ (4) (3, 81)

11. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax = b when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$$x = \begin{bmatrix}1\\1\\1\end{bmatrix}, x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix}, x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}, b_1 = \begin{bmatrix}1\\0\\0\end{bmatrix},$$

$b_2 = \begin{bmatrix}0\\2\\0\end{bmatrix}$ and $b_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal to :-

(1) $\frac{1}{2}$ (2) 4

(3) $\frac{3}{2}$ (4) 2

12. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$.

If $B = A + A^4$, then det(B) :

(1) is one (2) lies in (1, 2)

(3) is zero (4) lies in (2, 3)

VECTORS

1. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three units vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $\lambda = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $(\lambda,\vec{d})$ is equal to :

(1) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (2) $\left(-\frac{3}{2}, 3\vec{c}\times\vec{b}\right)$

(3) $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$ (4) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$

2. A vector $\vec{a} = \alpha\hat{i}+2\hat{j}+\beta\hat{k}(\alpha,\beta\in R)$ lies in the plane of the vectros $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then:

(1) $\vec{a}.\hat{i}+1=0$ (2) $\vec{a}.\hat{i}+3=0$

(3) $\vec{a}.\hat{k}+4=0$ (4) $\vec{a}.\hat{k}+2=0$

3. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}\cdot\vec{a}=0$, then $\vec{c}\cdot\vec{b}$ is equal to

(1) $\frac{1}{2}$ (2) –1

(3) $-\frac{1}{2}$ (4) $-\frac{3}{2}$

4. Let the volume of a parallelopiped whose coterminous edges are given by $\vec{u}=\hat{i}+\hat{j}+\lambda\hat{k}, \vec{v}=\hat{i}+\hat{j}+3\hat{k}$ and $\vec{w}=2\hat{i}+\hat{j}+\hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then cosθ can be

(1) $\frac{7}{6\sqrt{3}}$ (2) $\frac{5}{7}$

(3) $\frac{7}{6\sqrt{6}}$ (4) $\frac{5}{3\sqrt{3}}$

5. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{3}$, $|\vec{b}|=5$, $\vec{b}.\vec{c}=10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $|\vec{a}\times(\vec{b}\times\vec{c})|$ is equal to _____.

6. If the vectors, $\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$, $\vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and $\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$ $(a\in R)$ are coplanar and $3(\vec{p}.\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2=0$, then the value of λ is _______.

7. The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is _______.

8. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a}-\vec{b}|^2+|\vec{a}-\vec{c}|^2=8$. Then $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$ is equal to ________.

9. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio λ : 1 (λ > 0). If O is the origin and $\overrightarrow{OB}\cdot\overrightarrow{OP}-3|\overrightarrow{OA}\times\overrightarrow{OP}|^2=6$, then λ is equal to______.

10. The lines $\vec{r}=(\hat{i}-\hat{j})+\ell(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+m(\hat{i}+\hat{j}-\hat{k})$

(1) Intersect when ℓ = 1 and m = 2

(2) Intersect when ℓ = 2 and m = $\frac{1}{2}$

(3) Do not intersect for any values of ℓ and m

(4) Intersect for all values of ℓ and m

11. Let a plane P contain two lines $\vec{r}=\hat{i}+\lambda(\hat{i}+\hat{j})$, $\lambda \in R$ and $\vec{r}=-\hat{j}+\mu(\hat{j}-\hat{k})$, $\mu \in R$. If Q(α, β, γ) is the foot of the perpendicular drawn from the point M(1, 0, 1) to P, then 3(α + β + γ) equals _______.

12. Let x_0 be the point of local maxima of $f(x)=\vec{a}.(\vec{b}\times\vec{c})$, where $\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x = x_0$ is :

(1) –30 (2) 14
(3) –4 (4) –22

13. If $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$, then the value of $|\hat{i}\times(\vec{a}\times\hat{i})|^2+|\hat{j}\times(\vec{a}\times\hat{j})|^2+|\hat{k}\times(\vec{a}\times\hat{k})|^2$ is equal to ____.

14. If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a}=\hat{i}+\hat{j}+n\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-n\hat{k}$ and $\vec{c}=\hat{i}+n\hat{j}+3\hat{k}$ $(n \geq 0)$, is 158 cu. units, then :

(1) $\vec{a}\cdot\vec{c}=17$ (2) $\vec{b}\cdot\vec{c}=10$
(3) n = 7 (4) n = 9

15. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}|=2, |\vec{b}|=4$ and $|\vec{c}|=4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a}+\vec{b}-\vec{c}|$ is __________.

16. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is _.

17. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}|=|\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is __________.

3D

1. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to_____

2. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is :

(1) (6, 5, –2) (2) (4, 3, 2)
(3) (3, 4, –2) (4) (6, 5, 2)

3. The mirror image of the point (1,2,3) in a plane is $\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$. Which of the following points lies on this plane ?

(1) (–1, –1, –1) (2) (–1, –1, 1)
(3) (1, 1, 1) (4) (1, –1, 1)

4. The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is

(1) $\frac{7}{2}\sqrt{30}$ (2) $3\sqrt{30}$
(3) 3 (4) $2\sqrt{30}$

5. If the distance between the plane, $23x-10y-2z+48=0$ and the plane containing the lines $\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$ and $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}(\lambda\in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______.

6. If for some α and β in R, the intersection of the following three places

$x + 4y - 2z = 1$

$x + 7y - 5z = \beta$

$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to :

(1) 10 (2) –10

(3) 2 (4) 0

7. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, 2x = 3y, z = 1 also passes through the point :

(1) (0, 6, –2) (2) (–2, 0, 1)

(3) (0, –6, 2) (4) (2, 0, –1)

8. A plane passing through the point (3, 1,1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α, –3, 5), then α is equal to:

(1) –10 (2) 5

(3) 10 (4) –5

9. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, –2, 3) and (1, 1, 0) lies on the plane :

(1) x + 2y – z = 1 (2) x – 2y + z = 1

(3) x – y – 2z = 1 (4) 2x + y – z = 1

10. The plane which bisects the line joining the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point :

(1) (4, 0, –1) (2) (4, 0, 1)

(3) (0, 1, –1) (4) (0, –1, 1)

11. If the equation of a plane P, passing through the intesection of the planes, $x+4y-z+7=0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some a, b $\in$ R, then the distance of the point (3, 2, –1) from the plane P is ________.

12. The distance of the point (1, –2, 3) from the plane x–y+z = 5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is:

(1) 7 (2) 1

(3) $\frac{1}{7}$ (4) $\frac{7}{5}$

13. If (a, b, c) is the image of the point (1, 2, –3) in the line, $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$, then a + b + c is equal to

(1) –1 (2) 2

(3) 3 (4) 1

14. If for some $\alpha \in$ R, the lines

$L_1 : \frac{x+1}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$ and

$L_2 : \frac{x+2}{\alpha}=\frac{y+1}{5-\alpha}=\frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point :

(1) (–2, 10, 2) (2) (10, 2, 2)

(3) (10, –2, –2) (4) (2, –10, –2)

15. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and x + y + z + 1 = 0, 2x – y + z + 3 = 0 is :

(1) $\frac{1}{2}$ (2) 1

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

16. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1, 1, 2). Then the equation of the line through this centroid and perpendicular to the plane P is:

(1) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$

(2) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

(3) $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

(4) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$

PARABOLA

1. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to :

(1) 128 (2) –64
(3) –128 (4) –32

2. Let a line $y = mx$ $(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area $(\Delta OPQ) = 4$ sq. units, then m is equal to __________.

3. The locus of a point which divides the line segment joining the point (0,–1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2 is-

(1) $9x^2 - 3y = 2$ (2) $9x^2 - 12y = 8$
(3) $x^2 - 3y = 2$ (4) $4x^2 - 3y = 2$

4. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2}, -2\right)$, then the equation of the tangent to it at B is :

(1) $2x + y - 24 = 0$ (2) $x - 2y + 8 = 0$
(3) $2x - y - 24 = 0$ (4) $x + 2y + 8 = 0$

5. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is :

(1) $64\sqrt{3}$ (2) $256\sqrt{3}$
(3) $192\sqrt{3}$ (4) $128\sqrt{3}$

6. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then :

(1) $MQ = \frac{1}{3}$ (2) $PN = 3$
(3) $MQ = \frac{1}{4}$ (4) $PN = 4$

7. Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is :

(1) 8 (2) $4\sqrt{5}$
(3) 12 (4) $8\sqrt{5}$

8. The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is :

(1) $\frac{4}{3\sqrt{3}}$ (2) $\frac{1}{3\sqrt{3}}$
(3) $\frac{4}{3}$ (4) $\frac{2}{3\sqrt{3}}$

9. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$
(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4}$

10. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :

(1) $x + 3 = 0$ (2) $x + 2y = 0$
(3) $2x + 1 = 0$ (4) $x + 2 = 0$

11. The centre of the circle passing through the point (0, 1) and touching the parabola $y = x^2$ at the point (2, 4) is :

(1) $\left(\frac{3}{10}, \frac{16}{5}\right)$ (2) $\left(\frac{-16}{5}, \frac{53}{10}\right)$
(3) $\left(\frac{6}{5}, \frac{53}{10}\right)$ (4) $\left(\frac{-53}{10}, \frac{16}{5}\right)$

ELLIPSE

1. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is :

(1) 4 (2) $2\sqrt{7}$

(3) $2\sqrt{5}$ (4) $2\sqrt{2}$

2. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is :

(1) $\sqrt{3}$ (2) $2\sqrt{3}$

(3) $3\sqrt{2}$ (4) $\frac{3}{\sqrt{2}}$

3. Let the line y = mx and the ellipse $2x^2 + y^2 = 1$ intersect at a ponit P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}},0\right)$ and $(0,\beta)$, then β is equal to

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2\sqrt{2}}{3}$

(3) $\frac{2}{3}$ (4) $\frac{\sqrt{2}}{3}$

4. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, x + 6y = 8 ; then its eccentricity is :

(1) $\sqrt{\frac{5}{6}}$ (2) $\frac{1}{2}\sqrt{\frac{11}{3}}$

(3) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (4) $\frac{1}{2}\sqrt{\frac{5}{3}}$

5. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1 (b<5)$ and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to :

(1) (8, 10) (2) (8, 12)

(3) $\left(\frac{20}{3}, 12\right)$ (4) $\left(\frac{24}{5}, 10\right)$

6. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ $(a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to :

(1) 126 (2) 135

(3) 145 (4) 116

7. Let x = 4 be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If P $(1, \beta)$, $\beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is :-

(1) 7x – 4y = 1 (2) 4x – 2y = 1

(3) 4x – 3y = 2 (4) 8x – 2y = 5

8. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, –4), then PQ^2 is equal to:

(1) 21 (2) 36

(3) 48 (4) 29

9. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to :

(1) 8 (2) 6

(3) 16 (4) 9

10. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci ?

(1) $\left(-1,\sqrt{3}\right)$ (2) $\left(-1,\sqrt{2}\right)$

(3) $\left(-2,\sqrt{3}\right)$ (4) (1,2)

11. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies :

(1) $e^2 + 2e - 1 = 0$ (2) $e^2 + e - 1 = 0$

(3) $e^4 + 2e^2 - 1 = 0$ (4) $e^4 + e^2 - 1 = 0$

HYPERBOLA

1. If a hyperbola passes through the point P(10,16) and it has vertices at (±6,0), then the equation of the normal to it at P is

(1) $x + 2y = 42$ (2) $3x + 4y = 94$

(3) $2x + 5y = 100$ (4) $x + 3y = 58$

2. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola, $\frac{x^2}{9}-\frac{y^2}{4}=1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to :

(1) 15 (2) 14

(3) 17 (4) 16

3. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4}-\frac{y^2}{2}=1$ at the point (x_1, y_1). Then $x_1^2+5y_1^2$ is equal to :

(1) 5 (2) 6

(3) 8 (4) 10

4. For some $\theta\in\left(0,\frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2-y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is :

(1) $\sqrt{30}$ (2) $\frac{4\sqrt{5}}{3}$

(3) $2\sqrt{6}$ (4) $\frac{2\sqrt{5}}{3}$

5. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points ?

(1) $\left(1,-\frac{1}{\sqrt{2}}\right)$ (2) $\left(\sqrt{\frac{3}{2}},\frac{1}{\sqrt{2}}\right)$

(3) $\left(\frac{1}{\sqrt{2}},0\right)$ (4) $\left(-\sqrt{\frac{3}{2}},1\right)$

6. Let P(3, 3) be a point on the hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal to it at P intesects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :

(1) $\left(\frac{9}{2},3\right)$ (2) $\left(\frac{9}{2},2\right)$

(3) $\left(\frac{3}{2},2\right)$ (4) (9, 3)

7. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100}-\frac{y^2}{64}=1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?

(1) $5m = 4$ (2) $4c^2 = 369$

(3) $c^2 = 369$ (4) $8m + 5 = 0$

COMPLEX NUMBER

1. If $\frac{3+i\sin\theta}{4-i\cos\theta}, \theta\in[0,2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is :

(1) $-\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$

(3) $\pi-\tan^{-1}\left(\frac{4}{3}\right)$ (4) $\pi-\tan^{-1}\left(\frac{3}{4}\right)$

2. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x,y) lies on a :

(1) circle whose centre is at $\left(-\frac{1}{2},-\frac{3}{2}\right)$

(2) circle whose diameter is $\frac{\sqrt{5}}{2}$

(3) straight line whose slope is $\frac{3}{2}$

(4) straight line whose slope is $-\frac{2}{3}$

3. Let $\alpha=\frac{-1+i\sqrt{3}}{2}$. If $a=(1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b=\sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation :

(1) $x^2 - 102x + 101 = 0$

(2) $x^2 + 101x + 100 = 0$

(3) $x^2 - 101x + 100 = 0$

(4) $x^2 + 102x + 101 = 0$

4. If the equation, $x^2 + bx + 45 = 0$ $(b \in R)$ has conjugate complex roots and they satisfy $|z+1|=2\sqrt{10}$, then

(1) $b^2 - b = 42$ (2) $b^2 + b = 12$

(3) $b^2 + b = 72$ (4) $b^2 - b = 30$

5. If z be a complex number satisfying $|\text{Re}(z)| + |\text{Im}(z)| = 4$, then $|z|$ cannot be

(1) $\sqrt{\frac{17}{2}}$ (2) $\sqrt{10}$

(3) $\sqrt{8}$ (4) $\sqrt{7}$

6. Let z be complex number such that $\left|\frac{z-i}{z+2i}\right|=1$ and $|z|=\frac{5}{2}$. Then the value of $|z + 3i|$ is :

(1) $\sqrt{10}$ (2) $2\sqrt{3}$

(3) $\frac{7}{2}$ (4) $\frac{15}{4}$

7. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is :

(1) $\frac{1}{2}\left(\sqrt{3}-i\right)$ (2) $-\frac{1}{2}\left(\sqrt{3}-i\right)$

(3) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$ (4) $\frac{1}{2}\left(1-i\sqrt{3}\right)$

8. The imaginary part of $\left(3+2\sqrt{-54}\right)^{1/2}$ z $-\left(3-2\sqrt{-54}\right)^{1/2}$ can be :

(1) $-2\sqrt{6}$ (2) 6

(3) $\sqrt{6}$ (4) $-\sqrt{6}$

9. If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}}=\left(\frac{1+i}{i-1}\right)^{\frac{n}{3}}=1$, $(m, n \in N)$ then the greatest common divisor of the least values of m and n is _______ .

10. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1-z_2)=\frac{\pi}{6}$, then $\text{Im}(z_1+z_2)$ is equal to :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $2\sqrt{3}$

11. If $A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$, $\left(\theta=\frac{\pi}{24}\right)$ and $A^5=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, where $i=\sqrt{-1}$, then which one of the following is not true?

(1) $0 \le a^2+b^2 \le 1$ (2) $a^2-d^2=0$

(3) $a^2-b^2=\frac{1}{2}$ (4) $a^2-c^2=1$

12. Let $u=\frac{2z+i}{z-ki}$, $z=x+iy$ and $k>0$. If the curve represented by $\text{Re}(u)+\text{Im}(u)=1$ intersects the y-axis at the points P and Q where $PQ=5$, then the value of k is :

(1) 3/2 (2) 4

(3) 2 (4) 1/2

13. If a and b are real numbers such that $(2+\alpha)^4=a+b\alpha$, where $\alpha=\frac{-1+i\sqrt{3}}{2}$, then $a+b$ is equal to:

(1) 57 (2) 33

(3) 24 (4) 9

14. If the four complex numbers z, $\bar{z}$, $\bar{z}-2\text{Re}(\bar{z})$ and $z-2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to :

(1) 4 (2) 2

(3) $4\sqrt{2}$ (4) $2\sqrt{2}$

15. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is :

(1) $2^{15}i$ (2) -2^{15}

(3) $-2^{15}i$ (4) 6^5

16. The region represented by $\{z=x+iy \in C : |z|-\text{Re}(z)\le 1\}$ is also given by the inequality:

(1) $y^2 \ge x+1$ (2) $y^2 \ge 2(x+1)$

(3) $y^2 \le x+\frac{1}{2}$ (4) $y^2 \le 2\left(x+\frac{1}{2}\right)$

17. Let $z=x+iy$ be a non-zero complex number such that $z^2=i|z|^2$, where $i=\sqrt{-1}$, then z lies on the:

(1) imaginary axis

(2) real axis

(3) line, $y=x$

(4) line, $y=-x$

PROBABILITY

1. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to :

(1) $\frac{17}{2}$ (2) 4

(3) $\frac{17}{8}$ (4) $\frac{17}{4}$

2. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5 otherwise X takes the value –1. Then the expected value of X, is :

(1) $\frac{3}{16}$ (2) $-\frac{3}{16}$

(3) $\frac{1}{8}$ (4) $-\frac{1}{8}$

3. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is

(1) 0.02 (2) 0.01

(3) 0.20 (4) 0.10

4. Let A and B be two independent events such that $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{6}$. Then, which of the following is TRUE ?

(1) $P(A/B)=\frac{2}{3}$

(2) $P(A/(A\cup B))=\frac{1}{4}$

(3) $P(A/B')=\frac{1}{3}$

(4) $P(A'/B')=\frac{1}{3}$

5. A random variable X has the following probability distribution :

X :	1	2	3	4	5
P(X) :	K^2	2K	K	2K	$5K^2$

Then $P(X > 2)$ is equal to :

(1) $\frac{7}{12}$ (2) $\frac{23}{36}$

(3) $\frac{1}{36}$ (4) $\frac{1}{6}$

6. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :

(1) $\frac{945}{2^{11}}$ (2) $\frac{965}{2^{11}}$

(3) $\frac{945}{2^{10}}$ (4) $\frac{965}{2^{10}}$

7. In a box, there are 20 cards, out of which 10 are lebelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is :

(1) $\frac{11}{16}$ (2) $\frac{13}{16}$

(3) $\frac{9}{16}$ (4) $\frac{15}{16}$

8. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is :

(1) $\frac{8}{17}$ (2) $\frac{2}{3}$

(3) $\frac{4}{17}$ (4) $\frac{2}{5}$

9. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1\cap E_2\cap E_3) = 0$. Then $P(E_2^C \cap E_3^C/E_1)$ is equal to :

(1) $P(E_3^C) - P(E_2)$ (2) $P(E_2^C) + P(E_3)$

(3) $P(E_3^C) - P(E_2^C)$ (4) $P(E_3) - P(E_2^C)$

10. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is :

(1) $\frac{1}{8}$ (2) $\frac{1}{9}$

(3) $\frac{1}{3}$ (4) $\frac{1}{4}$

11. The probability that a randomly chosen 5-digit number is made from exactly two digits is :

(1) $\frac{121}{10^4}$ (2) $\frac{150}{10^4}$

(3) $\frac{135}{10^4}$ (4) $\frac{134}{10^4}$

12. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ________.

13. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six The game stops as soon as either of the players wins. The probability of A winning the game is :

(1) $\frac{31}{61}$ (2) $\frac{5}{6}$

(3) $\frac{5}{31}$ (4) $\frac{30}{61}$

14. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ____.

15. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _______.

16. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

(1) 2!3!4! (2) $(3!)^3.(4!)$

(3) $(3!)^2.(4!)$ (4) $3!(4!)^3$

17. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is :

(1) $\frac{15}{101}$ (2) $\frac{5}{101}$

(3) $\frac{5}{33}$ (4) $\frac{10}{99}$

18. The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval:

(1) [0.36, 0.40] (2) [0.35, 0.36]

(3) [0.25, 0.35] (4) [0.20, 0.25]

STATISTICS

1. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x·y is equal to_____

2. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then m + n is equal to ________.

3. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is

(1) 3.99 (2) 3.98

(3) 4.02 (4) 4.01

4. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 resepectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to

(1) –20 (2) 10

(3) –10 (4) –5

5. Let the observations $x_i (1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, ..., x_{10} - 3$, then the ordered pair (μ, λ) is equal to :

(1) (6, 6) (2) (3, 6)

(3) (6, 3) (4) (3, 3)

6. Let $X = \{x \in N : 1 \le x \le 17\}$ and $Y = \{ax + b: x \in X$ and $a, b \in R, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then a + b is equal to :

(1) –7 (2) 7
(3) 9 (4) –27

7. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, \ldots b_{11}$ is 90, then the common difference of this A.P. is_______.

8. For the frequency distribution :

Variate (x) :	x_1	x_2	$x_3 \ldots\ldots x_{15}$
Frequency (f) :	f_1	f_2	$f_3 \ldots f_{15}$

where $0 < x_1 < x_2 < x_3 < \ldots < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :

(1) 2 (2) 1
(3) 4 (4) 6

9. Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \ne p \in R$, then the standard deviation of these observations is :

(1) $\sqrt{\frac{3}{5}}$ (2) $\frac{7}{10}$
(3) $\frac{9}{10}$ (4) $\frac{4}{5}$

10. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

(1) 7 (2) 3
(3) 5 (4) 9

11. If the variance of the following frequency distribution:

Class :	10–20	20–30	30–40
Frequency :	2	x	2

is 50, then x is equal to ____

12. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is :

(1) 2 (2) 4
(3) 3 (4) 1

13. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation :

(1) $2x^2 - 20x + 19 = 0$
(2) $x^2 - 10x + 19 = 0$
(3) $x^2 - 10x + 18 = 0$
(4) $x^2 - 20x + 18 = 0$

14. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n, a > 1)$ then the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is

(1) $n\sqrt{a-1}$
(2) $\sqrt{a-1}$
(3) $a - 1$
(4) $\sqrt{n(a-1)}$

15. Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to ___________.

MATHEMATICAL REASONING

1. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is :

(1) If $A \subseteq C$, then $B \subset A$ or $D \subset B$
(2) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$
(3) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$
(4) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

2. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) p
(2) q
(3) ~p
(4) ~q

3. Which of the following statements is a tautology?

(1) $\sim(p \vee \sim q) \rightarrow p \vee q$

(2) $\sim(p \wedge \sim q) \rightarrow p \vee q$

(3) $\sim(p \vee \sim q) \rightarrow p \wedge q$

(4) $p \vee (\sim q) \rightarrow p \wedge q$

4. Which one of the following is a tautology ?

(1) $P \wedge (P \vee Q)$ (2) $P \vee (P \wedge Q)$

(3) $Q \rightarrow (P \wedge (P \rightarrow Q))$ (4) $(P \wedge (P \rightarrow Q)) \rightarrow Q$

5. If $p \rightarrow (p \wedge \sim q)$ is false, then the truth values of p and q are respectively :

(1) F, T (2) T, T

(3) F, F (4) T, F

6. Negation of the statement :

$\sqrt{5}$ is an integer or 5 is irrational is :

(1) $\sqrt{5}$ is irrational or 5 is an integer.

(2) $\sqrt{5}$ is not an integer and 5 is not irrational.

(3) $\sqrt{5}$ is an integer and 5 is irrational.

(4) $\sqrt{5}$ is not an integer or 5 is not irrational.

7. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is :

(1) If I will catch the train, then I reach the station in time.

(2) If I do not reach the station in time, then I will not catch the train.

(3) If I will not catch the train, then I do not reach the station in time.

(4) If I do not reach the station in time, then I will catch the train.

8. Which of the following is a tautology ?

(1) $(\sim p) \wedge (p \vee q) \rightarrow q$

(2) $(q \rightarrow p) \vee \sim(p \rightarrow q)$

(3) $(p \rightarrow q) \wedge (q \rightarrow p)$

(4) $(\sim q) \vee (p \wedge q) \rightarrow q$

9. The proposition $p \rightarrow \sim(p \wedge \sim q)$ is equivalent to:

(1) $(\sim p) \vee q$

(2) q

(3) $(\sim p) \wedge q$

(4) $(\sim p) \vee (\sim q)$

10. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively :

(1) T, F, T

(2) F, T, F

(3) T, T, F

(4) T, T, T

11. Given the following two statements :

(S_1) : $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.

(S_2) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy.

Then :

(1) only (S_1) is correct.

(2) both (S_1) and (S_2) are correct.

(3) both (S_1) and (S_2) are not correct.

(4) only (S_2) is correct.

12. Contrapositive of the statement:

'If a function f is differentiable at a, then it is also continuous at a', is :-

(1) If a function f is continuous at a, then it is not differentiable at a.

(2) If a function f is not continuous at a, then it is differentiable at a.

(3) If a function f is not continuous at a, then it is not differentiable at a.

(4) If a function f is continuous at a, then it is differentiable at a.

13. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :

(1) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

(2) $(x \wedge \sim y) \vee (\sim x \wedge y)$

(3) $(x \wedge y) \vee (\sim x \wedge \sim y)$

(4) $(x \wedge y) \wedge (\sim x \vee \sim y)$

14. The statement $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$ is :

(1) a contradiction

(2) equivalent to $(p \wedge q) \vee (\sim q)$

(3) a tautology

(4) equivalent to $(p \vee q) \wedge (\sim p)$

15. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p \vee \sim q$ (2) $\sim p \vee q$

(3) $\sim p \wedge \sim q$ (4) $p \wedge \sim q$

16. Consider the statement :

"For an integer n, if $n^3 - 1$ is even, then n is odd."

The contrapositive statement of this statement is :

(1) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

(2) For an integer n, if n is even, then $n^3 - 1$ is odd.

(3) For an integer n, if n is odd, then $n^3 - 1$ is even.

(4) For an integer n, if n is even, then $n^3 - 1$ is even.

ANSWER KEY

Logarithm

Que.	1
Ans.	4

Compound Angle

Que.	1	2	3	4
Ans.	2	1	3	1

Quadratic Equation

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	4	8.00	2	3	4	1	3	3	2
Que.	**11**	**12**	**13**	**14**	**15**	**16**				
Ans.	3	4	2	1	4	3				

Sequence & Progression										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	1	3	3	2	504	1540.00	3	4	14
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	1	1	4	2	3	4	3	39	2	3
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**		
Ans.	1	1	3	4	2	2	3	2		

Trigonometric Equation			
Que.	**1**	**2**	**3**
Ans.	8.00	4	1

Solution of Triangle	
Que.	**1**
Ans.	3

Height & Distance				
Que.	**1**	**2**	**3**	**4**
Ans.	4	1	80.00	1

Determinant										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	13.00	4	4	4	3	1	3	8	3	5
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans.	3	4	1	2	2	4	1	3.00		

Straight Line										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	5	3	2	4	3	4	30	2	3

Circle								
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Ans.	4	2	36	9.00	3	4	7	2

Permutation & Combination										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	2454	4	490.00	1	309.00	3	3	54	135
Que.	**11**	**12**	**13**							
Ans.	240	4	120.00							

Binomial Theorem										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	2	30	3	4	51	615.00	2	118	2
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans.	4	2	8	3	13	120.00	1	3		

Logarithm							
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Ans.	29.00	1	8	4	4	4	28.00

Relation		
Que.	**1**	**2**
Ans.	2	4

Function									
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Ans.	2	4	2	3	1	4	19.00	2	5

Inverse trigonometry function			
Que.	**1**	**2**	**3**
Ans.	1	3	4

Limit										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	36	4	40.00	4	2	8	1	4	1	4
Que.	**11**									
Ans.	1									

Continuity				
Que.	**1**	**2**	**3**	**4**
Ans.	5.00	2	4	8

Differentiability							
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Ans.	3	4	10	1	1	5.00	1

Method of differentiation										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	2	1	3	1	Bonus	3	91	1	2	2

Indefinite Integration

Que.	1	2	3	4	5	6	7
Ans.	1	1	1	3	4	1	4

Definite Integration

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	3	1	1	1	4	3	1.50	1.0	1
Que.	11	12	13	14	15	16	17	18		
Ans.	1	4	4	3	21	4	1	4		

Tangent & Normal

Que.	1	2	3	4	5	6	7	8
Ans.	2	4.00	3	2	1	4	0.50	4

Monotonicity

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	2	2	2	1	3	1	2	3	1
Que.	11									
Ans.	3									

Maxima & Minima

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	3	1	3	4	4	3	1	5.00	4

Differential Equation

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	4	1	2	4	3	3.00	4	2	1
Que.	11	12	13	14	15	16	17	18		
Ans.	2	1	3	2	1	2	1	1		

Logarithm

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	2	4	1	2	2	4	3	1	4
Que.	11									
Ans.	2									

Matrices

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	1	2	672.00	3	4	4	2	10	3
Que.	11	12								
Ans.	4	2								

Vectors

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	4	3	1	30	1.00	8.00	2.00	0.8	3
Que.	11	12	13	14	15	16	17			
Ans.	5	4	18	2	6.00	4.00	1.00			

3D

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4.00	1	4	2	3	1	2	2	4	1
Que.	11	12	13	14	15	16				
Ans.	3	2	2	4	4	2				

Parabola

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	0.50	2	2	3	3	1	1	3	1
Que.	11									
Ans.	2									

Ellipse

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	3	4	2	1	1	2	2	1	1
Que.	11									
Ans.	4									

Hyperbola

Que.	1	2	3	4	5	6	7
Ans.	3	4	2	2	2	1	2

Complex Number

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	2	1	4	4	3	2	1	4	4
Que.	11	12	13	14	15	16	17			
Ans.	3	3	4	4	3	4	3			

Probability										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	3	4	3	2	3	1	1	1	2
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans.	3	3	4	11	11.00	2	3	3		

Statistics										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	54.00	18	1	1	4	1	3.00	4	3	1
Que.	**11**	**12**	**13**	**14**	**15**					
Ans.	4	1	2	2	6.00					

Mathematical Reasoning										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	2.00	3	1	4	2	2	3	1	1	3
Que.	**11**	**12**	**13**	**14**	**15**	**16**				
Ans.	3	3	3	3	3	2				

IMPORTANT NOTES

ALLEN® CAREER INSTITUTE KOTA (RAJASTHAN)

path to success

JEE (MAIN) JANUARY & SEPTEMBER 2020

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

Chapter Contents 01

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & SEPTEMBER 2020

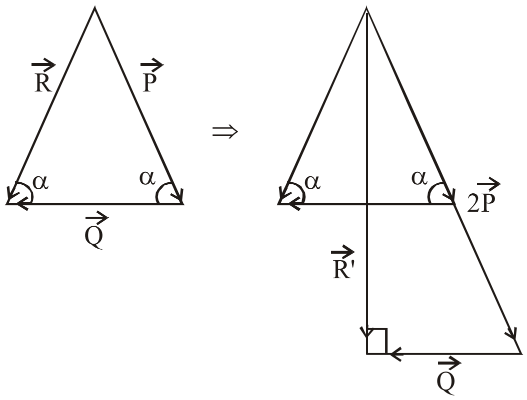
PHYSICS

JANUARY & SEPTEMBER 2020 ATTEMPT (PHYSICS)

BASIC MATHS & VECTOR

1. NTA Ans. (90)

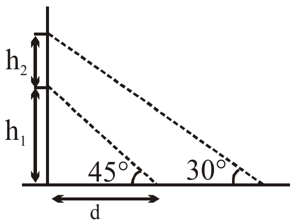
Sol.



Hence angle 90°

2. Official Ans. by NTA (1)

Sol.



$\frac{h_1}{d} = \tan 45° \Rightarrow h_1 = d$ ... (1)

$\frac{h_1 + h_2}{d + 2.464\,d} = \tan 30°$

$\Rightarrow (h_1 + h_2) \times \sqrt{3} = 3.46$ d

$(h_1 + h_2) = \frac{3.46\,d}{\sqrt{3}}$

$\Rightarrow d + h_2 = \frac{3.46\,d}{\sqrt{3}}$

$h_2 = d$

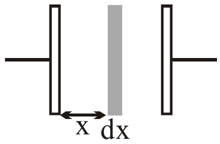
CAPACITOR

1. NTA Ans. (1)

Sol. As K is variable we take a plate element of Area A and thickness dx at distance x

Capacitance of element

$$dC = \frac{(A)K(1+\alpha x)\varepsilon_0}{dx}$$



Now all such elements are is series so equivalent capacitance

$$\frac{1}{C} = \int \frac{1}{dC} = \int_0^d \frac{dx}{AK\varepsilon_0(1+\alpha x)}$$

$$\frac{1}{C} = \frac{1}{\alpha AK\varepsilon_0} \ln\left(\frac{1+\alpha d}{1}\right)$$

$$= \frac{1}{C} = \frac{1}{\alpha AK\varepsilon_0}\left(\alpha d - \frac{(\alpha d)^2}{2} + \frac{(\alpha d)^3}{3} + ...\right)$$

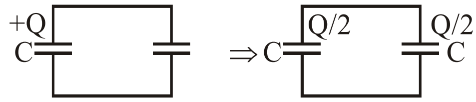
$$\Rightarrow \frac{1}{C} = \frac{\alpha d}{\alpha AK\varepsilon_0}\left(1 - \frac{\alpha d}{2} + \frac{(\alpha d)^2}{3} + ...\right)$$

$$\frac{1}{C} = \frac{d}{AK\varepsilon_0}\left(1 - \frac{\alpha d}{2}\right)$$

$$C = \frac{AK\varepsilon_0}{d}\left(1 + \frac{\alpha d}{2}\right)$$

2. NTA Ans. (6)

Sol.



$Q = CV$

$$\Delta Q_L = \frac{Q^2}{2C} - \left[\frac{(Q/2)^2}{2C} \times 2\right] = \frac{Q^2}{4C}$$

$= \frac{1}{4} CV^2$

$= \frac{1}{4} \times 60 \times 10^{-12} \times 4 \times 10^2$

$= 6$nJ

3. NTA Ans. (3)

Sol. $C_1 + C_2 = 10$ ....(i)

$\frac{1}{2} C_2 V^2 = 4 \times \frac{1}{2} C_1 V^2$

$\therefore\ C_2 = 4C_1$ ....(ii)

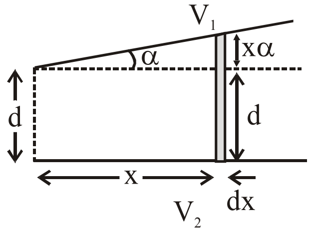
$\therefore\ C_1 = 2\ \&\ C_2 = 8$

For series combination

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 1.6$$

4. NTA Ans. (4)

Sol. Assume small element dx at a distance x from left end



Capacitance for small element dx is

$$dC = \frac{\varepsilon_0 a\, dx}{d + x\alpha}$$

$$C = \int_0^a \frac{\varepsilon_0 a\, dx}{d + x\alpha}$$

$$= \frac{\varepsilon_0 a}{\alpha} \ln\left(\frac{1+\alpha a}{d}\right)\Bigg|_0^a \qquad \left(\ln(1+x) \approx x - \frac{x^2}{2}\right)$$

$$= \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$$

5. Official Ans. by NTA (36)
Official Ans. by ALLEN (4 Actual 4.033)

Sol. $u_i = \frac{1}{2} \times 5 \times 10^{-6} (220)^2$

Final common potential

$$v = \frac{220 \times 5 + 0 \times 2.5}{5 + 2.5} = 220 \times \frac{2}{3}$$

$$u_f = \frac{1}{2}(5+2.5) \times 10^{-6}\left(220 \times \frac{2}{3}\right)^2$$

$\Delta u = u_f - u_i$

$\Delta u = -403.33 \times 10^{-4}$

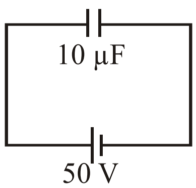
$$\Rightarrow -403.33 \times 10^{-4} = \frac{X}{100}$$

X = –4.03

or magnitude or value of X is approximate 4

6. Official Ans. by NTA (2)

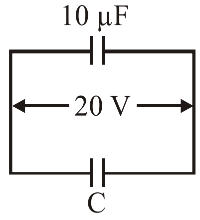
Sol. Initially



- Charge on capacitor 10 μF

Q = CV = (10 μF) (50V)

Q = 500 μC



- Final Charge on 10 μF capacitor

Q = CV = (10 μF) (20V)

Q = 200 μC

- From charge conservation,

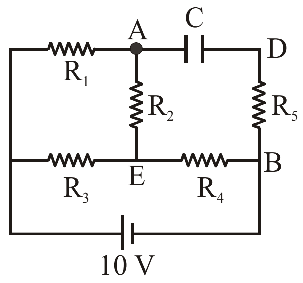
Charge on unknown capacitor

C = 500 μC – 200 μC = 300 μC

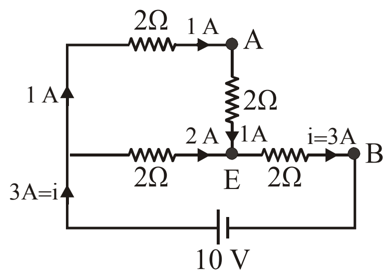
$$\Rightarrow \text{Capacitance (C)} = \frac{Q}{V} = \frac{300\ \mu C}{20\ V} = 15\ \mu F$$

7. Official Ans. by NTA (8.00)

Sol.



- R_1 to R_5 → each 2Ω
- Cap. is fully charged
- So no current is there in branch ADB
- Effective circuit of current flow :



$$R_{eq} = \left(\frac{4 \times 2}{4 + 2}\right) + 2$$

$$R_{eq} = \frac{4}{3} + 2 = \frac{10}{3}\Omega$$

$$i = \frac{10}{10/3} = 3A$$

So potential different across AEB

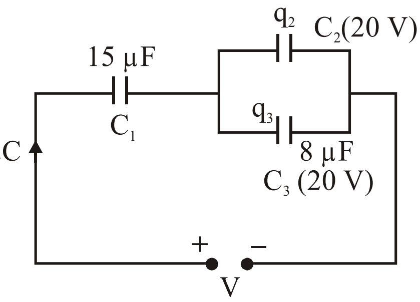
⇒ 2 × 1 + 2 × 3 = 8V

Hence potential difference across

Capacitor = $\Delta V = V_{AEB}$ = 8V

8. Official Ans. by NTA (1)

Sol.



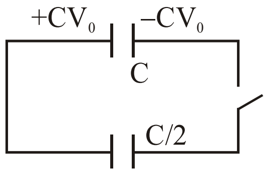
$q_3 = 20 \times 8 = 160\ \mu C$

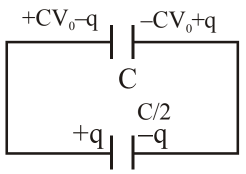
$\therefore\ q_2 = 750 - 160 = 590\ \mu C$

9. Official Ans. by NTA (4)

Official Ans. by ALLEN (1)

Sol.





$$\frac{CV_0 - q}{C} = \frac{q}{C/2} = \frac{2q}{C}$$

$$V_0 = \frac{3q}{C} \quad \Rightarrow q = \frac{CV_0}{3}$$

$$U_i = \frac{1}{2}CV_0^2$$

$$U_f = \frac{\left(\frac{2CV_0}{3}\right)^2}{2C} + \frac{\left(\frac{CV_0}{3}\right)^2}{2\left(\frac{C}{2}\right)}$$

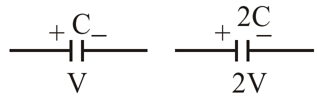
$$= \frac{1}{2}CV_0^2\left[\frac{4}{9} + \frac{2}{9}\right] = \frac{1}{2}CV_0^2\left(\frac{2}{3}\right)$$

$$\text{Heat loss} = \frac{1}{2}CV_0^2 - \left(\frac{2}{3}\right)\left(\frac{1}{2}CV_0^2\right)$$

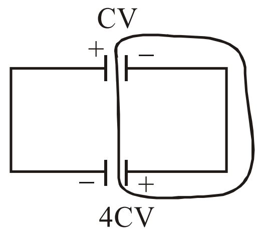
$$= \frac{1}{6}CV_0^2$$

10. Official Ans. by NTA (4)

Sol.



$Q_1 = CV \qquad Q_2 = 2C \times 2V = 4CV$



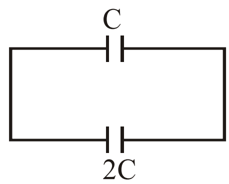
$\Rightarrow$ By conservation of charge

$q_i = q_f$

$Q_1 + Q_2 = q_1 + q_2$

$4CV - CV = (C + 2C)\ V_C$

$$V_C = \frac{3CV}{3C} \Rightarrow V$$

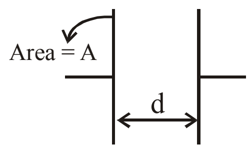


$$\Rightarrow \frac{1}{2} \times (3C) \times V_C^2$$

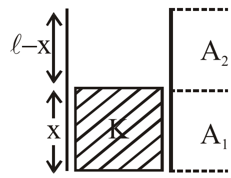
$$= \frac{1}{2} \times 3C \times V^2 = \frac{3}{2}CV^2$$

11. Official Ans. by NTA (3)

Sol.



Before inserting slab



After inserting dielectric slab

$$C_i = \frac{\varepsilon_0 A}{d} \qquad C_f = C_1 + C_2$$

$$C_i = \frac{\varepsilon_0 \ell w}{d} \qquad C_f = \frac{K\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d}$$

$$C_f = \frac{K\varepsilon_0 wx}{d} + \frac{\varepsilon_0 w(\ell - x)}{d}$$

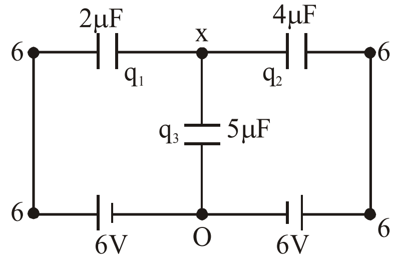
$$C_f = 2C_i \Rightarrow \frac{K\varepsilon_0 wx}{x} + \frac{\varepsilon_0 w(\ell - x)}{d} = \frac{2\varepsilon_0 \ell w}{d}$$

$$4x + \ell - x = 2\ell$$

$$\boxed{x = \frac{\ell}{3}}$$

12. Official Ans. by NTA (2)

Sol.



Let potential of point O $v_0 = 0$

Now, using junction analysis

We can say, $q_1 + q_2 + q_3 = 0$

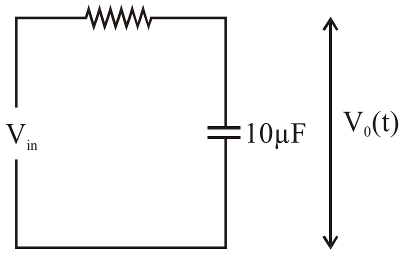
$2(x - 6) + 4(x - 6) + 5(x) = 0$

$x = \frac{36}{11} \qquad q_3 = \frac{36(5)}{11} = \frac{180}{11}$

$q_3 = 16.36\ \mu C$

13. Official Ans. by NTA (1)

Sol.



$$V_0(t) = V_{in}\left(1 - e^{-\frac{t}{RC}}\right)$$

at t = 5μs

$$V_0(t) = 5\left(1 - e^{-\frac{5\times10^{-6}}{10^3\times10\times10^{-9}}}\right)$$

$= 5\ (1 - e^{-0.5}) = 2V$

Now $V_{in} = 0$ means discharging

$$V_0(t) = 2e^{-\frac{t}{RC}} = 2e^{-0.5}$$

= 1.21 V

Now for next 5 μs

$$V_0(t) = 5 - 3.79e^{-\frac{t}{RC}}$$

after 5 μs again

$V_0(t) = 2.79$ Volt $\approx 3V$

Most approperiate Ans. (1)

$$U\left(r = \left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A}{2B/A} + \frac{B}{4B^2/A^2}$$

$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \frac{-A^2}{4B}$$

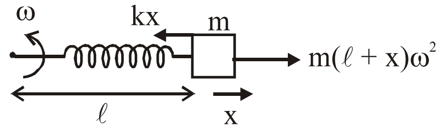
CIRCULAR MOTION

1. NTA Ans. (4)

Sol. $W = 196 - m\omega^2 R$

2. NTA Ans. (2)

Sol.

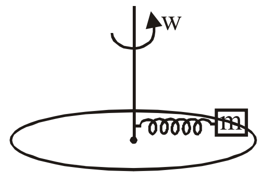


$kx = m\ell\omega^2 + mx\omega^2$

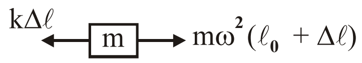
$$x = \frac{m\ell\omega^2}{k - m\omega^2}$$

3. NTA Ans. (4)

Sol.



FBD of m in frame of disc/-



$k\Delta\ell = m\omega^2(\ell_0 + \Delta\ell)$

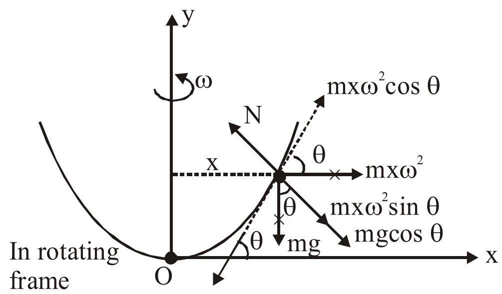
$$\Delta\ell = \frac{m\omega^2\ell_0}{k - m\omega^2} \approx \frac{m\omega\ell_0}{k}$$

$$\frac{\Delta\ell}{\ell_0} = \text{Relative change} = \frac{m\omega^2}{k}$$

∴ Correct answer (4)

4. Official Ans. by NTA (2)

Sol.



$mx\omega^2 \cos\theta = mg \sin\theta$

$x\omega^2 = g \tan\theta$

$$x\omega^2 = g.\frac{dy}{dx}$$

$x\omega^2 = g.(8cx)$

$\omega^2 = 8\ gc$

$$\omega = 2\sqrt{2gc}$$

5. Official Ans. by NTA (1)

Sol. R = 0.1 m

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105 \text{ rad/sec}$$

$a = \omega^2 R$

$= (0.105)^2\ (0.1)$

$= 0.0011$

$= 1.1 \times 10^{-3}$

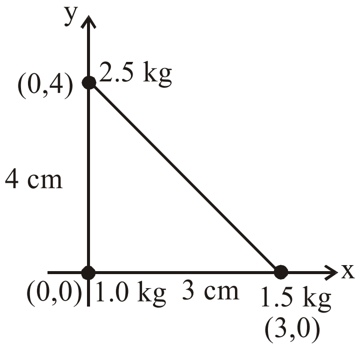
Average acceleration is of the order of 10^{-3}

$\therefore$ correct option is (1)

CENTRE OF MASS & COLLISION

1. NTA Ans. (2)

Sol.



Let 1 kg as origin and x-y axis as shown

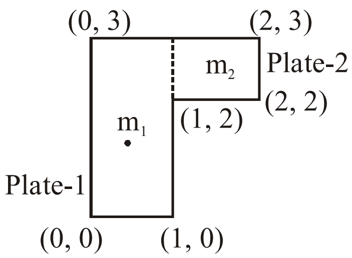
$$x_{cm} = \frac{1(0)+1.5(3)+2.5(0)}{5} = 0.9 \text{ cm}$$

$$y_{cm} = \frac{1(0)+1.5(0)+2.5(4)}{5} = 2 \text{ cm}$$

2. NTA Ans. (4)

Sol. $m_1 = 3\text{kg}$

$m_2 = 1\text{kg}$



Mass of plate-1 is assumed to be concentrated at (0.5, 1.5)

Mass of plate-2 is assumed to be concentrated at (1.5, 2.5).

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{3\times 0.5 + 1\times 1.5}{4} = 0.75$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{3\times 1.5 + 1\times 2.5}{4} = 1.75$$

3. NTA Ans. (1.00)

Sol. By conservation of linear momentum :

$$(0.1)(3\hat{i}) + (0.1)(5\hat{j}) = (0.1)(4)(\hat{i}+\hat{j}) + (0.1)\vec{v}$$

$\Rightarrow\ \vec{v} = -\hat{i} + \hat{j}$

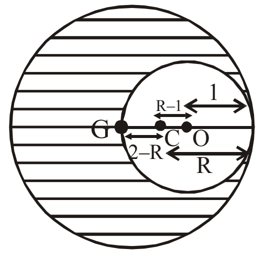
$\therefore$ Speed of B after collision $|\vec{v}| = \sqrt{2}$

Now, kinetic energy $= \frac{1}{2}mV^2 = \frac{1}{2}(0.1)(2) = \frac{1}{10}$

$\therefore$ x = 1

4. NTA Ans. (3)

Sol.



By concept of COM

$m_1R_1 = m_2R_2$

Remaining mass × (2–R) = cavity mass × (R–1)

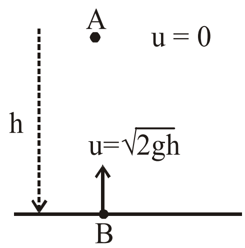
$$\left(\frac{4}{3}\pi R^3\rho - \frac{4}{3}\pi 1^3\rho\right)(2-R) = \frac{4}{3}\pi 1^3\rho \times (R-1)$$

$(R^3 - 1)\ (2 - R) = R - 1$

$(R^2 + R + 1)\ (2 - R) = 1$

5. NTA Ans. (4)

Sol.

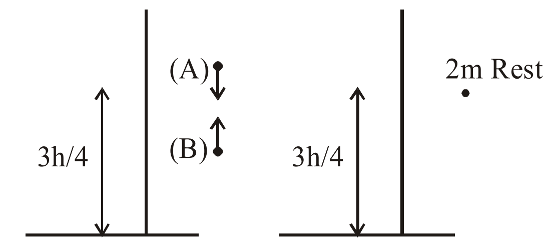


Particles will collide after time $t_0 = \frac{h}{\sqrt{2gh}}$

at collision, $v_A = gt_0$ $\qquad v_B = u_B - gt_0$

$\Rightarrow\ v_A = -v_B$

Before collision $\qquad$ After collision



Time taken by combined mass to reach the ground

$$\text{time} = \sqrt{\frac{2\times 3h/4}{g}} = \sqrt{\frac{3h}{2g}}$$

6. NTA Ans. (2)

Sol. From momentum conservation

$$mu\hat{i} + mu\left(\frac{\hat{i}+\hat{j}}{2}\right) = (m+m)\,\bar{v}$$

$$\Rightarrow \bar{v} = \frac{3}{4}u\hat{i} + \frac{u}{4}\hat{j}$$

$$\Rightarrow |v| = \frac{u}{4}\sqrt{10}$$

Final kinetic energy = $\frac{1}{2}2m\left(\frac{u}{4}\sqrt{10}\right)^2 = \frac{5}{8}mu^2$

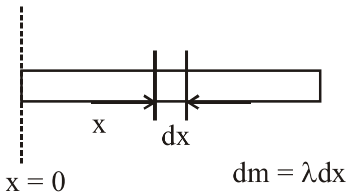
Initial kinetic energy

$$= \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{6}{8}mu^2$$

Loss in K.E. = $k_i - k_f = \frac{1}{8}mu^2$

7. NTA Ans. (4)

Sol.



$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int(\lambda dx)x}{\int dm}$$

$$= \frac{\int_0^L\left(a+\frac{bx^2}{L^2}\right)x\,dx}{\int_0^L\left(a+\frac{bx^2}{L^2}\right)dx}$$

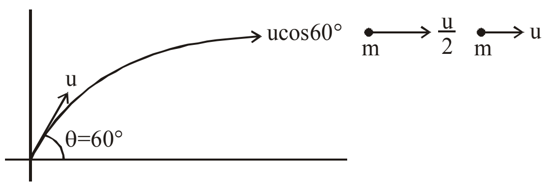
$$= \frac{\frac{aL^2}{2}+\frac{b}{L^2}\cdot\frac{L^4}{4}}{aL+\frac{b}{L^2}\cdot\frac{L^3}{3}}$$

$$= \frac{\left(\frac{4a+2b}{8}\right)L}{\frac{(3a+b)}{3}} = \frac{3}{4}\frac{(2a+b)L}{(3a+b)}$$

∴ correct answer 4

8. NTA Ans. (3)

Sol.



By momentum conservation,

$$\frac{mu}{2} + mu = 2mv'$$

$$v' = \frac{3v}{4}$$

Range after collision = $\frac{3v}{4}\sqrt{\frac{2H}{g}}$

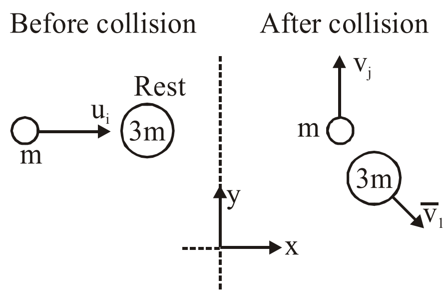
$$= \frac{3v}{4}\sqrt{\frac{2\cdot u^2\sin^2 60°}{g\,2g}}$$

$$= \frac{3}{4}\frac{\sqrt{3}}{2}\cdot\frac{u^2}{g} = \frac{3\sqrt{3}u^2}{8g}$$

∴ Correct answer (3)

9. Official Ans. by NTA (4)

Sol.



From momentum conservation

$\vec{P}_i = \vec{P}_f$

$m(ui) + 3m(0) = mvj + 3m\,\bar{v}_1$

$mui - mvj = 3m\,\bar{v}_1$

$$\bar{v}_1 = \frac{ui - vj}{3}$$

or $|v_1| = \frac{\sqrt{u^2+v^2}}{3}$

or $v_1^2 = \frac{u^2+v^2}{9}$ ....(1)

As collision is perfectely elastic hence

$k_i = k_j$

$$\frac{1}{2}mu^2 + \frac{1}{2}3m0^2 = \frac{1}{2}mv^2 + \frac{1}{2}3mv_1^2$$

$\Rightarrow u^2 = v^2 + 3v_1^2$

$$u^2 = v^2 + 3\frac{(u^2+v^2)}{9}$$

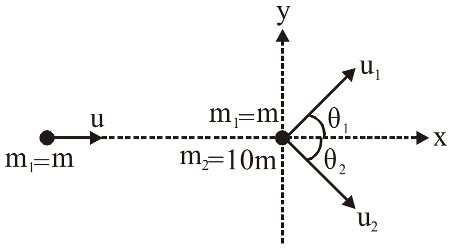
$\Rightarrow 3u^2 = 3v^2 + u^2 + v^2$

$\Rightarrow 2u^2 = 4v^2$

$$v = \frac{u}{\sqrt{2}}$$

10. Official Ans. by NTA (10.00)

Sol.



By momentum conservation along y :

$m_1u_1\sin\theta_1 = m_2u_2\sin\theta_2$

i.e. $mu_1\sin\theta_1 = 10mu_2\sin\theta_2$

$\Rightarrow \boxed{u_1\sin\theta_1 = 10u_2\sin\theta_2}$ .....(i)

$kf_{m_1} = \frac{1}{2}ki_{m_1}$ i.e. $\frac{1}{2}mu_1^2 = \frac{1}{2}\times\frac{1}{2}mu^2$

i.e. $\boxed{u_1 = \frac{u}{\sqrt{2}}}$ .....(ii)

Also collision is elastic : $k_i = k_f$

$\frac{1}{2}mu^2 = \frac{1}{2}mu_1^2 + \frac{1}{2}.10m.u_2^2$

$\frac{1}{2}mu^2 = \frac{1}{2}\times\frac{1}{2}mu^2 + \frac{1}{2}\times 10m.u_2^2$

$\frac{1}{4}mu^2 = \frac{1}{2}\times 10\times mu_2^2$

$\boxed{u_2 = \frac{u}{\sqrt{20}}}$ ....(iii)

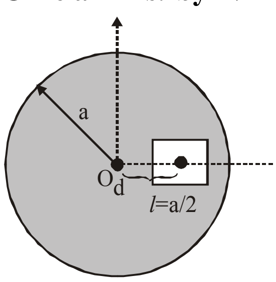
Putting (ii) & (iii) in (i)

$\frac{u}{\sqrt{2}}\sin\theta_1 = 10.\frac{u}{\sqrt{20}}\sin\theta_2$

$\boxed{\sin\theta_1 = \sqrt{10}\sin\theta_2}$ → Hence n = 10

11. Official Ans. by NTA (23.00)

Sol.



$X_{com} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$

where :

- m_1 = mass of complete disc
- m_2 = removed mass
- Let σ = surface mass density of disc material

wrt 'O' : $X_{com} = \frac{\sigma\pi a^2(O) - \sigma.\frac{a^2}{4}.d}{\sigma\pi a^2 - \sigma\frac{a^2}{4}} = \frac{-\frac{a^2}{4}d}{\pi a^2 - \frac{a^2}{4}}$

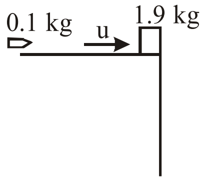
$= \frac{-d}{4\pi - 1} = -\frac{a}{2(4\pi - 1)}$

So, X = 2(4π–1) = (8π–2) = 23.12

So, nearest integer value of X = 23

12. Official Ans. by NTA (1)

Sol.



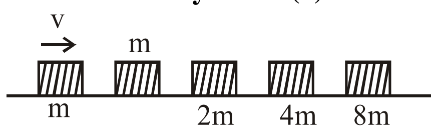
$p_i = p_f \Rightarrow 0.1 \times 20 = 2v$

$\therefore$ v = 1 m/s

$KE_f = mgh + \frac{1}{2}mv^2 = 213$

13. Official Ans. by NTA (4)

Sol.



All collisions are perfectly inelastic, so after the final collision, all blocks are moving together. So let the final velocity be v', so on applying momentum conservation:

mv = 16m v' ⇒ v' = v/16

Now initial energy $E_i = \frac{1}{2}mv^2$

Final energy : $E_f = \frac{1}{2}\times 16m\times\left(\frac{v}{16}\right)^2$

$\Rightarrow E_f = \frac{1}{2}m\frac{v^2}{16}$

Energy loss : $E_i - E_f \Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{16}$

$\Rightarrow \frac{1}{2}mv^2\left[1 - \frac{1}{16}\right] \Rightarrow \frac{1}{2}mv^2\left[\frac{15}{16}\right]$

$\%p = \frac{\text{Energy loss}}{\text{Original energy}}\times 100$

$= \frac{\frac{1}{2}mv^2\left[\frac{15}{16}\right]}{\frac{1}{2}mv^2}\times 100 = 93.75\%$

⇒ Value of P is close to 94.

14. Official Ans. by NTA (4)

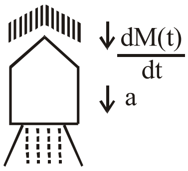
Sol. $\frac{dm(t)}{dt} = bv^2$

$F_{thast} = v\frac{dm}{dt}$

Force on statellile = $-\vec{v}\frac{dm(t)}{dt}$

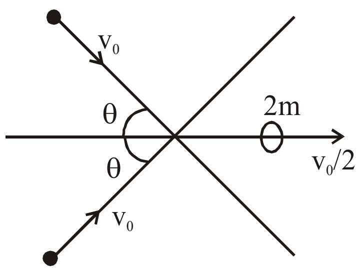
M(t) a = – v (bv²)

$\boxed{a = a\frac{bv^3}{M(t)}}$



15. Official Ans. by NTA (120.00)

Sol.



Momentum conservation along x

$2mv_0\cos\theta = 2m\frac{v_0}{2}$

$\cos\theta = \frac{1}{2}$

θ = 60

Angle is 2θ = 120

Ans. 120.00

16. Official Ans. by NTA (4)

Sol. $\vec{v}_{01} = (\sqrt{3}\hat{i} + \hat{j})\ m/s$

$\vec{v}_{02} = \vec{0}$

$m_1 = 2m_2$

After collision, $\vec{v}_1 = (\hat{i} + \sqrt{3}\hat{j})\ m/s$

$\vec{v}_2 = ?$

Applying conservation of linear momentum,

$m_1\vec{v}_{01} + m_2\vec{v}_{02} = m_1\vec{v}_1 + m_2\vec{v}_2$

$2m_2(\sqrt{3}\hat{i} + \hat{j}) + 0 = 2m_2(\hat{i} + \sqrt{3}\hat{j}) + m_2\vec{v}_2$

$\vec{v}_2 = 2(\sqrt{3}\hat{i} + \hat{j}) - 2(\hat{i} + \sqrt{3}\hat{j})$

$= 2(\sqrt{3}\hat{i} - \hat{j}) + 2(\hat{i} - \sqrt{3}\hat{j})$

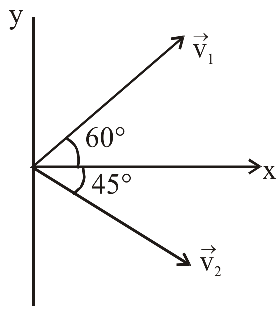
$\vec{v}_2 = 2(\sqrt{3} - 1)(\hat{i} - \hat{j})$

for angle between $\vec{v}_1$ & $\vec{v}_2$,

$\cos\theta = \frac{\vec{v}_1, \vec{v}_2}{\vec{v}_1\vec{v}_2} = \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{2\times 2\sqrt{2}(\sqrt{3}-1)}$

$\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}}$ ⇒ θ = 105°

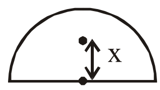
or



17. Official Ans. by NTA (3.00)

Sol. $x = \frac{3R}{8} = 3cm$

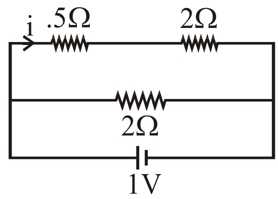
x = 3



CURRENT ELECTRICITY

1. NTA Ans. (2)

Sol. Equivalent resistance of upper branch of circuit R = 2.5 Ω



Voltage across upper branch = 1 V

⇒ $i = \frac{1}{2.5} = .4\ A$

⇒ $I_1 = 0.2\ A$

2. NTA Ans. (4)

Sol. $220\,I = P = 15\times 45 + 15\times 100 + 15\times 10 + 2\times 10^3$

$I = \frac{4325}{220} = 19.66$

I ≃ 20 A

3. NTA Ans. (12)

Sol. $r = R\left(\dfrac{x - x'}{x'}\right)$

$= 10 \times \dfrac{60}{500} = 12$

4. NTA Ans. (4)

Sol. $5 = \lambda\ell$

where λ is potential gradient & L is total length of wire.

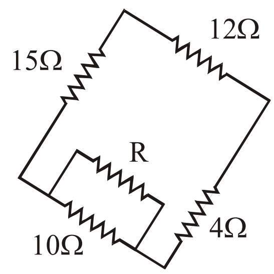
$5 = \dfrac{\Delta V}{L}\ell$

$\Delta V = \dfrac{5 \times L}{\ell} = 5 \times \dfrac{12}{10} = 6V$ = 60 mA × R

R = 100Ω

5. NTA Ans. (10.00)

Sol.



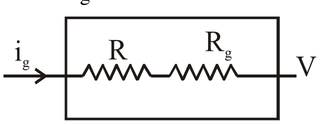
Let the resistance to be connected is R.
For balanced wheatstone bridge,

$15 \times 4 = 12 \times \dfrac{10R}{10 + R}$

⇒ R = 10Ω

6. NTA Ans. (1)

Sol. i_g = 1 mA , R_g = 100 Ω



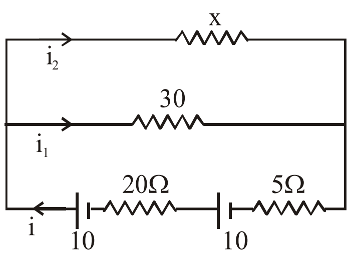
$V = i_g(R + R_g)$

$10 = 1 \times 10^{-3}\,(R + 100)$

R = 9.9 kΩ

7. NTA Ans. (30)

Sol.



$E_1 = E - ir$
$= 10 - i20 = 0$

$E_2 = E - ir$
$= 10 - 0.5 \times 5$
$= 7.5$ V

i = 0.5 A

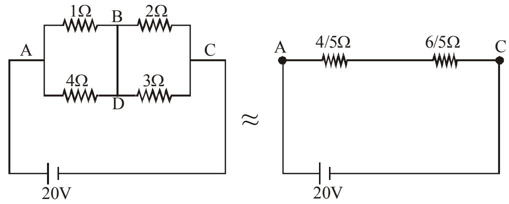
$E_{net} = E_1 + E_2 = 7.5V$

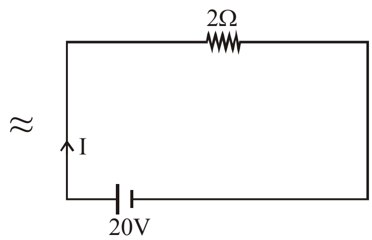
$i = i_1 + i_2$

$0.5 = \dfrac{7.5}{x} + \dfrac{7.5}{30}$ x = 30 Ω

8. NTA Ans. (2)

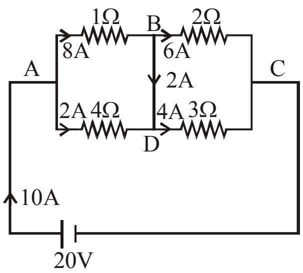
Sol.





$\Rightarrow I = \dfrac{20}{2} = 10A$

Now,



9. NTA Ans. (40.00)

Sol. In balancing

$\dfrac{R}{S} = \dfrac{25}{75}$

New resistance $R' = \dfrac{\rho\ell}{A}$

$= \dfrac{\rho \times \dfrac{\ell}{2}}{\dfrac{A}{4}} = \dfrac{\rho\ell}{2} \times 4A$

R' = 2R

$\dfrac{2R}{S} = \dfrac{\ell'}{100 - \ell'}$

$2 \times \dfrac{1}{3} = \dfrac{\ell'}{100 - \ell'} = 3\ell' = 200 - 2\ell'$

$5\ell' = 200$

$\ell' = 40$

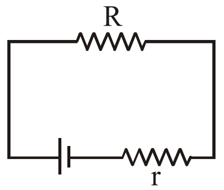
∴ Correct answer 40

10. Official Ans. by NTA (4)

Sol. $\rho_M > \rho_A > \rho_C$

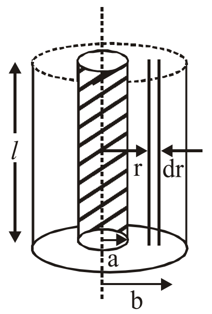
11. Official Ans. by NTA (4)

Sol. Maximum power in external resistance is generated when it is equal to internal resistance of battery.



$$P_R = \left(\frac{\in}{r+R}\right)^2 R$$

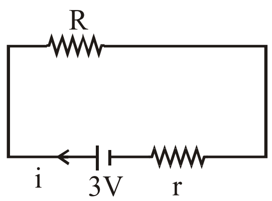
P_R is max. when r = R



$$\int dr = \int_a^b \frac{\rho dr}{2\pi r l} \Rightarrow r = \frac{\rho}{2\pi l} ln\frac{b}{a}$$

12. Official Ans. by NTA (4)

Sol.



$P_R = 0.5W$

$\Rightarrow i^2R = 0.5W$

Also, V = E – ir

2.5 = 3 – ir

$\Rightarrow$ ir = 0.5

Power dissipated across 'r' : $P_r = i^2r$

Now iR = 2.5

ir = 0.5

On dividing : $\frac{R}{r} = 5$

Now $\frac{P_R}{P_r} = \frac{i^2R}{i^2r} \Rightarrow \frac{P_R}{P_r} = \frac{R}{r} \Rightarrow \frac{P_R}{P_r} = 5$

$\Rightarrow P_r = \frac{P_R}{5}$

$\Rightarrow P_r = \frac{0.50}{5} \Rightarrow P_r = 0.10$ W

option (4) is correct.

13. Official Ans. by NTA (4)

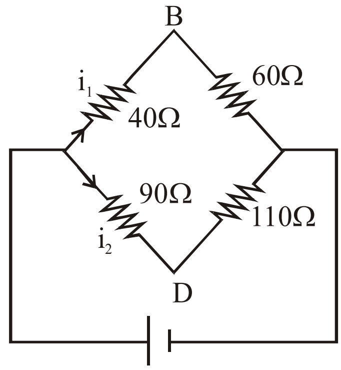
Sol. Voltage across AC = 8V

$R_{AC} = 4 + 4 = 8\Omega$

$$i_1 = \frac{V}{R_{AC}} = \frac{8}{8} = 1 \text{ A}$$

14. Official Ans. by NTA (2)

Sol.



$$i_1 = \frac{40}{40+60} = 0.4$$

$$i_2 = \frac{40}{90+110} = \frac{1}{5}$$

$v_B + i_1 (40) - i_2 (90) = v_D$

$$v_B - v_D = \frac{1}{5}(90) - \frac{4}{10}\times 40$$

$v_B - v_D = 18 - 16 = 2$

15. Official Ans. by NTA (2)

Sol. vi = 10^3

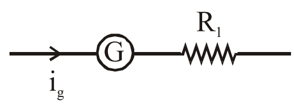
$$i = \frac{1000}{220}$$

$$\text{loss} = i^2R = \left(\frac{50}{11}\right)^2 \times 2$$

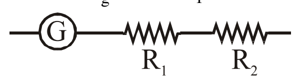
$$\text{efficiency} = \frac{1000}{1000+i^2R}\times 100 = 96\%$$

16. Official Ans. by NTA (2)

Sol.



$\Rightarrow 1 = i_g(G + R_1)$ ... (1)



$\Rightarrow 2 = i_g(R_1 + R_2 + G)$ ...(2)

(1) % (2)

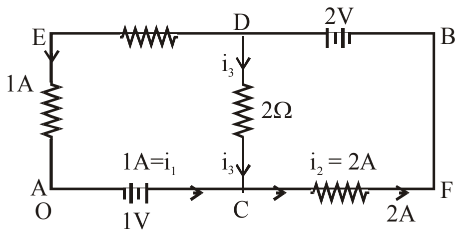
$$\Rightarrow \frac{1}{2} = \frac{G+R_1}{G+R_1+R_2}$$

$G + R_1 + R_2 = 2G + 2h_1$

$(R_2 = G + R_1)$

17. Official Ans. by NTA (1)

Sol.



Let us asssume the potential at A = $V_A = 0$

Now at junction C, According to KCL

$i_1 + i_3 = i_2$

$1A + i_3 = 2A$

$i_3 = 2A$

Now Analyse potential along ACDB

$v_A + 1 + i_3(2) - 2 = v_B$

$0 + 1 + 2(1) - 2 = v_B$

$v_B = 3 - 2$

$v_B = 1$ Amp

18. Official Ans. by NTA (1)

Sol. Figure of Merit = C = $\frac{i}{\theta}$

$$= C = \frac{6\times10^{-3}}{2} = 3\times10^{-3}\,Am^2$$

19. Official Ans. by NTA (1)

Sol. Conceptual

Option (1) is correct

Ammeter :- In series connection, the same current flows through all the components. It aims at measuring the curent flowing through the circuit and hence, it is connected in series.

Voltmeter :- A voltmeter measures voltage change between two points in a circuit, So we have to place the voltmeter in parallal with the circuit component.

20. Official Ans. by NTA (3)

Sol. $E_{eq} = \frac{20\times10}{17} = \frac{200}{17}$

and $R_{eq} = \frac{7\times10}{17} = \frac{70}{17}$

21. Official Ans. by NTA (1)

Sol. Balancing length is measured from P.

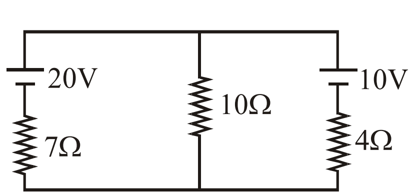
So 100 – 49 = 51 cm

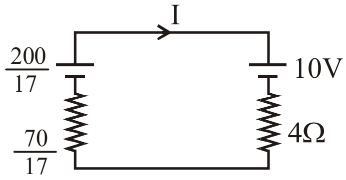
$E_2 = \phi \times 51$

Where ϕ = Potential gradient

$1.02 = \phi \times 51$

$\phi = 0.02$ V/cm





$$\therefore\quad I = \frac{\frac{20}{17} - 10}{4 + \frac{70}{17}} = 0.21\text{ A}$$

from +ve to –ve terminal

ELASTICITY

1. NTA Ans. (4.00)

Sol. $T = m\omega^2\ell$

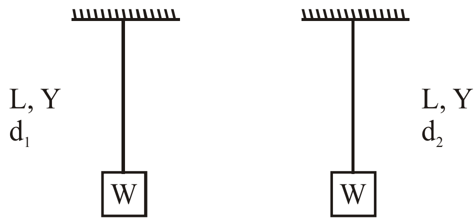
Breaking stress = $\frac{T}{A} = \frac{m\omega^2\ell}{A}$

$$\Rightarrow \omega^2 = \frac{4.8\times10^7\times(10^{-2}\times10^{-4})}{10\times0.3} = 16$$

$$\Rightarrow \omega = 4$$

2. NTA Ans. (4)

Sol.



$$\frac{\text{Energy stored}}{\text{Volume}} = \frac{1}{2}\frac{(\text{Stress})^2}{Y}$$

$$\frac{u_1}{u_2} = \frac{1}{4} \Rightarrow 4u_1 = u_2$$

$$4\frac{1}{2Y}\left[\frac{W\cdot4}{\pi d_1^2}\right]^2 = \frac{1}{2Y}\left[\frac{W\cdot4}{\pi d_2^2}\right]^2$$

$$4 = \left(\frac{d_1}{d_2}\right)^4$$

$$\Rightarrow \frac{d_1}{d_2} = \sqrt{2}:1$$

$\therefore$ Correct answer (4)

3. NTA Ans. (750.00)

Sol. The length of the screen used portion for 15 fringes, and also for ten fringes

$$15\times500\times\frac{D}{\lambda}=10\times\frac{\lambda D}{\lambda}$$

$15 \times 50 = \lambda$

$\lambda = 750$ nm

∴ Correct answer 750

4. Official Ans. by NTA (2)

Sol. $B = \frac{-\Delta P}{\frac{\Delta V}{V}}$

$$\left|\frac{\Delta V}{V}\right|=\frac{\Delta P}{B}$$

$$=\frac{4\times10^9}{8\times10^{10}}=\frac{1}{20}$$

$$\frac{\Delta\ell}{\ell}=\frac{1}{3}\times\frac{\Delta V}{V}=\frac{1}{60}$$

Percentage change = $\frac{\Delta\ell}{\ell}\times100\%$

$$=\frac{100}{60}\%=1.67\%$$

5. Official Ans. by NTA (1)

Sol. An elastic wire can be treated as a spring with

$$k=\frac{YA}{\ell}$$

$$T=2\pi\sqrt{\frac{m}{k}}$$

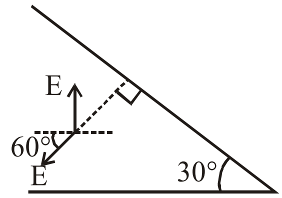
$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{YA}{m\ell}}$$

Ans. (1)

ELECTROSTATICS

1. NTA Ans. (2)

Sol. Electric field due to each sheet is uniform and equal to $E=\frac{\sigma}{2\varepsilon_0}$



Now net electric field between plates

$$\vec{E}_{net}=E\cos60°(-\hat{x})+(E-E\sin60°)(\hat{y})$$

$$=\frac{\sigma}{2\varepsilon_0}\left[-\frac{\hat{x}}{2}+\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}\right].$$

2. NTA Ans. (4)

Sol. $|\vec{E}|$ should be constant on the surface and the surface should be equipotential.

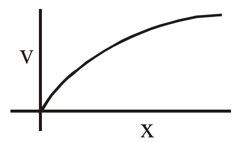
3. NTA Ans. (4)

Sol. $E_x=\frac{K(4q)}{R^2}\cos30°+\frac{K(2q)}{R^2}\cos30°+\frac{K(2q)}{R^2}\cos30°$

4. NTA Ans. (3)

Sol. $v^2 = u^2 + 2as$

$v^2 = 0 + 2\left(\frac{qE}{m}\right)x$



$v^2 = \frac{2qE}{m}x$

5. NTA Ans. (4)

Sol. $E_1 = \frac{KQ_1}{R_1^2}$ $\quad E_2 = \frac{KQ_2}{R_2^2}$

Given,

$$\frac{E_1}{E_2}=\frac{R_1}{R_2}$$

$$\frac{\frac{KQ_1}{R_1^2}}{\frac{KQ_2}{R_2^2}}=\frac{R_1}{R_2} \Rightarrow \boxed{\frac{Q_1}{Q_2}=\frac{R_1^3}{R_2^3}}$$

$$\frac{V_1}{V_2}=\frac{KQ_1/R_1}{KQ_2/R_2}=\frac{R_1^2}{R_2^2}$$

6. NTA Ans. (4)

Sol. Fill the empty space with $+\rho$ and $-\rho$ charge density.

$$|E_A| = 0 + \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{R}{2}\right)^2} = k\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)$$

$$|E_B| = \frac{k\rho.\frac{4}{3}\pi R^3}{R^2} - \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$= k\rho\frac{4}{3}\pi R - k\rho\frac{4}{3}\pi\frac{R}{18} = k\rho.\frac{4}{3}\pi\left(\frac{17R}{18}\right)$$

$$\frac{E_A}{E_B} = \frac{9}{17} = \frac{18}{34}$$

7. NTA Ans. (3)

Sol. Since $\vec{r}$ and $\vec{p}$ are perpendicular to each other therefore point lies on the equitorial plane. Therefore electric field at the point will be antiparallel to the dipole moment.

i.e. $\vec{E} \parallel -\vec{p}$

$\vec{E} \parallel (\hat{i} + 3\hat{j} - 2\hat{k})$

8. NTA Ans. (-48.00)

Sol. The flux passes through ABCD (x – y) plane is zero, because electric field parallel to surface.

Flux of the electric field through surface BCGF (y - z)

At BCGF (electric field) $\Rightarrow \vec{E} = 12\hat{i} - (y^2+1)\hat{j}$

(x = 3m)

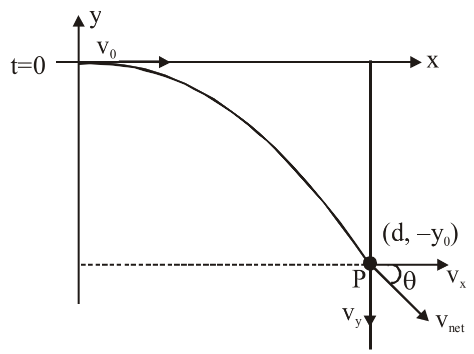
Flux $\phi_{II} = 12 \times 4 = 48$ Nm²/C

So $\phi_I - \phi_{II} = 0 - 48 = -48$ Nm²/C

$\therefore$ Correct answer –48

9. Official Ans. by NTA (1)

Sol.



Let particle have charge q and mass 'm'

Solve for (q,m) mathematically

$F_x = 0$, $a_x = 0$, $(v)_x$ = constant

time taken to reach at 'P' = $\frac{d}{v_0} = t_0$ (let) ...(1)

(Along –y), $y_0 = 0 + \frac{1}{2}.\frac{qE}{m}.t_0^2$ ....(2)

$v_x = v_0$

$v = u + at$ (along -ve 'y')

speed $v_{y0} = \frac{qE}{m}.t_0$

$$\tan\theta = \frac{v_y}{v_x} = \frac{qEt_0}{m.v_0}, (t_0 = \frac{d}{v_0})$$

$$\tan\theta = \frac{qEd}{m.v_0^2}$$

$$\boxed{\text{slope} = \frac{-qEd}{mv_0^2}}$$

Now we have to find eq^n of straight line whose slope is $\frac{-qEd}{mv_0^2}$ and it pass through point $\rightarrow$ (d, –y_0)

Because after x > d

No electric field $\Rightarrow F_{net} = 0$, $\vec{v}$ = const.

$$y = mx + c \;, \left\{\begin{matrix} m = \frac{qEd}{mv_0^2} \\ (d, -y_0) \end{matrix}\right\}$$

$$-y_0 = \frac{-qEd}{mv_0^2}.d + c \;\Rightarrow\; c = -y_0 + \frac{qEd^2}{mv_0^2}$$

Put the value

$$y = \frac{-qEd}{mv_0^2}x - y_0 + \frac{qEd^2}{mv_0^2}$$

$$y_0 = \frac{1}{2}.\frac{qE}{m}\left(\frac{d}{v_0}\right)^2 = \frac{1}{2}\frac{qEd^2}{mv_0^2}$$

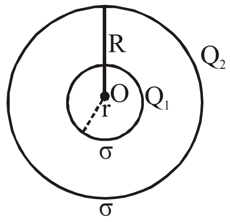
$$y = \frac{-qEdx}{mv_0^2} - \frac{1}{2}\frac{qEd^2}{mv_0^2} + \frac{qEd^2}{mv_0^2}$$

$$y = \frac{-qEd}{mv_0^2}x + \frac{1}{2}\frac{qEd^2}{mv_0^2}$$

$$\boxed{y = \frac{qEd}{mv_0^2}\left(\frac{d}{2} - x\right)}$$

10. Official Ans. by NTA (3)

Sol. Let the charges on inner and outer spheres are Q_1 and Q_2.



Since charge density 'σ' is same for both spheres, so

$$\sigma = \frac{Q_1}{4\pi r^2} = \frac{Q_2}{4\pi R^2} \Rightarrow \frac{Q_1}{Q_2} = \frac{r^2}{R^2}$$

$$Q_1 + Q_2 = Q \Rightarrow \frac{Q_2 r^2}{R^2} + Q_2 = Q$$

$$\Rightarrow Q_2 = \frac{QR^2}{(r^2 + R^2)}$$

$$Q_1 = \frac{r^2}{R^2} \cdot \frac{QR^2}{(R^2 + r^2)} = \frac{Qr^2}{(R^2 + r^2)}$$

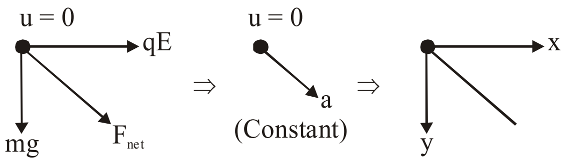
Potential at centre 'O' = $\frac{kQ_1}{r} + \frac{kQ_2}{R}$

$$= k\left[\frac{Qr^2}{r(R^2 + r^2)} + \frac{QR^2}{R(R^2 + r^2)}\right]$$

$$= \frac{kQ(r + R)}{(R^2 + r^2)} = \frac{1}{4\pi \epsilon_0} \frac{(R + r)}{(R^2 + r^2)} Q$$

11. Official Ans. by NTA (4)

Sol.



Since initial velocity is zero and acceleration of particle will be constant, so particle will travel on a straight line path.

12. Official Ans. by NTA (1)

Sol. Now

$Q_1 + Q_2 = Q'_1 + Q'_2 = 12\ \mu C - 3\ \mu C = 9\ \mu C$

$$\&\ V_1 = V_2 \Rightarrow \frac{KQ'_1}{\frac{2R}{3}} = \frac{KQ'_2}{\frac{R}{3}}$$

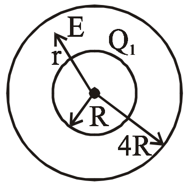
$$Q'_1 = 2Q'_2 \Rightarrow 2Q'_2 + Q'_2 = 9\mu C$$

$$\Rightarrow Q'_2 = 3\ \mu C$$

$\&\ Q'_1 = 6\ \mu C$

13. Official Ans. by NTA (1)

Sol.



$$E = \frac{KQ_1}{r^2}$$

$$\Delta V = \int_R^{4R} E\, dr = \frac{3KQ_1}{4R}$$

14. Official Ans. by NTA (1)

Sol. (1) Multimeter shows deflection when it connects with capacitor

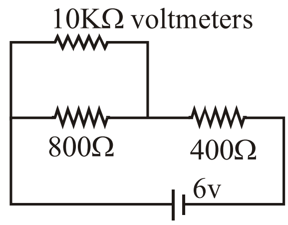
(2) If we assume that LED has negligiable resistance then multimeter shows no deflection for the forward blas but when it connects in reverse direction, it break down occurs so splash of light out.

(3) The resistance of metal wire may be taken zero, so no deflection in multimeter

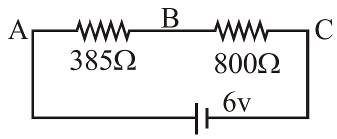
(4) No matter, how we connect the resistance across multimeter It shows same deflection.

15. Official Ans. by NTA (2)

Sol.



⇒



So the potential difference in voltmeter across the points A and B is $\frac{6}{1185} \times 385$ = 1.949 V

16. Official Ans. by NTA (3)

Sol. Potential of –q is same as initial and final point of the path therefore potential due to 4q will only change and as potential is decreasing the energy will decrease

Decrease in potential energy = q (V_i – V_f)

Decrease in potential energy

$$= q\left[\frac{k4q}{d/2} - \frac{k4q}{3d/2}\right] = \frac{4q^2}{3\pi\varepsilon_0 d}$$

Therefore correct answer is 3.

17. Official Ans. by NTA (1)

Sol. Thin infinite uniformly charged planes produces uniform electric field therefore option 2 and option 3 are obviously wrong.
And as positive charge density is bigger in magnitude so its field along Y direction will be bigger than field of negative charge in X direction and this is evident in option 1 so it is correct.

18. Official Ans. by NTA (4)

Sol. $E = E_0\,(1 - ax^2)$

$$W = \int qE\,dx = qE_0 \int_0^{x_0} (1-ax^2)\,dx$$

$$= qE_0 \left[x_0 - \frac{ax_0^3}{3}\right]$$

For $\Delta KE = 0$, $W = 0$

Hence $x_0 = \sqrt{\frac{3}{a}}$

19. Official Ans. by NTA (1)

Sol. $\frac{kQq}{R} + mgy$

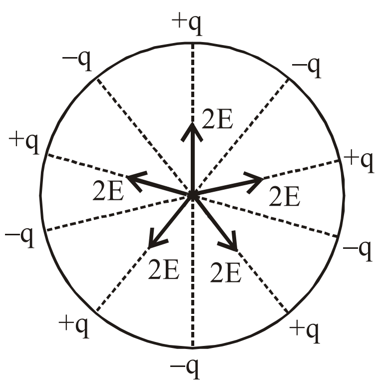
$$= \frac{kQq}{R+y} + \frac{1}{2}mv^2$$

$$v^2 = 2gy + \frac{2kQqy}{mR(R+y)}$$

20. Official Ans. by NTA (3)

Sol. Potential of centre = V = $\Sigma\left(\frac{kq}{R}\right)$

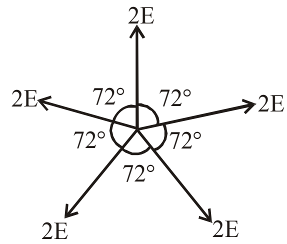
$$V_C = \frac{K(\Sigma q)}{R}$$



$$V_C = \frac{K(0)}{R} = 0$$

Electric field at centre $\vec{E}_B = \Sigma\vec{E}$

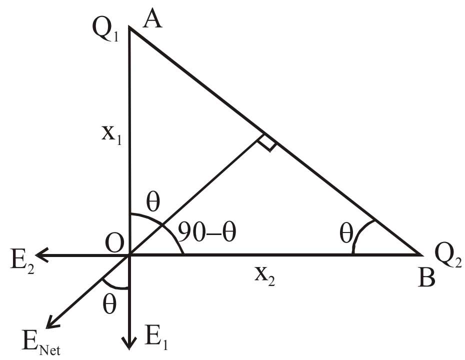
Let E be electric field produced by each charge at the centre, then resultant electric field will be



$E_C = 0$, Since equal electric field vectors are acting at equal angle so their resultant is equal to zero.

21. Official Ans. by NTA (3)

Sol.



E_2 = electric field due to Q_2

$$= \frac{kQ_2}{x_2^2}$$

$$E_1 = \frac{kQ_1}{x_1^2}$$

From diagram

$$\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2}$$

$$\frac{kQ_2}{x_2^2 \times \frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$$

$$\frac{Q_2x_1^2}{Q_1x_2^2} = \frac{x_1}{x_2}$$

$$\frac{Q_2}{Q_1} = \frac{x_2}{x_1}$$

$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

Ans. (3)

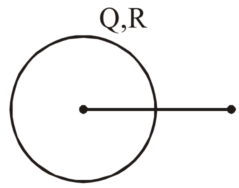
22. Official Ans. by NTA (1)

Sol. Inside the shell

$E = 0$

hence $F = 0$

Oustside the sheell



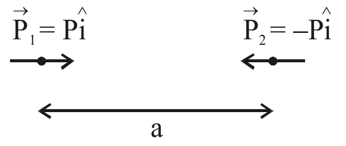
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

hence $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ for r > R

23. Official Ans. by NTA (3)

Sol. Using energy conservation:

$KE_i + PE_i = KE_f + PE_f$



$$O + \frac{2KP}{a^3} \times P = \frac{1}{2}mv^2 \times 2 + 0$$

$$V = \sqrt{\frac{2P^2}{4\pi\varepsilon_0 a^3 m}} = \frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 am}}$$

EM WAVE

1. NTA Ans. (1)

Sol. $\vec{E} \times \vec{B} = \vec{C} = -\hat{i}$

where $\vec{B}$ is along $\hat{j}$

$$\frac{E}{B} = C$$

$E = 3 \times 10^{-8} \times 3 \times 10^{8} = 9$ V/m.

2. NTA Ans. (3)

Sol. $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$$\vec{E} = E_0\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\cos\pi$$

$$= -E_0 \frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

as $\vec{E} \times \vec{B} = \vec{c}$

$$+E_0\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) \times \vec{B} = c\hat{k}$$

$$\Rightarrow \vec{B} = -\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\frac{E_0}{c}$$

$$\vec{F} = q\left(-E_0\frac{(\hat{i}+\hat{j})}{\sqrt{2}} - \frac{v_0\hat{k}}{c} \times (\hat{i}-\hat{j})E_0\right)$$

since $\frac{v_0}{c} << 1$

$\Rightarrow$ F is antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

3. NTA Ans. (2)

Sol. $E = \vec{B} \times \vec{V}$

$= (5 \times 10^{-8}\hat{j}) \times (3 \times 10^{8}\hat{k})$

$= 15\,\hat{i}$ V / m

4. NTA Ans. (3)

Sol. $\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx)$

Its corresponding magnetic field will be

$$\vec{B}_1 = \frac{E_0}{c}\hat{k}\cos(\omega t - kx)$$

$$\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$$

$$\vec{B}_2 = \frac{E_0}{c}\hat{i}\cos(\omega t - ky)$$

Net force on charge particle

$$= q\vec{E}_1 + q\vec{E}_2 + q\vec{v} \times B_1 + q\vec{v} \times \vec{B}_2$$

$$= qE_0\hat{j} + qE_0\hat{k} + q(0.8c\hat{j}) \times \left(\frac{E_0}{c}\hat{k}\right) + q(0.8c\hat{j}) \times \left(\frac{E_0}{c}\hat{i}\right)$$

$$= qE_0\hat{j} + qE_0\hat{k} + 0.8qE_0\hat{i} - 0.8qE_0\hat{k}$$

$$\vec{F} = qE_0[0.8\hat{i} + 1\hat{j} + 0.2\hat{k}]$$

5. Official Ans. by NTA (2)

Sol. Energy density $\frac{dU}{dV} = \frac{B_0^2}{2\mu_0}$

$$1.02 \times 10^{-8} = \frac{B_0^2}{2 \times 4\pi \times 10^{-7}}$$

$$B_0^2 = (1.02 \times 10^{-8}) \times (8\pi \times 10^{-7})$$

$B_0 = 16 \times 10^{-8}$ T = 160 nT

6. Official Ans. by NTA (1)

Sol. $\hat{E} = \hat{k}$

$$\vec{B} = 2\hat{i} - 2\hat{j} \Rightarrow \hat{B} = \frac{\vec{B}}{|B|} = \frac{2\hat{i} - 2\hat{j}}{2\sqrt{2}}$$

$$\Rightarrow \hat{B} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$$

Direction of wave propagation $= \hat{C} = \hat{E} \times \hat{B}$

$$\hat{C} = \hat{k} \times \left[\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})\right]$$

$$\hat{C} = \frac{1}{\sqrt{2}}(\hat{k} \times \hat{i} - \hat{k} \times \hat{j})$$

$$\hat{C} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$

7. Official Ans. by NTA (2)

Sol. $\vec{B} = 3 \times 10^{-8} \sin[200\pi(y + ct)]\hat{i}$ T

$E_0 = CB_0 \Rightarrow E_0 = 3 \times 10^8 \times 3 \times 10^{-8}$

= 9 V/m

and direction of wave propagation is given as

$(\vec{E} \times \vec{B}) \parallel \vec{C}$

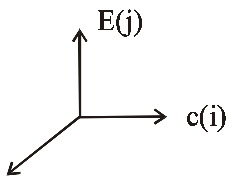
$\hat{B} = \hat{i}$ & $\hat{C} = -\hat{j}$

so $\hat{E} = -\hat{k}$

$\therefore \vec{E} = E_0 \sin[200\pi(y + ct)](-\hat{k})$ V / m

8. Official Ans. by NTA (3)

Sol.



$\therefore \vec{B}(\hat{k})$

$\Rightarrow \vec{B} = B_0 \cos(wt - kx)\hat{k}$

Now put t = 0.

9. Official Ans. by NTA (3)

Sol. Information based

$\lambda_{radiowaves} > \lambda_{microwaves} > \lambda_{visible} > \lambda_{x\text{-rays}}$

10. Official Ans. by NTA (2)

Sol. $\vec{E} = E_0(\hat{x} + \hat{y})\sin(kz - \omega t)$

direction of propagation $= +\hat{k}$

$$\hat{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

$$\hat{k} = \hat{E} \times \hat{B}$$

$$\hat{k} = \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) \times \hat{B} \qquad \Rightarrow \hat{B} = \frac{-\hat{i} + \hat{j}}{\sqrt{2}}$$

$$\therefore \vec{B} = \frac{E_0}{C}(-\hat{x} + \hat{y})\sin(kz - \omega t)$$

11. Official Ans. by NTA (2)

Sol. $\Rightarrow$E= $\vec{E} = 30\hat{j}\sin(1.5 \times 10^7 t - 5 \times 10^{-2} x)$V / m

$\Rightarrow B \Rightarrow E/V \Rightarrow \dfrac{30}{1.5 \times 10^7} \times 5 \times 10^{-2}$

$\Rightarrow 10^{-7}$ Tesla

$\Rightarrow F_{mag} = q(\vec{V} \times \vec{B}) = |qVB|$

$= 1.6 \times 10^{-19} \times 0.1 \times 3 \times 10^8 \times 10^{-7}$

$= 4.8 \times 10^{-19}$ N

12. Official Ans. by NTA (4)

Sol. Energes of given Radiation can have The following relation

$E_{\gamma\text{-Rays}} > E_{X\text{-Rays}} > E_{microwave} > E_{AM\ Radiowaves}$

$\therefore \lambda_{\gamma\text{-Rays}} < \lambda_{X\text{-Rays}} < \lambda_{microwave} < \lambda_{AM\ Radiowaves}$

According To tres.

(a) Microwave $\rightarrow 10^{-3}$m (iv)

(b) Gamma Rays $\rightarrow 10^{-15}$m (ii)

(c) AM Radio wave $\rightarrow$ 100 m (i)

(d) X-Rays $\rightarrow 10^{-10}$m (iii)

13. Official Ans. by NTA (275.00)

Allen Ans. (194.00)

Sol. $I = \epsilon_0 E_{rms}^2 C$

$$E_{rms}^2 = \frac{I}{\epsilon_0 C}$$

$$= \frac{315}{\pi \epsilon_0} \times \frac{1}{C}$$

$$= \frac{4 \times 315}{4\pi \epsilon_0} \times \frac{1}{3 \times 10^8}$$

$$= \frac{4 \times 315 \times 9 \times 10^9}{3 \times 10^8}$$

$E_{rms}^2 = 4 \times 315 \times 30$

$E_{rms} = 2\sqrt{315 \times 30}$

= 194.42

Ans. 194.00

14. Official Ans. by NTA (2)

Sol. $\vec{E}$ and $\vec{B}$ are perpendicular for EM wave

$E_0 = CB_0$

$= 3 \times 10^8 \times 1.2 \times 10^{-7}$

$= 36$

Having same phase

Propagation is along –x-axis, $\vec{B}$ is along z-axis

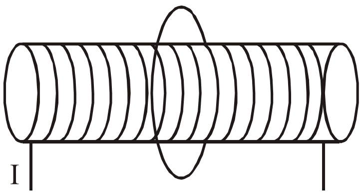
hence $\vec{E}$ must be along y-axis.

So, option (2) is correct

EMI & AC

1. NTA Ans. (1)

Sol.



Magnetic flux (ϕ) through ring is $\phi = \pi(R)^2.B$

$\phi = (\pi R^2)(\mu_0 nI) = (\pi R^2 \mu_0 nI_0)(t - t^2)$

Induced e.m.f. of $V_R = \frac{-d\phi}{dt}$

$= \left(\pi R^2 \mu_0 nI_0\right)(2t - 1)$

and induced current $I_R = \frac{\pi R^2 \mu_0 nI_0 (2t-1)}{R_R}$

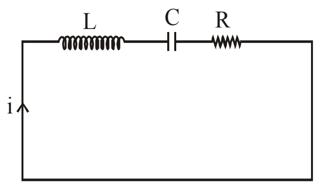
($R_R \rightarrow$ Resistance of Ring)

Clearly V_R and I_R are zero at $t = \frac{1}{2}$ = 0.5 sec.

and their sign also changes at t = 0.5 sec.

2. NTA Ans. (1)

Sol.



By kVL

$$-L\frac{di}{dt} - \frac{q}{C} - iR = 0$$

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q + R\frac{dq}{dt} = 0$$

for damped oscillator

net force = –kx – bv = ma

$$\frac{md^2x}{dt^2} + kx + \frac{bdx}{dt} = 0$$

by comparing ; Equivalence is

$$L \rightarrow m \; ; \; C \rightarrow \frac{1}{K} \; ; \; R \rightarrow b.$$

3. NTA Ans. (1)

ALLEN Ans. (2)

Sol. $i = i_0 (1 - e^{-Rt/L})$

$$\frac{i_0}{i} = \frac{1}{1 - e^{-2\times10^4}}$$

$$\frac{i_0}{i} \simeq 1$$

4. NTA Ans. (4)

Sol. Flux $\phi = \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos\omega t$

$$|\text{Induced emf}| = |e| = \left|\frac{d\phi}{dt}\right| = |BA\omega \sin \omega t|$$

$|e|$ will be maximum at $\omega t = \frac{\pi}{2}$

$$\left(\frac{2\pi}{T}\right)t = \frac{\pi}{2}$$

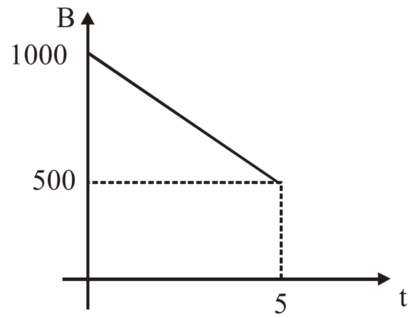
$$\left(\frac{2\pi}{10}\right)t = \frac{\pi}{2} \Rightarrow t = 2.5 \text{ sec}$$

$|e|$ will be minimum at $\omega t = \pi$

$$\left(\frac{2\pi}{10}\right)t = \pi \Rightarrow t = 5 \text{ sec}$$

5. NTA Ans. (3)

Sol.



$$\frac{dB}{dt} = 100$$

A = 16 ×4 – 4 × 2 = 56 cm²

$$\varepsilon = \frac{dB}{dt}A = 100 \times 10^{-4} \times 56 \times 10^{-4}$$

6. NTA Ans. (4)

Sol. $i = i_0 (1 - e^{-Rt/L}) = i_0(1 - e^{-t/T_C})$

$$q = \int_0^{T_C} i\,dt \qquad \Rightarrow = \int_0^{T_C} \frac{\varepsilon}{R}\left(1 - e^{-t/T_C}\right)$$

$$= \frac{\varepsilon}{R}\left(t - \frac{e^{-t/T_C}}{-1/T_C}\right)\Bigg|_0^{T_C}$$

$$= \frac{e}{R}\left(T_C - T_C e^{-1}\right) - \frac{e}{R}(0 + T_C) \Rightarrow q = \frac{e}{R} \times T_C \ e^{-1}$$

$$= \frac{\varepsilon}{R} \times \frac{L}{R}\,\frac{1}{e} \qquad \Rightarrow = \frac{\varepsilon L}{eR^2}$$

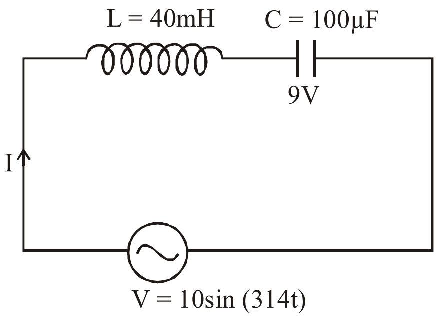
7. NTA Ans. (10.00)

Sol. $V = \left|L\frac{di}{dt}\right|$

$$\Rightarrow \quad L = \frac{V}{\left|\frac{di}{dt}\right|} = \frac{100}{\frac{0.25}{0.025 \times 10^{-3}}} = 10\text{mH}$$

8. NTA Ans. (1)

Sol.

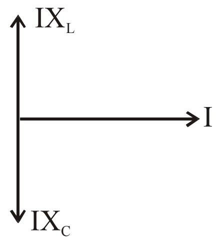


$X_L = \omega L = 314 \times 40 \times 10^{-3} = 12.56\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100 \times 10^{-6}}$$

$$= \frac{10^4}{314} = 31.84\Omega$$

Phasor



$V_m = I_m(X_C - X_L)$

$10 = I_m(31.84 - 12.56)$

$$I_m = \frac{10}{19.28} = 0.52\text{A}$$

$$I = 0.52 \sin\left(314t + \frac{\pi}{2}\right)$$

∴ Correct answer (1)

9. Official Ans. by NTA (15)

Sol. r = 0.1 m $\quad \frac{T}{2} = 0.2$ sec

$B = 3 \times 10^{-5}$ m $\qquad T = 0.4$ sec

At any time

flux $\phi = BA \cos \omega t$

$$|emf| = \left|\frac{d\phi}{dt}\right| = |BA\omega \ \sin \omega t|$$

$$(emf)_{max} = BA\omega = BA\,\frac{2\pi}{T}$$

$$= \frac{3 \times 10^{-5} \times \pi \times (0.1)^2 \times 2\pi}{0.4}$$

$$= \frac{6\pi^2}{4} \times 10^{-6} \qquad \left(\begin{matrix}\pi^2 \simeq 10 \\ \text{take}\end{matrix}\right)$$

$= 15 \times 10^{-6}$

$= 15\ \mu V$

10. Official Ans. by NTA (1)

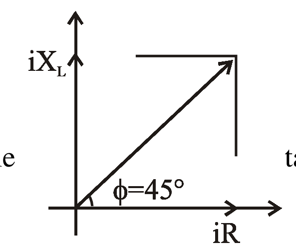
Sol.

- Reactance of inductance coil

$$= \sqrt{R^2 + x_L^2} = 100 \qquad .....(i)$$

- f = 1000 Hz of applied AC signal
- Voltage leads current by 45°

ie



$$\tan 45° = \frac{iX_L}{iR} = \frac{\omega L}{R}$$

ie $R = X_L = \omega L$

Putting in eqn (i) : $\sqrt{X_L^2 + X_L^2} = 100$

$$\sqrt{2}X_L = 100 \Rightarrow X_L = 50\sqrt{2}$$

ie $\omega L = 50\sqrt{2}$

$$L = \frac{50\sqrt{2}}{\omega} = \frac{50\sqrt{2}}{2\pi f} = \frac{25\sqrt{2}}{\pi \times 1000} \quad \text{H}$$

$= 1.125 \times 10^{-2}$ H

11. Official Ans. by NTA (3)

Sol. f = 750 Hz, V_{rms} = 20V,

R = 100 Ω, L = 0.1803 H,

C = 10 μF, S = 2 J/°C

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$= \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$

Putting values

|Z| = 834Ω

In AC power P = $V_{rms}\, i_{rms} \cos\phi$

$$\text{Cos}\phi = \frac{R}{|Z|} \qquad i_{rms} = \frac{V_{rms}}{|Z|}$$

$$= \frac{V_{rms}^2 R}{(|Z|)^2}$$

$$= \left(\frac{20}{834}\right)^2 \times 100 = 0.0575 \text{ J/s}$$

H = Pt = SΔθ

$$t = \frac{2(10)}{0.0575} = 348 \text{ sec}$$

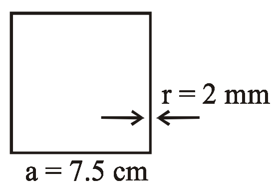
12. Official Ans. by NTA (1)

Sol. $\in = NAB\omega\cos\omega t$ $\boxed{N=1}$

$$P_{avg} = \left\langle \frac{\in^2}{R} \right\rangle = \left\langle \frac{(AB\omega\cos\omega t)^2}{R} \right\rangle$$

$$= \frac{A^2B^2\omega^2}{R}\frac{1}{2} = \frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$$

13. Official Ans. by NTA (1)

Sol. 

$$q_i = \frac{d(Ba^2)}{dt} = a^2\frac{dB}{dt}$$

$$i = \frac{q}{R} = \frac{a^2 dB/dt}{\frac{\rho(40)}{\pi r^2}}$$

14. Official Ans. by NTA (3)

Sol. When bar magnet is entering with constant speed, flux will change and an e.m.f. is induced, so galvanometer will deflect in positive direction.

When magnet is completely inside, flux will not change, so reading of galvanometer will be zero.

When bar magnet is making on exit, again flux will change and on e.m.f. is induced in opposite direction to not of (a), so galvanometer will deflect in negative direction.

Looking at options, option (3) is correct.

15. Official Ans. by NTA (3)

Sol. $U_{max} = \frac{1}{2}LI_{max}^2$

$$i = I_{max}\left(1 - e^{-Rt/L}\right)$$

For U to be $\frac{U_{max}}{n}$; i has to be $\frac{I_{max}}{\sqrt{n}}$

$$\frac{I_{max}}{\sqrt{n}} = I_{max}(1 - e^{-Rt/L})$$

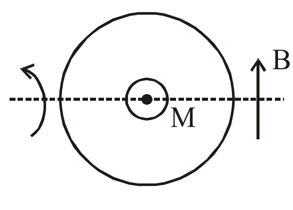
$$e^{-Rt/L} = 1 - \frac{1}{\sqrt{n}} = \frac{\sqrt{n}-1}{\sqrt{n}}$$

$$-\frac{Rt}{L} = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

$$t = \frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$$

16. Official Ans. by NTA (1)
Official Ans. by ALLEN (BONUS)

Sol. I_{dia} = 0.8 kg/m²
M = 20 Am²



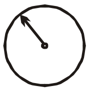
$U_i + K_i = U_f + K_f$

$$0 + 0 = -MB\cos 30° + \frac{1}{2}I\omega^2$$

$$20 \times 4 \times \frac{\sqrt{3}}{2} = \frac{1}{2}(0.8)\,\omega^2$$

$$\omega = \sqrt{100\sqrt{3}} = 10(3)^{1/4}$$

17. Official Ans. by NTA (5.00)

Sol. 

$$B = \frac{\mu_0 NI}{2R}$$

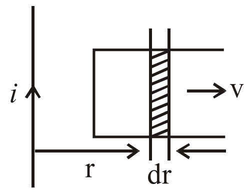
$$\phi = \frac{\mu_0 NN'I}{2R}\pi r^2$$

$$\varepsilon = \frac{d\phi}{dt} = \frac{2\pi\times10^{-7}\times10^{5}\times\pi\times10^{-4}}{0.2}$$

$= 8 \times 10^{-4} = 0.8$ mV

18. Official Ans. by NTA (2)

Sol. $B = \dfrac{\mu_0 i}{2\pi r}$



$$\phi = \frac{\mu_0 i}{2\pi r}\ell dr$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{\mu_0 i\ell}{2\pi r}\cdot\frac{dr}{dt}$$

$$\Rightarrow e = \frac{\mu_0}{2\pi}\cdot\frac{iv\ell}{r}$$

$$i = \frac{e}{R} = \frac{\mu_0}{2\pi}\cdot\frac{iv\ell}{Rr}$$

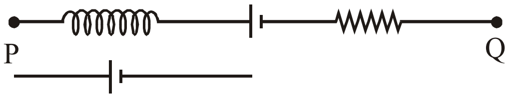
19. Official Ans. by NTA (2)

Sol. $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{100}\sqrt{\dfrac{80\times10^{-3}}{2\times10^{-6}}}$

$$= \frac{1}{100}\sqrt{40\times10^{3}}$$

$$= \frac{200}{100} = 2$$

20. Official Ans. by NTA (33.00)

Sol. 

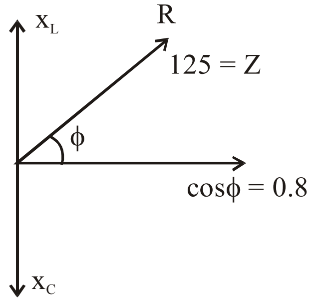
$$\frac{Ldi}{dt} = 5$$

$V_P - 5 - 30 + 2 \times 1 = VQ$

$V_P - V_Q = 33$ volt

Ans. 33.00

21. Official Ans. by NTA (400.00)

Sol. 

$$P = \frac{E_{rms}^2}{Z}\cos\phi$$

$$400 = \frac{(250)^2\times0.8}{Z}$$

$Z = 25 \times 5 = 125$

$X_L = 125 \sin\phi = 125 \times 0.6 = 75$

ERROR & MEASURMENT

1. NTA Ans. (3)

Sol. $T = 2\pi\sqrt{\dfrac{\ell}{g}}$

$$g = \frac{4\pi^2\ell}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$$

$$= \frac{0.1}{25} + \frac{2\times1}{50}$$

$$\frac{\Delta g}{g} = 4.4\%$$

2. NTA Ans. (2)

Sol. Given on six rotation, reading of main scale changes by 3mm.

$\therefore$ 1 rotation corresponds to $\dfrac{1}{2}$mm

Also no. of division on circular scale = 50.

$\therefore$ Least count of the screw gauge will be $\dfrac{0.5}{50}$mm = 0.001 cm.

3. NTA Ans. (BONUS)

Sol. $A_1 + B_1 + C_1 = 24.36 + 0.0724 + 256.2$
= 280.6324
= 280.6 (After rounding off)
$A_2 + B_2 + C_2 = 24.44 + 16.082 + 240.2$
= 280.722
= 280.7 (After rounding off)
$A_3 + B_3 + C_3 = 25.2 + 19.2812 + 236.183$
= 280.6642
= 280.7 (After rounding off)
$A_4 + B_4 + C_4 = 25 + 236.191 + 19.5$
= 280.691
= 281 (After rounding off)
$A_4 + B_4 + C_4 > A_3 + B_3 + C_3 = A_2 + B_2 + C_2 > A_1 + B_1 + C_1$

No option is matching Question should be (BONUS)

Best possible option is (2)

$\therefore$ Correct answer (2)

4. Official Ans. by NTA (4)

Sol. Least count = 1 mm or 0.01 cm
Zero error = 0 + 0.01 × 7 = 0.07 cm
Reading = 3.1 + (0.01 × 4) – 0.07
= 3.1 + 0.04 – 0.07
= 3.1 – 0.03
= 3.07 cm

5. Official Ans. by NTA (4)

Sol. $LC = \frac{\text{pitch}}{\text{CSD}} = \frac{0.1\,\text{cm}}{50} = 0.002\,\text{cm}$

So any measurement will be integral Multiple of LC.
So ans. will be 2.124 cm

6. Official Ans. by NTA (2)

Sol. $\frac{\Delta Z}{Z} = \frac{2\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{3\Delta d}{d} = 14.5\%$

7. Official Ans. by NTA (2)

Sol. Least count of screw gauge

$= \frac{\text{Pitch}}{\text{no. of division on circular scale}}$

$= \frac{0.5}{50}\text{mm} = 1\times10^{-5}\,\text{m}$

= 10 μm
Zero error in positive
Ans. (2)

8. Official Ans. by NTA (1050.00)

Sol. $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$

$\rho = \frac{6}{\pi}M\ D^{-3}$

taking log

$\ell n\rho = \ell n\left(\frac{6}{\pi}\right) + \ell nM - 3\ell mD$

Differentiates

$\frac{d\rho}{\rho} = 0 + \frac{dM}{M} - 3\frac{d(D)}{D}$

for maximum error

$100\times\frac{d\rho}{\rho} = \frac{dM}{M}\times100 + \frac{3dD}{D}\times100$

= 6 + 3 × 1.5
= 10.5 %

$= \frac{1050}{100}\%$ so x = 1050.00

9. Official Ans. by NTA (3)

Sol. Use significant figures. Answer must be upto three significant figures.
Ans. (3)

FLUIDS

1. NTA Ans. (4)

Sol. $A_1v_1 = A_2v_2$

$\frac{v_{min}}{v_{max}} = \frac{A_{min}}{A_{max}} \Rightarrow \frac{v_{min}}{v_{max}} = \left(\frac{4.8}{6.4}\right)^2$

$\frac{v_{min}}{v_{max}} = \frac{9}{16}$

2. NTA Ans. (4)

Sol. In case of minimum density of liqued, sphere will be floating while completely submerged
So mg = B

$m = \int_0^R \rho(4\pi r^2 dr) = B$

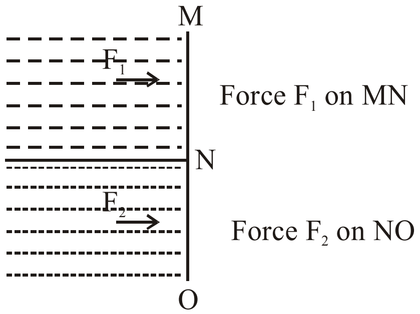
$= \rho_0\int_0^R\left(1-\frac{r^2}{R^2}\right)4\pi r^2dr = \frac{4}{3}\pi R^3\rho_\ell g$

On Solving

$\rho_\ell = \frac{2\rho_0}{5}$

3. NTA Ans. (1)

Sol.



$$F_1 = \frac{\rho g h}{2} \times A$$

$$F_2 = \left(\rho g h + \frac{2\rho g h}{2}\right)A$$

$$\frac{F_1}{F_2} = \frac{1}{4}$$

4. NTA Ans. (3)

Sol. Rate of flow of water = $A_A V_A = A_B V_B$

$(40)V_A = (20)V_B$

$V_B = 2V_A$ .... (1)

Using Bernoulli's theorem

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$700 = \frac{1}{2} \times 1000(4V_A^2 - V_A^2)$$

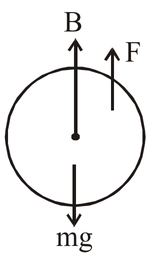
$V_A = 0.68$ m/s = 68 cm/s

Rate of flow = $A_A V_A$

= (40) (68) = 2720 cm^3/s

5. NTA Ans. (2)

Sol. FBD of droplet



B + F = mg

$$B = \left(\frac{2}{3}\pi R^3\right)\rho g$$

F = T(2πR)

$$m = d\left(\frac{4}{3}\pi R^3\right)$$

$$\left(\frac{2}{3}\pi R^3\right)\rho g + T(2\pi R) = d\left(\frac{4}{3}\pi R^3\right)g$$

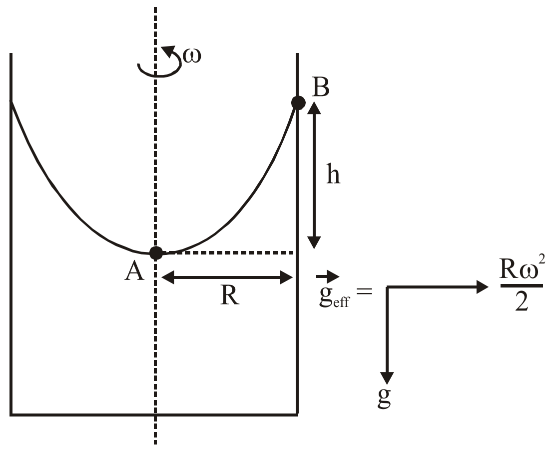
$$T(2\pi R) = \left(\frac{2}{3}\pi R^3\right)g[2d - \rho]$$

$$R = \sqrt{\frac{3T}{(2d-\rho)g}}$$

∴ Correct answer (2)

6. Official Ans. by NTA (1)

Sol.



Applying pressure equation from A to B

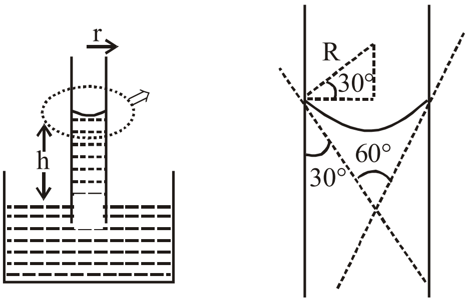
$$P_0 + \rho.\frac{R\omega^2}{2}.R - \rho g h = P_0$$

$$\frac{\rho R^2\omega^2}{2} = \rho g h$$

$$h = \frac{R^2\omega^2}{2g} = (5)^2\frac{\omega^2}{2g} = \frac{25}{2}\frac{\omega^2}{g}$$

7. Official Ans. by NTA (3)

Sol.



r → radius of capillary
R → Radius of meniscus.

From figure, $\frac{r}{R} = \cos 30°$

$$R = \frac{2r}{\sqrt{3}} = \frac{2 \times 0.15 \times 10^{-3}}{\sqrt{3}}$$

$$= \frac{0.3}{\sqrt{3}} \times 10^{-3} \text{m}$$

Height of capillary

$$h = \frac{2T}{\rho g R} = 2\sqrt{3}\ T$$

$$h = \frac{2 \times 0.05}{667 \times 10 \times \left(\frac{0.3 \times 10^{-3}}{\sqrt{3}}\right)}$$

h = 0.087 m

8. Official Ans. by NTA (1)

Sol. $\Delta P_1 = 0.01 = 4T/R_1$ .....(1)

$\Delta P_2 = 0.02 = 4T/R_2$ .....(2)

Equation (1) ÷ (2)

$$\frac{1}{2} = \frac{R_2}{R_1}$$

$R_1 = 2R_2$

$$\frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = 8$$

9. Official Ans. by NTA (101)

Sol. Capillary rise

$$h = \frac{2S\cos\theta}{\rho g r} \quad \Rightarrow S = \frac{\rho g r h}{2\cos\theta}$$

$$= \frac{(900)(10)(15 \times 10^{-5})(15 \times 10^{-2})}{2}$$

$S = 1012.5 \times 10^{-4}$

$S = 101.25 \times 10^{-3} = 101.25$ mN/m

In question closest integer is asked so closest integer = 101.00 Ans.

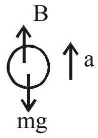
10. Official Ans. by NTA (3)

Sol. Volume $V = \frac{4\pi}{3} r^3 = \frac{4\pi}{3} \times (1)^3 = 4.19 \text{cm}^3$

$a = 9.8$ cm/s^2

B – mg = ma

$$m = \frac{B}{g + a}$$

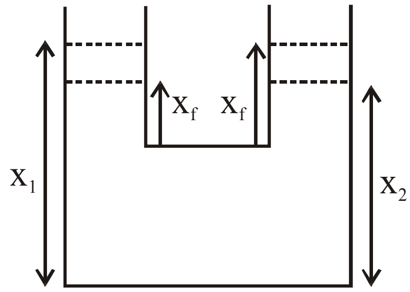


$$\Rightarrow m = \frac{(V\rho_\omega g)}{g + a} = \frac{V\rho_\omega}{1 + \frac{a}{g}}$$

$$= \frac{(4.19) \times 1}{1 + \frac{9.8}{980}} = \frac{4.19}{1.01} = 4.15 \text{gm}$$

11. Official Ans. by NTA (3)

Sol.



$$U_i = (\rho S x_1) g \cdot \frac{x_1}{2} + (\rho S x_2) g \cdot \frac{x_2}{2}$$

$$U_f = (\rho S x_f) g \cdot \frac{x_f}{2} \times 2$$

By volume conservation

$Sx_1 + Sx_2 = S(2x_f)$

$$x_f = \frac{x_1 + x_2}{2}$$

$$\Delta U = \rho S g \left[\left(\frac{x_1^2}{2} + \frac{x_2^2}{2}\right) - x_f^2\right]$$

$$= \rho S g \left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - \left(\frac{x_1 + x_2}{2}\right)^2\right]$$

$$= \frac{\rho S g}{2}\left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - x_1 x_2\right]$$

$$= \frac{\rho S g}{4}(x_1 - x_2)^2$$

12. Official Ans. by NTA (2)

Sol. $\frac{4}{3}\pi\left(R^3-r^3\right)\ \rho_m\ g = \frac{4}{3}\pi R^3\rho_w g$

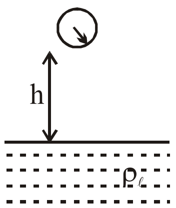
$$1-\left(\frac{r}{R}\right)^3=\frac{8}{27}$$

$$\Rightarrow \frac{r}{R}=\left(\frac{19}{27}\right)^{1/3}=\frac{19^{1/3}}{3}$$

$$= 0.88 \simeq \frac{8}{9}$$

13. Official Ans. by NTA (2)

Sol.



After falling through h, the velocity be equal to terminal velocity

$$\sqrt{2gh}=\frac{2}{9}\frac{r^2g}{\eta}(\rho_\ell-\rho)$$

$$\Rightarrow h = \frac{2}{81}\frac{r^4g\,(\rho_\ell-\rho)^2}{\eta^2}$$

$\Rightarrow h \propto r^4$

14. Official Ans. by NTA (2)

Sol. Applying Bernoulli's Equation

$$P_1+\frac{1}{2}\rho v_1^2+\rho g y_1 = P_2+\frac{1}{2}\rho v_2^2+\rho g y_2$$

$$P+\frac{1}{2}\rho v^2=\frac{P}{2}+\frac{1}{2}\rho V^2$$

$$\frac{2P}{2\rho}+\frac{1}{2}\frac{\rho v^2}{\rho}\times 2 = V^2$$

$$\sqrt{\frac{P}{\rho}+v^2}=V$$

Ans. (2)

GEOMETRICAL OPTICS

1. NTA Ans. (1)

Sol. $m=\frac{LD}{f_e\times f_0}=\frac{150\times 250}{f_e\times 25}=375$

f_e = 20 mm.

2. NTA Ans. (3)

Sol. Using $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{f}=\left(\frac{1.5}{1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ....(1)$$

and $\frac{1}{f_1}=\left(\frac{1.5}{1.42}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ ....(2)

equation (1)/(2),

we get $\frac{f_1}{f}=\frac{0.5}{0.056}$

$= 8.93 \approx 9$

3. NTA Ans. (4)

Sol. $L = f_0 + f_e = 60$ cm

$$M=\frac{f_0}{f_e}=5$$

$\Rightarrow f_0 = 5f_e$

$\therefore\ 6f_e = 60$ cm

$f_e = 10$ cm

4. NTA Ans. (4)

Sol. $\sin\theta_C=\frac{1}{\mu}=\frac{1}{\sqrt{3\times 4/3}}$

$\theta_C = 30°$

5. NTA Ans. (60.00)

Sol. Using Lens-Maker's formula :

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f}=(1.5-1)\left(\frac{1}{30}-0\right)$$

f = 60 cm

6. NTA Ans. (2)

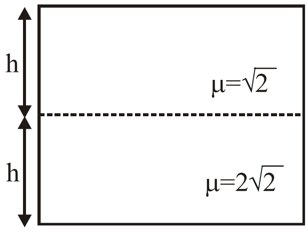
Sol. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

At focus m = ∞ x = f

At centre m = –1 x = 2f

7. NTA Ans. (2)

Sol.



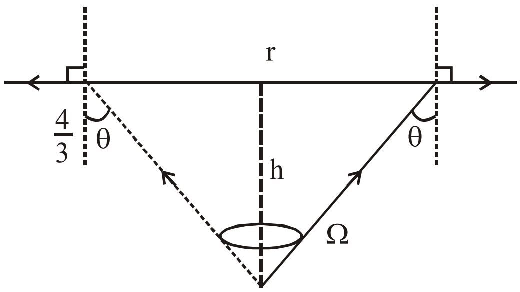
For near normal incidence,

$$h_{app} = \frac{h_{actual}}{\left(\frac{\mu_{in}}{\mu_{ref.}}\right)}$$

$$\therefore\ h_{apparent} = \frac{\frac{h}{\left(\frac{2\sqrt{2}}{\sqrt{2}}\right)} + h}{\frac{\sqrt{2}}{1}} = \frac{3h}{2\sqrt{2}} = \frac{3}{4}h\sqrt{2}$$

8. NTA Ans. (1)

Sol.



$$\frac{4}{3}\sin\theta = 1\sin 90°$$

$$\sin\theta = \frac{3}{4}$$

Area of sphere in which light spread = $4\pi R^2$

$\Omega = 2\pi\ (1 - \cos\ \theta)$

$$\Omega = 2\pi\left(1 - \frac{\sqrt{7}}{4}\right)$$

$P \to 4\pi$ steradians

$$P' \to \frac{P}{4\pi}(1 - \cos\theta)$$

$$\text{Ratio} = \frac{P'}{P} = \frac{2\pi(1-\cos\theta)}{4\pi} = \frac{(1-\cos\theta)}{2} = \frac{1 - \frac{\sqrt{7}}{4}}{2}$$

$$= \frac{0.33}{2} = 0.17$$

$\therefore$ Correct answer (1)

9. Official Ans. by NTA (1)
Official Ans. by ALLEN (4)

Sol. $f = \frac{-8}{2} = -4\text{cm}$

u = –10 cm

v = ?

as $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} + \left(\frac{1}{-10}\right) = \frac{1}{-4}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{4}$$

$$\frac{1}{v} = \frac{4-10}{40}$$

$$v = \frac{40}{-6}$$

$$v = \frac{-20}{3}$$

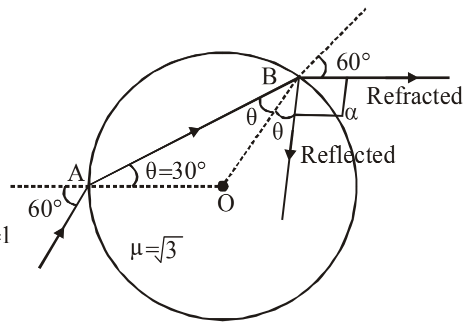
$$m = \frac{-v}{u}$$

$$m = \frac{-\left(-\frac{20}{3}\right)}{-10} \Rightarrow m = \frac{-2}{3}$$

or image will be real, inverted and unmagnified.

10. Official Ans. by NTA (90.00)

Sol.



By Snell's law at A :

$1 \times \sin 60° = \sqrt{3} \times \sin\theta$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin\theta$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

So at B :

$\theta + 60° + \alpha = 180°$

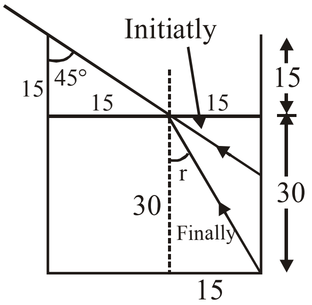
$30° + 60° + \alpha = 180°$

$\alpha = 90°$

11. Official Ans. by NTA (158)

Sol. $\tan r = \frac{15}{30} = \frac{1}{2}$

$\sin r = \frac{1}{\sqrt{5}}$



$1\sin 45° = \mu \sin r$

$\frac{1}{\sqrt{2}} = \mu\left(\frac{1}{\sqrt{5}}\right)$

$\boxed{\mu = \sqrt{\frac{5}{2}} = 1.581}$

$\frac{N}{100} = \mu$

N = 100 μ

N = 158.11

So integer value of N = 15800

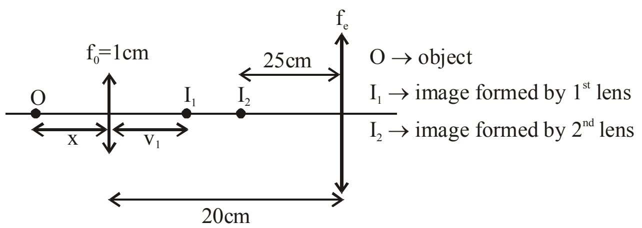
12. Official Ans. by NTA (1)

Sol. $\left|\left(\frac{dv}{dt}\right)\right| = \left|\frac{v^2}{4^2}\right|\left|\frac{du}{dt}\right|$

$= \left(\frac{10}{30}\right)2 \times 9 = 1 m/s$

13. Official Ans. by NTA (5)
Official Ans. by ALLEN (4.48)

Sol.



for first lens = $\frac{1}{v_1} - \frac{1}{-x} = \frac{1}{1} \Rightarrow v_1 = \frac{x}{x-1}$

also magnification $|m_1|$ = $\left|\frac{v_1}{u_1}\right| = \frac{1}{x-1}$

for 2nd lens this is acting as object

so $u_2 = -(20 - v_1) = -\left(20 - \frac{x}{x-1}\right)$

and $v_2 = -25$cm

angular magnification $|m_A| = \left|\frac{D}{u_2}\right| = \frac{25}{|u_2|}$

Total magnification $m = m_1 m_A = 100$

$$\left(\frac{1}{x-1}\right)\left(\frac{25}{20 - \frac{x}{x-1}}\right) = 100$$

$$\frac{25}{20(x-1)-x} = 100 \Rightarrow 1 = 80(x-1) - 4x$$

$\Rightarrow 76x = 81 \Rightarrow x = \frac{81}{76}$

$\Rightarrow u_2 = -\left(20 - \frac{81/76}{81/76 - 1}\right) = \frac{-19}{5}$

now by lens formula

$$\frac{1}{-25} - \frac{1}{-19/5} = \frac{1}{f_e} \Rightarrow f_e = \frac{25 \times 19}{106} \approx 4.48cm$$

14. Official Ans. by NTA (5)
Official Ans. by ALLEN (476)

Sol. Using displacement method

$f = \frac{D^2 - d^2}{4D}$

Here, D = 100 cm
d = 40 cm

$f = \frac{100^2 - 40^2}{4(100)} = 21$ cm

$P = \frac{1}{f} = \frac{100}{21}$ D $\qquad \frac{N}{100} = \frac{100}{21}$ N = 47

15. Official Ans. by NTA (4)

Sol. $v = \frac{uf}{u+f}$

Case-I

If v = u

⇒ f + u = f

⇒ u = 0

Case-II

If u = ∞

then v = f

Only option (4) satisfies this condition.

16. Official Ans. by NTA (50.00)

Sol. Final image at ∞

$\Rightarrow$ obj. for eye piece at 5cm

$\Rightarrow$ image for objective at 5 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad \Rightarrow \frac{1}{5}+\frac{1}{x}=1$$

$$\frac{1}{x}=1-\frac{1}{5}=\frac{4}{5} \Rightarrow x=\frac{5}{4}$$

17. Official Ans. by NTA (5.00)

Sol. $\delta_{min} = (\mu - 1)\,A$

$= (1.5 - 1)1$

$= 0.5$

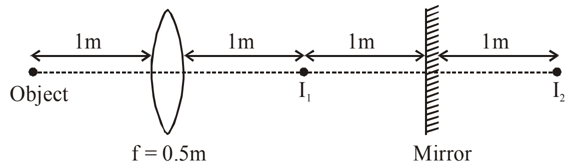
$$\delta_{min} = \frac{5}{10}$$

N = 5

18. Official Ans. by NTA (1,4)

Official Ans. by ALLEN (3)

Sol.



Object is at 2f. So image will also be at '2f'. (I_1).

Image of I_1 will be 1m behind mirror.

i.e. $\Rightarrow I_2$

Now I_2 will be object for lens.

$\therefore$ u $\Rightarrow$ – 3m

f $\Rightarrow$ +0.5 m

$$\frac{1}{v}\Rightarrow\frac{1}{f}+\frac{1}{u} \qquad \Rightarrow\frac{1}{+0.5}+\frac{1}{-3}$$

$$v\Rightarrow\frac{3}{5}\Rightarrow 0.6\text{m}$$

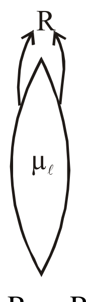
So total distance from mirror $\Rightarrow$ 2 + 0.6

$\Rightarrow$ 2.6 m and real image

Ans. (3)

19. Official Ans. by NTA (4)

Sol.



$R_1 = R_2 = R$

Power (P)

Refractive index is assume (μ_ℓ)

$$P=\frac{1}{f}=(\mu_\ell-1)\left(\frac{2}{R}\right) \qquad ....(i)$$

$$P'=\frac{1}{f'}=(\mu_\ell-1)\left(\frac{1}{R'}\right) \qquad ....(ii)$$

$$P'=\frac{3}{2}P$$

$$(\mu_\ell-1)\left(\frac{1}{R'}\right)=\mu\frac{3}{2}(\mu_\ell-1)\left(\frac{2}{R}\right)$$

$$\therefore\ R'=\frac{R}{3}$$

GRAVITATION

1. NTA Ans. (2)

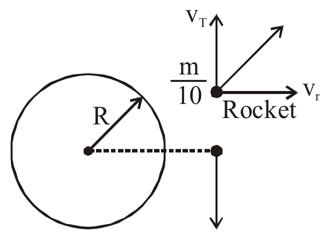
Sol. Applying energy conservation

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mu^2+\left(-\frac{GMm}{R}\right)=\frac{1}{2}mv^2-\frac{GMm}{2R}$$

$$v=\sqrt{u^2-\frac{GM}{R}} \qquad ....(i)$$

By momentum conservation, we have



$$\frac{m}{10}v_T=\frac{9m}{10}\sqrt{\frac{GM}{2R}} \qquad ....(ii)$$

$$\& \quad \frac{m}{10}v_r=mv$$

$$\Rightarrow \frac{m}{10}v_r=m\sqrt{u^2-\frac{GM}{R}} \qquad ....(iii)$$

Kinetic energy of rocket

$$=\frac{1}{2}m\left(v_T^2+v_r^2\right)$$

$$=\frac{m}{20}\left(81\frac{GM}{2R}+100u^2-100\frac{GM}{R}\right)$$

$$=\frac{m}{20}\left(100u^2-\frac{119GM}{2R}\right)$$

$$=5m\left(u^2-\frac{119GM}{200R}\right).$$

2. NTA Ans. (4)

Sol. Gravitational field on the surface of a solid sphere $I_g = \frac{GM}{R^2}$

By the graph

$$\frac{GM_1}{(1)^2} = 2 \quad \text{and} \quad \frac{GM_2}{(2)^2} = 3$$

On solving

$$\frac{M_1}{M_2} = \frac{1}{6}$$

3. NTA Ans. (16)

Sol. $U_1 + K_1 = U_2 + K_2$

$$-\frac{GM_e m}{10R} + \frac{1}{2}mv_0^2 = -\frac{GM_e m}{R} + \frac{1}{2}mv^2$$

$$+\frac{9}{10} \times \frac{GM_e m}{R} + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$$

$$\frac{9}{10} \times \frac{1}{2}M \times v_e^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$$

$$v^2 = \frac{9}{10}v_e^2 + v_0^2 \qquad \Rightarrow = \frac{9}{10} \times (11.2)^2 + (12)^2$$

$v^2 = 112.896 + 144$

$v = 16.027$

$v = 16$ km/s

4. NTA Ans. (1)

Sol. Initially, the body of mass m is moving in a circular orbit of radius R. So it must be moving with orbital speed.

$$v_0 = \sqrt{\frac{GM}{R}}$$

After collision, let the combined mass moves with speed v_1

$$mv_0 + \frac{m}{2}\frac{v_0}{2} = \left(\frac{3m}{2}\right)v_1 \qquad \Rightarrow v_1 = \frac{5v_0}{6}$$

Since after collision, the speed is not equal to orbital speed at that point. So motion cannot be circular. Since velocity will remain tangential, so it cannot fall vertically towards the planet.

Their speed after collision is less than escape speed $\sqrt{2}v_0$, so they cannot escape gravitational field.

So their motion will be elliptical around the planet.

5. NTA Ans. (1)

Sol. $V_e = \sqrt{\frac{2GM}{R}}$ (Escape velocity)

$$V_A = \sqrt{\frac{2GM}{R}}$$

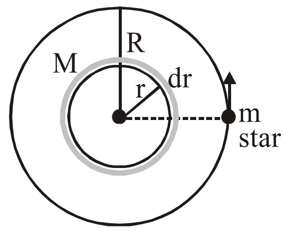
$$V_B = \sqrt{\frac{2G[M/2]}{R/2}} = \sqrt{\frac{2GM}{R}}$$

$$\frac{V_A}{V_B} = 1 = \frac{n}{4} \Rightarrow n = 4$$

$\therefore$ Correct answer (1)

6. Official Ans. by NTA (3)

Sol.



$dm = \rho dv$

$$dm = \left(\frac{k}{r}\right)(4\pi r^2 dr)$$

$dm = 4\pi k r dr$

$$M = \int_0^R dm = \int_0^R 4\pi k r dr$$

$$M = 4\pi k \left.\frac{r^2}{2}\right|_0^R$$

$M = 2\pi k(R^2 - 0)$

$M = 2\pi k R^2$

for circular motion gravitational force will provide required centripital force or

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\frac{G(2\pi kR^2)m}{R^2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{2\pi GkR}$$

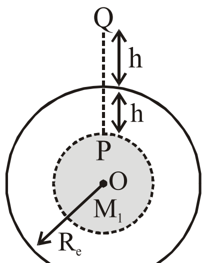
Time period $T = \frac{2\pi R}{v}$

$$T = \frac{2\pi R}{\sqrt{2\pi GkR}} \propto \sqrt{R}$$

or $T^2 \propto R$

7. Official Ans. by NTA (1)

Sol.



- M = mass of earth

M_1 = mass of shaded portion

R = Radius of earth

- $M_1 = \frac{M}{\frac{4}{3}\pi R^3}.\frac{4}{3}\pi(R-h)^3$

$= \frac{M(R-h)^3}{R}$

- Weight of body is same at P and Q

i.e. $mg_P = mg_Q$

$g_P = g_Q$

$\frac{GM_1}{(R-h)^2} = \frac{GM}{(R+h)^2}$

$\frac{GM(R-h)^3}{(R-h)^2 R^3} = \frac{GM}{(R+h)^2}$

$(R-h)(R+h)^2 = R^3$

$R^3 - hR^2 - h^2R - h^3 + 2R^2h - 2Rh^2 = R^3$

$R^2 - Rh^2 - h^3 = 0$

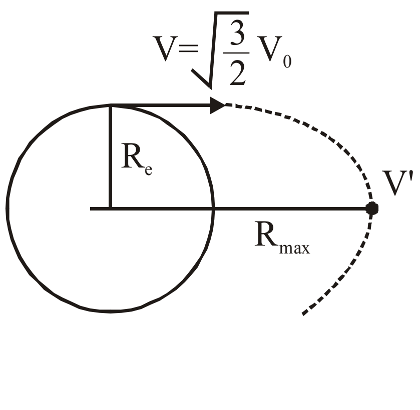
$R^2 - Rh - h^2 = 0$

$h^2 + Rh - R^2 = 0 \Rightarrow h = \frac{-R \pm \sqrt{R^2 + 4R^2}}{2}$

ie $h = \frac{-R + \sqrt{5}R}{2} = \left(\frac{\sqrt{5}-1}{2}\right)R$

8. Official Ans. by NTA (2)

Sol.



$V_0 = \sqrt{\frac{GM}{R_e}}$

$\frac{-GMm}{R_e} + \frac{1}{2}mv^2 = \frac{-GMm}{R_{max}} + \frac{1}{2}mv'^2$ ....(i)

$VR_e = V'R_{max}$ ....(ii)

Solving (i) & (ii)

$\boxed{R_{max} = 3R_e}$

9. Official Ans. by NTA (2)

Sol. $E\,4\pi r^2 = \int \rho_0 4\pi r^2 dr$

$\Rightarrow Er^2 = 4\pi G\int_0^r \rho_0\left(1-\frac{r^2}{R^2}\right)r^2 dr$

$\Rightarrow E = 4\pi G\rho_0\left(\frac{r^3}{3}-\frac{r^5}{5R^2}\right)$

$\frac{dE}{dr} = 0 \therefore r = \sqrt{\frac{5}{9}}R$

10. Official Ans. by NTA (1)

Sol. Given $E_G = \frac{Ax}{(x^2+a^2)^{3/2}}, V_\infty = 0$

$\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G.\vec{d}_x$

$V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2+a^2)^{3/2}}dx$

put $x^2 + a^2 = z$

$2x\,dx = dz$

$V_x - 0 = -\int_\infty^x \frac{A\,dz}{2(z)^{3/2}} = \left[\frac{A}{z^{1/2}}\right]_\infty^x = \left[\frac{A}{(x^2+a^2)^{1/2}}\right]_\infty^x$

$V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$

11. Official Ans. by NTA (3)

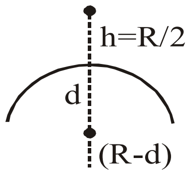
Sol. $V_{orbit} = \sqrt{\frac{GM}{R}}$

$V_{escape} = \sqrt{\frac{2GM}{R}}$

$\frac{V_{orbit}}{V_{escape}} = \frac{1}{\sqrt{2}}$

12. Official Ans. by NTA (4)

Sol.



$$g_1 = \frac{GM}{\left(R + \frac{R}{2}\right)^2} \quad \ldots (1)$$

$$g_2 = \frac{GM(R-d)}{R^3} \quad \ldots (2)$$

$g_1 = g_2$

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM(R-d)}{R^3}$$

$$\Rightarrow \frac{4}{9} = \frac{(R-d)}{R}$$

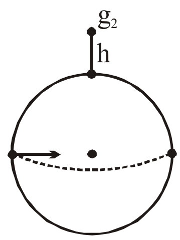
$4R = 9R - 9d$

$5R = 9d \Rightarrow \frac{d}{R} = \frac{5}{9}$

13. Official Ans. by NTA (4)

Sol. $g_e = g - R\omega^2$

$g_2 = g\left(1 - \frac{2h}{R}\right) \quad g_1 = ge$



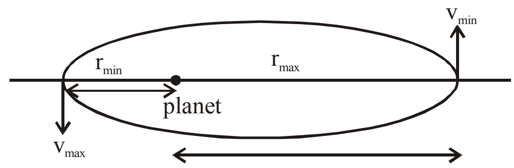
$g_2 = g - \frac{2gh}{R}$

Now $R\omega^2 = \frac{2gh}{R}$

$$h = \frac{R^2\omega^2}{2g}$$

14. Official Ans. by NTA (1)

Sol.



By angular momentum conservation

$r_{min}v_{max} = r_{max}v_{min} \quad \ldots (i)$

Given $v_{min} = \frac{v_{max}}{6}$

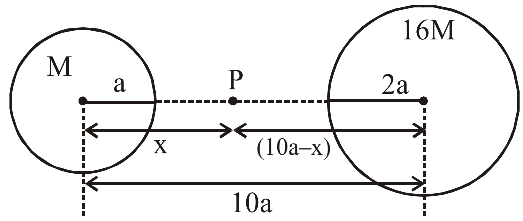
from equation (i)

$$\frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$$

Ans. (1)

15. Official Ans. by NTA (2)

Sol.



$$\frac{GM}{x^2} = \frac{G(16M)}{(10a-x)^2}$$

$\frac{1}{x} = \frac{4}{(10a-x)} \quad \Rightarrow 4x = 10a - x$

$x = 2a \quad \ldots.(i)$

COME

$$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE$$

$$= -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$$

$$KE = GMm\left[\frac{1}{8a} + \frac{16}{2a} - \frac{1}{2a} - \frac{16}{8a}\right]$$

$$KE = GMm\left[\frac{1+64-4-16}{8a}\right]$$

$$\frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right]$$

$$V = \sqrt{\frac{90GM}{8a}}$$

$$V = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

HEAT & THERMODYNAMICS

1. NTA Ans. (1)

Sol. $w = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{P_1V_1 - P_2V_2}{0.4}$

$= \frac{100 - \frac{100}{4.6555} \times 3}{0.4} = 88.90$

2. NTA Ans. (2)

Sol. $C_{Peq} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1 + n_2}$

$C_{Veq} = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$

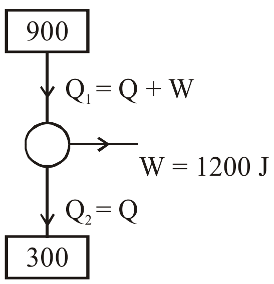
$\gamma_{eq} = \frac{C_{P_{eq}}}{C_{V_{eq}}} = \frac{2 \times \frac{5R}{2} + 3 \times \frac{8R}{2}}{2 \times \frac{3R}{2} + 3 \times \frac{6R}{2}}$

$= \frac{5+12}{3+9} = \frac{17}{12} \simeq 1.42$

Correct Answer : 2

3. NTA Ans. (600)

Sol.



for carnot engine

$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$\frac{Q + 1200}{Q} = \frac{900}{300}$

Q + 1200 = 3Q

Q = 600 J.

4. NTA Ans. (60)

Sol. $\gamma = \alpha_x + \alpha_y + \alpha_z$

$= 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$

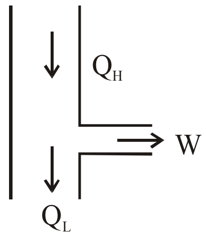
$= (50 + 5 + 5) \times 10^{-6}$

$\gamma = 60 \times 10^{-6}$

C = 60.

5. NTA Ans. (3)

Sol.



$\frac{Q_H}{Q_L} = \frac{T_1}{T}$ and $W = Q_H - Q_L$ ....(1)

$\frac{Q_L}{Q'_L} = \frac{T}{T_2}$ and $W = Q_L - Q'_L$ ....(2)

6. NTA Ans. (1)

Sol. $t \propto \frac{V}{\sqrt{T}}$ ....(1)

$TV^{\gamma-1}$ = constant ....(2)

$\therefore t \propto V^{\frac{\gamma+1}{2}}$

7. NTA Ans. (40)

Sol. $M \times 540 + M + 60 = 200 \times 80 + 200 \times 1 \times (40 - 0)$

$\Rightarrow M = 40$

8. NTA Ans. (4)

Sol. Mean free time = $\frac{\text{Mean free path}}{\text{Average speed}}$

$= \frac{\frac{1}{\sqrt{2}\pi D^2 n}}{\sqrt{\frac{8RT}{\pi M_w}}}$

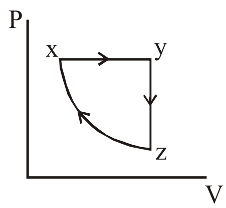
$t \propto \frac{1}{\sqrt{T}}$

9. NTA Ans. (4)

Sol. $x \to y \Rightarrow$ Isobaric

$y \to z \Rightarrow$ Isochoric

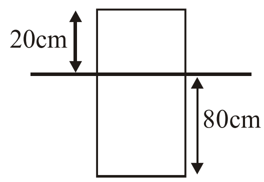
$z \to x \Rightarrow$ Isothermal



10. NTA Ans. (1)

Sol. $m = \rho_0 A$ (80) ....(i)

$m = \rho A$ (79) ....(ii)



11. NTA Ans. (3)

Sol. Refrigerator cycle is :

$$\eta = \frac{W}{Q_+} = \frac{W}{W + Q_-}$$

$$\frac{1}{10} = \frac{10}{10 + Q_-}$$

$Q_- = 90$ J

Heat absorbed from the reservoir at lower temperature is 90 J

12. NTA Ans. (2)

Sol. $$\frac{C_P}{C_V}\text{mix} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1C_{V_1} + n_2C_{V_2}}$$

$$\frac{C_P}{C_V}\text{mix} = \frac{n \times \left(\frac{5R}{2}\right) + 2n\left(\frac{7R}{2}\right)}{n \times \frac{3R}{2} + 2n\left(\frac{5R}{2}\right)}$$

$$\frac{C_P}{C_V} = \frac{19}{13}$$

13. NTA Ans. (50)

Sol. According to table and applying law of calorimetry

$1T_1 + 2T_2 = (1 + 2)60°$ ....(1)

$= 180$

$1T_2 + 2T_3 = (1 + 2)30°$ ....(2)

$= 90$

$2T_1 + 1T_3 = (1 + 2)60$

$= 180$ ....(3)

Adding (1) + (2) + (3)

$3(T_1 + T_2 + T_3) = 450$

$T_1 + T_2 + T_3 = 150°$

Hence,

$T_1 + T_2 + T_3 = (1 + 1 + 1)\theta$

$150 = 3\theta$

$\theta = 50°C$

14. NTA Ans. (2)

Sol. Degree of freedom of a diatomic molecule if vibration is absent = 5

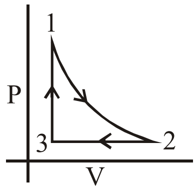
Degree of freedom of a diatomic molecule if vibration is present = 7

$$\therefore\ C_v^A = \frac{f_A}{2}R = \frac{5}{2}R \ \&\ C_v^B = \frac{f_B}{2}R = \frac{7}{2}R$$

$$\therefore\ \frac{C_v^A}{C_v^B} = \frac{5}{7}$$

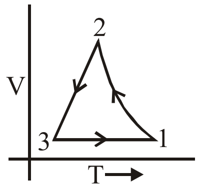
15. NTA Ans. (4)

Sol.



In process 2 to 3 pressure is constant & in process 3 to 1 volume is constant which is correct only in option 4.

Correct graph is



16. NTA Ans. (1)
ALLEN Ans. (3)

Sol. $$\lambda = \frac{1}{\sqrt{2}\pi n_v d^2}$$

$$\tau = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\pi n_v d^2 v} = \frac{1}{\sqrt{2}\pi n_v d^2}\sqrt{\frac{M}{3RT}}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{M_1}{M_2}}\frac{d_2^2}{d_1^2}$$

$$= \sqrt{\frac{40}{140}}\frac{(0.1)^2}{(0.07)^2}$$

$= 1.09$

$\therefore$ Nearest possible answer (3)

17. NTA Ans. (1816.00 to 1820)

Sol. PV^γ = constant

$TV^{\gamma - 1} = C$

$$300 \times V^{\frac{7}{5}-1} = T_2\left(\frac{V}{16}\right)^{\frac{7}{5}-1}$$

$$300 \times 2^{4\times\frac{2}{5}} = T_2$$

Isobaric process

$$V = \frac{nRT}{P}$$

$V_2 = kT_2$ ... (1)

$2V_2 = KT_f$ ... (2)

$$\frac{1}{2} = \frac{T_2}{T_f} \Rightarrow T_f = 2T_2$$

$T_f = 2 \times 300 \times 2^{\frac{8}{5}} = 1818.85$

$\therefore$ Correct answer 1819

18. Official Ans. by NTA (2)

Sol. $u = \frac{f_1 n_1 RT}{2} + \frac{f_2 n_2 RT}{2}$

$u = \frac{5}{2} \times 3RT + \frac{3 \times 5RT}{2} = 15RT$

19. Official Ans. by NTA (46)
Official Ans. by ALLEN (46 Actual 45.78)

Sol. Diatomic :

f = 5

$\gamma = 7/5$

$T_i = T = 273 + 20 = 293$ K

$V_i = V$

$V_f = V/10$

Adiabatic $TV^{\gamma-1}$ = constant

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$T.V^{7/5-1} = T_2 \left(\frac{V}{10}\right)^{7/5-1}$

$\Rightarrow T_2 = T.\ 10^{2/5}$

$\Delta U = \frac{nfR(T_2 - T_1)}{2} = \frac{5 \times 5 \times \frac{25}{3} \times (T.10^{2/5} - T)}{2}$

$= \frac{25 \times 25 \times}{6} T\ (10^{2/5} - 1)$

$= \frac{625 \times 293 \times (10^{2/5} - 1)}{6}$

$= 4.033 \times 10^3 \approx 4kJ$

20. Official Ans. by NTA (3)

Sol. $\eta = \frac{\text{Work done}}{\text{Heat supplied}}$

$\frac{1}{2} = \eta = \frac{1915 - 40 + 125 - Q}{1915 + 125}$

$\frac{1}{2} = \frac{2000 - Q}{2040}$ $\Rightarrow 2040 = 4000 - 2Q$

2Q = 1960

Q = 980 J

21. Official Ans. by NTA (4)

Sol. The mean free path of molecules of an ideal gas is given as:

$\lambda = \frac{V}{\sqrt{2}\pi d^2 N}$

where : V = Volume of container
N = No of molecules

Hence with increasing temp since volume of container does not change (closed container), so mean free path is unchanged.

Average collision time

$= \frac{\text{mean free path}}{V_{av}} = \frac{\lambda}{\text{(avg speed of molecules)}}$

$\because$ avg speed $\alpha \sqrt{T}$

$\therefore$ Avg coll. time $\alpha \frac{1}{\sqrt{T}}$

Hence with increase in temperature the average collision time decreases.

22. Official Ans. by NTA (2)

Sol. Given $\frac{\Delta L}{L} = 0.02\%$

$\therefore \Delta L = L\alpha\Delta T \Rightarrow \frac{\Delta L}{L} = \alpha\Delta T = 0.02\%$

$\therefore \beta = 2\alpha$ (Areal coefficient of expansion)

$\Rightarrow \beta\Delta T = 2\alpha\Delta T = 0.04\%$

Volume = Area × Length

$\text{Density}(\rho) = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Area} \times \text{Length}} = \frac{M}{AL}$

$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} - \frac{\Delta A}{A} - \frac{\Delta L}{L}$ (Mass remains constant) [$\frac{\Delta M}{M}$ struck through → 0]

$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right) = \frac{\Delta A}{A} + \frac{\Delta L}{L} = \beta\Delta T + \alpha\Delta T$

$= 0.04\% + 0.02\%$

$= 0.06\%$

23. Official Ans. by NTA (2)

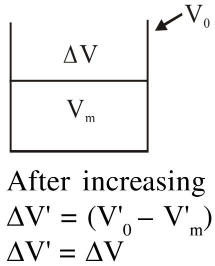
Sol. Bursting of helium balloon is irreversible & adiabatic.

24. Official Ans. by NTA (4)

Sol. DOF = 3 + 3 = 6

$U = \frac{f}{2} nRT = 3RT$

25. Official Ans. by NTA (20)

Sol. 

$\Rightarrow \boxed{\Delta V = (V_0 - V_m)}$

After increasing temperature

$\Delta V' = (V'_0 - V'_m)$

$\Delta V' = \Delta V$

$V_0 - V_m = V_0(1+\gamma_b\Delta T) - V_m\ (1+\gamma_M\Delta T)$

$V_0\gamma_b = V_m\gamma_m$

$V_m = \frac{V_0\gamma_b}{\gamma_m} = \frac{(500)(6 \times 10^{-6})}{(1.5 \times 10^{-4})}$

= 20 CC

26. Official Ans. by NTA (3)

Sol. $nC_p(50) = 160$

$nC_v\,(100) = 240$

$\Rightarrow \dfrac{C_p}{2C_v} = \dfrac{160}{240} = \dfrac{\gamma}{2}$

$\therefore\ \gamma = \dfrac{4}{3}$ and $f = \dfrac{2}{\gamma - 1} = 6$

27. Official Ans. by NTA (1)

Sol. $\dfrac{50-40}{300} = \beta\left(\dfrac{50+40}{2} - 20\right)$

$\dfrac{40-T}{300} = \beta\left(\dfrac{40+T}{2} - 20\right)$

$\therefore\ T = \dfrac{100}{3}$

28. Official Ans. by NTA (2)

Sol.

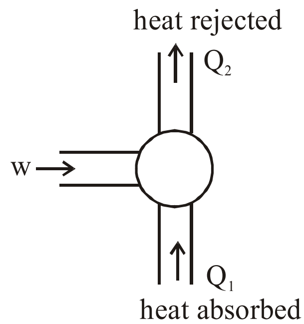
Cal	H_2O	Sterm
20gm	180gm	m
$25^\circ C$	$25^\circ C$	$100^\circ C$

$200 \times 1 \times (31 - 25)$

$= m \times 540 + m \times 1 \times (100 - 31)$

29. Official Ans. by NTA (8791)

Sol.



$w + Q_1 = Q_2$

$w = Q_2 - Q_1$

$C.O.P. = \dfrac{Q_1}{w} = \dfrac{Q_1}{Q_2 - Q_1} = \dfrac{273}{300-273} = \dfrac{Q_1}{W}$

$w = \dfrac{27}{273} \times 80 \times 100 \times 4.2$

$Q_2 = w + \theta_1$

$Q_2 = \dfrac{27}{273} \times 80 \times 100 \times 4.2 + 80 \times 100 \times 4.2$

$Q_2 = \dfrac{300}{273} \times 80 \times 100 = 8791.2$ cal

30. Official Ans. by NTA (1)

Sol. $\gamma = \dfrac{C_p}{C_v} = 1 + \dfrac{2}{f}$

where 'f' is degree of freedom

(A) Monoatomic f = 3, $\gamma = 1 + \dfrac{2}{3} = \dfrac{5}{3}$

(B) Diatomic rigid molecules,

$f = 5, \gamma = 1 + \dfrac{2}{3} = \dfrac{7}{5}$

(C) Diatomic non-rigid molecules

$f = 7, \gamma = 1 + \dfrac{2}{7} = \dfrac{9}{7}$

(D) Triatomic rigid molecules

$f = 6, \gamma = 1 + \dfrac{2}{6} = \dfrac{4}{3}$

31. Official Ans. by NTA (4)

Sol. $\because\ \dfrac{d\theta}{dt} = kA\dfrac{dT}{dx}$

$k = \dfrac{\left(\dfrac{d\theta}{dt}\right)}{A\left(\dfrac{dT}{dx}\right)}$

$[k] = \dfrac{[ML^2T^{-3}]}{[L^2][KL^{-1}]} = [MLT^{-3}K^{-1}]$

32. Official Ans. by NTA (1)

Sol. Here the water will provide heat for ice to melt therefore

$m_w\, s_w\, \Delta\theta = m_{ice}\, L_{ice}$

$m_{ice} = \dfrac{0.2 \times 4200 \times 25}{3.4 \times 10^5}$

$= 0.0617$ kg

$= 61.7$ gm

Remaining ice will remain un-melted

so correct answer is 1

33. Official Ans. by NTA (266)

Official Ans. by ALLEN (266.67)

Sol. As work done on gas and heat supplied to the gas are zero,

total internal energy of gases remain same

$u_1 + u_2 = u_1' + u_2'$

$(0.1)\ C_v\ (200) + (0.05)\ C_v\ (400) = (0.15) C_v\, T$

$T = \dfrac{800}{3} k = 266.67\, k$

34. Official Ans. by NTA (1)

Sol. (I) Adiabatic process $\Rightarrow \Delta Q = 0$

No exchange of heat takes place with surroundings

(II) Isothermal proess $\Rightarrow$ Temperature remains constant $(\Delta T = 0)$

$$\Delta u = \frac{F}{2} nR\Delta T \Rightarrow \Delta u = 0$$

No change in internal energy $[\Delta u = 0]$

(III) Isochoric process Volume remains constant

$$\Delta V = 0$$

$$W = \int P.dV = 0$$

Hence work done is zero.

(IV) Isobaric process $\Rightarrow$ Pressure remains constant

$$W = P.\ \Delta V \neq 0$$

$$\Delta u = \frac{F}{2} nR\Delta T = \frac{F}{2}[P\Delta V] \neq 0$$

$$\Delta Q = nC_P\Delta T \neq 0$$

35. Official Ans. by NTA (150)

Sol. PV = nRT

$P\Delta V + V\Delta P = 0$ (for constant temp.)

$P\Delta V = nR\Delta T$ (for constant pressure)

$$\Delta T = \frac{P\Delta V}{nR}$$

$$\Delta P = -\frac{P\Delta V}{V} \quad (\Delta V \text{ is same in both cases})$$

$$\frac{\Delta T}{\Delta P} = \frac{P\Delta V}{nR}\frac{V}{-P\Delta V} = \frac{-V}{nR} = -\frac{T}{P}$$

(PV = nRT)

$$\left(\frac{V}{nR} = \frac{T}{P}\right) \qquad \left|\frac{\Delta T}{\Delta P}\right| = \left|\frac{-300}{2}\right| = 150$$

36. Official Ans. by NTA (1)

Sol. $\Delta U = nC_v\ \Delta T$ = same

AB $\rightarrow$ volume is increasing $\Rightarrow W > 0$

AD $\rightarrow$ volume is decreasing $\Rightarrow W < 0$

AC $\rightarrow$ volume is constant $\Rightarrow W = 0$

37. Official Ans. by NTA (2)

Sol. $$\frac{1}{2}mv^2 \times \frac{1}{2} = ms\Delta T$$

$$\Delta T = \frac{v^2}{4\times5} = \frac{210^2}{4\times30\times4.200}$$

= 87.5°C

38. Official Ans. by NTA (3)

Sol. $$n = \frac{PV}{RT}, \frac{3}{2}kT = 4\times10^{-14}$$

$$N = \frac{PV}{RT}\times Na$$

$$= \frac{2\times13.6\times980\times4}{\frac{8}{3}\times10^{-14}} = 3.99 \times 10^{18}$$

39. Official Ans. by NTA (4)

Sol. In adiabatic process

PV^γ = constant

$$P\left(\frac{m}{\rho}\right)^\gamma = \text{constant}$$

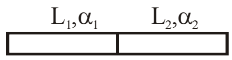
as mass is constant

$P \propto \rho^\gamma$

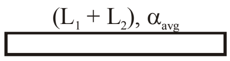
$$\frac{P_f}{P_i} = \left(\frac{\rho_f}{\rho_i}\right)^\gamma = (32)^{7/5} = 2^7 = 128$$

40. Official Ans. by NTA (4)

Sol. At T°C $\quad L = L_1 + L_2$



At T + ΔT $\quad L'_{eq} = L'_1 + L'_2$



where $L'_1 = L_1(1+\alpha_1\Delta T)$

$$L'_2 = L_2(1+\alpha_2\Delta T)$$

$$L'_{eq} = (L_1+L_2)\ (1+\alpha_{avg}\Delta T)$$

$\Rightarrow (L_1+L_2)(1+\alpha_{avg}\Delta T) = L_1 + L_2 + L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$

$\Rightarrow (L_1 + L_2)\ \alpha_{avg} = L_1\alpha_1 + L_2\alpha_2$

$$\Rightarrow \alpha_{avg} = \frac{L_1\alpha_1 + L_2\alpha_2}{L_1+L_2}$$

41. Official Ans. by NTA (41.00)

Official Ans. by ALLEN (40.93)

Sol. $$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{N_2} = V_{H_2}$$

$$\sqrt{\frac{3RT_{N_2}}{M_{N_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\frac{573}{28} = \frac{T_{H_2}}{2} \Rightarrow T_{H_2} = 40.928$$

42. Official Ans. by NTA (4)

Sol. Total degree of freedom = 3 + 2 = 5

$$U = \frac{nfRT}{2} \Rightarrow \frac{5RT}{2}$$

$$\gamma \Rightarrow \frac{C_P}{C_V} \Rightarrow 1 + \frac{2}{f} \Rightarrow 1 + \frac{2}{5} \Rightarrow \frac{7}{5}$$

Ans. (4)

43. Official Ans. by NTA (5.00)

Sol. PV = nRT

P_1V_1 = nR 250

$$P_2(2V_1) = \frac{5n}{4} R \times 2000$$

Divide

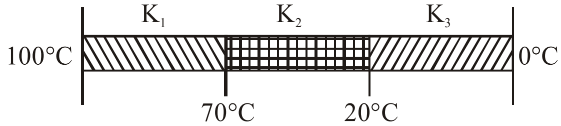
$$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000}$$

$$\frac{P_1}{P_2} = \frac{1}{5} \qquad \Rightarrow \frac{P_2}{P_1} = 5$$

Ans. 5.00

44. Official Ans. by NTA (1)

Sol.



Rods are identical have same length (ℓ) and area of cross-section (A)

Combination are in series, so heat current is same for all Rods

$$\left(\frac{\Delta Q}{\Delta t}\right)_{AB} = \left(\frac{\Delta Q}{\Delta t}\right)_{BC} = \left(\frac{\Delta Q}{\Delta t}\right)_{CD} = \text{Heat current}$$

$$\frac{(100-70)K_1A}{\ell} = \frac{(70-20)K_2A}{\ell} = \frac{(20-0)K_3A}{\ell}$$

$30K_1 = 50K_2 = 20K_3$

$3K_1 = 2K_3$

$$\frac{K_1}{K_3} = \frac{2}{3} = 2:3$$

$5K_2 = 2K_3$

$$\frac{K_2}{K_3} = \frac{2}{5} = 2:5$$

45. Official Ans. by NTA (3)

Sol. $v_{avg} \propto \sqrt{T}$

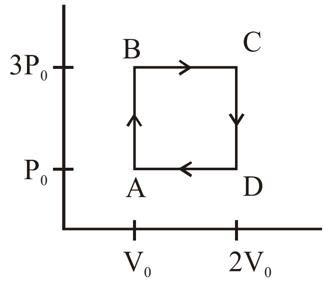
t_0 : mean time

λ : mean free path

$$t_0 = \frac{\lambda}{v_{avg}} \propto \frac{1}{\sqrt{T}}$$

46. Official Ans. by NTA (19.00)

Sol.



$W_{ABCDA} = 2P_0V_0$

$Q_{in} = Q_{AB} + Q_{BC}$

$Q_{AB} = nC(T_B - T_A)$

$$= \frac{n3R}{2}(T_B - T_A)$$

$$= \frac{3}{2}(P_BV_B - P_AV_A)$$

$$= \frac{3}{2}(3P_BV_0 = P_0V_0) = 3P_0V_0$$

$Q_{BC} = nC_P(T_C - T_B)$

$$= \frac{n5R}{2}(T_C - T_B)$$

$$= \frac{5}{2}(P_CV_C - P_BV_B)$$

$$= \frac{5}{2}(6P_0V_0 - 3P_0V_0) = \frac{15}{2}P_0V_0$$

$$\eta = \frac{W}{Q_{in}} \times 100 = \frac{2P_0V_0}{3P_0V_0 + \frac{15}{2}P_0V_0} \times 100$$

$$\eta = \frac{400}{21} = 19.04 \approx 19$$

$\eta = 19$

KINEMATICS

1. NTA Ans. (580.00)

Sol. $x = 10 + 8t - 3t^2$

$v_x = 8 - 6t$

$(v_x)_{t=1} = 2\hat{i}$

$y = 5 - 8t^3$

$v_y = -24t^2$

$(v_y)_{t=1} = -24\hat{j}$

Now

$$\sqrt{v} = \sqrt{(24)^2 + (2)^2} = \sqrt{580}$$

$\therefore$ v = 580 m^2/s^2

2. NTA Ans. (1)

Sol. $\vec{r}(t) = \cos\omega\hat{i} + \sin\omega t\,\hat{j}$

On diff. we get

$$\vec{v} = -\omega\sin\omega t\;\hat{i} + \omega\cos\omega t\;\hat{j}$$

$$\vec{a} = -\omega^2\vec{r}$$

$$\vec{v}\,.\,\vec{r} = 0$$

3. NTA Ans. (8 or 2888)

Sol. Time to travel 81 m is t sec.

Time to travel 100 m is $t + \frac{1}{2}$ sec.

$$81 = \frac{1}{2}\times a\times t^2 \qquad \Rightarrow t = 9\sqrt{\frac{2}{a}}$$

$$100 = \frac{1}{2}\times a\times\left(t+\frac{1}{2}\right)^2 \qquad \Rightarrow t + \frac{1}{2} = 10\sqrt{\frac{2}{a}}$$

$$9\sqrt{\frac{2}{a}} + \frac{1}{2} = 10\sqrt{\frac{2}{a}}$$

$$\frac{1}{2} = \sqrt{\frac{2}{a}}$$

$$\boxed{a = 8\ m/s^2}$$

4. NTA Ans. (3.00)

Sol. $x = \sqrt{at^2 + 2bt + c}$

Differentiating w.r.t. time

$$\frac{dx}{dt} = v = \frac{1}{2\sqrt{at^2+2bt+c}}\times(2at+2b)$$

$$\Rightarrow v = \frac{at+b}{x}$$

$\Rightarrow$ vx = at + b

Differentiating w.r.t. x

$$\Rightarrow \frac{dv}{dx}\times x + v = a\times\frac{dt}{dx}$$

Multiply both side by v

$$\Rightarrow \left(v\frac{dv}{dx}\right)x + v^2 = a$$

$\Rightarrow a'x = a - v^2$ [Here a' is acceleration]

$$\Rightarrow a'x = a - \left(\frac{at+b}{x}\right)^2$$

$$\Rightarrow a'x = \frac{ax^2 - (at+b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{a(at^2+2bt+c) - (at+b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{ac - b^2}{x^2}$$

$$\Rightarrow a' = \frac{ac-b^2}{x^3}$$

$\therefore a' \propto \frac{1}{x^3}$ $\quad\therefore$ n = 3

5. NTA Ans. (3)

Sol. $x = u_x t + \frac{1}{2}a_x t^2$

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$32 = 0\times t + \frac{1}{2}(4)(t)^2$$

$t^2 = 16$

t = 4 sec

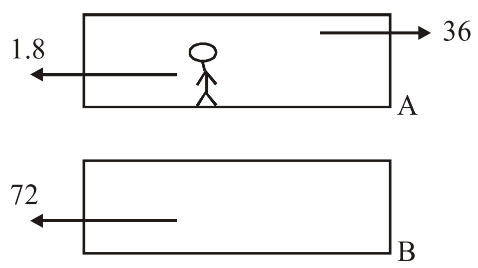
$$x = 3\times 4 + \frac{1}{2}\times 6\times 4^2$$

= 12 + 48 = 60 m

$\therefore$ Correct answer (3)

6. Official Ans. by NTA (2)

Sol.



Velocity of man with respect to ground

$\vec{V}_{m/g} = \vec{V}_{m/A} + \vec{V}_A$ = –1.8 + 36

Velocity of man w.r.t. B

$\vec{V}_{m/B} = \vec{V}_m - \vec{V}_B$

= –1.8 + 36 – (–72)

= 106.2 km/hr

= 29.5 m/s

7. Official Ans. by NTA (3)

Sol. Given $\vec{u} = 5\hat{j}\,m/s,\ \vec{a} = 10\hat{i} + 4\hat{j}$, final coordinate (20, y_0) in time t

$$S_x = 4_x t + \frac{1}{2}a_x t^2$$

$$20 - 0 = 0 + \frac{1}{2} \times 10 \times t^2$$

t = 2sec

$$S_y = u_y \times t + \frac{1}{2}a_y t^2$$

$$y_0 = 5 \times 2 + \frac{1}{2}4 \times 2^2 = 18m$$

2 sec and 18 m

8. Official Ans. by NTA (3)

Sol. Velocity at ground (means zero height) is non-zero therefore one is incorrect and velocity versus height is non-linear therefore two is also incorrect.

$v^2 = 2gh$

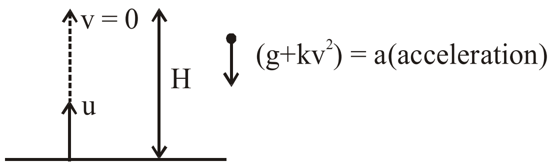
$$v\frac{dv}{dh} = 2g = \text{const.}$$

$$\frac{dv}{dh} = \frac{\text{constant}}{v}$$

Here we can see slope is very high when velocity is low therefore at Maximum height the slope should be very large which is in option 3 and as velocity increases slope must decrease there for option 3 is correct.

9. Official Ans. by NTA (2)

Sol.



$$\vec{F} = mkv^2 - mg$$

$$\vec{a} = \frac{\vec{F}}{m} = -\left[kv^2 + g\right]$$

$$\Rightarrow v.\frac{dv}{dh} = -[kv^2 + g]$$

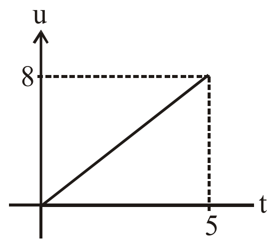
$$\Rightarrow \int_u^0 \frac{v.dv}{kv^2 + g} = -\int_0^H dh$$

$$\frac{1}{2K}\ln\left[kv^2 + g\right]_u^0 = -H$$

$$\Rightarrow \frac{1}{2K}\ln\left[\frac{ku^2 + g}{g}\right] = H$$

10. Official Ans. by NTA (20)

Sol.

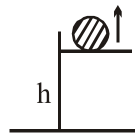


Distance = $\int v\,dt$

Area under graph = $\frac{1}{2} \times 5 \times 8$ = 20

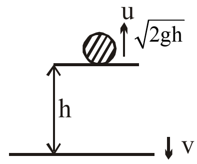
11. Official Ans. by NTA (3)

Sol.



$u^2 = 0 + 2gh$

$\Rightarrow u = \sqrt{2gh}$



$v^2 = u^2 + 2as$

$v^2 = 2gh + 2gh$

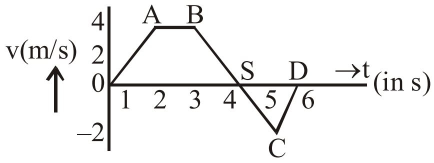
$v = \sqrt{4gh}$

$$\Rightarrow \sqrt{4gh} = \sqrt{2gh} + gt$$

$$\Rightarrow t = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}} \Rightarrow 3.4\sqrt{\frac{h}{g}}$$

12. Official Ans. by NTA (4)

Sol.



$OS = 4 + \frac{1}{3} = \frac{13}{3}$

$SD = 2 - \frac{1}{3} = \frac{5}{3}$

Area of OABS is A_1

Area of SCD is A_2

Distance = $|A_1| + |A_2|$

$A_1 = \frac{1}{2}\left[\frac{13}{3} + 1\right]4 = \frac{32}{3}$

$A_2 = \frac{1}{2} \times \frac{5}{3} \times 2 = \frac{5}{3}$

Distance = $|A_1| + |A_2|$

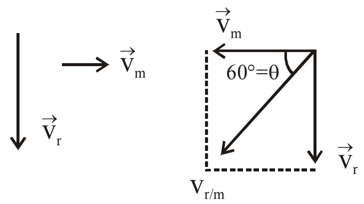
$= \frac{32}{3} + \frac{5}{3}$

$= \frac{37}{3}$

13. Official Ans. by NTA (4)

Sol. Rain is falling vertically downwards.

$\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$

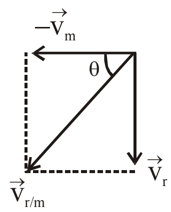


$\tan 60° = \frac{v_r}{v_m} = \sqrt{3}$

$v_r = v_m\sqrt{3} = v\sqrt{3}$

Now, $v_m = (1 + B)v$

and $\theta = 45°$



$\tan 45 = \frac{v_r}{v_m} = 1$

$v_r = v_m$

$v\sqrt{3} = (1 + \beta)v$

$\sqrt{3} = 1 + \beta$

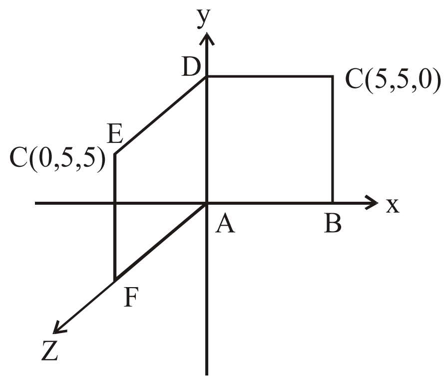
$\Rightarrow \beta = \sqrt{3} - 1 = 0.73$

MAGNETISM

1. NTA Ans. (1)

2. NTA Ans. (175)

Sol.



$\vec{A}_{ABCD} = 25\hat{k}$

$\vec{A}_{ADEF} = 25\hat{i}$

$\vec{A}_{net} = 25\hat{i} + 25\hat{k}$

$\vec{B} = 3\hat{i} + 4\hat{k}$

$\phi = \vec{B}.\vec{A}$

$= 25 \times 3 + 25 \times 4$

$\phi = 175\ W_{b.}$

3. NTA Ans. (3)

Sol. $(2V_0)^2 = \nu_0^2 + \upsilon_x^2$

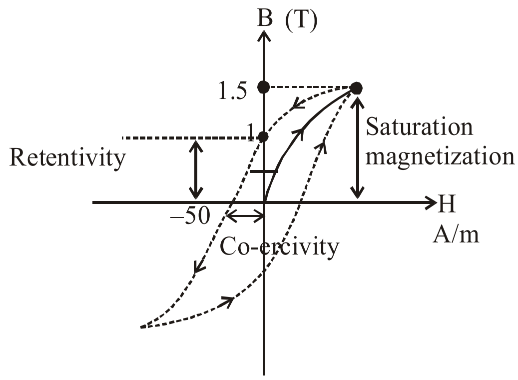
$\nu_x = \sqrt{3}\, v_0$

$\sqrt{3}\, v_0 = 0 + \frac{qE_0}{m} t$

$t = \frac{\sqrt{3}\nu_0 m}{qE_0}$

4. NTA Ans. (2)

Sol.



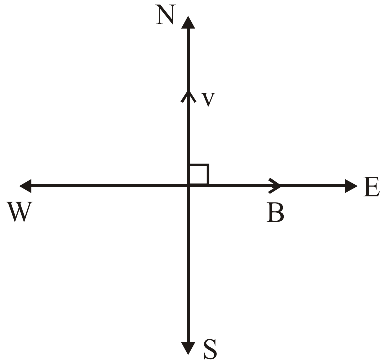
Retentivity = 1.0 T

Co-ercivity = 50 A/m

Saturation = 1.5 T

5. NTA Ans. (4)

Sol. $a = \frac{qvB}{m}$



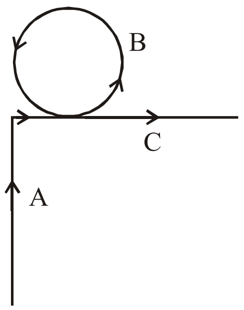
$B = \frac{ma}{qv} = \frac{ma\sqrt{m}}{\sqrt{2k}}$

$= \frac{m^{3/2}a}{e\sqrt{2k}} = \frac{(1.6\times10^{-27})^{3/2}\times10^{12}}{1.6\times10^{-19}\sqrt{2\times1\times10^{6}\times1.6\times10^{-19}}}$

$= 0.71$ mT

6. NTA Ans. (3)

Sol. We say we have 3 parts (A, B, C)



$B = B_A + B_B + B_C$

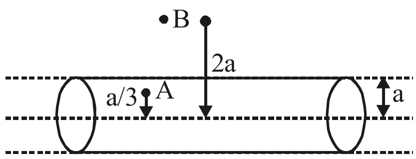
$= \frac{\mu_0 I}{4\pi R}(\sin 90° - \sin 45°)\otimes + \frac{\mu_0 I}{2R}\odot + \frac{\mu_0 I}{4\pi R}(\sin 45° + \sin 90°)\odot$

$= \frac{\mu_0 I}{2\pi R}(\sin 45° + \pi)$

$= \frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$

7. NTA Ans. (1)

Sol.



Let current density be J.

$\therefore$ Applying Ampere's law.

$\oint \vec{B}.d\vec{\ell} = \mu_0 i \Rightarrow B_A 2\pi\frac{a}{3} = \mu_0 J\pi\left(\frac{a}{3}\right)^2$

$\therefore\ B_A = \frac{\mu_0 Ja}{6}$

Similarly, $B_B = \frac{\mu_0 Ja}{4}$

$\therefore\ \frac{B_A}{B_B} = \frac{\mu_0 Ja \times 4}{\mu_0 J6a} = \frac{2}{3}$

8. NTA Ans. (2)

Sol. Option (A)

$W = k_f - k_i$

$qE(2a - 0) = \frac{1}{2}m(2V)^2 - \frac{1}{2}mV^2$

$qE2a = \frac{3}{2}mV^2 \qquad \Rightarrow E = \frac{3}{4}\frac{mv^2}{qa}$

Option (B)

Rate of work done $P = \vec{F}.\vec{V} = FV\cos\theta = FV$

Power = qEV

Power = $q\left(\frac{3}{4}\frac{mV^2}{qa}\right)V$

Power = $q\frac{3}{4}\frac{mV^3}{qa}$

Power = $\frac{3}{4}\frac{mV^3}{a}$

Option (C)

Angle between electric force and velocity is 90º, hence rate of work done will be zero at Q.

Option (D)

Initial angular momentum L_i = mVa

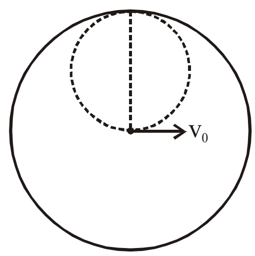
Final angular momentum L_f = m(2V) (2a)

Change in angular momentum $L_f - L_i$ = 3mVa

(Note : angular momentum is calculated about O)

9. NTA Ans. (2)

Sol. Top view of solenoid



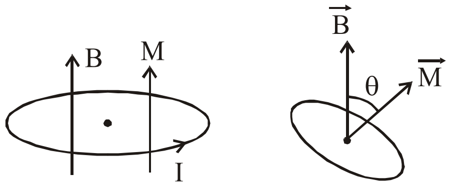
Maximum possible radius of electron = $\frac{R}{2}$

$\therefore \frac{R}{2} = \frac{mv}{qB} = \frac{mv_{max}}{e(\mu_0 ni)} \qquad \Rightarrow v_{max} = \frac{R}{2}\frac{e\mu_0 ni}{m}$

$\therefore$ Correct answer = 2

10. NTA Ans. (2)

Sol.



$\vec{T} = \vec{M} \times \vec{B} = -MB\sin\theta$

$I\alpha = -MB\sin\theta$

for small θ,

$\alpha = -\frac{MB}{I}\theta$

$\omega = \sqrt{\frac{MB}{I}} = \sqrt{\frac{(i)(\pi R^2)B}{\left(\frac{mR^2}{2}\right)}}$

$\omega = \sqrt{\frac{2i\pi B}{m}}$

$\therefore = T = \frac{2\pi}{\omega} = \sqrt{\frac{2\pi m}{iB}}$

∴ Correct answer (2)

11. Official Ans. by NTA (2)

Sol. Pitch = $\frac{2\pi m}{qB} v\cos\theta$

Pitch = $\frac{2(3.14)(1.67\times10^{-27})\times4\times10^5\times\cos 60}{(1.69\times10^{-19})(0.3)}$

Pitch = 0.04m = 4 cm

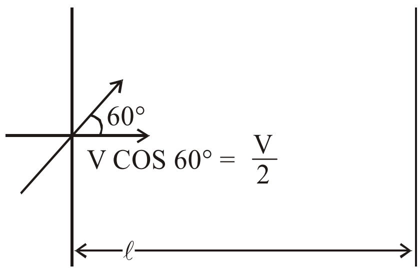
12. Official Ans. by NTA (4)

Sol. As for permanent magnet large retentivity and large coercivity required

13. Official Ans. by NTA (3)

Sol. $T = \frac{2\pi m}{qB}$

total time t = 10 T



Kinematics

$\ell = \frac{V}{2}t \qquad \Rightarrow \ell = \frac{V}{2} 10 \times \frac{2\pi m}{qB}$

$= 4\times10^5 \times 10 \times \frac{3.14\times1.67\times10^{-27}}{1.6\times10^{-19}\times0.3}$

= 0.439

14. Official Ans. by NTA (1)

Sol. M = NIA

N = 1

For ABCD

$\vec{M}_1 = abI\,\hat{K}.$

For DEFA

$\vec{M}_2 = abI\,\hat{j}$

$\vec{M} = \vec{M}_1 + \vec{M}_2$

$= ab\,I\,(\hat{k}+\hat{j}) \qquad \Rightarrow = ab\,I\,\sqrt{2}\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

15. Official Ans. by NTA (3)

Sol. $\vec{F} = 9(\vec{V}\times\vec{B})$ (Force on charge particle moving in magnetic field)

$\vec{V}\times\vec{B} = (2\hat{i}+3\hat{j}+4\hat{k})\times(5\hat{i}+3\hat{j}-6\hat{k})\times10^{-3}$

$= \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 3 & -6 \end{pmatrix} \times 10^{-3}$

$= \left[\hat{i}[-18-12] - \hat{j}[-12-20] + \hat{k}[6-15]\right]\times10^{-3}$

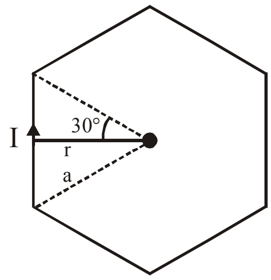
$= \left[\hat{i}[-30] + \hat{j}[32] + \hat{k}[-9]\right]\times10^{-3}$

$\text{Force} = 10^{-6}[-30\hat{i}+32\hat{j}-9\hat{k}]\times10^{-3}$

$= 10^{-9}[-30\hat{i}+32\hat{j}-9\hat{k}]$

16. Official Ans. by NTA (3)

Sol.



r = acos30°

$B = \frac{6\mu_0 I}{4\pi a\cos30^\circ}\times 2\sin30^\circ\times50$

$= \frac{\mu_0 I}{\pi}\frac{150}{\sqrt{3}a} = \frac{50\sqrt{3}}{0.1}\frac{\mu_0 I}{\pi}$

$= 500\sqrt{3}\frac{\mu_0 I}{\pi}$

17. Official Ans. by NTA (1)

Sol. A perfect diamagnetic substance will completely expel the magnetic field. Therefore, there will be no magnetic field inside the cavity of sphere. Hence the paramagnetic substance kept inside the cavity will experience no force.

18. Official Ans. by NTA (20)

Sol. $\vec{\tau} = \vec{m} \times \vec{B}$

$\tau = NI \times A \times B$

$105 = 500 \times 3\times10^{-4} \times \frac{1}{2} \times B$

B = 20

19. Official Ans. by NTA (3)

Sol. Given $i_A = 2$, $r_A = 2$ cm, $\theta_A = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

$i_B = 3$, $r_B = 4$ cm, $\theta_B = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$B = \frac{\mu_0 I\theta}{4\pi R} \quad \Rightarrow \frac{B_A}{B_B} = \frac{I_A}{I_B} \times \frac{\theta_A R_B}{\theta_B R_A} = \frac{6}{5}$

20. Official Ans. by NTA (4)

Sol. Torque on a bar magnet : I = MB sin θ

Here, θ = 30°, I = 0.018 N-m, B = 0.06 T

$\Rightarrow 0.018 = M \times 0.06 \times \sin 30°$

$\Rightarrow 0.018 = M \times 0.06 \times \frac{1}{2}$

$\Rightarrow M = 0.6$ A-m^2

Now v = – MB cos θ

Position of stable equilibrium (θ = 0°) : $u_i = -MB$

Position of unstable equilibrium (θ = 180°) : $u_f = MB$

⇒ work done : ΔU

⇒ W = 2MB

⇒ W = 2 × 0.6 × 0.06

⇒ W = 7.2×10^{-2} J

option (4) is correct

21. Official Ans. by NTA (2)

Sol. For paramagnetic material

According to curies law

$\chi \propto \frac{1}{T}$

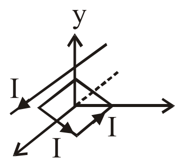
$\chi \propto \frac{1}{T} \quad \Rightarrow \quad \chi_1 T_1 = \chi_2 T_2$

$\Rightarrow \frac{6}{0.4} \times 4 = \frac{I}{0.3} \times 24$

$I = \frac{0.3}{0.4} = 0.75$ A / m

22. Official Ans. by NTA (1)

Sol.



$\vec{\tau} = \vec{M} \times \vec{B}$

$= 4a^2 I \times \frac{\mu_0 I}{2\pi b}$

23. Official Ans. by NTA (4)

Sol. $M = \mu_r NiA$

Here

μ_r = Relative permeability

N = No. of turns

i = Current

A = Aea of cross section

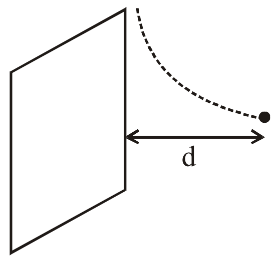
$M = \mu_r NiA = \mu_r n\ell iA$

$M = \mu_r niV = 1000(1000)\ 0.5\ (10^{-3})$

$= 500 = 5 \times 10^2$ Am^2

24. Official Ans. by NTA (2)

Sol.



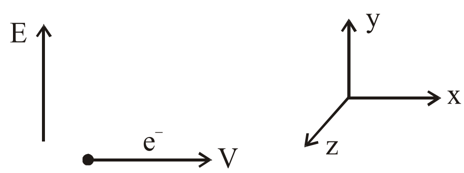
In uniform magnetic field particle moves in a circular path, if the radius of the circular path is 'd', particle will not hit the screen.

$d = \frac{mv}{qB_0} \qquad \Rightarrow v = \frac{qB_0 d}{m}$

∴ correct option is (2)

25. Official Ans. by NTA (1)

Sol.



$\vec{B}$ must be in +z axis.

$\vec{V} = 6 \times 10^6 \hat{i}$

$\vec{E} = 300\hat{j}$ V/cm = 3×10^4 V/m

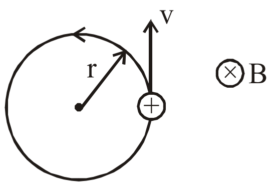
$q\vec{E} + q\vec{V} \times \vec{B} = 0$

E = VB

$B = \frac{E}{V} = \frac{3\times10^4}{6\times10^6} = 5\times10^{-3}$T

26. Official Ans. by NTA (4)

Sol.



Magnetic moment

M = iA

$$M=\left(\frac{q}{T}\right)\times\pi r^2=\frac{q\pi r^2}{\left(\frac{2\pi r}{v}\right)}=\frac{qvr}{2}$$

$$M=\frac{qv}{2}\times\frac{vm}{qB}$$

$$M=\frac{mv^2}{2B}$$

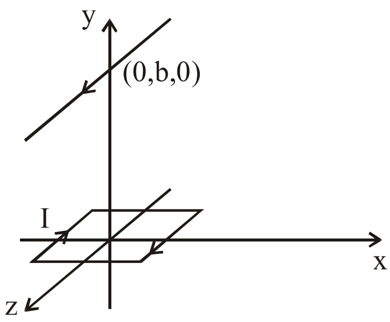
As we can see from the figure, direction of magnetic moment (M) is opposite to magnetic field.

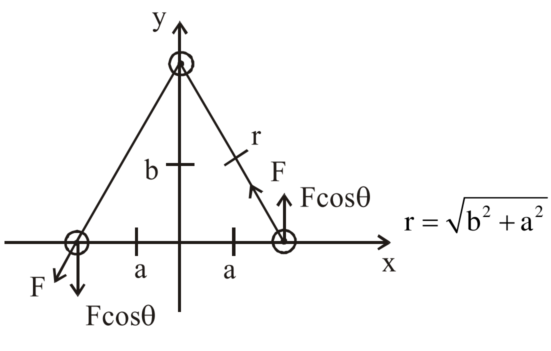
$$\vec{M}=-\frac{mv^2}{2B}\hat{B}$$

$$=-\frac{mv^2}{2B^2}\vec{B}$$

27. Official Ans. by NTA (1)

Sol.





$$F=BI2a=\frac{\mu_0 I}{2\pi r}I\times 2a$$

$$F=\frac{\mu_0 I^2 a}{\pi\sqrt{b^2+a^2}}$$

$$\tau=F\cos\theta\times 2a$$

$$=\frac{\mu_0 I^2 a}{\pi\sqrt{b^2+a^2}}\times\frac{b}{\sqrt{b^2+a^2}}\times 2a$$

$$\tau=\frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$$

If b >> a then $\tau=\frac{2\mu_0 I^2 a^2}{\pi b}$

But among the given options (1) is most appropriate

MODERN PHYSICS

1. NTA Ans. (3)

Sol. Time period of revolution of electron in n^{th} orbit

$$T=\frac{2\pi r}{V}=\frac{2\pi a_0\left(\frac{n^2}{Z}\right)}{V_0\left(\frac{Z}{n}\right)}$$

$$\Rightarrow T\propto\frac{n^3}{Z^2}$$

$$\frac{T_2}{T_1}=\frac{(2)^3}{(1)^3}=8\Rightarrow T_2=8\times 1.6\times 10^{-16}$$

Now frequency $f_2=\frac{1}{T_2}=\frac{10^{16}}{8\times 1.6}\approx 7.8\times 10^{14}$ Hz.

2. NTA Ans. (11)

Sol. Power incident P = I × A

n = no. of photons incident/second

$nE_{ph}=IA$

$$n=\frac{IA}{E_{ph}}$$

$$n=\frac{IA}{\left(\frac{hc}{\lambda}\right)}=\frac{6.4\times 10^{-5}\times 1}{\frac{1240}{310}\times 1.6\times 10^{-19}}$$

$n=10^{+14}$ per second

Since efficiency = 10^{-3}

no. of electrons emitted = 10^{+11} per second.

x = 11.

3. NTA Ans. (4)

Sol. $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$500 = 700 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$0.7 \approx \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$\left(\frac{1}{2}\right)^{1/2} \approx \frac{t}{T_{1/2}}$

$\frac{30}{T_{1/2}} \approx \frac{1}{2} \Rightarrow T_{1/2} = 60$

4. NTA Ans. (2)

Sol. $\frac{\lambda_{electron}}{\lambda_{photon}} = ?$

$E = \frac{hc}{\lambda_{photon}}$ ....(1)

$\lambda_{electron} = \frac{h}{\sqrt{2mE}}$ ....(2)

from (1) and (2)

$\frac{\lambda_{electron}}{\lambda_{photon}} = \frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

5. NTA Ans. (3)

Sol. $\lambda_B = 2\lambda_A$

$\Rightarrow \frac{h}{\sqrt{2T_B m}} = \frac{2h}{\sqrt{2T_A m}}$

$T_A = 4T_B$ ....(i)

and $T_B = (T_A - 1.5)$ eV ....(ii)

from (i) and (ii)

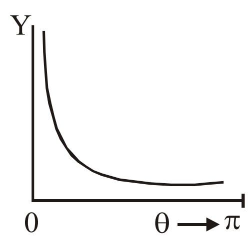
$3T_B$ 1.5 eV $\Rightarrow T_B = 0.5$ eV

$T_B = 0.5$ eV $= 4.5$ eV $-\phi_B$

$\phi = 4$eV

6. NTA Ans. (3)

Sol.



$Y \propto \frac{1}{\left(\sin\frac{\theta}{2}\right)^4}$

7. NTA Ans. (3)

Sol. By de-Broglie hypothesis

$\lambda = \frac{h}{mv}$

$\lambda_0 = \frac{h}{m\sqrt{2}v_0}$ ....(1)

$\lambda' = \frac{h}{\sqrt{v_0^2 + v_0^2 + \left(\frac{eE_0 t}{m}\right)^2}}$

$= \frac{h}{m\sqrt{2v_0^2 + \frac{e^2E_0^2t^2}{m^2}}}$ ....(2)

By (1) and (2)

$\lambda' = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{2m^2v_0^2}}}$

8. NTA Ans. (486)

Sol. For Balmer series,

$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$

$\frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{\left(\frac{1}{2^2} - \frac{1}{4^2}\right)}$

$\frac{\lambda_2}{6561} = \frac{5/36}{3/16}$

$\lambda_2 = \frac{20}{27} \times 6561$

$\lambda_2 = 4860$ Å $= 486$ nm

9. NTA Ans. (3)

Sol. Given, de-Broglie wavelength = $\frac{h}{\sqrt{2mE}} = \lambda$

Also, $\frac{h}{\sqrt{2m(E+\Delta E)}} = \frac{\lambda}{2}$

$\therefore \frac{E+\Delta E}{E} = 4 \Rightarrow \Delta E = 3E.$

10. NTA Ans. (2)

Sol. Let the work function be ϕ.

$\therefore \quad KE_{max} = \frac{hc}{\lambda} - \phi$

Again, $R_{max} = \frac{\sqrt{2mKE_{max}}}{qB} = \frac{\sqrt{2m\left(\frac{hc}{\lambda}-\phi\right)}}{qB}$

$\therefore \frac{R_{max}^2 q^2 B^2}{2m} = \frac{hc}{\lambda} - \phi$

$\therefore \phi = \frac{hc}{\lambda} - \frac{R_{max}^2 q^2 B^2}{2m}$ = 1.0899 eV $\approx$ 1.1eV

11. NTA Ans. (4)

Sol. 1 Rydberg energy = 13.6 eV

So, ionisation energy = (13.6 Z^2)eV

= 9 × 13.6eV

Z = 3

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 1.09 \times 10^7 \times 9 \times \frac{8}{9}$

λ = 11.4 nm

12. NTA Ans. (1)

Sol. $a = \frac{eE}{m}$

$v = u + at = \left(\frac{eE}{m}\right)t$

$\lambda = \frac{h}{mv}$

$\frac{d\lambda}{dt} = \frac{-(hm)\cdot\frac{dv}{dt}}{(mv)^2} = -\frac{ah}{mv^2} = -\frac{h}{|e|Et^2}$

$\therefore$ Correct answer (1)

13. Official Ans. by NTA (2)

Sol. Number of uranium atoms in 2kg

$= \frac{2 \times 6.023 \times 10^{26}}{235}$

energy from one atom is 200 × 10^6 e.v. hence total energy from 2 kg uranium

$= \frac{2 \times 6.023 \times 10^{26}}{235} \times 200 \times 10^6 \times 1.6 \times 10^{-19}$ J

2 kg uranium is used in 30 days hence this energy is recieved in 30 days hence energy recived per second or power is

$\text{Power} = \frac{2 \times 6.023 \times 10^{26} \times 200 \times 10^6 \times 1.6 \times 10^{-19}}{235 \times 30 \times 24 \times 3600}$

Power = 63.2 × 10^6 watt or 63.2 Mega Watt

14. Official Ans. by NTA (9)

Sol. $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV$ .....(i)

$\frac{hc}{3\lambda} = \frac{hc}{\lambda_0} + \frac{e \cdot V}{4}$ .....(ii)

(multiply by 4)

$\frac{4hc}{3\lambda} = \frac{4hc}{\lambda_0} + eV$ ....(iii)

From (i) & (iii)

$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{4hc}{3\lambda} - \frac{4hc}{\lambda_0}$

$-\frac{hc}{3\lambda} = -\frac{3hc}{\lambda_0}$

$\boxed{9\lambda = \lambda_0}$

n = 9

15. Official Ans. by NTA (2)

Sol. In hydrogen atom,

$E_n = \frac{-E_0}{n^2}$

Where E_0 is Ionisation Energy of H.

→ For transition from (n + 1) to n, the energy of emitted radiation is equal to the difference in energies of levels.

$$\Delta E = E_{n+1} - E_n$$

$$\Delta E = E_0\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$\Delta E = h\nu = E_0\left(\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right)$$

$$h\nu = E_0\left[\frac{2n+1}{n^4\left(1+\frac{1}{n}\right)^2}\right]$$

$$h\nu = E_0\left[\frac{n\left(2+\frac{1}{n}\right)}{n^4\left(1+\frac{1}{n}\right)^2}\right]$$

Since n >>> 1

Hence, $\frac{1}{n} \simeq 0$

$$h\nu = E_0\left[\frac{2}{n^3}\right]$$

$$\nu \propto \frac{1}{n^3}$$

16. Official Ans. by NTA (4)

Sol. Let mass of particle = m

Let speed of e^- = V

⇒ speed of particle = 5V

Debroglie wavelength $\lambda_d = \frac{h}{P} = \frac{h}{mv}$

$$\Rightarrow (\lambda_d)_P = \frac{h}{m(5V)} \quad .....(1)$$

$$\Rightarrow (\lambda_d)_e = \frac{h}{m_e.V} \quad .....(2)$$

According to question

$$\frac{(1)}{(2)} = \frac{m_e}{5m} = 1.878\times10^{-4}$$

$$\Rightarrow m = \frac{m_e}{5\times1.878\times10^{-4}}$$

$$\Rightarrow m = \frac{9.1\times10^{-31}}{5\times1.878\times10^{-4}}$$

$\Rightarrow m = 9.7 \times 10^{-28}$ kg

17. Official Ans. by NTA (1)

Sol. $\frac{3}{1} = \frac{\frac{hc}{200\text{ nm}} - \phi}{\frac{hc}{500\text{ nm}} - \phi}$, hc = 1240 eV-nm

On solving $\phi = 0.61$ eV

18. Official Ans. by NTA (1)

Sol. First order decay

$N(t) = N_0 e^{-\lambda t}$

Given $N(t) / N_0 = 9/16 = e^{-\lambda t}$

Now, $N(t/2) = N_0\, e^{-\lambda t/2}$

$$\frac{N(t/2)}{N_0} = \sqrt{e^{-\lambda t}} = \sqrt{9/16}$$

$N(t/2) = 3/4\ N_0$

19. Official Ans. by NTA (3)

Sol.

$$\rho_{nucleus} = \frac{mass}{volume} = \frac{A}{(4/3)\pi r_0^3 A} = \frac{3}{4\pi r_0^3} = 2.3\times10^{17}\,kg/m^3$$

20. Official Ans. by NTA (4)

Sol. $q\Delta V = \frac{1}{2}mV^2 \Rightarrow v = \sqrt{\frac{2q\Delta V}{m}}$

$$\therefore \frac{V_1}{V_2} = \sqrt{\frac{e}{m}\frac{4m}{e}} = 2$$

21. Official Ans. by NTA (1)

Sol. $P = \frac{nhc}{\lambda t}$

$$\therefore \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1}{5}$$

22. Official Ans. by NTA (1)

Sol. Graph of V_s and f given (B 5.5, 0)

$h\nu = \phi + eV_s$

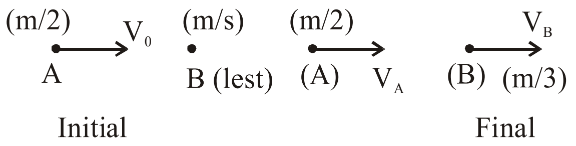
at B $V_s = 0$, $\nu = 5.5$

⇒ $h \times 5.5 \times 10^{14} = \phi$

$$\phi = \frac{6.62\times10^{-34}\times5.5\times10^{14}}{1.6\times10^{-19}}eV = 2.27\,eV$$

23. Official Ans. by NTA (1)

Sol.



Applying momentum conservation

$$\frac{m}{2}\times V_0+\frac{m}{3}\times(0)=\frac{m}{2}V_A+\frac{m}{3}V_B$$

$$=\frac{V_0}{2}=\frac{V_A}{2}+\frac{V_B}{3} \quad .... (1)$$

Since, collision is elastic (e = 1)

$$e=1=\frac{V_B-V_A}{V_0} \Rightarrow V_0=V_B-V_A \quad ... (2)$$

On solving (1) & (2) : $V_A=\frac{V_0}{5}$

Now, De-Broglie wavelength of A before collision :

$$\lambda_0=\frac{h}{m_A V_0}=\frac{h}{\left(\frac{m}{2}\right)V_0}$$

$$\Rightarrow \lambda_0=\frac{2h}{mV_0}$$

Final De-Broglie wavelength :

$$\lambda_f=\frac{h}{m_A V_0}=\frac{h}{\frac{m}{2}\times\frac{V_0}{5}} \Rightarrow \lambda_f=\frac{10h}{mV_0}$$

Now $\Delta\lambda=\lambda_f-\lambda_0$

$$\Delta\lambda=\frac{10h}{mV_0}-\frac{2h}{mV_0}$$

$$\Rightarrow \Delta\lambda=\frac{8h}{mv_0} \Rightarrow \Delta\lambda=4\times\frac{2h}{mv_0}$$

$$\Rightarrow \Delta\lambda=4\lambda_0$$

option (1) is correct.

24. Official Ans. by NTA (10553)
Official Ans. by ALLEN (10553.14)

Sol. $\lambda=\frac{c}{\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}$

for lyman series

$$\lambda_1=\frac{c}{\frac{1}{1^2}-\frac{1}{\infty^2}}=c\ (n=\infty \text{ to } n=1)$$

$$\lambda_2=\frac{c}{\frac{1}{1^2}-\frac{1}{2^2}}=\frac{4c}{3}\ (n=2 \text{ to } n=1)$$

$$\Delta\lambda=\lambda_2-\lambda_1=\frac{c}{3}=304\text{Å}\Rightarrow c=912\text{Å}$$

for paschen series

$$\lambda_1=\frac{c}{\frac{1}{3^2}-\frac{1}{\infty^2}}=9c\ (n=\infty \text{ to } n=3)$$

$$\lambda_2=\frac{c}{\frac{1}{3^2}-\frac{1}{4^2}}=\frac{144c}{7}\ (n=4 \text{ to } n=3)$$

$$\Delta\lambda=\lambda_2-\lambda_1=\frac{144c}{7}-9c=\frac{81c}{7}=\frac{81\times912}{7}$$

= 10553.14 Å

25. Official Ans. by NTA (3)

Sol. $eV=\frac{hc}{\lambda}-\phi$

$$V=\left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right)-\phi$$

Slope of the line in above equation and all other terms are independent of intensity.
The graph does not change.

26. Official Ans. by NTA (3)

Sol. $R=R_0\, e^{-\lambda t}$

$\ln R=\ln R_0-\lambda t$

$$\lambda_A=\frac{6}{10}\Rightarrow T_A=\frac{10}{6}\ln 2$$

$$\lambda_B=\frac{6}{5}\Rightarrow T_B=\frac{5\ln 2}{6}$$

$$\lambda_C=\frac{2}{5}\Rightarrow T_C=\frac{5\ln 2}{2}$$

$$\frac{10}{6}:\frac{5}{6}:\frac{15}{6}::2:1:3$$

27. Official Ans. by NTA (51.00)

Sol. $mV_0 = MV = p$

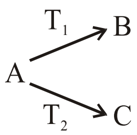
$$10.2 = \frac{p^2}{2m} - \frac{p^2}{2M} = \frac{p^2}{2m}\left(1 - \frac{m}{M}\right)$$

$$= \frac{p^2}{2m}(1 - 0.2)$$

$$\Rightarrow \frac{p^2}{2m} = K = \frac{10.2}{0.8}$$

28. Official Ans. by NTA (1)

Sol.



$$\frac{1}{T_{eff}} = \frac{1}{T_1} + \frac{1}{T_2}$$

$$T_{eff} = \frac{T_1T_2}{T_1 + T_2} = \frac{1000}{110} = \frac{100}{11} = 9.09$$

$$T_{eff} \cong 9$$

29. Official Ans. by NTA (2.00)

Sol. $E_1 = \phi + K_1$ ...(1)

$E_2 = \phi + K_2$ ...(2)

$E_1 - E_2 = K_1 - K_2$

Now $\frac{V_1}{V_2} = 2 \quad \Rightarrow \frac{K_1}{K_2} = 4$

$K_1 = 4K_2$

Now from equation (2)

$\Rightarrow 4–2.5 = 4K_2 - K_2$

$1.5 = 3K_2$

$K_2 = 0.5eV$

Now putting This

Value in equation (2)

$2.5 = \phi + 0.5eV$

$$\boxed{\phi = 2ev}$$

30. Official Ans. by NTA (4)

Sol. $$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2m(KE)}}$$

$$\lambda \propto \frac{1}{\sqrt{m}} \Rightarrow \lambda = \frac{C}{\sqrt{m}}$$

$$m_{He^{++}} > m_P > m_e$$

$$\therefore \lambda_{He^{++}} < \lambda_P < \lambda_e$$

$\therefore$ correct option is (4)

31. Official Ans. by NTA (2)

Sol. ${}^7_3Li + {}^1_1H \rightarrow 2\left({}^4_2He\right)$

$\Delta m \Rightarrow [m_{Li} + m_H] - 2[M_{He}]$

Energy released in 1 reaction $\Rightarrow \Delta mc^2$.

In use of 7.016 u Li energy is Δmc^2

In use of 1gm Li energy is $\frac{\Delta mc^2}{m_{Li}}$

In use of 20 gm energy is $\Rightarrow \frac{\Delta mc^2}{m_{Li}} \times 20gm$

$$\Rightarrow \frac{[(7.016 + 1.0079) - 2 \times 4.0026]u \times c^2}{7.016 \times 1.6 \times 10^{-24} gm} \times 20gm$$

$$\Rightarrow \left(\frac{0.0187 \times 1.6 \times 10^{-19} \times 10^9}{7.016 \times 1.6 \times 10^{-24} gm} \times 20gm\right) \text{ Joule}$$

$\Rightarrow 0.05 \times 10^{+14}$ J

$\Rightarrow 1.4 \times 10^{+6}$ kwh

[1 J $\Rightarrow 2.778 \times 10^{-7}$ kwh]

32. Official Ans. by NTA (1)

Sol. Only in case-I, $M_{LHS} > M_{RHS}$ i.e. total mass on reactant side is greater then that on the product side. Hence it will only be allowed.

33. Official Ans. by NTA (2)

Sol. $$v_{rms} = \sqrt{\frac{3KT}{m}}$$

$m \rightarrow$ mass of one molecule (in kg) $= \frac{\text{molar mass}}{NA}$

de-Broglie wavelenth,

$$\lambda = \frac{h}{mv}$$

given, $v = v_{rms}$

$$\lambda = \frac{h}{m\sqrt{\frac{3KT}{m}}} \quad \Rightarrow \lambda = \frac{h}{\sqrt{3KTm}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.38 \times 10^{-23} \times 400 \times \left(\frac{28 \times 10^{-3}}{6.023 \times 10^{-23}}\right)}}$$

$$\lambda = \frac{6.63 \times 10^{-11}}{2.77} = 2.39 \times 10^{-11} m$$

$\lambda = 0.24$ Å

34. Official Ans. by NTA (1)

Sol. B.E. = $[\Delta m].c^2$

$M_{expected} = ZM_p + (A - Z)M_n$

$= 50\ [1.00783] + 70\ [1.00867]$

$M_{actual} = 119.902199$

$$B.E. = \left[50[1.00783] + 70[1.00867] - 119.902199\right] \times 931$$

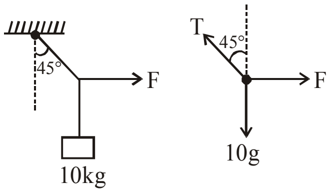
$= 1020.56$

$$\frac{BE}{nucleon} = \frac{1020.56}{120} = 8.5 \text{ MeV}$$

NLM & FRICTION

1. NTA Ans. (1)

Sol.



For equilibrium,

T sin 45° = F ....(1)

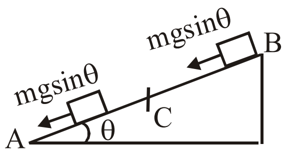
and T cos 45° = 10g ....(2)

equation (1)/(2)

we get F = 10g = 100 N

2. Official Ans. by NTA (3)

Sol.



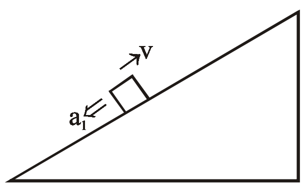
Apply work energy theorem

$mg\sin\theta\ (AC + 2AC) - \mu mg \cos\theta AC = 0$

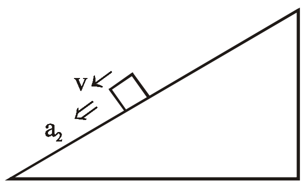
$\mu = 3\tan\theta$

3. Official Ans. by NTA (346)

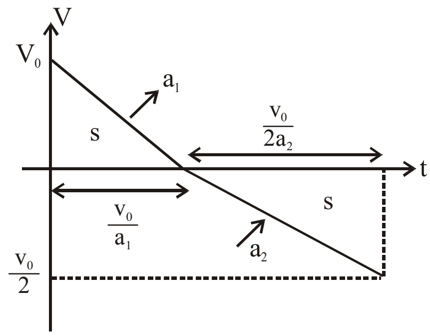
Sol.



$a_1 = g(\sin\theta + \mu \cos\theta)$



$a_2 = g(\sin\theta + \mu \cos\theta)$



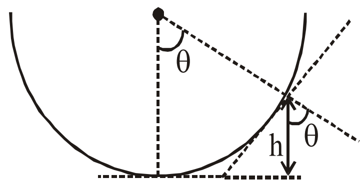
$$\therefore \frac{1}{2} v_0 \frac{v_0}{a_1} = \frac{1}{2}\left(\frac{v_0}{2}\right)\left(\frac{v_0}{2a_1}\right)$$

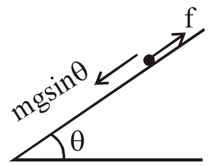
$\Rightarrow 3 \sin \theta = 5 \mu \cos \theta$

$\therefore \mu = \sqrt{3}/5$

4. Official Ans. by NTA (4)

Sol.



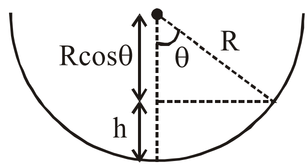


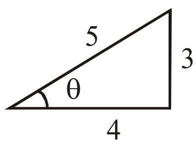
For balancing $mg\sin\theta = f$

$mg\sin\theta = \mu mg\cos\theta$

$\tan\theta = \mu$

$$\tan\theta = \frac{3}{4}$$





$h = R - R\cos\theta$

$$= R - R\left(\frac{4}{5}\right) = \frac{R}{5}$$

$$h = \frac{R}{5} = 0.2\text{m}$$

$\therefore$ correct option is (4)

PRINCIPAL OF COMUNICATION

1. Official Ans. by NTA (3)

Official Ans. by ALLEN

(Close Option is 3 Amax. = 8, Amin. = 2)

Sol. $V_m = 5(1+0.6 \cos 6280t) \sin (2\pi \times 10^4 t)$

$V_m = [5+3\cos 6820t] \sin (2\pi \times 10^4 t)$

$V_{max.} = 5 + 3 = 8$

$V_{min.} = 5 - 3 = 2$

ROTATIONAL MECHANICS

1. NTA Ans. (3)

Sol. $mgh = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}mr^2 \times \frac{v^2}{r^2} = \frac{3}{4}mv^2$

$u = \sqrt{\frac{4}{3}gh}$

$\omega = \frac{v}{r}$

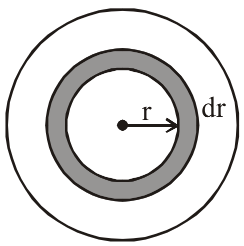
2. NTA Ans. (2)

Sol. $m\frac{l^2}{12} + m\frac{l^2}{16} = mk^2$

$\frac{7l^2}{48} = k^2$

3. NTA Ans. (1)

Sol.



$dI = dmr^2$

$dI = \sigma 2\pi r \, dr \, r^2$

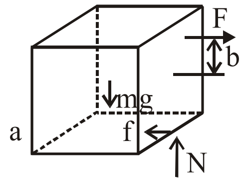
$dI = 2\pi(A + Br) r^3 \, dr$

$\int dI = 2\pi \int_0^a (Ar^3 + Br^4) dr$

$I = 2\pi a^4 \left(\frac{A}{4} + \frac{B9}{5}\right)$

4. NTA Ans. (75)

Sol.



$F = \mu mg$ ...(1)

$F\left(b + \frac{a}{2}\right) = mg \frac{a}{2}$ ...(2)

$\mu mg \left(b + \frac{a}{2}\right) = mg \times \frac{a}{2}$

$\left(b + \frac{a}{2}\right) \mu = \frac{a}{2}$

$0.4 = \mu = \frac{a}{2b+a}$

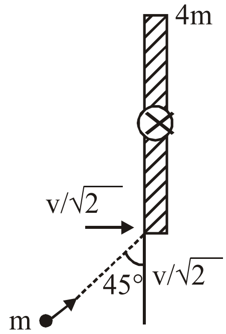
$0.8b + 0.4a = a$

$0.8 b = 0.6 a$

$\frac{b}{a} = \frac{3}{4}$

5. NTA Ans. (2)

Sol.



Let angular velocity of the system after collision be ω.

By conservation of angular momentum about the hinge :

$m\left(\frac{v}{\sqrt{2}}\right)\left(\frac{\ell}{2}\right) = \left[\frac{4m\ell^2}{12} + \frac{m\ell^2}{4}\right]\omega$

On solving

$\omega = \frac{3\sqrt{2}}{7}\left(\frac{v}{\ell}\right)$

6. NTA Ans. (1)

Sol. m = 0.5 kg, v = 5 cm/s

KE in rolling = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$

$= 8.75 \times 10^{-4}$ J

7. NTA Ans. (1)

Sol. From parallel axis theorem

$$I_0 = 3\times\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$$

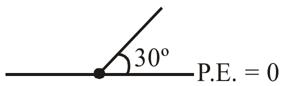
$$I_A = I_0 + 3M\left(\frac{d}{\sqrt{3}}\right)^2$$

$$= \frac{13}{10}Md^2 + Md^2$$

$$= \frac{23}{10}Md^2 \qquad \Rightarrow \frac{I_0}{I_A} = \frac{13}{23}$$

8. NTA Ans. (15.00)

Sol.



From mechanical energy conservation,

$U_i + K_i = U_f + K_f$

$$\Rightarrow mg\frac{\ell}{2}\sin 30° + 0 = 0 + \frac{1}{2}I\omega^2$$

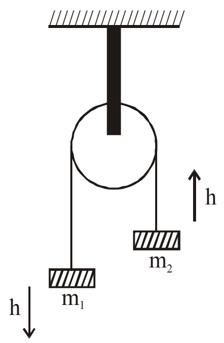
$$\Rightarrow mg \times \frac{1}{2} \times \frac{1}{2} + 0 = 0 + \frac{1}{2} \times \frac{m(1)^2}{3}\omega^2$$

$$\Rightarrow \omega^2 = \frac{3g}{2} \Rightarrow \omega = \sqrt{15}$$

$\therefore$ n = 15

9. NTA Ans. (2)

Sol.



by using work energy theorem

$W_g = \Delta KE$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1+m_2)V^2 + \frac{1}{2}I\omega^2$$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1+m_2)(\omega R)^2 + \frac{1}{2}I\omega^2$$

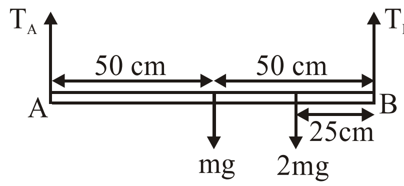
$$(m_1 - m_2)gh = \frac{\omega^2}{2}\left[(m_1+m_2)R^2 + I\right]$$

$$\omega = \sqrt{\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2 + I}}$$

$\therefore$ Correct answer (2)

10. Official Ans. by NTA (4)

Sol.



$\tau_B = 0$ (torque about point B is zero)

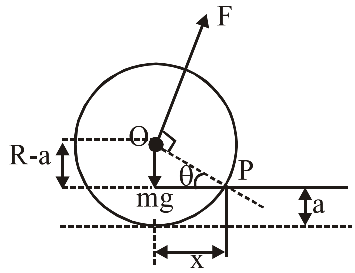
$(T_A) \times 100 - (mg) \times 50 - (2mg) \times 25 = 0$

$100\ T_A = 100$ mg

$T_A = 1$ mg

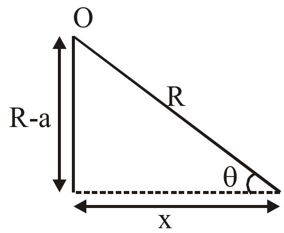
11. Official Ans. by NTA (4)

Sol.



$(\tau)_P = 0$

F.R. – mgx = 0



$$x = \sqrt{R^2 - (R-a)^2}$$

$$F = mg\frac{x}{R}$$

$$F = mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$$

= minimum value of force to pull

12. Official Ans. by NTA (4)

Sol. • Both discs are rotating in same sense

• Angular momentum conserved for the system

i.e. $L_1 + L_2 = L_{final}$

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

$0.1 \times 10 + 0.2 \times 5 = (0.1+0.2) \times \omega_f$

$$\omega_f = \frac{20}{3}$$

• Kinetic energy of combined disc system

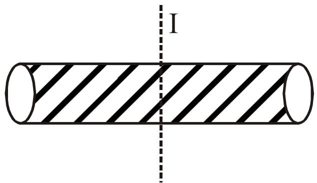
$$\Rightarrow \frac{1}{2}(I_1 + I_2)\omega_f^2$$

$$= \frac{1}{2}(0.1 + 0.2).\left(\frac{20}{3}\right)^2$$

$$= \frac{0.3}{2} \times \frac{400}{9} = \frac{120}{18} = \frac{20}{3}J$$

13. Official Ans. by NTA (3)

Sol.



$$I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right) \quad .....(1)$$

as mass is constant $\Rightarrow m = \rho V$ = constant

V = constant

$\pi^2 R l$ = constant $\Rightarrow R^2L$ = constant

$$2RL + R^2\frac{dL}{dR} = 0 \quad .....(2)$$

From equation (1)

$$\frac{dI}{dR} = M\left(\frac{2R}{4} + \frac{2L}{12} \times \frac{dL}{dr}\right) = 0$$

$$\frac{R}{2} + \frac{L}{6}\frac{dL}{dR} = 0$$

Substituting value of $\frac{dL}{dR}$ from eqution (2)

$$\frac{R}{2} + \frac{L}{6}\left(\frac{-2L}{R}\right) = 0$$

$$\frac{R}{2} = \frac{L^2}{3R} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

14. Official Ans. by NTA (2)

Sol. Angular momentum conservation

$$mvl = \frac{Ml^2}{3}\omega + ml^2\omega$$

$$\Rightarrow \omega = \frac{1 \times 6 \times 1}{\frac{2}{3} + 1} = \frac{18}{5}$$

Now using energy consevation

$$\frac{1}{2}\left(M\frac{l^2}{3}\right)\omega^2 + \frac{1}{2}(ml^2)\omega^2$$

$$= (m + M)r_{cm}(1 - \cos\theta)$$

$$= (m + M)\left(\frac{ml + \frac{Ml}{2}}{m + M}\right)g(1 - \cos\theta)$$

$$\frac{5}{6} \times \left(\frac{18}{5}\right)^2 = 20(1 - \cos\theta)$$

$$\Rightarrow 1 - \cos\theta = \frac{18}{5} \times \frac{3}{20}$$

$$\cos\theta = 1 - \frac{27}{50}$$

$$\cos\theta = \frac{23}{50} \Rightarrow \theta \simeq 63°$$

15. Official Ans. by NTA (9)

Sol. $L_i = L_f$

$$\left(80R^2 + \frac{200R^2}{2}\right)\omega = \left(0 + \frac{200R^2}{2}\right)\omega_1$$

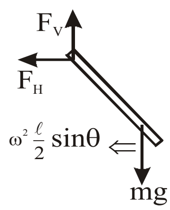
$180\omega_0 = 100\omega_1$

$\omega_1 = 1.8\omega_0 = 1.8 \times 5$

$= 9$ rpm

16. Official Ans. by NTA (2)

Sol.



$F_V = mg$

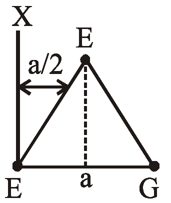
$$F_H = m\omega^2\frac{\ell}{2}\sin\theta$$

$$mg\frac{\ell}{2}\sin\theta - m\omega^2\frac{\ell}{2}\sin\theta\frac{\ell}{2}\cos\theta = \frac{m\ell^2}{12}\omega^2\sin\theta\cos\theta$$

$$\cos\theta = \frac{3}{2}\frac{g}{\omega^2\ell} \quad .....(ii)$$

17. Official Ans. by NTA (25)

Sol.

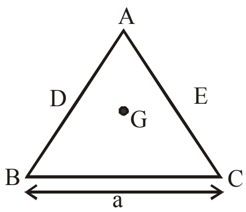


$$I = 0 + m\left(\frac{a}{2}\right)^2 + ma^2$$

$$= \frac{5}{4}ma^2$$

18. Official Ans. by NTA (11)

Sol. Let side of triangle is a and mass is m



MOI of plate ABC about centroid

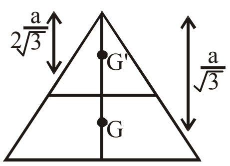
$$I_0 = \frac{m}{3}\left(\left(\frac{a}{2\sqrt{3}}\right)^2 \times 3\right) = \frac{ma^2}{12}$$

triangle ADE is also an equilateral triangle of side a/2.

Let moment of inertia of triangular plate ADE about it's centroid (G') is I_1 and mass is m_1

$$m_1 = \frac{m}{\frac{\sqrt{3}a^2}{4}} \times \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{m}{4}$$

$$I_1 = \frac{m_1}{12}\left(\frac{a}{2}\right)^2 = \frac{m}{4 \times 12}\frac{a^2}{4} = \frac{ma^2}{192}$$



distance GG' $= \frac{a}{\sqrt{3}} - \frac{a}{2\sqrt{3}} = \frac{a}{2\sqrt{3}}$

so MOI of part ADE about centroid G is

$$I_2 = I_1 + m_1\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{ma^2}{192} + \frac{m}{4}.\frac{a^2}{12}$$

$$= \frac{5ma^2}{192}$$

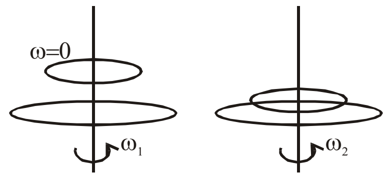
now MOI of remaining part

$$= \frac{ma^2}{12} - \frac{5ma^2}{192} = \frac{11ma^2}{12 \times 16} = \frac{11I_0}{16}$$

$\Rightarrow$ N = 11

19. Official Ans. by NTA (20)

Sol.



Let moment of inertia of bigger disc is $I = \frac{MR^2}{2}$

$\Rightarrow$ MOI of small disc $I_2 = \frac{M\left(\frac{R}{2}\right)^2}{2} = \frac{I}{4}$

by angular momentum conservation

$I\omega_1 + \frac{I}{4}(0) = I\omega_2 + \frac{I}{4}\omega_2 \Rightarrow \omega_2 = \frac{4\omega_1}{5}$

initial kinetic energy $K_1 = \frac{1}{2}I\omega_1^2$

final kinetic energy K_2

$$= \frac{1}{2}\left(I + \frac{I}{4}\right)\left(\frac{4\omega_1}{5}\right)^2 = \frac{1}{2}I\omega_1^2\left(\frac{4}{5}\right)$$

$$P\% = \frac{K_1 - K_2}{K_1} \times 100\% = \frac{1 - 4/5}{1} \times 100 = 20\%$$

20. Official Ans. by NTA (2)

Sol. $I_1 = \frac{MR^2}{2} = \frac{\rho(\pi R^2)t.R^2}{2}$

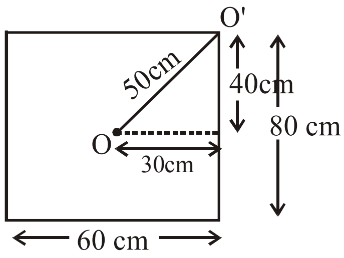
$I \propto R^4$

$$\frac{I_1}{I_2} = \frac{R_1^4}{R_2^4} = \frac{1}{16}$$

$$\therefore \frac{R_1}{R_2} = \frac{1}{2}$$

21. Official Ans. by NTA (4)

Sol.



Rectangular sheet

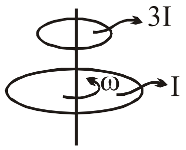
$$I_O = \frac{M}{12}[L^2+B^2] = \frac{M}{12}[80^2+60^2]$$

$$I_{O'} = I_0 + Md^2 \{\text{parallel axis theorem}\}$$
$$= \frac{M}{12}\left[80^2+60^2\right] + M\,[50]^2$$

$$\frac{I_O}{I_{O'}} = \frac{M/12[80^2+60^2]}{\frac{M}{12}[80^2+60^2]+M[50]^2} = \frac{1}{4}$$

22. Official Ans. by NTA (3)

Sol.



By anglar momentum conservation

$$\omega I + 3I \times 0 = 4I\omega' \Rightarrow \omega' = \frac{\omega}{4}$$

$$(KE)_i = \frac{1}{2} I\omega^2$$

$$(KE)_f = \frac{1}{2} \times (4I) \times \left(\frac{\omega}{4}\right)^2 = \frac{I\omega^2}{8}$$

$$\Delta KE = \frac{3}{8} I\omega^2$$

$$\text{fractional loss} = \frac{\Delta KE}{KE_1} = \frac{\frac{3}{8}I\omega^2}{\frac{1}{2}I\omega^2} = \frac{3}{4}$$

23. Official Ans. by NTA (195)

Sol. $\vec{\tau} = \left(\vec{r}_2 - \vec{r}_1\right) \times \vec{F}$

$$= [(4\hat{i}+3\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+\hat{k})]\times\vec{F}$$

$$= \left(3\hat{i}+\hat{j}-2\hat{k}\right)\times\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$

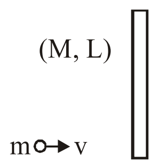
$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= 7\hat{i} - 11\hat{j} + 5\hat{k}$$

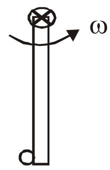
$$|\vec{\tau}| = \sqrt{195}$$

24. Official Ans. by NTA (20.00)

Sol.



Before collision



After collision

$$\vec{L}_i = \vec{L}_f$$

$$mvL = I\omega$$

$$mvL = \left(\frac{ML^2}{3} + mL^2\right)\omega$$

$$0.1 \times 80 \times 1 = \left(\frac{0.9\times1^2}{3} + 0.1\times1^2\right)\omega$$

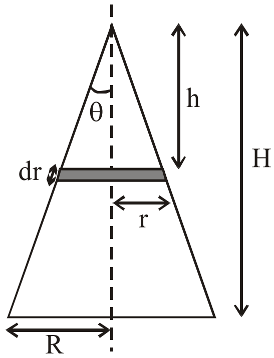
$$8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega$$

$$8 = \frac{4}{10}\omega$$

$$\omega = 20\ \text{rad}\,\frac{\text{rad}}{\text{sec}}$$

25. Official Ans. by NTA (1)

Sol.



Area = $\pi R\ell = \pi R\left(\sqrt{H^2+R^2}\right)$

Area of element dA = $2\pi r d\ell = 2\pi r\dfrac{dh}{\cos\theta}$

mass of element $dm = \dfrac{M}{\pi R\sqrt{H^2+R^2}}\times\dfrac{2\pi r dh}{\cos\theta}$

$dm = \dfrac{2Mh\tan\theta dh}{R\sqrt{H^2+R^2}\cos\theta}$ (here r = h tanθ)

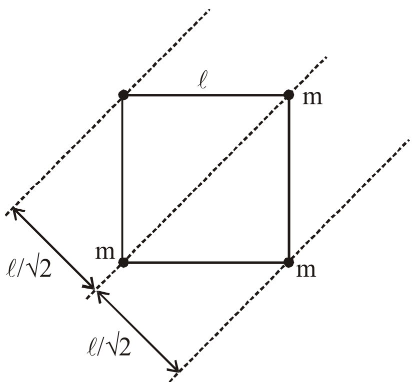
$I = \int (dm)r^2 = \int \dfrac{h^2\tan^2\theta}{\cos\theta}\left(\dfrac{2m}{R}\dfrac{h\tan\theta}{\sqrt{R^2+H^2}}\right)dh$

$= \dfrac{2M}{\cos\theta R}\dfrac{\tan^3\theta}{\sqrt{R^2+H^2}}\int_0^H h^3 dh = \dfrac{MR^2H^4}{2RH^3\sqrt{R^2+H^2}\cos\theta}$

$= \dfrac{MR^2H\sqrt{R^2+H^2}}{2\sqrt{R^2+H^2}\times H} \Rightarrow = \dfrac{MR^2}{2}$

26. Official Ans. by NTA (2)

Sol.



$I = m(0)^2 + m\left(\dfrac{\ell}{\sqrt{2}}\right)^2\times 2 + m\left(\sqrt{2}\ell\right)^2$

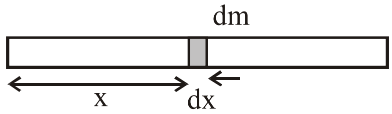
$= \dfrac{2m\ell^2}{2} + 2m\ell^2 = 3m\ell^2$

Angular momentum L = Iω

$= 3m\ell^2\omega$

27. Official Ans. by NTA (4)

Sol.



$I = \int r^2 dm = \int x^2\lambda dx \Rightarrow I = \int_0^L x^2\lambda_0\left(1+\dfrac{x}{L}\right)dx$

$I = \lambda_0\int_0^L\left(x^2+\dfrac{x^3}{L}\right)dx$

$I = \lambda\left[\dfrac{L^3}{3}+\dfrac{L^3}{4}\right]$

$I = \dfrac{7L^3\lambda_0}{12}$ ....(i)

$M = \int_0^L \lambda dx = \int_0^L \lambda_0\left(1+\dfrac{x}{L}\right)dx$

$M = \lambda_0\left(L+\dfrac{L}{2}\right) = \lambda_0\dfrac{3L}{2}$

$\dfrac{2}{3}M = (\lambda_0 L)$ ....(ii)

From (i) & (ii) $I = \dfrac{7}{12}\left(\dfrac{2}{3}M\right)L^2 = \dfrac{7ML^2}{18}$

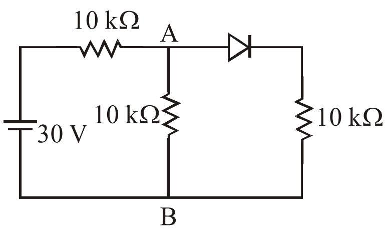
Ans. (4)

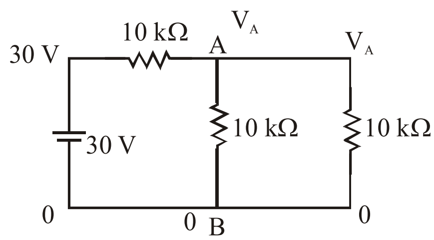
SEMICONDUCTOR

1. Official Ans. by NTA (2)

2. Official Ans. by NTA (2)

Sol.





$\dfrac{30-V_A}{10}+\dfrac{0-V_A}{10}+\dfrac{0-V_A}{10}=0$

$3 = \dfrac{3V_A}{10}$

$V_A = 10$ V

3. Official Ans. by NTA (3)

Sol.

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

4. Official Ans. by NTA (2)

Sol. $Y = \overline{\overline{AB}.A}$

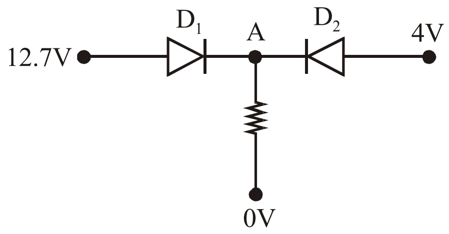
$= \overline{\overline{AB}} + \overline{A}$

$= 0 + 0$

$= 0$

5. Official Ans. by NTA (12.00)

Sol.

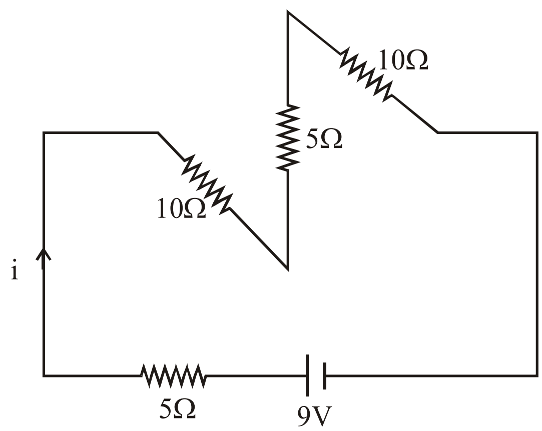


Diode D_1 is forward biased and D_2 is reverse biased.

$\therefore$ $V_A = 12.7 - 0.7 = 12V.$

6. Official Ans. by NTA (3)

Sol.

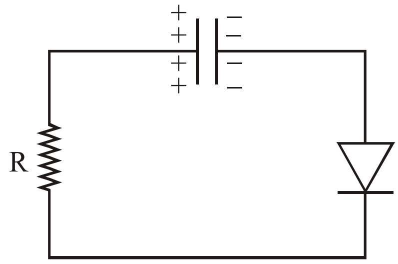


$$i = \frac{9}{(5+10+5+10)} = \frac{9}{30}A$$

$\therefore$ Correct answer (3)

7. Official Ans. by NTA (1)

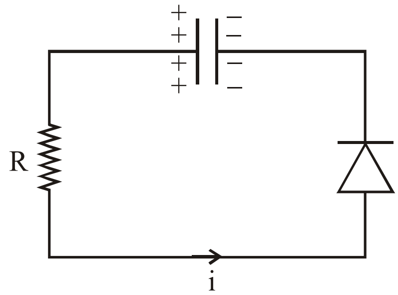
Sol. For (A)



No current flows

Hence $Q_A = CV$

For (B)



$$i = \frac{V}{R}e^{-\frac{t}{RC}}$$

$$q = CVe^{-\frac{t}{RC}}$$

at t = CR

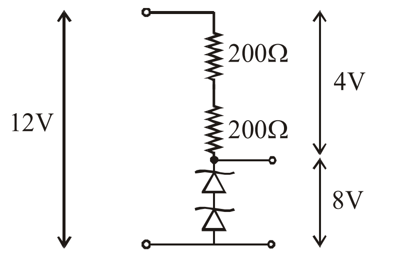
$$Q_B = CVe^{-1} = \frac{CV}{e}$$

$\therefore$ Correct answer (1)

8. Official Ans. by NTA (12.00)

ALLEN Ans. (40.00)

Sol.



Current in circuit $= \frac{4}{400} = \frac{1}{100}A$

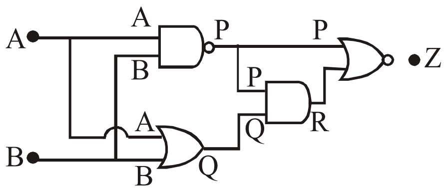
So power dissipated in each diode = VI

$= 4 \times \frac{1}{100} W = 40 \times 10^{-3}$ mW

$\therefore$ Correct answer 40

9. Official Ans. by NTA (3)

Sol.



$Z = \overline{(P+R)}$

$Z = \overline{(P+PQ)}$

$Z = \overline{(P(1+Q))}$

$Z = (\overline{P})$ [Using Identity $(1+A)=1$]

$Z = \overline{(\overline{AB})}$

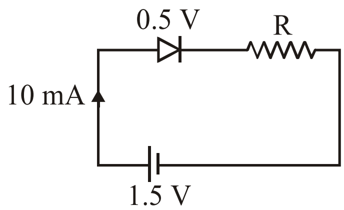
Z = AB

Truth table for Z = AB

A	B	Z
1	0	0
0	0	0
1	1	1

10. Official Ans. by NTA (1)

Sol.



$1.5 - 0.5 - R \times 10 \times 10^{-3} = 0$

$\therefore R = 100\ \Omega$

11. Official Ans. by NTA (3)

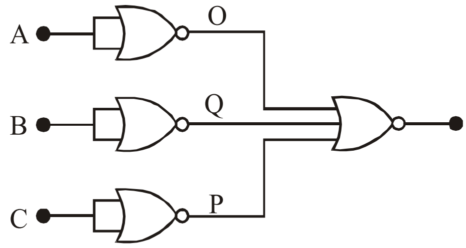
Sol. $\Delta E = \frac{\lambda c}{\lambda e} = 3.1 eV$

12. Official Ans. by NTA (2)

Sol. As there are two zener diodes in reverse polarity so if one is in forward bias the other will be in reverse bias and above 6V the reverse bias will too be in conduction mode. Therefore when voltage is more than 6V the output will be constant. And when it is less than 6V it will follow the input voltage so correct answer is two.

13. Official Ans. by NTA (1)

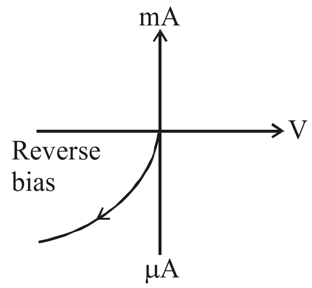
Sol.



A	B	C	
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
1	0	1	0
0	1	1	0
1	1	1	1

14. Official Ans. by NTA (1)

Sol. I-V characteristic of a photodiode is as follows:



On increasing the potential difference the current first increases and then attains a saturation.

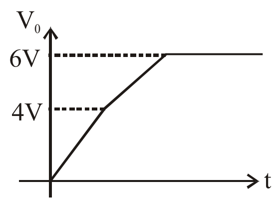
15. Official Ans. by NTA (4)

Official Ans. by ALLEN (2)

Sol. Till input voltage Reaches 4V No zener is in Breakdown Region So $V_0 = V_i$ Then Now when V_i changes between 4V to 6V One Zener with 4V will Breakdown are P.D. across This zener will become constant and Remaining Potential will drop. acro

Resistance in series with 4V Zener.

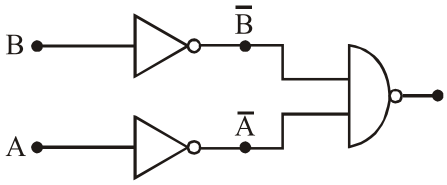
Now current in circuit increases Abruptly and source must have an internal resistance due to which. Some potential will get drop across the source also so correct graph between V_0 and t. will be



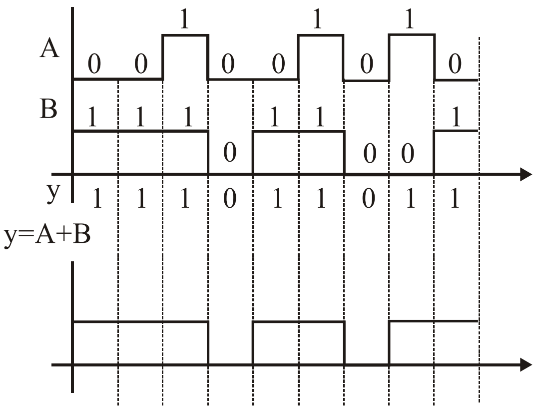
We have to Assume some resistance in series with source.

16. Official Ans. by NTA (3)
Official Ans. by ALLEN None (Approx Ans can be 2)

Sol.



$y = \overline{\overline{A}.\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$



17. Official Ans. by NTA (150.00)

Sol. $\Delta I_B = (30 - 20) = 10\mu A$

$\Delta I_C = (4.5 - 3)\ mA = 1.5mA$

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{1.5mA}{10\mu A} = 150$$

$\beta_{ac} = 150$

SIMPLE HARMONIC MOTION

1. Official Ans. by NTA (4)

Sol. (A) F = ma $\qquad a = -\omega^2 x$

at $\frac{3T}{4}$ displacement zero (x = 0), so a = 0

F = 0

(B) at t = T $\quad$ displacement (x) = A

x maximum, So acceleration is maximum.

(C) $V = \omega\sqrt{A^2 - x^2}$

V_{max} at x = 0

$V_{max} = A\omega$

at $t = \frac{T}{4}$, x = 0, So $V_{max.}$

(D) KE = PE

$\therefore$ at $x = \frac{A}{\sqrt{2}}$.

at $t = \frac{T}{2}$ $\quad x = -A$ $\quad$ (So not possible)

2. Official Ans. by NTA (4)

Sol. At equilibrium position

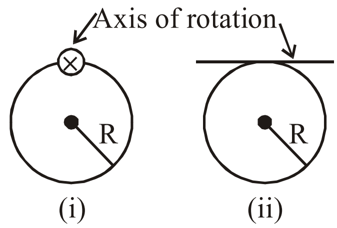
$$V_0 = \omega_0 A = \sqrt{\frac{K}{m}}A \qquad .....(i)$$

$$V = \omega A^1 = \sqrt{\frac{K}{\frac{m}{2}}}A^1 \qquad .....(ii)$$

$$\therefore \quad A^1 = \frac{A}{\sqrt{2}}$$

3. Official Ans. by NTA (1)

Sol.



Moment of inertia in case (i) is I_1

Moment of inertia in case (ii) is I_2

$I_1 = 2MR^2$

$I_2 = \frac{3}{2}MR^2$

$T_1 = 2\pi\sqrt{\frac{I_1}{Mgd}}$; $T_2 = 2\pi\sqrt{\frac{I_2}{Mgd}}$

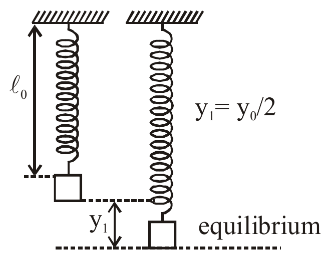
$$\frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}} = \frac{2}{\sqrt{3}}$$

4. Official Ans. by NTA (2)

Sol. $y = y_0 \sin^2 \omega t$

$$y = \frac{y_0}{2}(1 - \cos 2\omega t) \Rightarrow y - \frac{y_0}{2} = -\frac{y_0}{2}\cos 2\omega t$$

Amplitude : $\frac{y_0}{2}$



$$\frac{y_0}{2} = \frac{mg}{K} \Rightarrow 2\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{2g}{y_0}}$$

$$\omega = \sqrt{\frac{g}{2y_0}}$$

Ans. (2)

UNIT & DIMENSION

1. NTA Ans. (1)

Sol. Magnetic energy stored per unit volume is

$\frac{B^2}{2\mu_0}$ ⇒ Dimension is $ML^{-1}T^{-2}$

2. NTA Ans. (BONUS)

Sol. $v_0 = h^x c^y G^z A^w$

$$\frac{ML^2T^{-2}}{AT} = (ML^2T^{-1})^x (LT^{-1})^y (M^{-1}L^3T^{-2})^z A^w$$

$\Rightarrow w = -1$

$(x - z = 1)$

$2x + y + 3x = 2$

$- x - y - 2z = -3$

————————————

$2x = 0$

$x = 0$

$z = -1$

$2 \times 0 + y + 3x - 1 = 2$

$y = 5 \quad \Rightarrow v_0 = h^0c^5G^{-1}A^{-1}$

So Bonus

3. NTA Ans. (3)

Sol. $[h] = M^1L^2T^{-1}$

$[C] = L^1T^{-1}$

$[G] = M^{-1}L^3T^{-2}$

$$[f] = \sqrt{\frac{M^1L^2T^{-1} \times L^5T^{-5}}{M^{-1}L^3T^{-2}}} = M^1L^2T^{-2}$$

4. Official Ans. by NTA (1)

Sol. $Y = F^xA^yV^z$

$M^1L^{-1}T^{-2} = [MLT^{-2}]^x[L^2]^y[LT^{-1}]^z$

$M^1L^1T^{-2} = [M]^x[L]^{x+2y+z}[T]^{-2x-z}$

comparing power of ML and T

$x = 1$...(1)

$x + 2y + z = -1$ ....(2)

$-2x - z = -2$ ...(3)

after solving

$x = 1$

$y = -1$

$z = 0$

$Y = FA^{-1}V^0$

5. Official Ans. by NTA (2)

Sol. Let $[E] = [P]^x [A]^y [T]^z$

$ML^2T^{-2} = [MLT^{-1}]^x [L^2]^y [T]^z$

$ML^2T^{-2} = M^x L^{x+2y} T^{-x+z}$

$\rightarrow x = 1$

$\rightarrow x + 2y = 2$

$1 + 2y = 2$

$y = \frac{1}{2}$

$\rightarrow -x + z = -2$

$-1 + z = -2$

$z = -1$

$[E] = [PA^{1/2} T^{-1}]$

6. Official Ans. by NTA (4)

Sol. $S = \frac{P}{A} = \frac{ML^2T^{-3}}{L^2} = MT^{-3}$

7. Official Ans. by NTA (3)

Sol. $x = \frac{IFV^2}{WL^4}$

$$[x] = \frac{[ML^2][MLT^{-2}][LT^{-1}]^2}{[ML^2T^{-2}][L]^4}$$

$[x] = [ML^{-1}T^{-2}]$

$$[\text{Energy density}] = \left[\frac{E}{V}\right]$$

$$= \left[\frac{ML^2T^{-2}}{L^3}\right]$$

$= [ML^{-1}T^{-2}]$

Same as x

8. Official Ans. by NTA (2)

Sol. $x = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \text{speed} \Rightarrow [x] = [L^1T^{-1}]$

$y = \frac{E}{B} = \text{speed} \Rightarrow [y] = [L^1T^{-1}]$

$z = \frac{\ell}{RC} = \frac{\ell}{\tau} \Rightarrow [z] = [L^1T^{-1}]$

So, x, y, z all have the same dimensions.

WAVE MOTION

1. NTA Ans. (3)

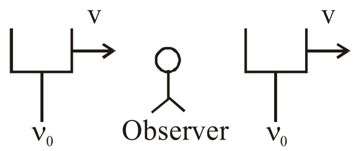
Sol. $v = \sqrt{\frac{T}{\mu}}$

$$90 = \sqrt{\frac{\frac{YA}{l}\Delta l}{\frac{m}{l}}} = \sqrt{\frac{16\times10^{11}\times10^{-6}\times\Delta l}{6\times10^{-3}}}$$

$$= \frac{8100\times3}{8}\times10^{-8} = \Delta l$$

2. NTA Ans. (4)

Sol.



$\nu_1 = \left(\frac{c}{c-v}\right)\nu_0$

$\nu_2 = \left(\frac{c}{c+v}\right)\nu_0$

beat frequency = $\nu_1 - \nu_2$

$= c\nu_0 \left(\frac{1}{c-v} - \frac{1}{c+v}\right)$

$= c\nu_0\left(\frac{c+v-c+v}{c^2-v^2}\right) = \frac{2c\nu_0^2 v}{c^2-v^2}$

$\approx \frac{2c\nu_0 v}{c^2} = \frac{2\nu_0 v}{c} = 2$

$\Rightarrow \frac{2\times1400\times v}{350} = 2$

$\Rightarrow v = \frac{1}{4}$ m/s

3. NTA Ans. (106.00 to 107.20)

Sol. $v_s = \sqrt{\frac{\gamma P}{\rho}}$

$\frac{v_{gas}}{v_{air}} = \sqrt{\frac{\rho_{air}}{\rho_{gas}}}$ $\quad\Rightarrow \frac{v_{gas}}{300} = \frac{1}{\sqrt{2}}$

$\Rightarrow v_{gas} = \frac{300}{\sqrt{2}}$ $\quad\Rightarrow \therefore\ v_{gas} = 150\sqrt{2}$

Now $n_2 - n_1 = \frac{v_{gas}}{2\ell} = \frac{150\sqrt{2}}{2(1)} = 75\sqrt{2}$

$\Rightarrow$ Δn = 106.06 Hz

4. NTA Ans. (2)

Sol. Velocity of transverse wave $V \propto \sqrt{T}$

$V \to \frac{V}{2} \Rightarrow T \to T' = \frac{T}{4}$

$T' = \frac{2.06\times10^4}{4} = 5.15\times10^3 N$

5. NTA Ans. (1)

Sol. Let amplitude of each wave is A.
Resultant wave equation

$= A \sin \omega t + A \sin\left(\omega t - \frac{\pi}{4}\right) + A \sin\left(\omega t + \frac{\pi}{4}\right)$

$= A \sin \omega t + \sqrt{2} A \sin \omega t$

$= \left(\sqrt{2}+1\right) A \sin \omega t$

Resultant wave amplitude = $\left(\sqrt{2}+1\right)A$

as $I \propto A^2$

so $\frac{I}{I_0} = \left(\sqrt{2}+1\right)^2$

$I = 5.8\ I_0$

6. NTA Ans. (4)

Sol. $\frac{nv}{2\ell} = 420$

$\frac{(n+1)v}{2\ell} = 490$

$\frac{v}{2\ell} = 70$

$\ell = \frac{v}{140} = \frac{1}{140}\sqrt{\frac{540}{6\times10^{-3}}} = \frac{1}{140}\sqrt{90\times10^3}$

$\ell = \frac{300}{140} = 2.142$

$\therefore$ Correct answer (4)

7. Official Ans. by NTA (3)

Sol. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

For identical string l and μ will be same

$f \propto \sqrt{T}$

$\frac{450}{300} = \sqrt{\frac{T_x}{T_y}}$

$\frac{T_x}{T_y} = \frac{9}{4} = 2.25$

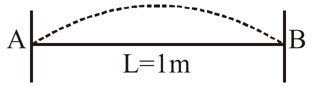
8. Official Ans. by NTA (35.00)

Sol. $\rho_{wire} = 9\times10^{-3}\dfrac{kg}{cm^3} = \dfrac{9\times10^{-3}}{10^{-6}} kg/m^3$

$= 9000$ kg/m^2

(A = CSA of wire)

(Y = 9 × 10^{10} Nm^2)



(Strain = 4.9 × 10^{-4})

$\Rightarrow L = 1m = \dfrac{\lambda}{2} \Rightarrow \lambda = 2m$

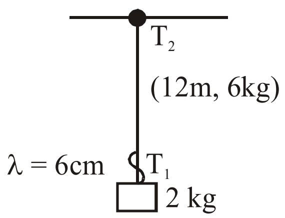
$\Rightarrow v = f\lambda \Rightarrow \sqrt{\dfrac{T}{\mu}} = f\,\lambda$

Where $Y = \dfrac{T/A}{strain} \Rightarrow T = Y.A.\,strain$

9. Official Ans. by NTA (2)

Sol. $V \propto \lambda$ $T_2 = 8g$

$T_1 = 2g$



$\dfrac{V_1}{V_2} = \dfrac{\lambda_1}{\lambda_2}$

$\lambda_2 = \dfrac{V_2}{V_1}\lambda_1 = \sqrt{\dfrac{T_2}{T_1}} \times \lambda_1$

$= \sqrt{\dfrac{8g}{2g}}\,\lambda_1 = 2\times6 = 12$ cm

10. Official Ans. by NTA (4)

Sol. Given T to C 1.5 m

C to C 5m

T to C = $(2n_1 + 1)\dfrac{\lambda}{2}$

C to C = $n_2\lambda$

$\dfrac{1.5}{5} = \dfrac{(2n_1+1)}{2n_2} \Rightarrow 3n_2 = 10n_1 + 5$

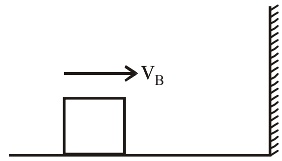
$n_1 = 1, n_2 = 5 \rightarrow \lambda = 1$

$n_1 = 4, n_2 = 15 \rightarrow \lambda = 1/3$

$n_1 = 7, n_2 = 25 \rightarrow \lambda = 1/5$

11. Official Ans. by NTA (1)

Sol.



$f_1 = \left(\dfrac{330}{330 - v_B}\right)420$

$f_2 = \left(\dfrac{330 + v_0}{330}\right)\left(\dfrac{330}{330 - v_B}\right)420$

$490 = \left(\dfrac{330 + v_B}{330 - v_B}\right)420$

$\dfrac{7}{6} = \dfrac{330 + v_B}{330 - v_B}$

$v_B = \dfrac{330}{13}$ m/s

$= \dfrac{330}{13} \times \dfrac{18}{5} \approx 91$ km/hr

12. Official Ans. by NTA (3)

Sol. $\Rightarrow \lambda = 2\,(l_2 - l_1) \Rightarrow 2 \times (24.5 - 17)$

$\Rightarrow 2 \times 7.5 = 15$ cm

& $v = f\lambda \Rightarrow 330 = \lambda \times 15 \times 10^{-2}$

$\lambda = \dfrac{330}{15}\times100 \Rightarrow \dfrac{1100\times100}{5}$

$\Rightarrow 2200$ Hz

13. Official Ans. by NTA (2)

Sol. $\Delta p = BkS_0$

$= \rho v^2 \times \dfrac{\omega}{v} \times S_0$

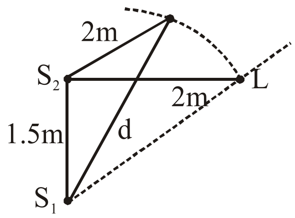
$\Rightarrow S_0 = \dfrac{\Delta p}{\rho v \omega}$

$\approx \dfrac{10}{1\times300\times1000}$ m

$= \dfrac{1}{30}$ mm $\approx \dfrac{3}{100}$ mm

14. Official Ans. by NTA (2)

Sol.



Initially $S_2L = 2m$

$S_1L = \sqrt{2^2 + (3/2)^2}$

$S_1L = \frac{5}{2} = 2.5$ m

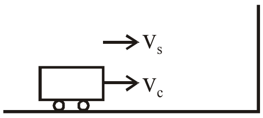
$\Delta x = S_1L - S_2L = 0.5$ m

So since $\lambda = 1m \quad \therefore \Delta x = \frac{\lambda}{2}$

So while listener moves away from S_1
Then, Δx (= $S_1L - S_2L$) increases
and hence, at $\Delta x = \lambda$ first maxima will appear.
$\Delta x = \lambda = S_1L - S_2L$
$1 = d - 2 \Rightarrow d = 3m$

15. Official Ans. by NTA (4)

Sol.



f_1 = frequency heard by wall = $f_s = \left(\frac{v_s}{v_s - v_c}\right)$

f_2 = frequency heard by driver after reflection from wall

$$f_2 = \left(\frac{v_s + v_c}{v_s}\right) f_1 = \left(\frac{v_s + v_c}{v_s - v_c}\right) f_0$$

$$\frac{f_2}{f_0} = \frac{v_s - v_c}{v_s + v_c}$$

$$\frac{48}{44} = \frac{v_s - v_c}{v_s + v_c}$$

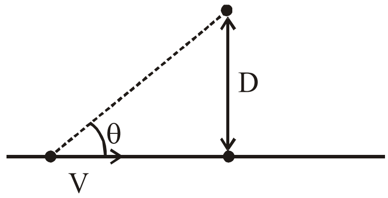
$12(v_s + v_c) = 11(v_s - v_c)$
$23v_c = v_s$

$$v_c = \frac{v_s}{23} = \frac{345}{23} = 15 m/s$$

$$= \frac{15 \times 18}{5} = 54 \text{ km/hr}$$

16. Official Ans. by NTA (4)

Sol.



$$f_{observed} \Rightarrow \left(\frac{v_{sound}}{v_{sound} - v\cos\theta}\right) f_0$$

initially θ will be less $\Rightarrow \cos\theta$ more
$\therefore f_{observed}$ more, then it will decrease.
$\therefore$ Ans. (4)

WAVE OPTICS

1. NTA Ans. (4)

Sol. $\sin\theta = \frac{2\lambda}{\omega}$

$$\sin 60° = \frac{2\lambda}{\omega}$$

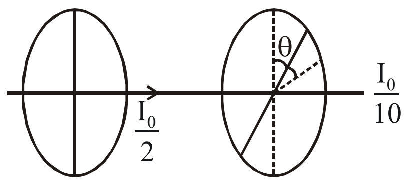
$$\sin\theta_1 = \frac{\lambda}{\omega} = \frac{\sqrt{3}}{4}$$

$\theta_1 = 25°$

2. NTA Ans. (1)

Sol. $\frac{I_0}{10} = I = \frac{I_0}{2} \times \cos^2\theta$

$$\cos\theta = \frac{1}{\sqrt{5}}$$



$\theta = 63.44°$
angle rotated = 90 – 63.44° = 26.56°
Closest is 1.

3. NTA Ans. (2)

Sol. Finge width, $\beta = \frac{D\lambda}{d} = \frac{1.5 \times 589 \times 10^{-9}}{0.15 \times 10^{-3}}$

$= 5.9 \times 10^{-3}$ m
$= 5.9$ mm

4. NTA Ans. (4)

Sol. $I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

$\frac{I}{I_0} = \cos^2\left(\frac{\Delta\phi}{2}\right)$

$\frac{I}{I_0} = \cos^2\left(\frac{\frac{2\pi}{\lambda}\times\frac{\lambda}{8}}{2}\right)$

$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{8}\right) \quad \Rightarrow \frac{I}{I_0} = 0.853$

5. NTA Ans. (3)

Sol. Let distance is x then

$d\theta = \frac{1.22\lambda}{D}$ (D = diameter)

$\frac{x}{d} = \frac{1.22\lambda}{D}$ (d = distance between earth & moon)

$x = \frac{1.22\times(5500\times10^{-10})\times(4\times10^{8})}{5} = 53.68\,m$

most appropriate is 60m.

6. NTA Ans. (1)

Sol. Direction of polarisation $= \hat{E} = \hat{k}$

Direction of propagation $= \hat{E}\times\hat{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

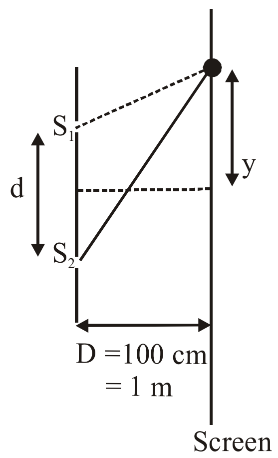
$\therefore \hat{E}\times\hat{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

$\hat{B} = \frac{\hat{i}-\hat{j}}{\sqrt{2}}$

Correct answer (1)

7. Official Ans. by NTA (1)

Sol.



$y = \frac{nD\lambda}{d}$

$n = \frac{yd}{D\lambda} = \frac{1.27\times10^{-3}\times10^{-3}}{1\times632.8\times10^{-9}} = 2$

Path difference $\Delta x = n\lambda$

$= 2 \times 632.8$ nm

$= 1265.6$ nm

$= 1.27$ μm

8. Official Ans. by NTA (1)

Sol. Let the length of segment is "ℓ"

Let N is the no. of fringes in "ℓ"

and w is fringe width.

→ We can write

$N w = \ell$

$N\left(\frac{\lambda D}{d}\right) = \ell$

$\frac{N_1\lambda_1 D}{d} = \ell$

$\frac{N_2\lambda_2 D}{d} = \ell$

$N_1\lambda_1 = N_2\lambda_2$

$16 \times 700 = N_2 \times 400$

$N_2 = 28$

9. Official Ans. by NTA (4)

Sol. $\Delta\theta_0 = \left(\frac{\lambda}{d}\times\frac{180}{\pi}\right)^0$

$= 0.57°$

10. Official Ans. by NTA (1)

Sol. $\Delta p = n_1 L_1 - n_2 L_2$

$\Delta\phi = \frac{2\pi}{\lambda}\Delta p$

11. Official Ans. by NTA (3)

Sol. Intensity, $I = 3.3\ Wm^{-2}$

Area, $A = 3 \times 10–4\ m^2$

Angular speed, $\omega = 31.4$ rad/s

$\because \langle\cos^2\theta\rangle = \frac{1}{2}$, in one time period

$\therefore$ Average energy $= I_0A \times \frac{1}{2}$

$= \frac{(3.3)(3\times10^{-4})}{2}$

$\simeq 5\times10^{-4}\,J$

12. Official Ans. by NTA (200)

Official Ans. by ALLEN (198)

Sol. Condition for minimum,

$d\sin\theta = n\lambda$

$\therefore\ \sin\theta = \frac{n\lambda}{d} < 1$

$n < \frac{d}{\lambda} = \frac{6\times10^{-5}}{6\times10^{-7}} = 100$

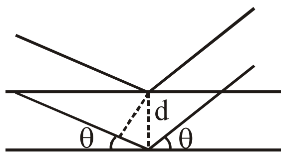
$\therefore$ Total number of minima on one side $= 99$

Total number of minima = 198

Correct Answer is 198

13. Official Ans. by NTA (50.00)

Sol.



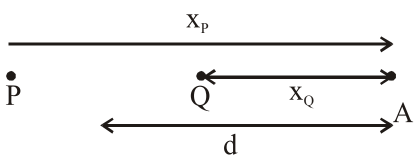
$$2d\sin\theta = \lambda = \frac{h}{\sqrt{2mE}}$$

$$2\times10^{-10}\times\frac{\sqrt{3}}{2} = \frac{6.6\times10^{-34}}{\sqrt{2mE}}$$

$$E = \frac{1}{2}\times\frac{6.64^2\times10^{-48}}{9.1\times10^{-31}\times3\times1.6\times10^{-19}} = 50.47$$

14. Official Ans. by NTA (4)

Sol. For (A)



$x_P - x_Q = (d + 2.5) - (d - 2.5)$

$= 5m$

$\Delta\phi$ due to path difference $= \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{20}(5)$

$= \frac{\pi}{2}$

At A, Q is ahead of P by path, as wave emitted by Q reaches before wave emitted by P.

Total phase difference at A

$= \frac{\pi}{2} - \frac{\pi}{2}$ (due to P being ahead of Q by 90°)

$= 0$

$I_A = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$

$= I + I + 2\sqrt{I}\sqrt{I}\cos(0)$

$= 4I$

For C

$x_Q - x_P = 5$ m

$\Delta\phi$ due to path difference $= \frac{2\pi}{\lambda}(\Delta x)$

$= \frac{2\pi}{20}(5) = \frac{\pi}{2}$

Total phase difference at C = $\frac{\pi}{2} + \frac{\pi}{2} = \pi$

$I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\Delta\phi)$

$= I + I + 2\sqrt{I}\sqrt{I}\cos(\pi) = 0$

For B

$x_P - x_Q = 0$,

$\Delta\phi = \frac{\pi}{2}$ (Due to P being ahead of Q by 90°)

$I_B = I + I + 2\sqrt{I}\sqrt{I}\cos\frac{\pi}{2} = 2I$

$I_A : I_B : I_C = 4I : 2I : 0$

$= 2 : 1 : 0$

$\therefore$ correct option is (4)

15. Official Ans. by NTA (9.00)

Sol. $I_{max} = k$

$I_1 = I_2 = K/4$

$\Delta x = \lambda/6 \quad \Rightarrow \Delta\phi = \pi/3$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$I = \frac{K}{4} + \frac{K}{4} + 2\times\frac{K}{4}\frac{1}{2}$

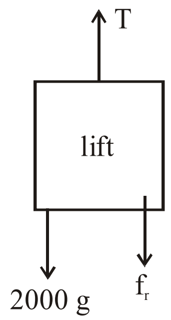
$= \frac{K}{2} + \frac{K}{4} = \frac{3K}{4} = \frac{9K}{12}$

$n = 9$

WORK POWER ENERGY

1. **NTA Ans. (3)**

Sol.



Let elevator is moving upward with constant speed V.

Tension in cable

T = 2000 g + f_r = 2000 + 4000

T = 24000 N

Power P = TV

$\Rightarrow$ 60 × 746 = (24000) V

$V = \frac{60 \times 746}{24000} = 1.865 \approx 1.9$ m/s.

2. **NTA Ans. (10)**

Sol. Mechanical energy conservation between A & P

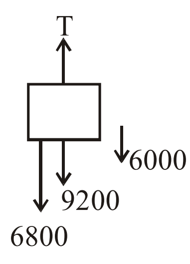
$U_1 + \cancel{K_1}^{\,0} = K_2 + U_2$

mg × 2 = mg × 1 + K_2

K_2 = mg × 1 = 10 J.

3. **NTA Ans. (3)**

Sol.



elevater moving with constant speed hence

T = 6800 + 9200 + 6000

T = 22000 N

power = T·v = 22000 × 3

= 66000 W

4. **NTA Ans. (2)**

Sol. $W = \int_{\vec{r}_i}^{\vec{r}_g} \vec{F} \cdot d\vec{r}$

$W = \int_1^0 -x\,dx + \int_0^1 y\,dy$

$W = \left.\frac{-x^2}{2}\right|_1^0 + \left.\frac{y^2}{2}\right|_0^1$

$= -\left(\frac{0^2}{2} - \frac{1^2}{2}\right) + \left(\frac{1^2}{2} - \frac{0^2}{2}\right)$

W = 1J

5. **Official Ans. by NTA (150)**

Sol. $W_F = \frac{1}{2}mv^2 = mgh$

F(S) = mgh

F(0.2) = (0.15) (10) (20)

$\boxed{F = 150N}$

6. **Official Ans. by NTA (3)**

Sol. $\frac{dK}{dE} = P = \cos t \Rightarrow K = Pt = \frac{1}{2}mV^2$

$\therefore\ V = \sqrt{\frac{2Pt}{m}} = \frac{ds}{dt} \therefore S = \sqrt{\frac{2P}{m}}\frac{2}{3}t^{\frac{3}{2}}$

7. **Official Ans. by NTA (2)**

Sol. F = 200 N for $0 \le x \le 15$

$= 200 - \frac{100}{15}(x - 15)$ for $15 \le x < 30$

$W = \int F\,dx$

$= \int_0^{15} 200\,dx + \int_{15}^{30}\left(300 - \frac{100}{15}x\right)dx$

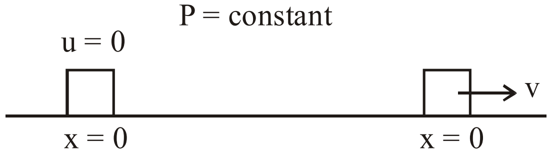
$= 200 \times 15 + 300 \times 15 - \frac{100}{15} \times \frac{(30^2 - 15^2)}{2}$

= 3000 + 4500 – 2250

= 5250 J

8. Official Ans. by NTA (18.00)

Sol.



$$P = mav$$

$$m\frac{dv}{dt}v = P$$

$$\int_0^v v\,dv = \frac{P}{m}\int_0^t dt$$

$$\frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$\frac{dx}{dt} = \sqrt{\frac{2P}{m}}\,t^{1/2}$$

$$\int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt$$

$$x = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{3/2} = \sqrt{\frac{2P}{m}} \times \frac{2}{3}t^{3/2}$$

$$= \sqrt{\frac{2\times 1}{2}} \times \frac{2}{3} \times 9^{3/2}$$

$$= \frac{2}{3} \times 27 = 18$$

9. Official Ans. by NTA (3)

Sol. $U = \frac{-A}{r^6} + \frac{B}{r^{12}}$

$$F = -\frac{dU}{dr} = -\left(A\left(-6r^{-7}\right)\right) + B\left(-12r^{-13}\right)$$

$$0 = \frac{6A}{r^7} - \frac{12B}{r^{13}}$$

$$\frac{6A}{12B} = \frac{1}{r^6} \Rightarrow r = \left(\frac{2B}{A}\right)^{1/6}$$

$$U\left(r = \left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A}{2B/A} + \frac{B}{4B^2/A^2}$$

$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \frac{-A^2}{4B}$$

10. Official Ans. by NTA (3)

Sol. $\frac{dv_x}{dt} = \frac{k}{m}v_y$

$$\frac{dv_y}{dt} = \frac{k}{m}v_x$$

$$\frac{dv_y}{dv_x} = \frac{v_x}{v_y} \Rightarrow \int v_y dv_y = \int v_x dv_x$$

$$v_y^2 = v_x^2 + C$$

$$v_y^2 - v_x^2 = \text{constant}$$

Option (3)

$$\vec{v} \times \vec{a} = (v_x\hat{i} + v_y\hat{j}) \times \frac{k}{m}(v_y\hat{i} + v_x\hat{j})$$

$$= (v_x^2\hat{k} - v_y^2\hat{k})\frac{k}{m}$$

$$= (v_x^2 - v_y^2)\frac{k}{m}\hat{k}$$

= Constant

IMPORTANT NOTES

Chapter Contents 02

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & SEPTEMBER 2020

CHEMISTRY

PHYSICAL CHEMISTRY

ORGANIC CHEMISTRY

INORGANIC CHEMISTRY

JANUARY & SEPTEMBER 2020 ATTEMPT (PC)

MOLE CONCEPT

1. NTA Ans. (3)

Sol. Option(3) is according to Gaylussac's law of volume combination.

2. NTA Ans. (3)

Sol. $NH_2CONH_2 + 2NaOH \rightarrow Na_2CO_3 + 2NH_3$

10 mmoles 20mmoles

Hence, NH_3 will require 20 meq.

3. NTA Ans. (4.95 to 4.97)

Sol. $FeSO_4.7H_2O$ (M = 277.85)

$$\text{ppm} = \frac{\text{wt. of Fe}}{\text{wt. of wheat}} \times 10^6$$

let the wt. of salt be = w gm

$$\text{moles} = \frac{w}{277.85}$$

$$\text{wt. of Fe} = \left(\frac{W}{277.85} \times 55.85\right)\text{gm}$$

$$10 = \frac{\frac{W}{277.85} \times 55.85}{10^5} \times 10^6$$

$$W = \frac{277.85}{55.85} = 4.97$$

4. NTA Ans. (2120 to 2140)

Sol. Mole of O_2 consumed $= \frac{1 \times 492}{0.082 \times 300} = 20$

Mole of $NaClO_3$ required = 20

Mass of $NaClO_3$ = 20 × 106.5 = 2130 gm

5. NTA Ans. (1)

Sol. IE values indicate, that the metal belongs to I[st] group since second IE is very high

($\because$ only one valence electron)

Metal hydroxide will be of type, MOH.

$MOH + HCl \rightarrow MCl + H_2O$

(1mol) (1mol)

$$MOH + \frac{1}{2}H_2SO_4 \rightarrow \frac{1}{2}M_2SO_4 + H_2O$$

(1mol) ($\frac{1}{2}$ mol)

So one mole of HCl required to react with one mole MOH.

So $\frac{1}{2}$ mole of H_2SO_4 required to react with one mole MOH.

6. NTA Ans. (4)

Sol. $Zn + 2HCl \longrightarrow ZnCl_2 + H_2$

$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2$

The ratio of the volume of H_2 is 1 : 1

7. Official Ans. by NTA (18)

Sol. $\underset{1\,mole}{C_3H_8} + \underset{5\,mole}{5O_2} \rightarrow 3CO_2 + 4H_2O$

For 1 mole propane combustion 5 mole O_2 required

$$C_4H_{10} + \frac{13}{2}O_2 \rightarrow 4CO_2 + 5H_2O$$

1 mole 6.5 mole

2 mole 13 mole

For 2 moles of butane 13 mole of O_2 is required

total moles = 13 + 5 = 18

8. Official Ans. by NTA (5.00)

Sol. C : H = 4 : 1

C : O = 3 : 4

Mass ratio

C : H : O = 12 : 3 : 16

Mole ratio

C : H : O = 1 : 3 : 1

Empirical formula = CH_3O

Molecular formula = $C_2H_6O_2$

(saturated acyclic organic compound)

$$C_2H_6O_2 + \frac{5}{2}O_2 \longrightarrow 2CO_2 + 3H_2O$$

2 mole 5 mol

Moles of O_2 required = 5 moles

9. Official Ans. by NTA (50.00)

CONCENTRATION TERMS

1. NTA Ans. (14.00)

Sol. 100 gm soln → 63 gm HNO_3

$\frac{100}{1.4}$ mL → 1 mole HNO_3

$$\text{Molarity} = \frac{1}{\frac{100}{1.4} \times \frac{1}{1000}} = 14M$$

2. NTA Ans. (10)

Sol. ppm = $\dfrac{10.3\times10^{-3}}{1030}\times10^{6}=10$

3. Official Ans. by NTA (100)

Sol. Volume strength of H_2O_2 at 1 atm

273 kelvin = M × 11.2 = 8.9 × 11.2 = 99.68

Ans : 100

4. Official Ans. by NTA (47)

Sol. $X_{C_6H_{12}O_6} = 0.1$

Let total mole is 1 mol then mole of glucose will be 0.1 and mole of water will be 0.9

so mass % of water = $\dfrac{0.9\times18}{0.1\times180+0.9\times18}\times100$

= 47.36

Ans : 47

5. Official Ans. by NTA (25)

Sol. moles = $\dfrac{\text{number of molecules}}{6\times10^{23}}=\dfrac{\text{given mass}}{\text{molar mass}}$

$\Rightarrow$ molar mass = $\dfrac{10\times6.023\times10^{23}}{6.023\times10^{22}}=100\text{g / mol}$

$\Rightarrow$ molarity = $\dfrac{\text{moles of solute}}{\text{volume of sol}^{n}(\ell)}=\dfrac{(5/100)}{2}$ = 0.025

6. Official Ans. by NTA (2)

Sol. Volume strength = 11.2 × molarity

$\Rightarrow$ molarity = $\dfrac{5.6}{11.2}=0.5$

Assuming 1 litre solution;

mass of solution = 1000 ml × 1 g/ml = 1000 g

mass of solute = moles × molar mass

= 0.5 mol × 34 g/mol

= 17 gm.

$\Rightarrow$ mass% = $\dfrac{17}{1000}\times100=1.7\%$

7. Official Ans. by NTA (4)

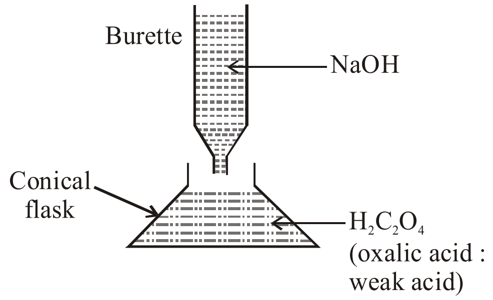
REDOX REACTIONS

1. NTA Ans. (3)

Sol. Potasisum has an oxidation of +1 (only) in combined state.

2. NTA Ans. (4)

Sol.



3. NTA Ans. (4)

Sol. (i) H_2O_2 act as oxidising agent as well as reducing agent depending on condition.

(ii) H_2SO_3 act as oxidising agent as well as reducing agent depending on condition.

(iii) HNO_2 act as oxidising agent as well as reducing agent depending on condition.

(iv) H_3PO_4 can not act both as oxidising and reducing agent.

H_3PO_4 can act as only oxidising agent.

$H_3PO_4 \rightleftharpoons 3H^+ + PO_4^{3-}$

4. NTA Ans. (100)

Sol. 1 Litre has 10^{-3} moles $MgSO_4$

So , 1000 litre has 1 mole $MgSO_4$

= 1 mole $CaCO_3$

= 100 ppm

5. Official Ans. by NTA (19)

Sol. $\left[Fe^{2+}\rightarrow Fe^{3+}+e^{-}\right]\times2$

$$\frac{H_2O_2+2e^-\rightarrow 2HO^{\ominus}}{2Fe^{2+}+H_2O_2\rightarrow 2Fe^{3+}+2HO^{\ominus}_{(aq)}}$$

x = 2 y = 2

$\left[8H^+ + MnO_4^- + 5e^- \rightarrow Mn^{2+} + 4H_2O\right]\times2$

$\left[H_2O_2 \rightarrow O_{2(g)} + 2H^+ + 2e^-\right]\times5$

$\Rightarrow 16H^+ + 2MnO_4^- + 5H_2O_2$

$\rightarrow 2Mn^{2+} + 8H_2O + 5O_{2(g)} + 10H^+$

$\Rightarrow 6H^+ + 2MnO_4^- + 5H_2O_2$

$\rightarrow 2Mn^{2+} + 8H_2O + 5O_{2(g)}$

So x' = 2 y' = 8 z' = 5

so x + y + x' + y' + z'

$\Rightarrow$ 2 + 2 + 2 + 8 + 5

$\Rightarrow$ 19

6. Official Ans. by NTA (10)

Sol. Molar mass of $Na_2CO_3 \cdot xH_2O$

$\Rightarrow 23 \times 2 + 12 + 48 + 18x$

$\Rightarrow 46 + 12 + 48 + 18x$

$\Rightarrow (106 + 18x)$

$$\text{Eqwt} = \frac{M}{2} = (53 + 9x)$$

As n_{factor} in dissolution will be determined from net cationic or anionic charge; which is 2 so

$$\text{Eqwt} = \frac{M}{2} = 53 + 9x$$

$$\text{Gmeq} = \frac{\text{wt}}{\text{Eqwt}} = \frac{1.43}{53 + 9x}$$

$$\text{Normality} = \frac{\text{Gmeq}}{V_{\text{litre}}}$$

$$\text{Normality} = 0.1 = \frac{\frac{1.43}{53+9x}}{0.1}$$

As volume = 100 ml

= 0.1 Litre

$$\text{So} \quad 10^{-2} = \frac{1.43}{53 + 9x}$$

53 + 9x = 143

9x = 90

x = 10.00

7. Official Ans. by NTA (85)

Sol. Eq of H_2O_2 = Eq of $KMnO_4$

$$x \times 2 = \frac{0.316}{158} \times 5$$

$x = 5 \times 10^{-3}$ mol

$$m_{H_2O_2} = 5 \times 10^{-3} \times 34 = 0.17\text{gm}$$

$$\%H_2O_2 = \frac{0.17}{0.2} \times 100 = 85$$

8. Official Ans. by NTA (10)

Sol. $H_3PO_2 + NaOH \rightarrow NaH_2PO_2 + H_2O$

$$\frac{n_{H_3PO_2}\text{ reacted}}{1} = \frac{n_{NaOH}\text{ reacted}}{1}$$

$$\Rightarrow \frac{0.1 \times 10}{1} = 0.1 \times V_{NaOH}$$

$\Rightarrow V_{NaOH} = 10$ ml.

9. Official Ans. by NTA (50.00)

Sol. $K_2Cr_2O_7 + FeC_2O_4 \longrightarrow Cr^{+3} + Fe^{+3} + CO_2$

n = 6   n = 3

$$\frac{0.02 \times 6 \times V(mL)}{1000} = \frac{0.288}{144} \times 3$$

$$\Rightarrow \boxed{V = 50mL}$$

10. Official Ans. by NTA (19.00)

Sol. $K_2Cr_2O_7$

2 (+1) + 2x + 7(–2) = 0

x = +6

In $K_2Cr_2O_7$, Transition metal (Cr) present in +6 oxidation state.

$KMnO_4$

(+1) + y + 4(–2) = 0

x = +7

In $KMnO_4$, transition metal (Mn) present in +7 oxidation state

K_2FeO_4

2 (+1) + z + 4(–2) = 0

x = +6

In K_2FeO_4, transition metal (Fe) present in +6 oxidation state

So, x = +6

y = +7

z = +6

x + y + z = 19

IDEAL GAS

1. NTA Ans. (4)

Sol. $V_{mp}\left(=\sqrt{\frac{2RT}{M}}\right) < V_{av}\left(=\sqrt{\frac{8RT}{\pi M}}\right) < V_{rms}\left(=\sqrt{\frac{3RT}{M}}\right)$

2. Official Ans. by NTA (3)

Sol. According to Dalton's law of partial pressure

$p_i = x_i \times P_T$

p_i = partial pressure of the i^{th} component

x_i = mole fraction of the i^{th} component

p_T = total pressure of mixture

$$\Rightarrow 2\text{ atm} = \left(\frac{n_{H_2}}{n_{H_2}+n_{H_e}+n_{O_2}}\right)\times p_T$$

$$\Rightarrow p_T = 2\text{ atm} \times \frac{3}{1} = 6\text{ atm}$$

3. Official Ans. by NTA (1)

Sol. $PM = dRT \Rightarrow d \propto \frac{1}{T}$

4. Official Ans. by NTA (750.00)

ATOMIC STRUCTURE

1. NTA Ans. (2)

Sol. No. of orbitals = $n^2 = 5^2 = 25$

For n = 5, no. of orbitals = n^2 = 25

Total number of orbitals is equal to no. of electrons having $m_s = \frac{1}{2}$

2. NTA Ans. (2)

Sol. For balmer : $n_1 = 2$, $n_2 = 3, 4, 5, \ldots \infty$

$$\bar{\nu} = \frac{1}{\lambda} = R_H\left[\frac{1}{2^2}-\frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda_{longest}} = R_H\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$$

Ans.(2)

3. NTA Ans. (4)

Sol. $r_n = \frac{n^2 \times a_0}{z}$

For 2^{nd} Bohr orbit of Li^{+2}

n = 2

z = 3

$$\Rightarrow r_n = \frac{2^2 \times a_0}{3} = \frac{4a_0}{3}$$

4. NTA Ans. (1)

Sol. $2\pi r = n\lambda$

for $n = 1, r = a_0$

$n = 4, r = 16a_0$

So, $2\pi \times 16a_0 = 4 \times \lambda$

$\lambda = 8\pi a_0$

5. Official Ans. by NTA (3)

Sol. As we know $\Delta E = \frac{hc}{\lambda}$

So $\lambda = \frac{hc}{\Delta E}$ for λ minimum i.e.

shortest; ΔE = maximum

for Lyman series n = 1 & for ΔE_{max}

Transition must be form $n = \infty$ to n = 1

$$\text{So } \frac{1}{\lambda} = R_H Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = R_H Z^2(1-0)$$

$$\frac{1}{\lambda} = R \times (1)^2 \Rightarrow \lambda_1 = \frac{1}{R}$$

For longest wavelength ΔE = minimum for Balmer series n = 3 to n = 2 will have ΔE minimum

for He^+ Z = 2

$$\text{So } \frac{1}{\lambda_2} = R_H \times Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda_2} = R_H \times 4\left(\frac{1}{4}-\frac{1}{9}\right)$$

$$\frac{1}{\lambda_2} = R_H \times \frac{5}{9}$$

$$\lambda_2 = \lambda_1 \times \frac{9}{5}$$

6. **Official Ans. by NTA (4)**

Sol. $\frac{\Delta R_1}{\Delta R_2}=\frac{(r_4-r_3)_{4^{2+}}}{(r_4-r_3)_{He^+}}=\frac{\frac{4^2}{3}-\frac{3^2}{3}}{\frac{4^2}{2}-\frac{3^2}{2}}=\frac{7/3}{7/2}=\frac{2}{3}$

7. **Official Ans. by NTA (1)**

8. **Official Ans. by NTA (222.00)**

Sol. $E = W + K{\cdot}E_{max}$

$K{\cdot}E_{max} = E - W$

$= \frac{hc}{\lambda} - 4.41 \times 10^{-19}$

$= \frac{6.63\times10^{-34}\times3\times10^{8}}{300\times10^{-9}} - 4.41 \times 10^{-19}$

$= 2.22 \times 10^{-19}$ J

$= 222 \times 10^{-21}$ J

CHEMICAL EQUILIBRIUM

1. **NTA Ans. (1)**

ALLEN Ans. (1 or Bonus)

Sol. Bonus (no reaction is given)

$A \rightleftharpoons B$ (Assume reaction)

$K = \frac{[B]}{[A]}=\frac{11}{6}\simeq 2$

2. **Official Ans. by NTA (1)**

Sol. $\Delta H^o > 0$ T↓ equation shifts back ward.

N_2 is treated as inert gas in this case hence no effect on equilibrium.

3. **Official Ans. by NTA (2)**

Sol. $N_2 + 3H_2 \rightleftharpoons 2NH_3 \rightarrow K_C = 64$

$2NH_3 \rightleftharpoons N_2 + 3H_2 \rightarrow K_C = \frac{1}{64}$

$NH_3 \rightleftharpoons \frac{1}{2}N_2 + \frac{3}{2}H_2 \rightarrow K_C = \left(\frac{1}{64}\right)^{1/2} = \frac{1}{8}$

IONIC EQUILIBRIUM

1. **NTA Ans. (10.60)**

Sol. 4 gm of NaOH in 100 L sol. ⇒ 10^{-3} M sol.

9.8 gm of H_2SO_4 in 100 L sol. ⇒ 10^{-3} M sol.

Mixture : 40L of 10^{-3} M NaOH and 10 L of 10^{-3} M H_2SO_4 sol.

Final Conc. of OH^-

$= \frac{10^{-3}(40\times1-10\times1\times2)}{40+10}=6\times10^{-4}M$

pOH = – log (6×10^{-4})

= 4 – log 6 = 4 – 0.60 = 3.40

pH = 14 – 3.40 = 10.60

2. **NTA Ans. (5.22 to 5.24)**

Sol. 3gm Acetic Acid + 250 ml 0.1 M HCl + Water

⟶ made to 500 ml solution.

⇒ 500 ml solution has 25 meq of HCl

50 meq of CH_3COOH

∴ 20ml solution has 1 meq of HCl

2 meq of CH_3COOH

We have added 2.5 meq. of NaOH $\left(5M, \frac{1}{2}ml\right)$

Finally , NaOH & HCl are completely consumed and we are left with 0.5 meq of CH_3COOH

and 1.5 meq of CH_3 COONa

$pH = pKa + \log\frac{1.5}{0.5}$

$= 4.75 + \log 3 = 4.75 + 0.4771$

$= 5.2271$

3. **NTA Ans. (3)**

Sol. From the graph & dimensions salt is : XY_2

$[X] = 1 \times 10^{-3}M$

$[Y] = 2 \times 10^{-3}M$

$XY_2(s) \rightleftharpoons X^{2+}_{(aq.)} + 2Y^-_{(aq.)}$

ksp = $[X^{2+}][Y^-]^2$

= $(10^{-3})(2 \times 10^{-3})^2$

= 4×10^{-9} M^3

4. **NTA Ans. (2)**

Sol. $H_2O(\ell) \rightleftharpoons H^+_{(aq)} + OH^-_{(aq)}$

For ionization of H_2O : $\Delta H > O$

$\Rightarrow$ ENDOTHERMIC

On temperature increase reaction shifts forward

$\Rightarrow$ both $[H^+]$ and $[OH^-]$ increase

$\Rightarrow$ pH & pOH decreases.

5. **NTA Ans. (2)**

Sol. $[Pb^{2+}] = \frac{300 \times 0.134}{400} = 1.005 \times 10^{-1}$ M

$[Cl^-] = \frac{100 \times 0.4}{400} = 10^{-1}$ M

$PbCl_{2(s)} \rightleftharpoons Pb^{+2}_{(aq.)} + 2Cl^-_{(aq.)}$

$Q = [Pb^{2+}] \times [Cl^-]^2 = 1.005 \times 10^{-3} > k_{sp}$

6. **NTA Ans. (1)**

Sol. $Cr(OH)_3(s) \rightleftharpoons Cr^{3+}(aq.) + 3OH^-(aq.)$

(s) (3s)

$k_{sp} = 27(s)^4 = 6 \times 10^{-31}$

$\Rightarrow [3(s)]^4 = 18 \times 10^{-31}$

$[OH^-] = 3(s) = [18 \times 10^{-31}]^{1/4}$

7. **Official Ans. by NTA (3)**

Sol. $\underset{\substack{10\text{ milli mol}\\ -}}{HCl} + \underset{\substack{20\text{ milli mol}\\ 10\text{ milli mol}}}{CH_3COONa} \rightarrow \underset{\substack{-\\ 10\text{ milli mol}}}{CH_3COOH} + \underset{\substack{-\\ 10\text{ milli mol}}}{NaCl}$

So finally we get mixture of

$CH_3COOH + CH_3COONa$ that will work like acidic buffer solution.

8. **Official Ans. by NTA (3)**

Sol. Steep rise in pH around the equivalence point for titration of strong acid with strong base.

9. **Official Ans. by NTA (37)**

Sol. $P_{CO_2} = K_H \times CO_2$

$\frac{3}{30} = \frac{K_H.n_{CO_2}}{K_H 1} \Rightarrow n_{CO_2=0.1}$mol

$pH = \frac{1}{2}(pka_1 - \log c) = \frac{1}{2}(6.4 \times 1) = 3.7$

$pH = 37 \times 10^{-1}$

10. **Official Ans. by NTA (2.00)**

Sol. $AB_2(s) \rightleftharpoons \underset{S}{A^{+2}_{(aq.)}} + \underset{2S}{2B^-_{(aq.)}} : K_{sp}$

$K_{SP} = S^1 \times (2s)^2 = 4s^3$

$3.2 \times 10^{-11} = 4 \times S^3$

$S = 2 \times 10^{-4}$ M/L

11. **Official Ans. by NTA (4)**

THERMODYNAMICS

1. **NTA Ans. (–2.70 to –2.71)**

Sol. $A(\ell) \longrightarrow 2B(g)$

$\Delta U = 2.1$ Kcal , $\Delta S = 20$ cal K^{-1} at 300 K

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta G = \Delta H - T\Delta S$

$\Delta G = \Delta U + \Delta n_g RT - T\Delta S$

$= 2.1 + \frac{2 \times 2 \times 300}{1000} - \frac{300 \times 20}{1000}$

($R = 2$ cal K^{-1} mol^{-1})

$= 2.1 + 1.2 - 6 = -2.70$ Kcal/mol

2. **NTA Ans. (48.00)**

Sol. Area enclosed under

P V curve = 48 = 48 Joule

3. **NTA Ans. (6.25)**

Sol. For ideal gas :

$\Delta U = nC_V[T_2 - T_1]$

$\Rightarrow$ $5000 = 4 \times C_V[500 - 300]$

$\Rightarrow$ $C_V = \frac{5000}{800} = 6.25$ J $mole^{-1}$ K^{-1}

4. **NTA Ans. (1)**

Sol. $ds = \int \frac{q_{rev.}}{T}$

5. **NTA Ans. (2.17 to 2.23)**

Sol. $0 - T_f' = 2 \times 0.5 = 1$

$T_f' = -1°C = 272$ K

for gas $P = \frac{0.1 \times 0.08 \times 272}{1}$

P = 2.176 atm

$P_1V_1 = P_2V_2$

$2.176 \times 1 = 1 \times V_2$

$V_2 = 2.176$ litre

6. **Official Ans. by NTA (4)**

Sol. As the expansion is done in vaccum that is in absence of p_{ext} so

W = zero

7. **Official Ans. by NTA (1)**

Sol. For ideal Gas

U = f(T), H = f(T)

Z = 1

$C_P - C_V = R$

$dU = C_V\, dT$

8. Official Ans. by NTA (–13538.00)
Official Ans. by ALLEN (–13537.57)

Sol. $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$= (\Delta U^\circ + \Delta n_g RT) - T\Delta S^\circ$

$$= \left[\left\{-20 + (-1))\frac{8.314}{1000} \times 298\right\} - \frac{298}{1000} \times (-30)\right] kJ$$

$= -13.537572$ kJ

$= -13537.57$ Joule

9. Official Ans. by NTA (189494.00)
Official Ans. by ALLEN (189494.39)

Sol. $H_2O(\ell) \rightleftharpoons H_2O(g)$ 90 gm of H_2O

$\Delta H = \Delta U + \Delta n_g RT$ $\Rightarrow$ 5 moles of H_2O

$5 \times 41000 \text{ J} = \Delta U + 1 \times 8.314 \times 373 \times 5$

$\Delta U = 189494.39$ Joule

10. Official Ans. by NTA (96500.00)

Sol. $\Delta G = \Delta G^o + RT \ln \left[\frac{Sn^{+2}}{Cu^{+2}}\right]$

$= -2 \times 96500 [(-0.16) - 0.34] + RT \ln \left(\frac{1}{1}\right)$

$= 96500$ J

11. Official Ans. by NTA (3)

THERMOCHEMISTRY

1. NTA Ans. (–192.50 or –85.00)

Sol. $2C(\text{graphite}) + 3H_2(g) \longrightarrow C_2H_6(g)$

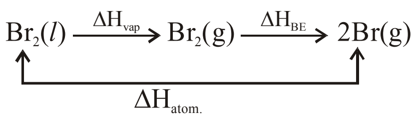
$\Delta_f H\,(C_2H_6) = 2\Delta H_{comb}\,(C_{graphite}) + 3\,\Delta H_{comb}(H_2) - \Delta H_{comb}\,(C_2H_6)$

$= -(286 \times 2) - (393.5 \times 3) - (-1560)$

$= -572 - 1180.5 + 1560 = -192.5$ kJ/mole

2. NTA Ans. (4)

Sol. Enthalpy of atomisation of $Br_2(l)$



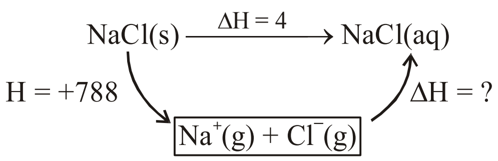
$\Delta H_{atom} = \Delta H_{vap} + \Delta H_{BE}$

$x = \Delta H_{vap} + y$

So, $x > y$

3. Official Ans. by NTA (2)

Sol.



$4 = 788 + \Delta H$

$\Delta H = -784$ kJ

4. Official Ans. by NTA (–326400.00)
Official Ans. by ALLEN (326400.00)

Sol. $C_2H_5OH_{(\ell)} + 3O_{2(g)} \longrightarrow 2CO_{2(g)} + 3H_2O_{(\ell)}$

$\Delta n_g = 2 - 3 = -1$

$\Delta_c H = \Delta_c U + (\Delta n_g)\,RT$

$\Delta_c H = \Delta_c U - RT$

$\Delta_c U = \Delta_c H + RT$

$= -327 \times 10^3 + 2 \times 300$

$= -326400$ cal.

$\therefore$ Heat evolved

$= 326400$ cal.

SOLID STATE

1. NTA Ans. (1)

Sol. Since AgBr has intermediate radius ratio

$\therefore$ it shows both schottky & Frenkel defects

ZnS $\rightarrow$ Frenkel defects

KBr, CsCl $\rightarrow$ Schottky defects

2. Official Ans. by NTA (143)

Sol. $d = \dfrac{z\left(\dfrac{M}{N_A}\right)}{a^3}$

$$2.7 \times 10^3 = z\frac{\left(\dfrac{2.7 \times 10^{-2}}{6 \times 10^{23}}\right)}{\left(405 \times 10^{-12}\right)^3}$$

$$2.7 \times 10^3 = z\frac{\left(2.7 \times 10^{-2}\right)}{6 \times 10^{23}\left(4.05 \times 10^{-10}\right)^3}$$

$$2.7 \times 10^3 = z\frac{\left(2.7 \times 10^{-2}\right)}{6 \times 10^{23} \times 66.43 \times 10^{-30}}$$

$3.98 = z$

$z \approx 4$ structure is fcc

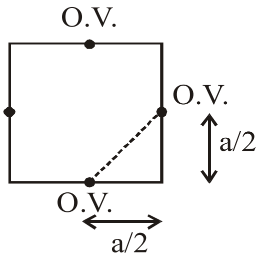
$\frac{a}{\sqrt{2}} = 2r$

$$r = \frac{a}{2\sqrt{2}} = \frac{\sqrt{2}a}{4} = \frac{1.414 \times 405 \times 10^{-12}}{4}$$

$r = 143.16 \times 10^{-12}$

3. Official Ans. by NTA (3)

Sol.



distance between nearest octahedral voids(O.V.)

$$= \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} \quad \Rightarrow \quad = \frac{a}{\sqrt{2}}$$

4. Official Ans. by NTA (1)

Sol. O^{-2} ions form ccp. O_4 ↓ (–8 charge)

M_1 = 50% of O.V. $\Rightarrow \frac{50}{100} \times 4 = 2 : (M_1)_2$

M_2 = 12.5% of T.V. $\Rightarrow \frac{12.5}{100} \times 8 = 1 : (M_2)_1$

So formula is : $(M_1)_2\ (M_2)_1 O_4$

This must be neutral. Both metals must have +8 charge in total.

From given options : $\left\{\begin{array}{r} \text{O.N. of } M_1 = +2 \\ M_2 = +4 \end{array}\right\}$

CHEMICAL KINETICS

1. NTA Ans. (4)

Sol. $$K_{eq} = \frac{k_f}{k_b} = \frac{[N_2][H_2O]^2}{[H_2]^2[NO]^2}$$

At equilibrium $r_f = r_b$

$$k_f \underbrace{[H_2][NO]^2}_{[\text{Given}]} = k_b \frac{[N_2][H_2O]^2}{[H_2]}$$

Hence, rate expression for reverse reaction.

$$= k_b \frac{[N_2][H_2O]^2}{[H_2]}$$

2. NTA Ans. (4)

Sol. $K = Ae^{\frac{-E_a}{RT}}$

$K' = Ae^{\frac{-E_a'}{RT}} = 10^6 K$

$Ae^{\frac{-E'}{RT}} = 10^6 \times Ae^{\frac{-E_a}{RT}}$

$\frac{-E_a'}{RT} = \frac{-E_a}{RT} + \ln 10^6$

$E_a' = E_a - RT \ln 10^6$

$E_a' - E_a = - RT \ln 10^6 = - 6RT \times 2.303$

3. NTA Ans. (3)

Sol. $\log K = \frac{-Ea}{2.303RT} + \log A$

Acrroding to Arrhenius equation plot of 'log K' Vs. $\frac{1}{T}$ is linear with.

Slope = $\frac{-Ea}{2.303R}$

From plot we conclude :

$\underset{(\text{magnitude})}{|\text{slope}|}$: c > a > d > b

$\therefore\ E_c > E_a > E_d > E_b$

4. NTA Ans. (4)

Sol. $K_1 = Ae^{-\frac{Ea}{R \times 700}}$

$K_2 = A \times e^{-\frac{(Ea-30)}{R \times 500}}$

For same rate

$K_1 = K_2$

$e^{-\frac{Ea}{700R}} = e^{-\frac{(Ea-30)}{R \times 500}}$

$\frac{Ea}{700R} = \frac{Ea - 30}{R \times 500}$

5Ea = 7Ea – 210

210 = 2Ea

E_a = 105 kJ/mole

E_a – 30 = 75

5. NTA Ans. (3.98 to 4.00 or –3.98 to –4.00)

Sol. $\ln\left(\frac{t_1}{t_2}\right) = \frac{-Ea}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$

$\ln\left(\frac{60}{40}\right) = \frac{-Ea}{8.3}\left[\frac{1}{400} - \frac{1}{300}\right]$

E = 0.4 × 1200 × 8.3

= 3.984 kJ/mole

6. Official Ans. by NTA (84297)

Official Ans. by ALLEN (84297.47 or 84297.48)

Sol. T_1 = 300K T_2 = 315K

As per question $K_{T_2} = 5K_{T_1}$ as molecules activated are increased five times so k will increases 5 times

Now

$$\ln\left(\frac{K_{T_2}}{K_{T_1}}\right) = \frac{Ea}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\ln 5 = \frac{Ea}{R}\left(\frac{15}{300 \times 315}\right)$$

So $Ea = \dfrac{1.6094 \times 8.314 \times 300 \times 315}{15}$

Ea = 84297.47 Joules/mole

7. Official Ans. by NTA (60)

Sol. $t_{0.75} = 2 \times \dfrac{\ln 2}{k} = 90$

$$k = \frac{\ln 2}{45}\min^{-1}$$

$$kt = \ln\frac{1}{1-0.6} = \ln 2.5$$

$$\frac{\ln 2}{45} \times t = \ln 2.5$$

$$t = 45 \times \frac{\log 2.5}{\log 2} = 45 \times \frac{0.4}{0.3} = 60\,\text{min}$$

8. Official Ans. by NTA (4)

Sol. Zero order reaction is multiple step reaction.

9. Official Ans. by NTA (4)

Sol. For aA + bB → cC;

$$\frac{-1}{a}\frac{d[A]}{dt} = \frac{-1}{b}\frac{d[B]}{dt} = \frac{1}{c}\frac{d[C]}{dt}$$

$$\therefore \frac{-1}{2}\frac{d[A]}{dt} = \frac{-1}{3}\frac{d[B]}{dt} = \frac{-2}{3}\frac{d[C]}{dt} = \frac{1}{3}\frac{d[P]}{dt}$$

10. Official Ans. by NTA (4)

Sol. $[A]_t = 4[B]_t$

$$[A]_0\ e^{-(\ln^2/300)^t} = 4[B]_0 e^{(-\ln 2/180)t}$$

$$e^{\left(\frac{\ln^2}{180} - \frac{\ln^2}{300}\right)} = 4$$

$$\left(\frac{\ln^2}{180} - \frac{\ln^2}{300}\right)t = \ln 4$$

$$\left(\frac{1}{180} - \frac{1}{300}\right)t = 2 \Rightarrow t = \frac{2 \times 180 \times 300}{120} = 900\,\text{sec}.$$

11. Official Ans. by NTA (1)

Sol. Slope = $-\dfrac{E_a}{R}$

$$-\frac{10}{5} = -\frac{E_a}{R}$$

E_a = 2R

12. Official Ans. by NTA (1)

Sol. From rate law

$$r = -\frac{1}{2}\frac{d[A]}{dt} = \frac{-d[B]}{dt}$$

$= K[A]^x [B]^y$

$6 \times 10^{-3} = K(0.1)^x (0.1)^y$ ……(1)

$2.4 \times 10^{-2} = K(0.1)^x (0.2)^y$ ……(2)

$1.2 \times 10^{-2} = K(0.2)^x (0.1)^y$ ……(3)

(3) ÷ (1) ⇒ x = 1

(2) ÷ (3) ⇒ x = 2

So, order with respect to A = 1

Order with respect to B = 2

(4) ÷ (3)

$$\left(\frac{x}{0.2}\right) \times \left(\frac{0.2}{0.1}\right)^2 = \frac{7.2 \times 10^{-2}}{1.2 \times 10^{-2}}$$

$$x = \frac{6 \times 0.2}{4}$$

x = 0.3 M

(5) ÷ (4)

$$\left(\frac{y}{0.2}\right)^2 = \frac{2.88 \times 10^{-1}}{7.2 \times 10^{-2}}$$

$y^2 = 4 \times 0.2^2$

y = 0.4 M

13. Official Ans. by NTA (3)

14. Official Ans. by NTA (100.00)

Official Ans. by ALLEN (99.98)

Sol. $\ell n\left(\frac{K_{T_2}}{K_{T_1}}\right)=\frac{E_a}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$

$T_1 = 303$ K ; $T_2 = 313$ K

$\frac{K_{T_2}}{K_{T_1}}=3.555$

$\ell n(3.555)=\frac{E_a}{8.314}\left[\frac{1}{303}-\frac{1}{313}\right]$

$E_a = 99980.715$

$E_a = 99.98 \frac{\text{kJ}}{\text{mole}}$

RADIOACTIVITY

1. NTA Ans. (23 to 23.03)

Sol. All nuclear decays follow first order kinetics

$t = \frac{1}{k}\ell n\frac{[A_0]}{[A]}$

$= \frac{(t_{1/2})}{0.693}\times 2.303 \;\; \log_{10}10 \;\; = 10 \times 2.303 \times 1$

$= 23.03$ years

SURFACE CHEMISTRY

1. NTA Ans. (0.36 to 0.38)

Sol. 1 L solution requires 30 m.mol HCl

250 ml sol. will require 7.5 m.mol HCl

or 3.75 m.mol H_2SO_4

$\Rightarrow \frac{3.75\times 98}{1000}$ gm H_2SO_4

$= 0.3675$ gm H_2SO_4

2. NTA Ans. (4)

Sol. Since, $Fe(OH)_3$ is positively charged sol, hence, anionic charge will flocculate

As per Hardy Schulze rules coagulation power of anion follows the order :

$Fe(CN)_6^{3-} > CrO_4^{2-} > Cl^- = Br^- = NO_3^-$

Higher the coagulation power lower will be its flocculation value

therefore order will be :

$Fe(CN)_6^{3-} < CrO_4^{2-} < Cl^- = Br^- = NO_3^-$

3. NTA Ans. (4)

4. NTA Ans. (4)

Sol. Adsorption of Gases will decreases

5. Official Ans. by NTA (3)

Sol. Foam - Froth

Gel → Jellies

Aerosol → Smoke

Sol → Cell fluids

Solid sol → rubber

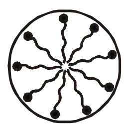
6. Official Ans. by NTA (3)

Sol. The diameter of disperseed particles is similar to wavelength of light used.

7. Official Ans. by NTA (3)

Sol.

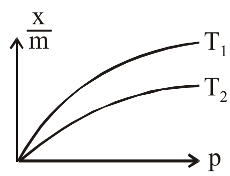
Polar head
more compatible
with polar aq. solution



Micelles formed at CMC.

8. Official Ans. by NTA (3)

Sol. $\frac{x}{m} = K.P.^{1/n}$



$(T_2 > T_1)$

9. Official Ans. by NTA (6.00)
Official Ans. by ALLEN (48.00)

Sol. $\frac{x}{m} = k\,p^x$ ...(1)

$\Rightarrow \underbrace{\log\frac{x}{m}}_{y} = \underbrace{\log k}_{c} + \underbrace{x}_{m}\underbrace{\log p}_{x}$

Given c = log k = 0.4771 or k = 3

slope x = 2

put in eq. (1) $\frac{x}{m} = 3 \times (4)^2 \Rightarrow 48$

10. Official Ans. by NTA (2)

Sol.(a) Since adsorption is exothermic process, as adsorption proceeds number of active sites present over adsorbent decreases, so less heat is evolved.

(b) Since NH_3 has higher force of attraction on adsorbent due to its polar nature (high value of 'a').

(c) As the adsorption increases, residual forces over surface decreases.

(d) Since process is exothermic, on increasing temperature it shift to backward direction, so concentration of adsorbate particle decreases.

11. Official Ans. by NTA (48.00)

Sol. $\frac{x}{m} = KP^{1/n}$

$$\log\left(\frac{x}{m}\right) = \frac{1}{n}\log P + \log K$$

$$\text{slope} = \frac{1}{n} = 2$$

intercept = log K = 0.4771

K = 3

mass of gas adsorbed per gm of adsorbent = $\frac{x}{m}$

$$\frac{x}{m} = 3\times(0.04)^2 = 48\times10^{-4}$$

ELECTROCHEMISTRY

1. NTA Ans. (1)

Sol.

	E^0	ΔG^0
$Cu^{2\oplus} + 2e^{\oplus} \longrightarrow Cu$	0.34	= – 2F (0.34)
$Cu^{\oplus} + e^{\oplus} \longrightarrow Cu$	0.522	= –F (0.522)
$Cu^{2\oplus} + e^{\oplus} \longrightarrow Cu^{+}$		

ΔG^0 = –2F (0.34) – (– F(0.522) = –F (0.68 – 0.522) = – F (0.158)

$$E^0 = \frac{-F(0.158)}{-F} = 0.158V$$

2. NTA Ans. (1)

Sol. Option (1) is incorrect.

According to Kohlrausch's law correct expression is

$$\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaI} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{KI}$$

The other statements are correct.

3. NTA Ans. (–0.93 to –0.94)

Sol. $2H_2O(l) \rightarrow O_2(g) + 4H^+ + 4e^-$; $E^0_{red.} = 1.23V$

From nernst equation

$$E_{cell} = E^0_{cell} - \frac{RT}{nF}\ln Q$$

at 1 bar & 298 K

$$\frac{2.303RT}{F} = 0.059$$

pH = 5 $\Rightarrow$ $[H^+] = 10^{-5}$ M

$E^\circ_{oxidation}$ = – 1.23 volt

$$E_{cell} = -1.23 - \frac{0.059}{4}\log[H^+]^4$$

$$E_{cell} = -1.23 - \frac{0.059}{4}\log(10^{-5})^4$$

= –1.23 + 0.059 × 5 = – 0.935 V

4. NTA Ans. (2.13 to 2.17)

Sol. Cell reaction is :

$Sn(s) + Pb^{+2}(aq) \longrightarrow Sn^{+2}(aq) + Pb(s)$

Apply Nernst equation :

$$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log\frac{[Sn^{+2}]}{[Pb^{+2}]} \quad ...(1)$$

E^0_{cell} = –0.13 + 0.14 = 0.01 V

At equilibrium : E_{cell} = 0

Substituting in (1)

$$0 = 0.01 - \frac{0.06}{2}\log\frac{[Sn^{+2}]}{[Pb^{+2}]}$$

$$\Rightarrow \frac{1}{3} = \log\frac{[Sn^{+2}]}{[Pb^{+2}]}$$

$$\Rightarrow \frac{[Sn^{+2}]}{[Pb^{+2}]} = 2.15$$

5. NTA Ans. (5.66 to 5.68)

Sol. gm eq. of Ag = $\frac{108}{108}$ = 1

gm eq. of $O_2(g) = 1$

Volume of $O_2(g) = 22.7 \times \frac{1}{4}$ = 5.675 litre

6. NTA Ans. (1)

Sol. Distilled water have lowest ionic conductance.

7. Official Ans. by NTA (4)

Sol. As voltage is '2V' so both Ag^+ & Au^+ will reduce and their equal gm equivalent will reduce so

gmeq Ag = gmeq of Au

$$\frac{Wt_{Ag}}{E_{qwt_{Ag}}} = \frac{Wt_{Au}}{E_{qwt_{Au}}}$$

So $\frac{wt_{Ag}}{wt_{Au}} = \frac{E_{qwt_{Ag}}}{E_{qwt_{Au}}} = \frac{At\,wt_{Ag}}{Atwt_{Au}}$

8. Official Ans. by NTA (1)

Sol. $E^o_{cell} = 0.34 - (-0.76)$

= 1.10 volt

If E_{ext} > 1.10 volt

Cu → Anode

Zn → Cathode

If E_{ext} = 1.10 volt

Zn → Anode

Cu → Cathode

9. Official Ans. by NTA (58)

Official Ans. by ALLEN (142)

Sol. $\frac{1}{2}H_2 \rightarrow H^+ + e^{\ominus}$

$e^{\ominus} + AgCl_{(s)} \rightarrow Ag_{(s)} + Cl^{\ominus}$

$\frac{1}{2}H_2 + AgCl_{(s)} \rightarrow H^+_{(aq)} + Ag_{(s)} + Cl^{\ominus}_{(aq)}$

$$E = \varepsilon^0 - \frac{.06}{1}\log\frac{[H^+][Cl^{\ominus}]}{P_{H_2}^{\frac{1}{2}}}$$

$$E = 0.22 - .06\log\frac{(10^{-1})(10^{-1})}{1^{\frac{1}{2}}}$$

E = 0.22 + .12 = .34 volt

⇒ total energy of photon will be (for Na)

= 2.3 + 0.34 = 2.64 eV

10. Official Ans. by NTA (60)

Sol. Moles of $e^{\ominus} = \left(\frac{8 \times 60 \times 2}{96000}\right)$

Using stoichiometry; theoritically

$$\frac{n_{e^{\ominus}}\text{ used}}{6} = \frac{n_{cr^{+3}}\text{ produced}}{2}$$

$$\Rightarrow n_{cr^{+3}}\text{ produced} = \frac{2}{6} \times \frac{8 \times 60 \times 2}{96000}$$

$$= \frac{0.02}{6}$$

⇒ $wt_{cr^{+3}}$ theoritically produced

$$= \left(\frac{0.02}{6} \times 52\right)g$$

$$\Rightarrow \%\text{ efficiency} = \frac{0.104g}{\left(\frac{0.02 \times 52}{6}\right)g} \times 100$$

= 60%

11. Official Ans. by NTA (6)

Sol. $\Delta G° = -AFE° = -3 \times 96500 \times E°$

$\Rightarrow E° = -6 \times 10^{-2}$ V

12. Official Ans. by NTA (1)

Sol. Its a weak electrolyte hence : CH_3COOH

13. Official Ans. by NTA (144.00)

Sol.

$Cu^+ \longrightarrow Cu^{2+} + e^-$

$Cu^+ + e^- \longrightarrow Cu(s)$

$2Cu^+ \longrightarrow Cu^{2+} + Cu$

$E^o_{cell} = E^o_{Cu^+/Cu} - E^o_{Cu^{2+}/Cu^+}$

= 0.52 – 0.16

= 0.36 V

At equilibrium → E_{cell} = 0

$$E^o_{cell} = \frac{RT}{nF}\ln K$$

$$\ln K = \frac{E^o_{cell} \times nF}{RT}$$

$$\ln K = \frac{0.36 \times 1}{0.025}$$

= 14.4 = 144 × 10^{-1}

14. Official Ans. by NTA (11.00)

15. Official Ans. by NTA (4)

Sol. $\Delta G = -n F E_{cell}$

ΔG is negative, if E_{cell} is positive

Anode : $Cu(s) \longrightarrow Cu^{+2}(C_1) + 2e^- : E°$

Cathode : $Cu^{+2}(C_2) + 2e^- \longrightarrow Cu(S) : -E°$

Cell reaction : $Cu^{+2}(C_2) \longrightarrow Cu^{+2}(C_1)\ E°_{cell} = 0$

$$E_{cell} = E^{\circ}_{cell} - \frac{2.303RT}{nF}\log Q$$

$$E_{cell} = 0 - \frac{2.303RT}{nF}\log\left(\frac{C_1}{C_2}\right)$$

$$E_{cell} > 0 : \text{ if } \frac{C_1}{C_2} < 1 \Rightarrow C_1 < C_2$$

LIQUID SOLUTION

1. NTA Ans. (3)

Sol. The vapour pressure of mixture (= 600 mm Hg) is greater than the individual vapour pressure of its constituents (Vapour pressure of CS_2 = 512 mm Hg, acetone = 344 mm Hg). Hence, the solution formed shows positive deviation from Raoult's law.

$\Rightarrow (1)\Delta_{Sol}H > 0$, (2) Raoult's law is not obeyed

(3) $\Delta_{sol.}$ Volume > 0

(4) CS_2 and Acetone are less attracted to each ether than to themselves.

2. NTA Ans. (3)

Sol. The pure solvent solution will try to maintain higher vapour pressure in the sealed container and in return the solvent vapour molecules will condense in the solution of non-volatile solute as it maintains an equilibrium with lower vapour pressure. (Lowering of vapour pressure is observed when a non volatile solute is mixed in a volatile solvent)

This will eventually lead to increase in the volume of solution and decrease in the volume of solvent.

3. NTA Ans. (3)

Sol. Order of B.P. is : Z > Y > X

Order of vapour pressure : Z < Y < X

order of intermolecular interaction : Z > Y > X.

4. NTA Ans. (1.74 to 1.76 or 0.03)

Sol. $\Delta T_f = i \times m \times K_f$

$$0.2 = 2 \times 2 \times \frac{w/58.5}{600/1000}$$

w = 1.755 gm

5. Official Ans. by NTA (167)

Sol. Osmotic pressure = $\pi = i \times C \times RT$

For NaCl i = 2 so

$\pi_{NaCl} = i \times C_{NaCl} \times RT$ $\quad C_{NaCl}$ = conc. of NaCl

$0.1 = 2 \times C_{NaCl} \times RT$

$C_{NaCl} = \frac{0.05}{RT}$ $\quad C_{glucose}$ = conc. of glucose

For glucose i = 1 so

$\pi_{Glucose} = i \times C_{glucose} \times RT$

$0.2 = 1 \times C_{glucose} \times RT$

$C_{Glucose} = \frac{0.2}{RT}$ $\quad \eta_{NaCl}$ = No. of moles NaCl

η_{NaCl} in 1 L = $C_{NaCl} \times V_{Litre}$

$= \frac{0.05}{RT}$ $\quad \eta_{glucose}$ = No. of moles glucose

$\eta_{glucose}$ in 2 L = $C_{glucose} \times V_{Litre}$

$= \frac{0.4}{RT}$

$V_{Total} = 1 + 2 = 3L$

so Final conc. NaCl = $\frac{0.05}{3RT}$

Final conc. glucose = $\frac{0.4}{3RT}$

$\pi_{Total} = \pi_{NaCl} + \pi_{glucose}$

$$= \left[i \times C_{NaCl} + C_{glucose}\right] \times RT$$

$$= \left(\frac{2 \times 0.05}{3RT} + \frac{0.4}{3RT}\right) \times RT$$

$$= \frac{0.5}{3}\text{atm}$$

= 0.1666 atm

= 166.6×10^{-3} atm

$\Rightarrow 167.00 \times 10^{-3}$ atm

so x = 167.00

6. **Official Ans. by NTA (600)**

Sol. $550 = P_A^o \times \frac{1}{4} + P_B^o \times \frac{3}{4}$

$2200 = P_A^o + 3P_B^o$ .....(i)

$2800 = P_A^o + 4P_B^o$ ....(ii)

$560 = P_A^o \times \frac{1}{5} + P_B^o \times \frac{4}{5}$

$P_B^o = 600, P_A^o = 400$

7. **Official Ans. by NTA (2)**

Sol. (1) $P_\gamma = K_H X_Y$

$$P_\gamma = 2\times10^{-15} \times \frac{55.5}{55.5+\frac{1000}{18}} = 2\times10^{-5} \text{K bar}$$

$= 2 \times 10^{-2}$ bar

(2) $P_\delta = K_H X_\delta$

$$P_\delta = 0.5 \times \frac{55.5}{55.5+\frac{1000}{18}} = .249 \text{ K bar} = 249 \text{ bar}$$

(3) On increasing temperature solubility of gases decreases

(4) $K_H \downarrow$ solubility $\uparrow$ and lowest K_H is for γ.

8. **Official Ans. by NTA (177)**

9. **Official Ans. by NTA (1)**

Sol. Relative lowering of V.P. = $\frac{\Delta P}{P^0} = x_{solute}$

$$\left(\frac{\Delta P}{P^0}\right)_A = \frac{\frac{10}{100}}{\frac{10}{100}+\frac{180}{18}} : \left(\frac{\Delta P}{P^0}\right)_B = \frac{\frac{10}{200}}{\frac{10}{200}+\frac{180}{18}}$$

$$\left(\frac{\Delta P}{P^0}\right)_C = \frac{\frac{10}{10,000}}{\frac{10}{10,000}+\frac{180}{18}} : \left(\frac{\Delta P}{P^0}\right)_A > \left(\frac{\Delta P}{P^0}\right)_B > \left(\frac{\Delta P}{P^0}\right)_C$$

CHEMICAL EQUILIBRIUM

1. **Official Ans. by NTA (2)**

Sol. $A \rightleftharpoons B + C \quad K_{eq}^{(1)} = \frac{[B][C]}{[A]}$ .....(1)

$B + C \rightleftharpoons P \quad K_{eq}^{(2)} = \frac{[P]}{[B][C]}$ .....(2)

For

$A \rightleftharpoons P \quad K_{eq} = \frac{[P]}{[A]}$

Multiplying equation (1) & (2)

$K_{eq}^{(1)} \times K_{eq}^{(2)} = \frac{[P]}{[A]} = K_{eq}$

2. **Official Ans. by NTA (3)**

Sol. at equilibrium

$r_a = r_b$

3. **Official Ans. by NTA (16)**

Sol.

	X	+ Y	= 2Z
t = 0	1	1.5	0.5
At eq.	0.75	1.25	1

;

$K_{eq.} = \frac{1^2}{\frac{3}{4}\times\frac{5}{4}} = \frac{16}{15}$

4. **Official Ans. by NTA (2)**

Sol. $N_2 + 3H_2 \rightleftharpoons 2NH_3 \rightarrow K_C = 64$

$2NH_3 \rightleftharpoons N_2 + 3H_2 \rightarrow K_C = \frac{1}{64}$

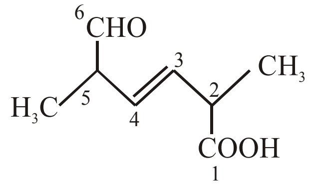
$NH_3 \rightleftharpoons \frac{1}{2}N_2 + \frac{3}{2}H_2 \rightarrow K_C = \left(\frac{1}{64}\right)^{1/2} = \frac{1}{8}$

5. **Official Ans. by NTA (4)**

JANUARY & SEPTEMBER 2020 ATTEMPT (OC)

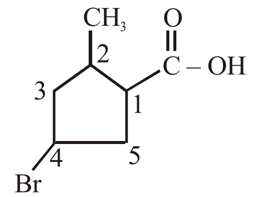
NOMENCLATURE

1. Official Ans. by NTA (4)

Soll.  IUPAC name

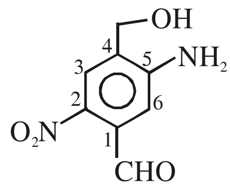
2, 5-dimethyl-6-oxo-hex-3-enoic acid

2. Official Ans. by NTA (1)

Sol. 

4-bromo-2-methyl cyclopentane carboxylic Acid

3. Official Ans. by NTA (4)

Sol. 

5-amino-4-hydroxymethyl-2-nitrobenzaldehyde

ACIDITY & BASICITY

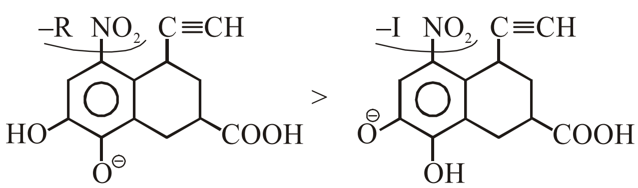
1. Official Ans. by NTA (1)

Sol. Acidic strength order :

$$R-\overset{\overset{O}{||}}{C}-OH > R-OH > R-C\equiv CH$$

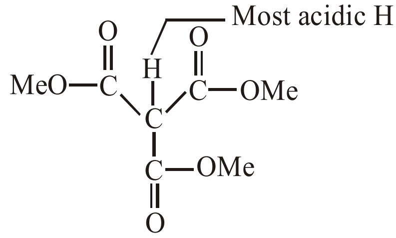
Reason : $R-\overset{\overset{O}{||}}{C}-O^{\ominus}$ stable by equivalent resonance.

Stable :



So answer is b > c > d > a.

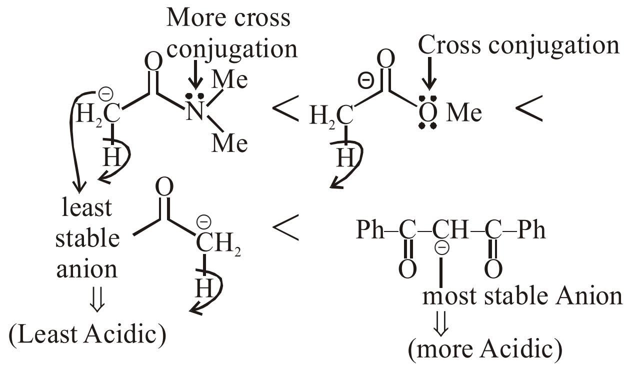
2. Official Ans. by NTA (4)

Sol. 

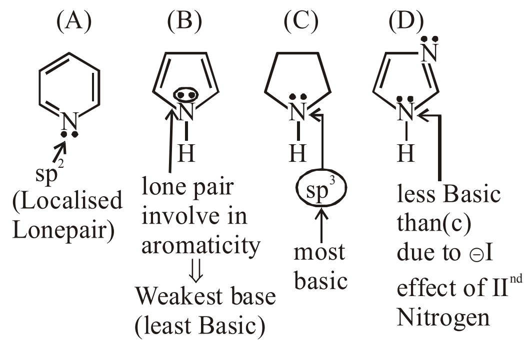
Due to presence of 3 (–R) groups

3. Official Ans. by NTA (4)

Sol. D < C < A < B



4. Official Ans. by NTA (4)

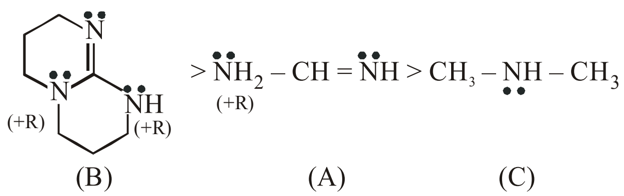
Sol. 

5. Official Ans. by NTA (1)

ELECTRONIC DISPLACEMENT EFFECT

1. NTA Ans. (3)

Sol. Base strength order



(B) (A) (C)

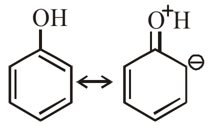
pk_b order (C > A > B)

2. NTA Ans. (1)

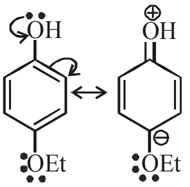
Sol. (C) > (B) > (A)

3. NTA Ans. (2)

Sol. $H_3C - OH$ (100% single bond)

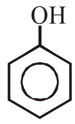


C–OH bond has partial double bond character



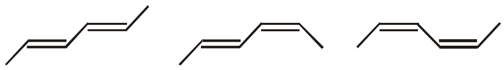
(C–OH bond has some double bond character but double bond character is less)

(+M)

Ans. CH_3OH >  (p-ethoxyphenol) > 

4. NTA Ans. (1)

Sol.



(a) (Trans, Trans) (b) (Trans, Cis) (c) (Cis, Cis)

∴ Generally trans is more stable then cis form.

Heat of combustion (HOC) $\propto \frac{1}{\text{Stability}}$

Stability : a > b > c

HOC : c > b > a

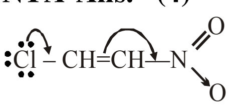
5. NTA Ans. (3)

Sol. (i) $CH_3-\overset{\ominus}{C}(CH_3)_2$ (ii) $CH_2=C(CH_3)-\overset{\ominus}{C}H_2$ (iii) $HC\equiv C^{\ominus}$ (iv) $CH_3^{\ominus}$ (v) $\overset{\ominus}{C}\equiv N$

Basic strength order : (i) > (iv) > (ii) > (iii) > (v)

6. NTA Ans. (4)

Sol.



Due to –M effect of $-NO_2$ and + M effect of Cl more D.B. character between C – Cl bond. So shortest bond length.

7. NTA Ans. (3)

8. Official Ans. by NTA (3)

Sol. $CH_2=CH-Cl$

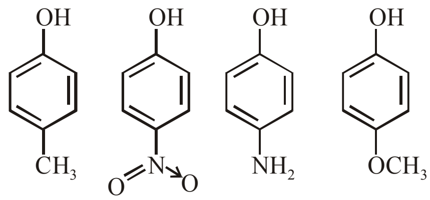
In option (3) C—Cl bond is shortest due to resonance of lone pair of –Cl.

Due to resonance C—Cl bond acquire partial double bond character.

Hence C—Cl bond length is least.

9. Official Ans. by NTA (1)

Sol.



	(I)	(II)	(III)	(IV)
BP value from net	≃202°C	≃279°	≃284°C	≃243°C

BP ∝ dipolemoment (μ)

Alter

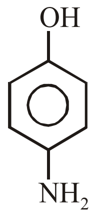
Increasing order of boiling point is :

I. 

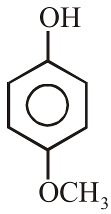
⇒ Shows hydrogen bonding from –O–H group only

II. 

⇒ Shows strongest hydrogen bonding from both sides of –OH group as well as $-NO_2$ group.

III. 

⇒ Shows stronger hydrogen from both side of –OH group as well as $-NH_2$ group.

IV. 

⇒ Shows stronger hydrogen bonding from one side –OH–group and another side of $-OCH_3$ group shows only dipole-dipole interaction.

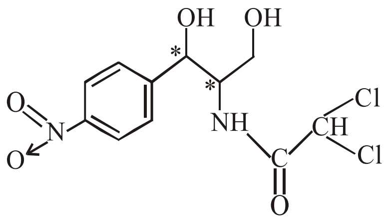
⇒ Hence correct order of boiling point is:

(I) < (IV) < (III) < (II)

ISOMERISM

1. **NTA Ans. (2)**

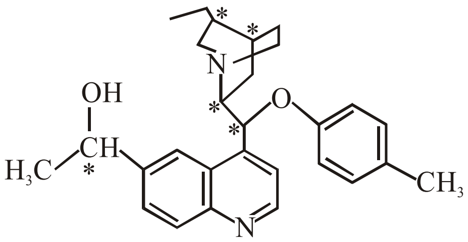
Sol.



Chloramphenicol

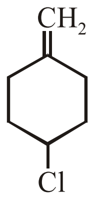
2. **Official Ans. by NTA (5.00)**

Sol. No. of chiral centres



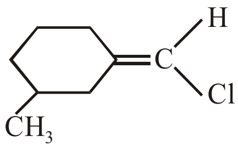
3. **Official Ans. by NTA (2)**
Official Ans. by ALLEN (2 & 4)

Sol. (1)



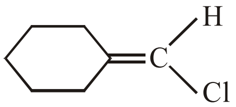
Not show GI

(2)



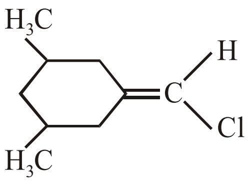
Show GI

(3)



Not show GI

(4)



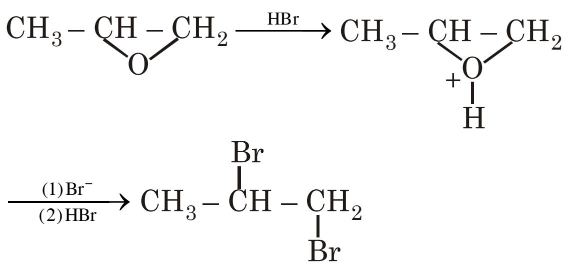
Show GI

4. **Official Ans. by NTA (3)**

HALOGEN DERIVATIVE

1. **NTA Ans. (4)**

Sol.

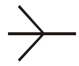


2. **NTA Ans. (3)**

Sol.

$$CH_3-CH_2-CH_2-Br + Z^{\ominus} \xrightarrow{K_s} CH_3-CH_2-CH_2-Z$$
$$CH_3-CH_2-CH_2-Br + Z^{\ominus} \xrightarrow{K_e} CH_3-CH=CH_2 + HZ + Br^-$$

(A) $CH_3-CH_2-O^- = Z^{\ominus} \longrightarrow$

(B)  $O^{\ominus} = Z^{\ominus}$

(B) with more steric crowding forms elimination product compared to substitution.

$K_e(B) > K_e(A)$

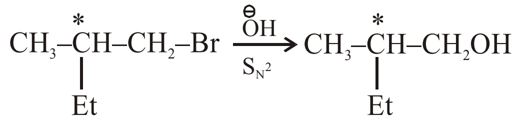
$$\mu_B = \frac{K_s(B)}{K_e(A)} < \mu_A = \frac{K_s(A)}{K_e(A)}$$

3. **NTA Ans. (3)**

Sol. Reactivity D > B > C > A
Carbocation formed from D is most stable
Carbocation formed from A is least stable

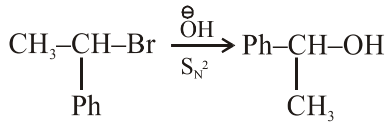
4. **Official Ans. by NTA (1)**

Sol. (1)

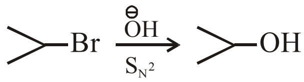


* (NCERT)

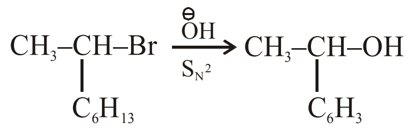
(2)



(3)



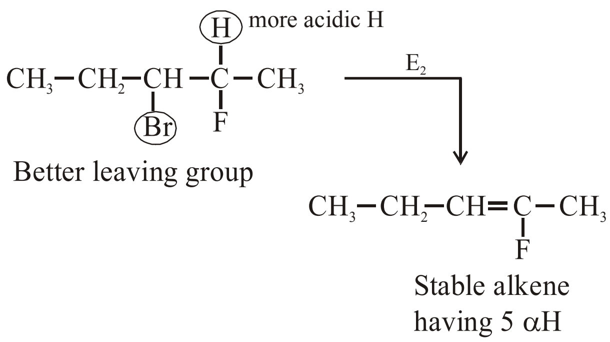
(4)



As language given, we have to go with option (1) as stereochemistry of chiral centre is not distorted.

5. **Official Ans. by NTA (4)**

Sol.



6. Official Ans. by NTA (1)

Sol. Reaction 1 : SN_1

Reaction 2 : E_2

SN_1 is independent of concentration of nucleophile/base

7. Official Ans. by NTA (2)

Sol. $S_N{}^1$ favours

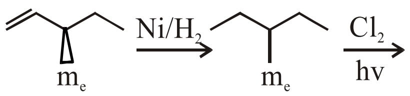
(a) The reaction is favoured by weak nucleophiles

(b) $R^{\oplus}$ would be easily formed if the substituents are bulky

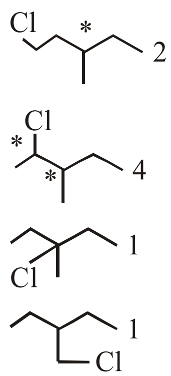
(c) The reaction is accompained by recemization

8. Official Ans. by NTA (8)

Sol.

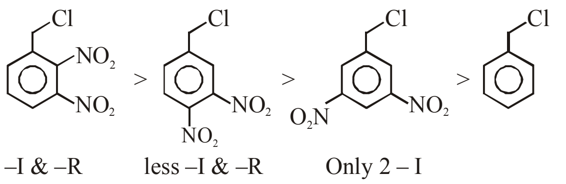


Simplest
O.A. Alkene



9. Official Ans. by NTA (2)

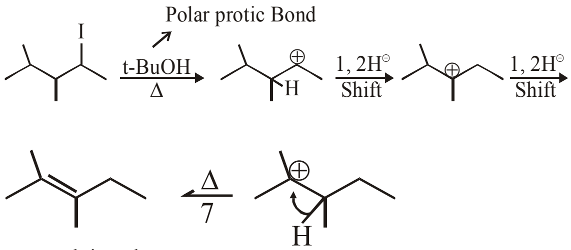
Sol.



rate of SN^2 reaction

10. Official Ans. by NTA (4)

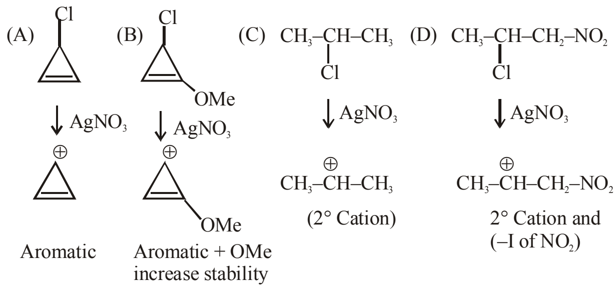
Sol.



more subtituted
Alkene (major)

11. Official Ans. by NTA (4)

Sol.



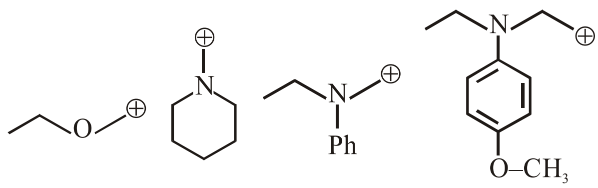
∴ Stability Cation B > A > C > D

12. Official Ans. by NTA (2)

Sol. $R-x+aq.AgNO_3 \xrightarrow{R.D.S} R^{\oplus} + \underset{(PPT)}{Agx}$ (1)

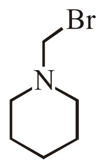
So rate of P.P.T formation of Agx depend's on stability of carbocation (R^+)

In given question formed carbocation will be



(a) (b) (c) (d)

Most stable carbocation is (b) so



give fastest P.P.T of AgBr with aq $AgNO_3$

13. Official Ans. by NTA (1)

Official Ans. by ALLEN (4)

Sol. $CH_3–CH{=}CH–\underset{CH_3}{CH}–CH_3$

$\Big\downarrow$ From HBr $H^{\oplus}$

$CH_3–\overset{\oplus}{C}H–\underset{H}{CH}–\underset{CH_3}{CH}–CH_3$

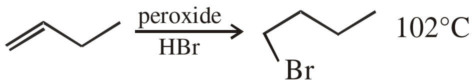
$\Big\downarrow Br^{\ominus}$

$CH_3–\underset{Br}{CH}–CH_2–\underset{CH_3}{CH}–CH_3$

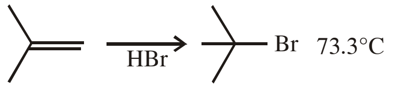
Addition of HBr according to M.R.

14. Official Ans. by NTA (2)

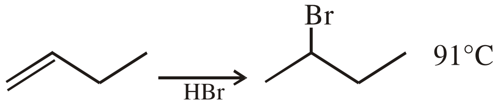
Sol. (a)



(b)



(c)



$$B.P. \propto \frac{1}{\text{Branching}} \qquad \therefore a > c > b \text{ (order of B.P.)}$$

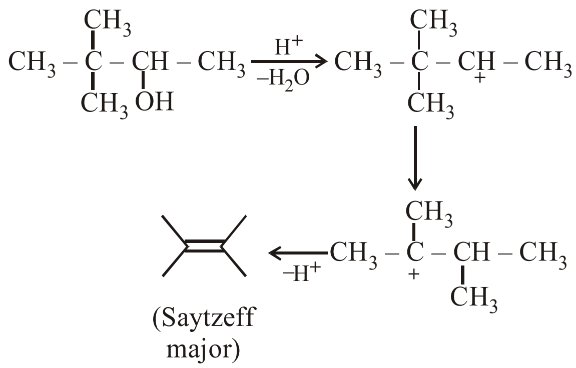
15. Official Ans. by NTA (2)

ALCOHOL & ETHER

1. NTA Ans. (1)

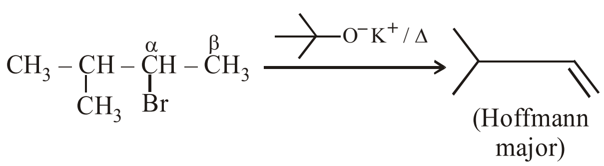
Sol.

(a)

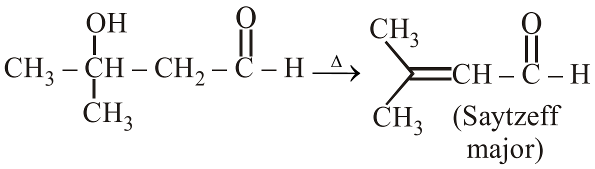


(b) $CH_3-\underset{CH_3}{\underset{|}{C}H}-\underset{Br}{\underset{|}{C}H}-CH_3 \xrightarrow[\Delta]{\text{alc.KOH}} CH_3-\underset{CH_3}{\underset{|}{C}}=CH-CH_3$ (Saytzeff major)

(c)



(d)

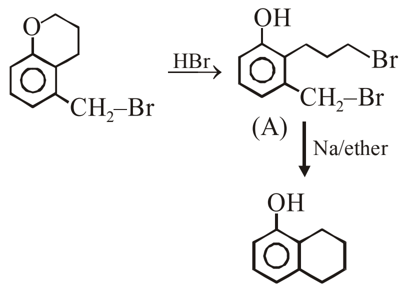


$(CH_3)_3O^-K^+$ is incorrect representation of potassium tert-butoxide $[(CH_3)_3CO^-K^+]$.

So it is possible that it can be given as **Bonus**

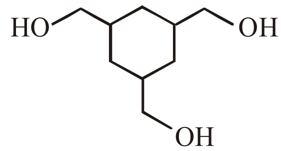
2. NTA Ans. (4)

Sol.

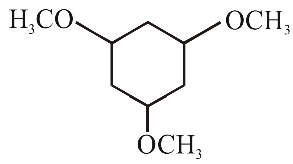


3. NTA Ans. (1)

Sol. Alcohol has more boiling point than ether (due to hydrogen bonding).
So,

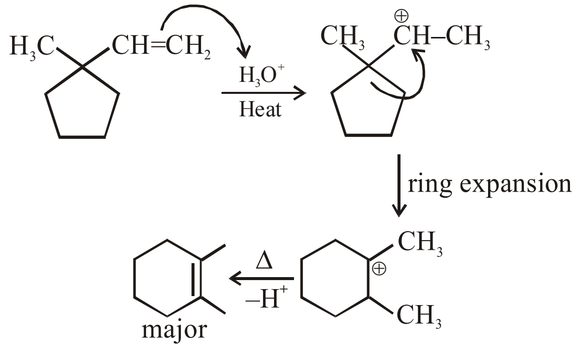


has more boiling point than



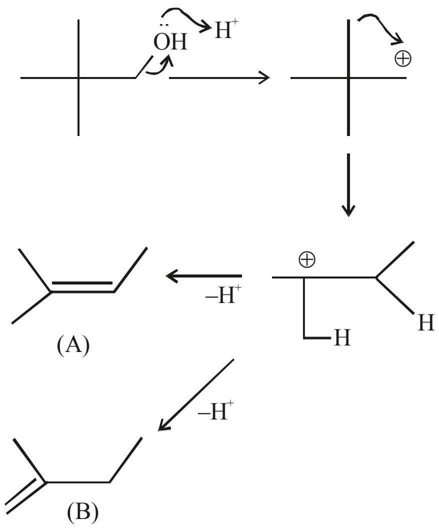
4. Official Ans. by NTA (3)

Sol.



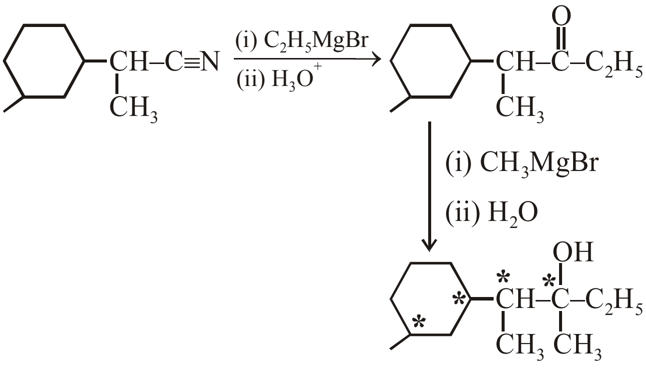
5. Official Ans. by NTA (4)

Sol.



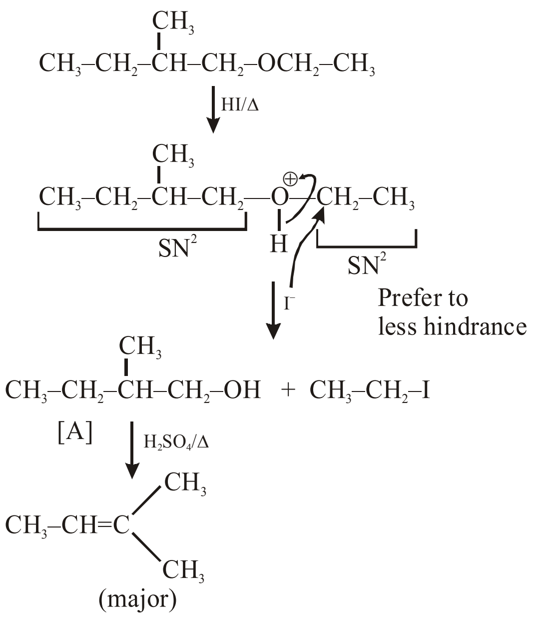
6. Official Ans. by NTA (4)

Sol.



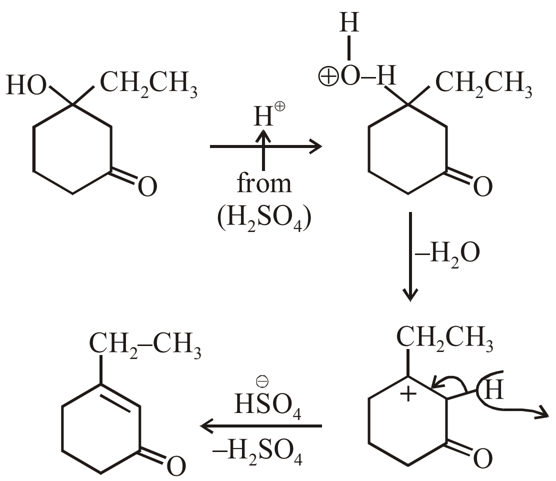
7. Official Ans. by NTA (4)

Sol.



8. Official Ans. by NTA (2)

Sol.

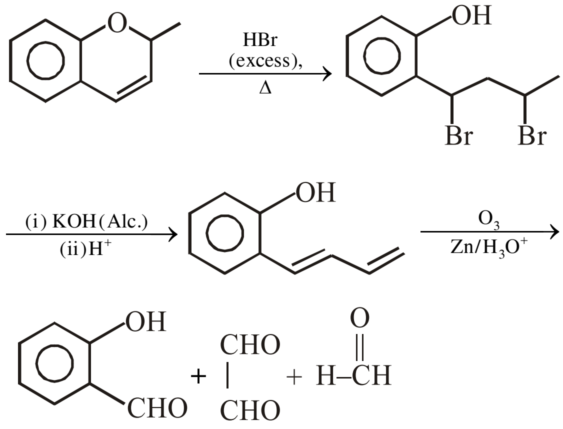


9. Official Ans. by NTA (3)

OXIDATION

1. Official Ans. by NTA (2)

Sol.

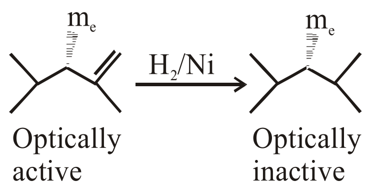


2. Official Ans. by NTA (1)

REDUCTION

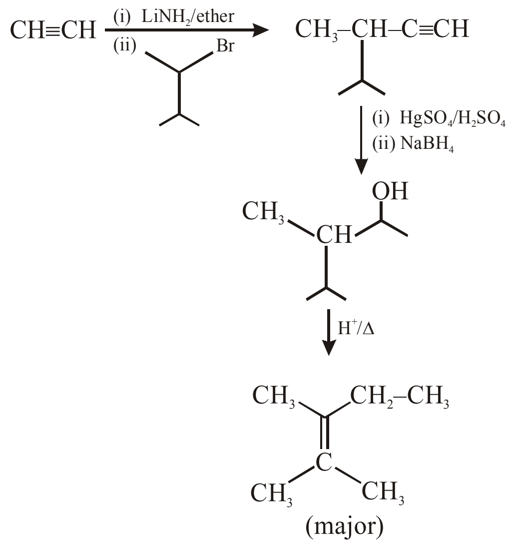
1. Official Ans. by NTA (2)

Sol.



2. Official Ans. by NTA (2)

Sol.



Now :- (i) $HgSO_4$/dil.H_2SO_4

(ii) $NaBH_4$

is convert triple bond into ketone and formed ketone is reduced by $NaBH_4$ and convert into Alcohol.

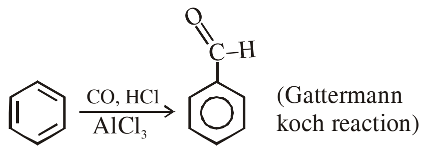
3. Official Ans. by NTA (2)

Sol. $CH_3-CH_2-C\equiv N \xrightarrow{?} CH_3-CH_2-CH_2-NH_2$

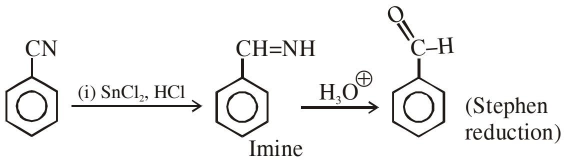
$CH_3-CH_2-C\equiv N \xrightarrow{LiAlH_4} CH_3-CH_2-CH_2-NH_2$

4. Official Ans. by NTA (3)

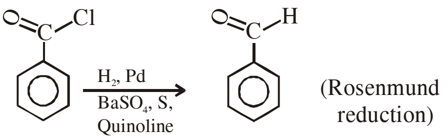
Sol.(i)



(ii)



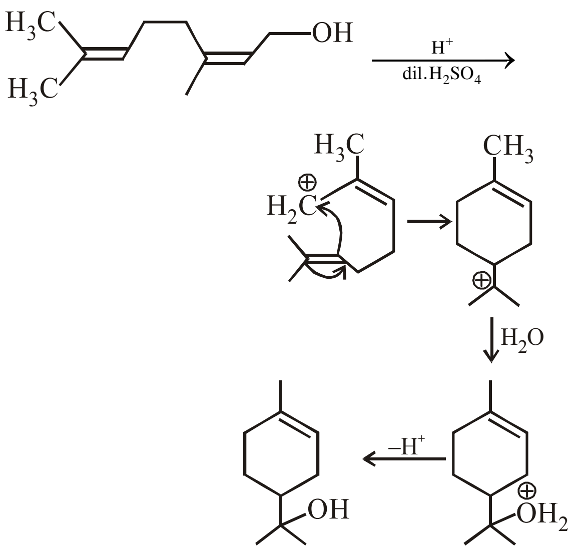
(iii)



HYDROCARBON

1. NTA Ans. (2)

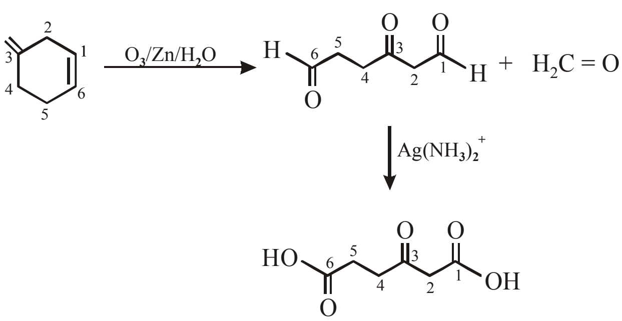
Sol.



Terpineol

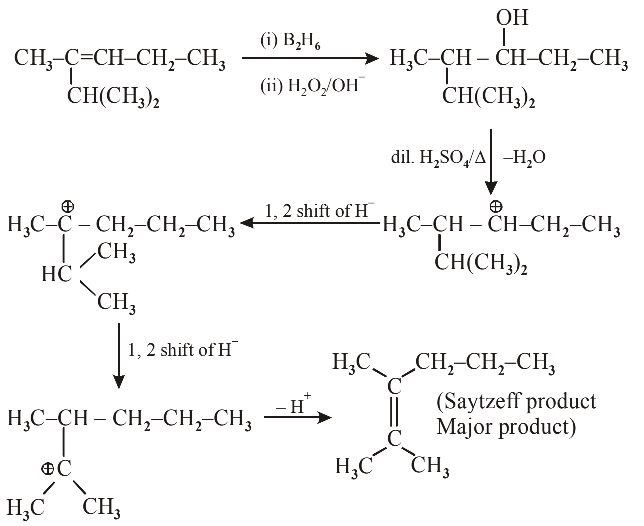
2. NTA Ans. (1)

Sol.



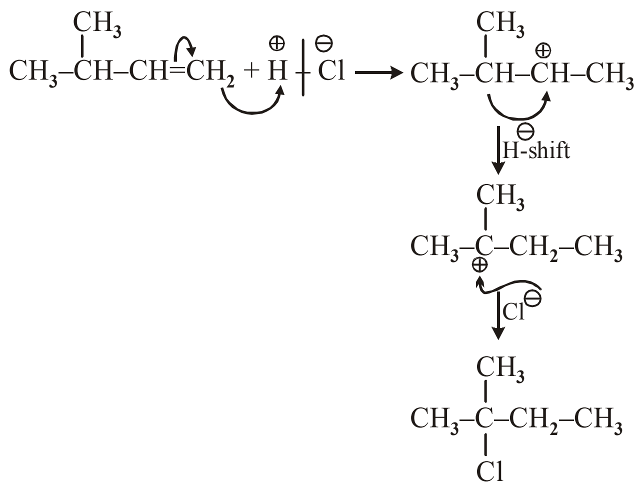
3. NTA Ans. (1)

Sol.



4. NTA Ans. (1)

Sol.

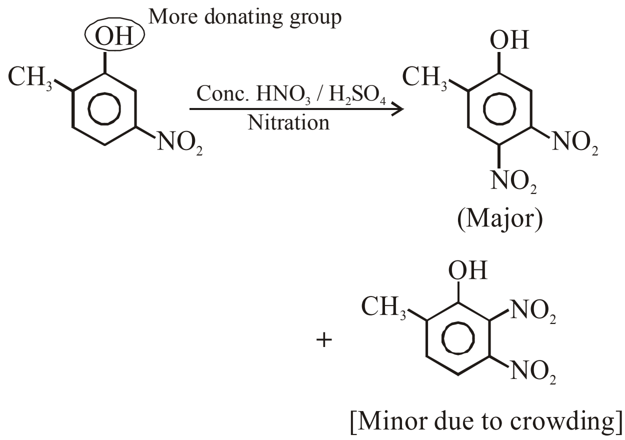


(No chiral centre, so no racemisation possible)

AROMATIC COMPOUND

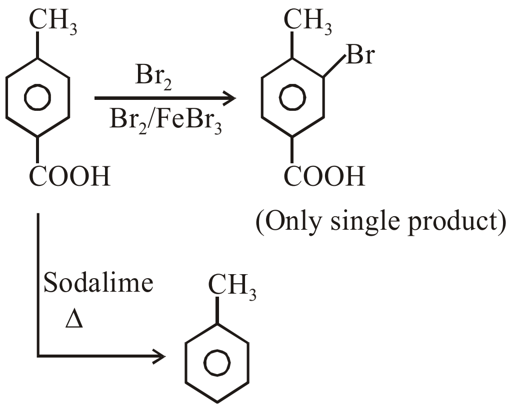
1. Official Ans. by NTA (3)

Sol.



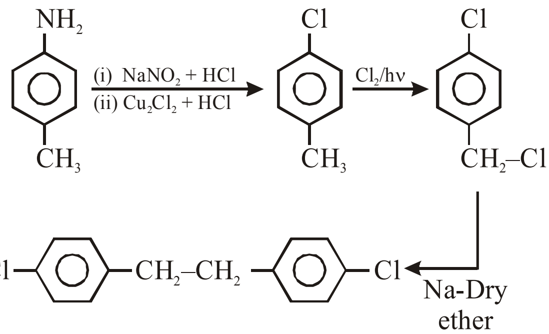
2. Official Ans. by NTA (4)

Sol.



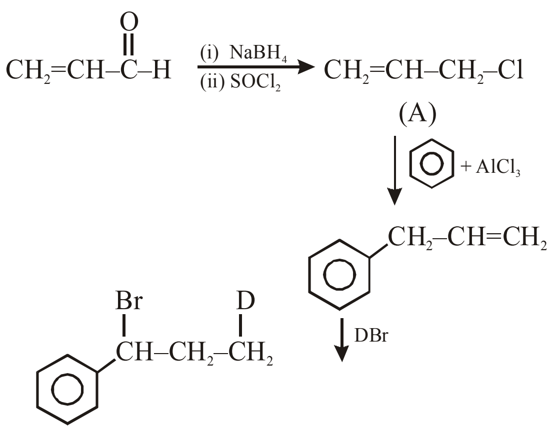
3. Official Ans. by NTA (3)

Sol.



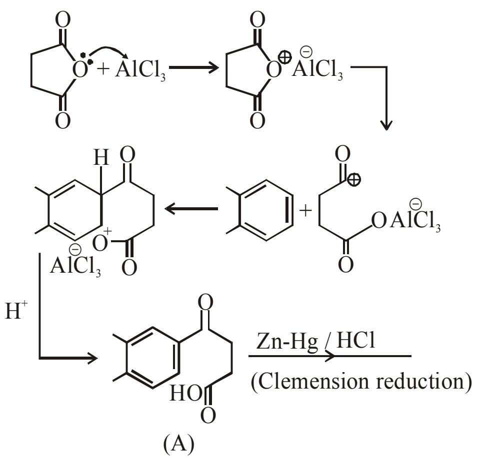
4. Official Ans. by NTA (3)

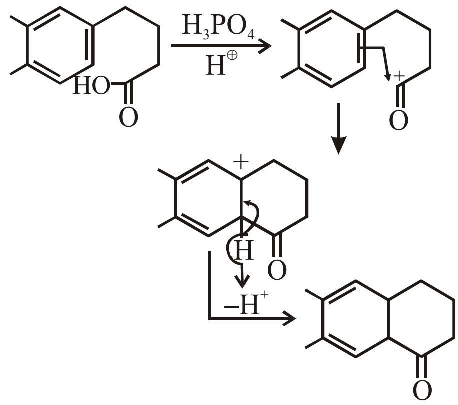
Sol.



5. Official Ans. by NTA (1)

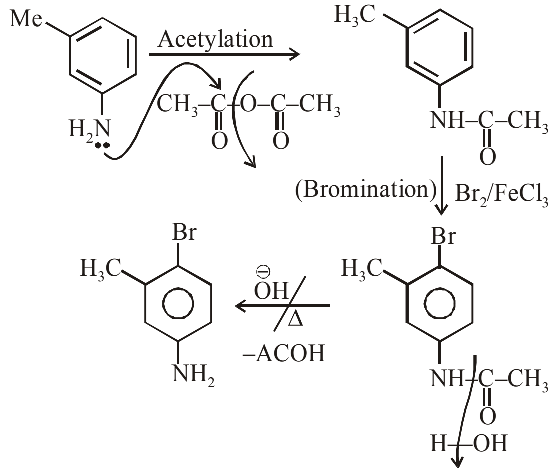
Sol.





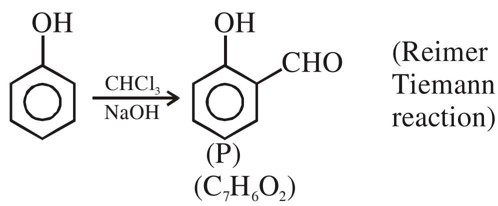
6. Official Ans. by NTA (1)

Sol.



7. Official Ans. by NTA (69.00)
Official Ans. by ALLEN (68.85)

Sol.

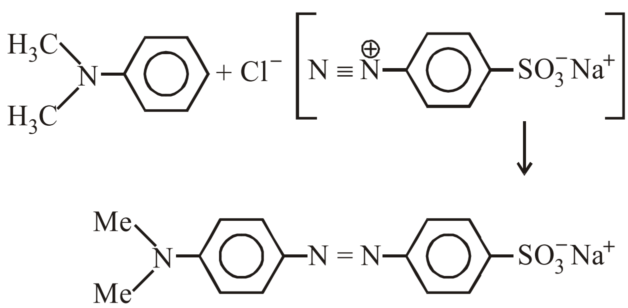


Molecular weight of $C_7H_6O_2$ = 122

$$\%C = \frac{12 \times 7 \times 100}{122} = 68.85 \approx 69$$

8. NTA Ans. (1)

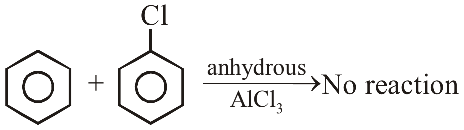
Sol.

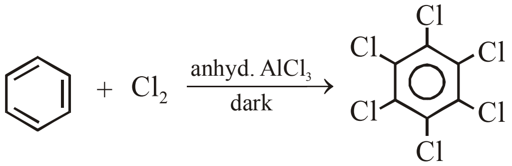


(Methyl orange)

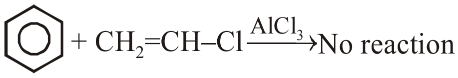
It is an acid base indicator

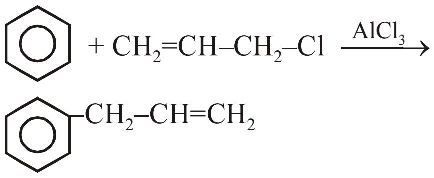
9. NTA Ans. (2)

Sol. (a)


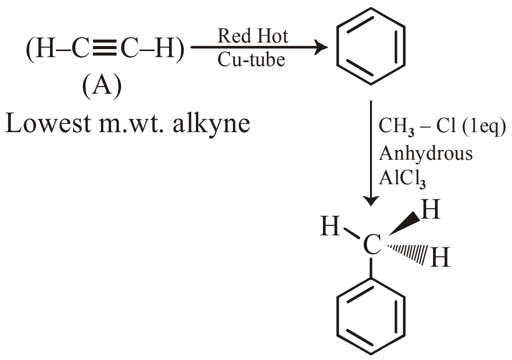
(b)


(electrophilic substitution)

(c)


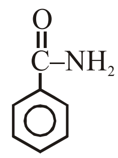
(d)


10. NTA Ans. (13)

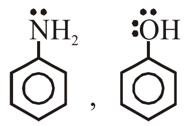
Sol.


Total 13 atom are present in same plane (7 carbon & 6 hydrogen atoms.)

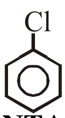
11. NTA Ans. (3)

Sol. ∴


(Deactivated ring due to –R effect of amide)

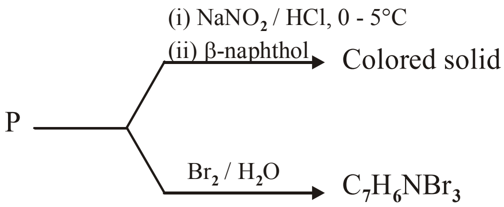


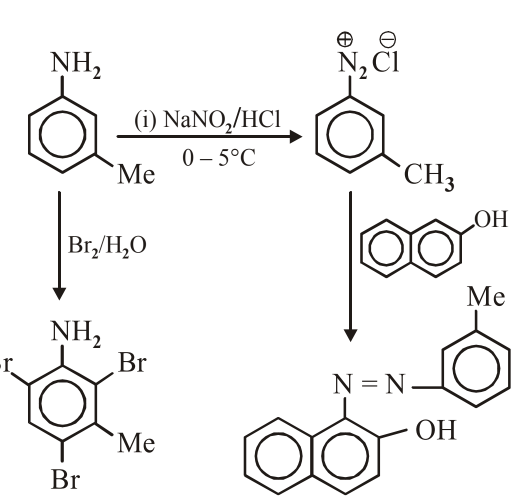
(l.p.e. of $-NH_2$ and $-\ddot{O}H$ group do acid-base reaction with lewis acid $AlCl_3$, so ring is deactivated)



(Highest yield produced)

12. NTA Ans. (2)

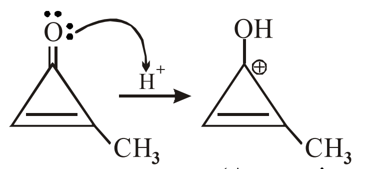
Sol.




(colored dye)

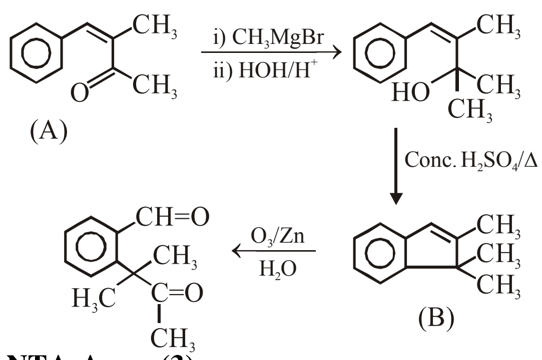
CARBONYL COMPOUNDS

1. NTA Ans. (2)

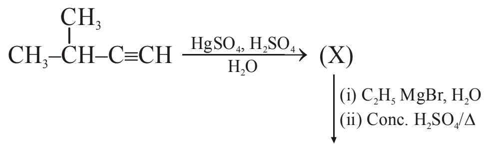
Sol.


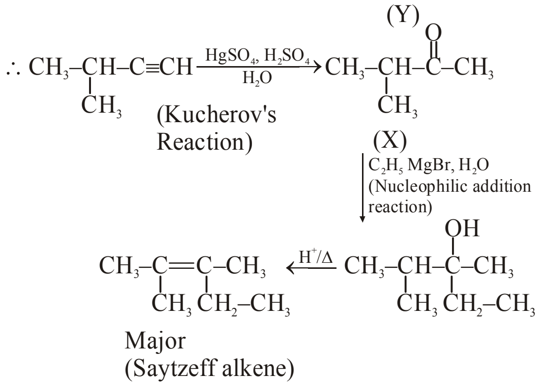
(Aromatic stable product)

2. NTA Ans. (4)

Sol.


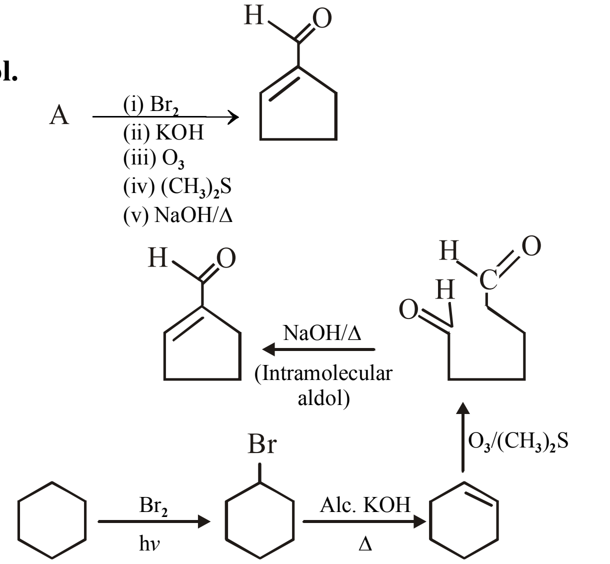
3. NTA Ans. (3)

Sol.




4. NTA Ans. (3)

Sol.



5. NTA Ans. (66.65 to 66.70)

Sol.

$$A \xrightarrow[H_3O^+]{CH_3MgBr} B \xrightarrow[573\ K]{Cu} CH_3\text{–}\underset{}{\overset{CH_3}{\overset{|}{C}}}\text{=CH–}CH_3$$

(2-methyl-2-butene)

$$\underset{\textbf{(A)}}{CH_3\text{–}\overset{}{\underset{O}{\underset{||}{C}}}\text{–}CH_2\text{–}CH_3} \xrightarrow[H_3O^+]{CH_3MgBr} \underset{\textbf{(B)}}{CH_3\text{–}\overset{CH_3}{\overset{|}{\underset{OH}{\underset{|}{C}}}}\text{–}CH_2\text{–}CH_3}$$

$$\xrightarrow[573\ K]{Cu} CH_3\text{–}\overset{CH_3}{\overset{|}{C}}\text{=CH–}CH_3$$

(2-methyl-2-butene)

C ⇒ 12 × 4 = 48

H ⇒ 8 × 1 = 8

O ⇒ 16 × 1 = 16

____

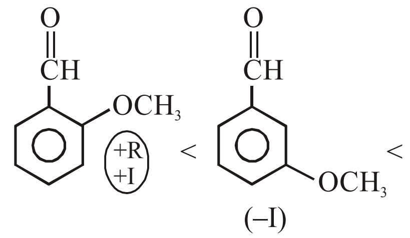
Total 72

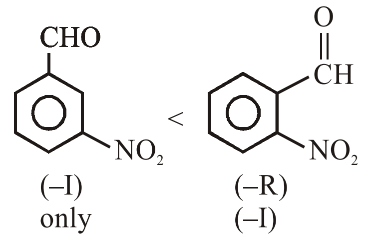
$$\%\text{ of C} = \frac{48}{72} \times 100 = 66.66\%$$

6. Official Ans. by NTA (3)

Sol. Increasing order of reactivity towards HCN addition

Greater the electrophilicity on $\overset{O}{\overset{||}{\text{–C–}}}$ group greater the reactivity in nucleophilic addition.

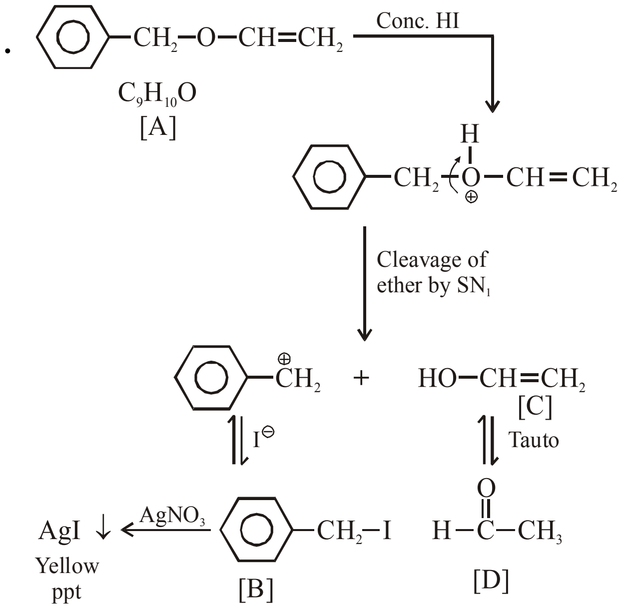




(iii) < (i) < (iv) < (ii)

7. Official Ans. by NTA (2)

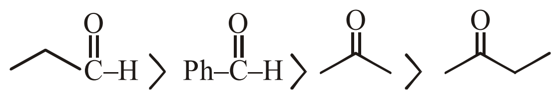
Sol



Iodoform test : ✓

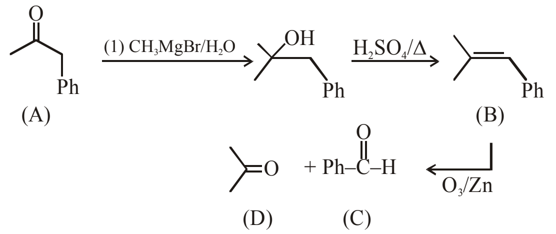
8. Official Ans. by NTA (1)

Sol. Reactivity order of various carbonyl compounds → Aldehydes > Ketones

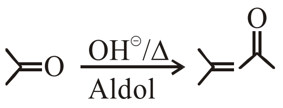


9. Official Ans. by NTA (3)

Sol.



$$Ph-\overset{O}{\overset{||}{C}}-H \xrightarrow[\text{NaOH (cannizaro)}]{\text{Conc.}} Ph-COOK + Ph-CH_2OH$$



10. Official Ans. by NTA (2)

CARBOXYLIC ACID AND THEIR DERIVATIVES

1. NTA Ans. (3)

Sol.

$$CH_3-CH_2-C\equiv C-CH_2-\overset{O}{\overset{||}{C}}-H \xrightarrow{NaBH_4} CH_3-CH_2-C\equiv C-CH_2-CH_2-OH$$

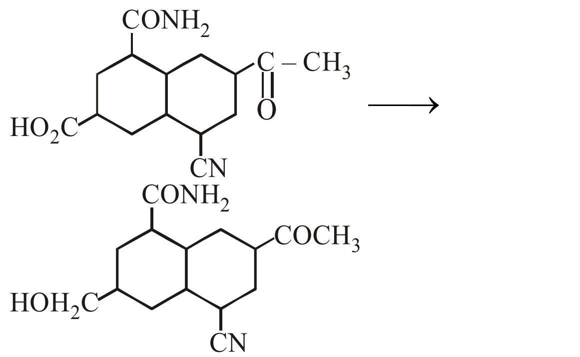
$$\xrightarrow{PBr_3} CH_3-CH_2-C\equiv C-CH_2-CH_2-Br$$

$$\xrightarrow{Mg/ether} CH_3-CH_2-C\equiv C-CH_2-CH_2-MgBr$$

$$\xrightarrow[(2)\ H_3O^+]{(1)\ CO_2} CH_3-CH_2-C\equiv C-CH_2-CH_2-\overset{O}{\overset{||}{C}}-OH$$

2. NTA Ans. (4)

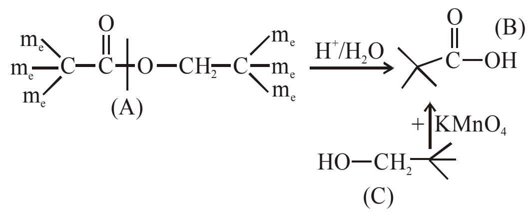
Sol.



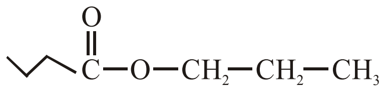
Most suitable reagent for given conversion is B_2H_6 (electrophilic reducing agent)

3. Official Ans. by NTA (3)
Official Ans. by ALLEN (2 & 3)

Sol. (1)

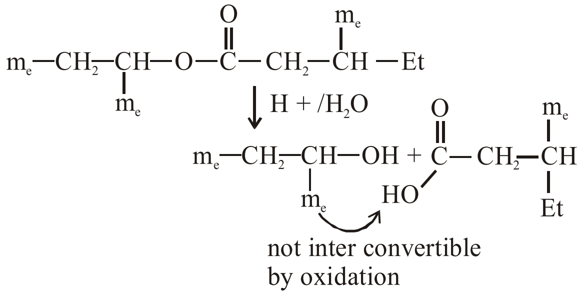


(2)

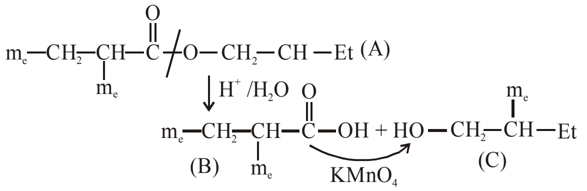


Total 8 'C' → so molecular formula not matched.

(3)

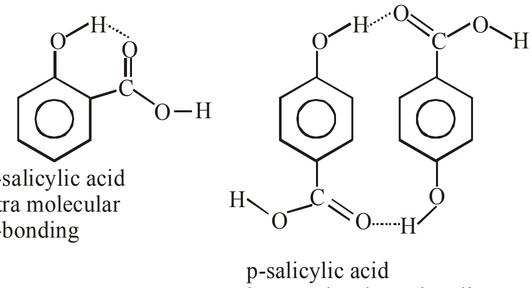


(4)



4. Official Ans. by NTA (3)
Official Ans. by ALLEN (2, 3 & 4)

Sol.



O-salicylic acid intra molecular H-bonding

p-salicylic acid inter molecular H-bonding

(a) B will be more crystalline due to more inter molecular interactions hence more efficient packing.

(b) B will have higher boiling point due to higher intermolecular interactions.

(c) B will be more soluble in water than A as B will have more extent of H-bonding in water

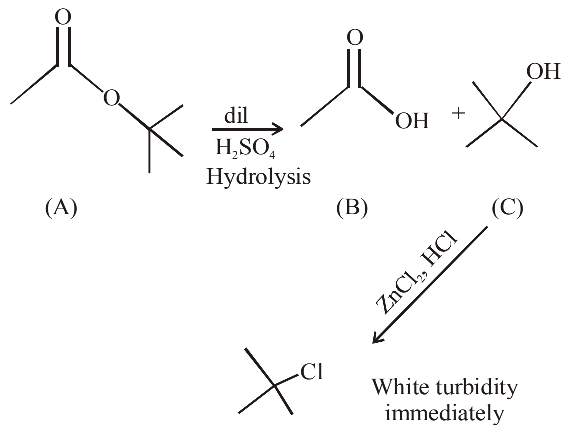
So all three statements are correct

{Solubility date ⇒ O-salicylic acid = 2g/L

P-salicylic acid = 5g/L}

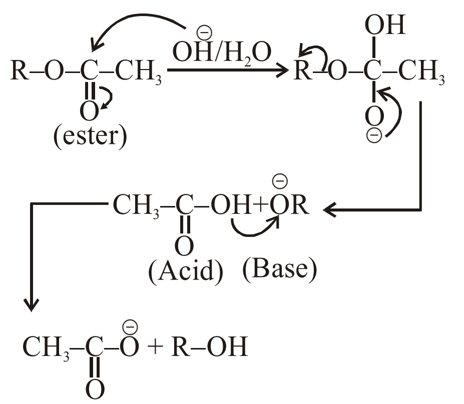
5. Official Ans. by NTA (1)

Sol.



6. Official Ans. by NTA (2)

Sol.

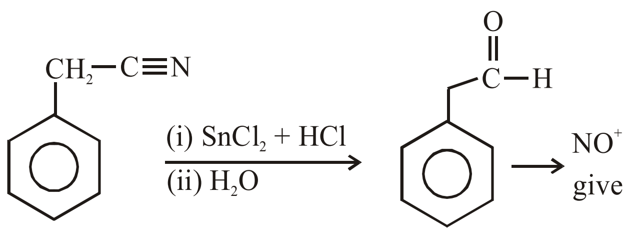


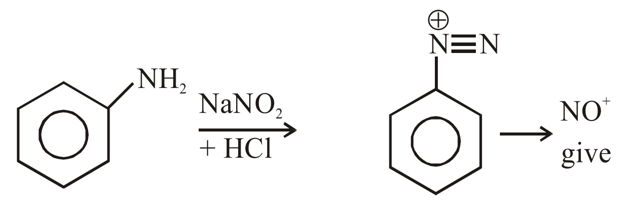
It is a hydrolysis of ester in basic medium.

AMINES

1. Official Ans. by NTA (2)

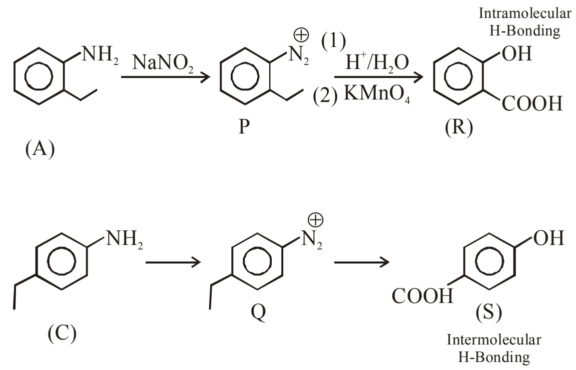
Sol. Kjeldahl method is used for N estimation But not given by 'Diazo' compounds

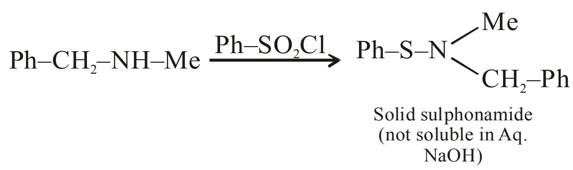




2. Official Ans. by NTA (2)

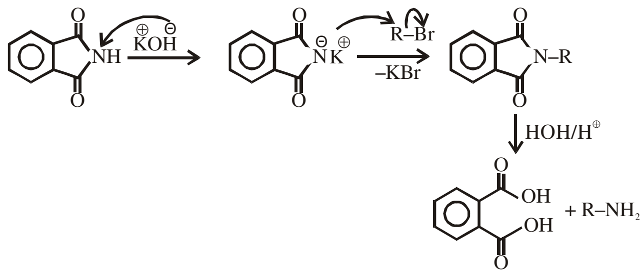
Sol.

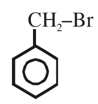


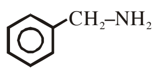


3. Official Ans. by NTA (1)

Sol. Gabriel phthalimide synthesis is used for preparation of 1° Aliphatic amine

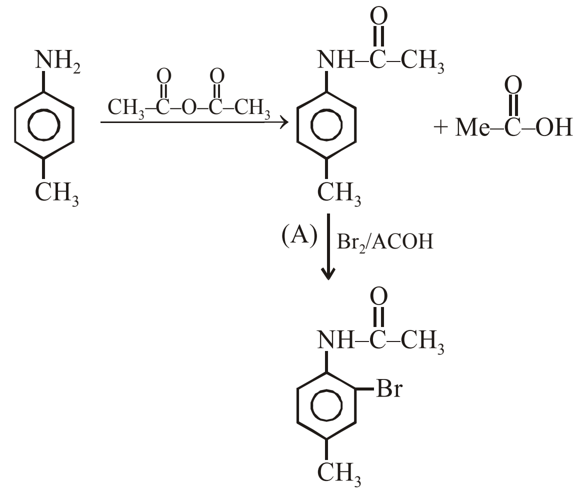


Here R–Br = 

R–NH_2 = 

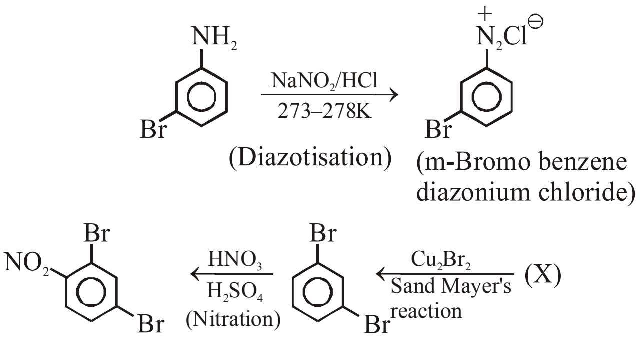
4. NTA Ans. (1)

Sol.



5. NTA Ans. (2)

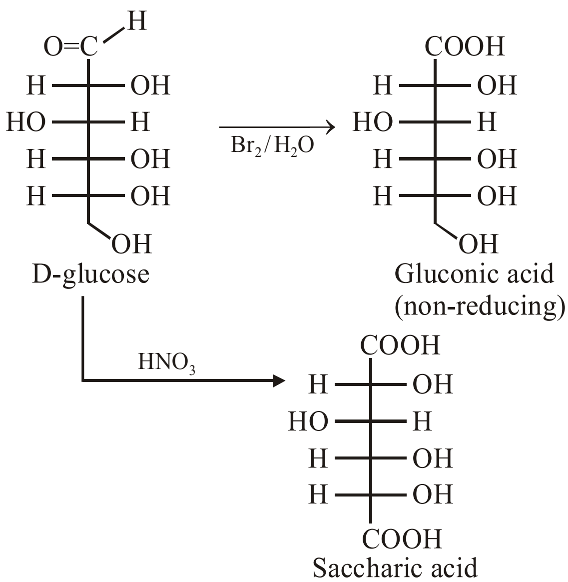
Sol.



BIOMOLECULES

1. NTA Ans. (2)

Sol.



2. NTA Ans. (2)

Sol. Glucose gives negative test with Schiff reagent

3. NTA Ans. (3)

Sol. Two monomers in maltose are α-D-glucose & α-D-glucose.

4. NTA Ans. (4)

Sol. Alanine does not show **Biuret test** because **Biuret test** is used for deduction of peptide linkage & alanine is amino acid.

Albumine is protein so have paptide linkage so it gives positive **Biuret test**.

Positive **Barfoed test** is shown by monosaccharide but not disaccharide. Positive **Molisch's test** is shown by glucose.

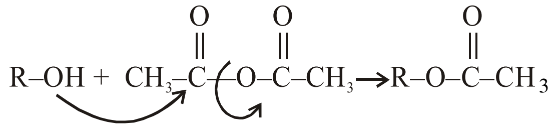
5. Official Ans. by NTA (4)

Sol. (i) Glucose + dry HCl $\xrightarrow{ROH}$ Acetal $\xrightarrow[(CH_3CO)_2O]{x\ Eq}$ acetyl derivative

(ii) Glucose $\xrightarrow{Ni/H_2}$ A $\xrightarrow[(CH_3CO)_2O]{y\ Eq}$ Acetyl derivative

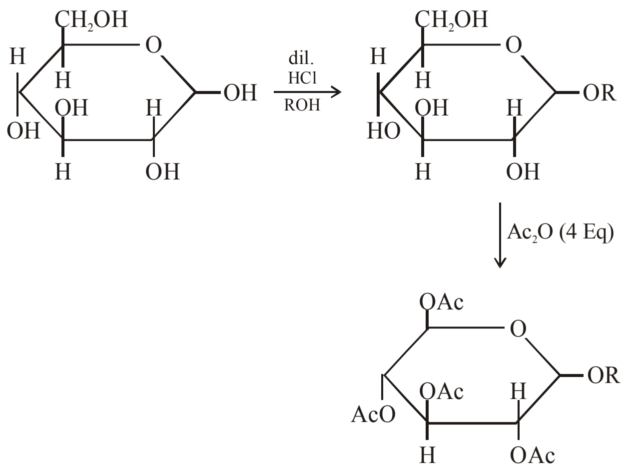
(iii) Glucose $\xrightarrow[(CH_3CO)_2O]{z\ Eq}$ Acetyl derivative

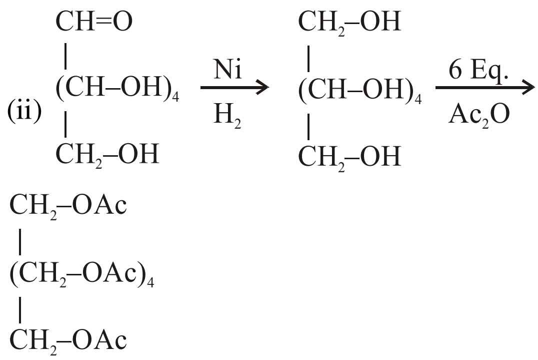
due to presence of –OH group in Glucose the reaction is

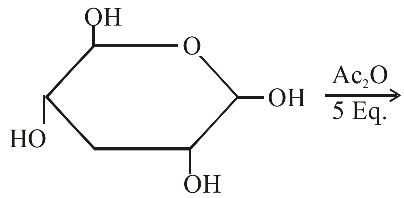


Acetyl derivative

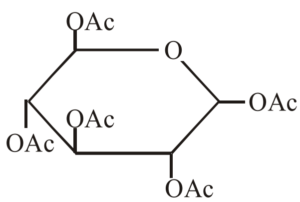
so for (i)







Glucose pentacetase



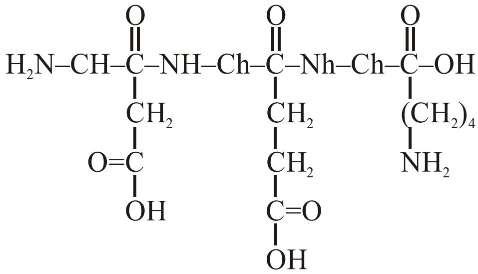
6. Official Ans. by NTA (3)

Sol. Seliwanoff's test is used to distinguish between aldose and ketose sugars; when added to a solution containing ketose, red colour is formed rapidly.

$$\text{Sucrose} \xrightarrow{\text{Hydrolysis}} \text{Glucose} + \text{Fructose} \xrightarrow[\text{reagent}]{\text{Seliwanoff's}} \text{Red colour}$$

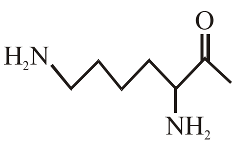
7. Official Ans. by NTA (5)

Sol. Structure of Tri peptide Asp – Glu – Lys

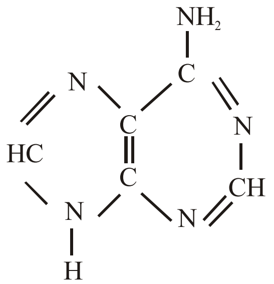


8. Official Ans. by NTA (1)

Sol. Adenine and lysine Both have primary amine react with $CHCl_3$ + alc. KOH



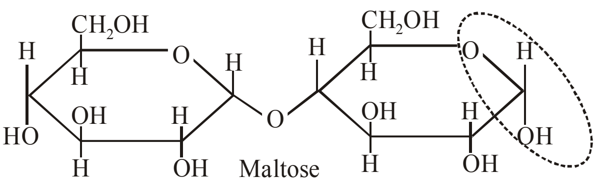
Lysine



Adenine

9. Official Ans. by NTA (2)

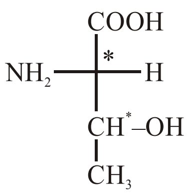
Sol.



Maltose Hemiacetal

10. Official Ans. by NTA (2)

Sol. Structure of Threonine is :



S. 2-chiral center is present

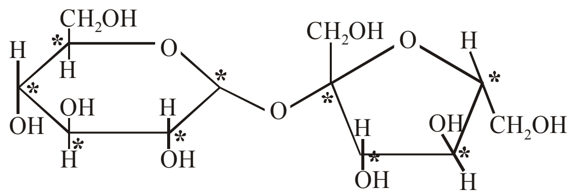
11. Official Ans. by NTA (4)

Sol. Tyrosine is not an essential amino acid.

12. Official Ans. by NTA (4)

13. Official Ans. by NTA (9)

Sol.

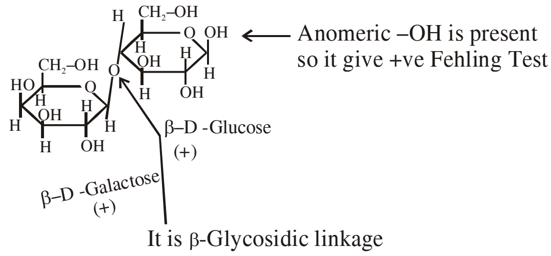


[α-D-Glucose] [β-D-Fructose]

Total no. of chiral carbon in sucrose = 9

14. Official Ans. by NTA (1)

Sol.



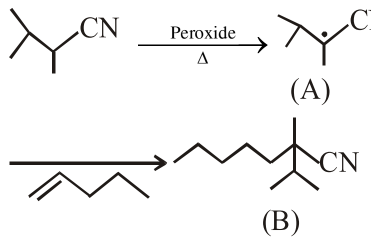
Structure of Lactose

structure of lactose

POLYMER

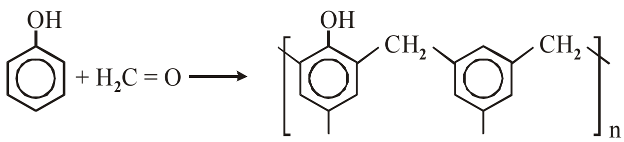
1. NTA Ans. (1)

Sol.



2. NTA Ans. (3)

Sol. Bakelite formation is example of electrophilic substitution and dehydration.



3. NTA Ans. (4)

Sol. PHBV :

Poly β-hydroxy butyrate-co-β-hydroxy valerate

$$CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2-COOH$$

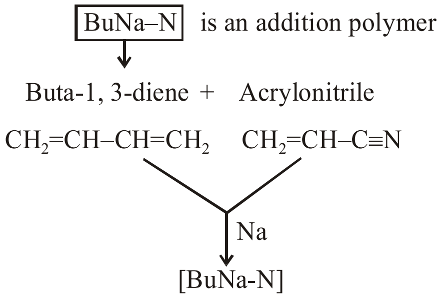
(3-hydroxy butanoic acid)

+

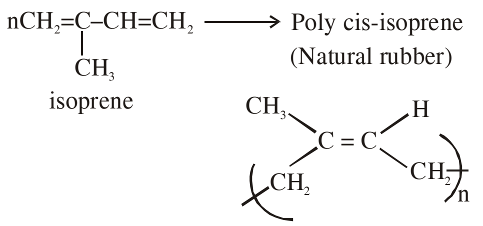
$$CH_3-CH_2-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2-\overset{\displaystyle O}{\overset{||}{C}}-OH$$

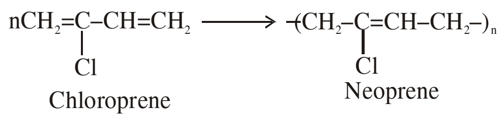
(3-hydroxy pentanoic acid)

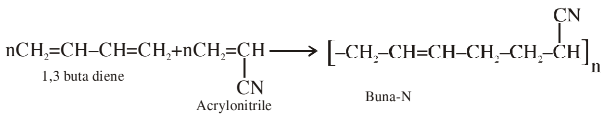
4. Official Ans. by NTA (1)

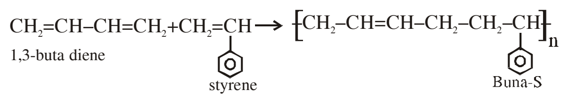
Sol.


5. Official Ans. by NTA (2)

Sol.(a)


(b)


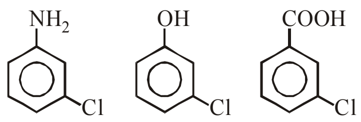
(c)


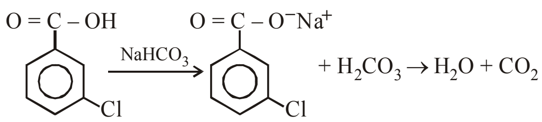
(d)


6. Official Ans. by NTA (3)

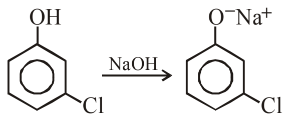
PRACTICAL ORGANIC CHEMISTRY (POC)

1. NTA Ans. (3)

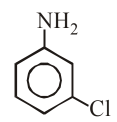
Sol.




Fraction-A



Fraction-B



Fraction-C

2. NTA Ans. (2)

Sol. (A) Benzanilide → Ph–NH–C(=O)–Ph (μ = 2.71 D)

(B) Aniline → $Ph–NH_2$ (μ = 1.59 D)

(C) Acetophenone → Ph–C(=O)–CH_3 (μ = 3.05 D)

Dipole moment : C > A > B

Hence the sequence of obtained compounds is (C), (A) and (B)

3. NTA Ans. (3)

Sol. Liquid which have less difference in boiling point can be isolated by fractional distillation and liquid with less boiling point will be isolated first.

4. NTA Ans. (1)

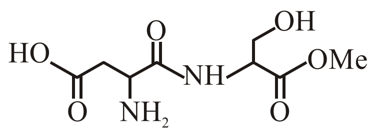
Sol. Kjeldahl's method for estimation of nitrogen is not applicable for nitrobenzene $C_6H_5NO_2$. because reaction with H_2SO_4, nitrobenzene can not give ammonia.

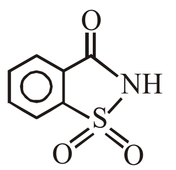
5. NTA Ans. (1)

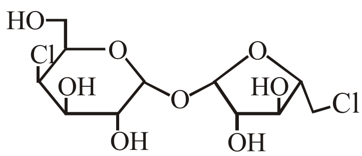
Sol. (i) Blue voilet color with Ninhydrine → amino acid derivative. So it cannot be saccharide or sucralose.

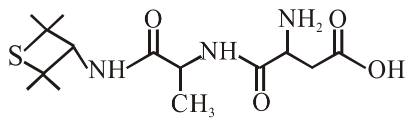
(ii) Lassaigne extract give +ve test with $AgNO_3$. So Cl is present, –ve test with $Fe_4[Fe(CN)_6]_3$ means N is absent. So it can't be Aspartame or Saccharine or Alitame, so C is sucralose.

(iii) Lassaigne solution of B and D given +ve sodium nitroprusside test, so it is having S, so it is Saccharine and Alitame.

(A) Aspartame


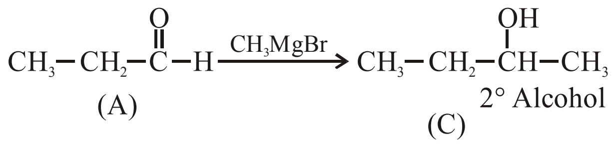
(B) Saccharine


(C) Sucralose


(D) Alitame


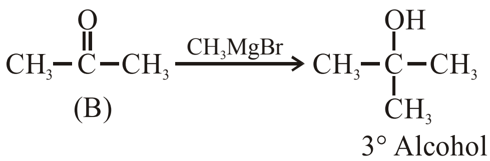
6. Official Ans. by NTA (3)

Sol.



CAN test for alcohol : ✓

Iodoform test : ✓

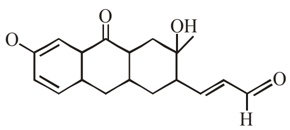


CAN test for alcohol : ✓

Lucas test : Immediately

Iodoform test : ×

7. Official Ans. by NTA (2)

Sol. Compound $\xrightarrow[\text{anhydride}]{\text{Chromic}}$ 

due to pressure of b

8. Official Ans. by NTA (1)

Sol.

Test	Correct reagent
(i) Lucas test ⟶	conc. HCl + $ZnCl_2$
(ii) Dumas method ⟶	CuO / CO_2
(iii) Kjeldahl's method ⟶	H_2SO_4
(iv) Hinsberg Test ⟶	$C_6H_5SO_2Cl$ + aq. KOH

PURIFICATION AND SEPRATION TECHNIQUE

1. Official Ans. by NTA (1)

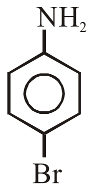
Sol. In Carius method

mass of organic compound = 0.172 gm

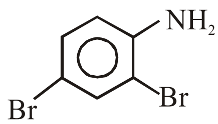
mass of Bromine = 0.08 gm

Hence % of Bromine = $\frac{0.08}{0.172} \times 100$

= 46.51%

(1)  C_6H_6NBr $\left[\%Br = \frac{80}{172} \times 100\right] = 46.51\%$

(2) CH_3CH_2Br C_2H_5Br $\%Br = \frac{80}{109} \times 100 = 73.33\%$

(3)  $C_6H_5NBr_2$

(4) CH_3Br

CHEMISTRY IN EVERYDAY LIFE

1. NTA Ans. (1)

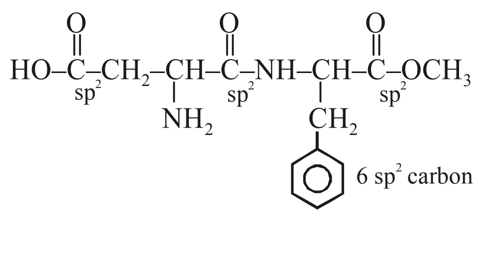
Sol. (i) Riboflavin ⟶ (c) Cheilosis

(ii) Thiamine ⟶ (a) Beriberi

(iii) Pyridoxin ⟶ (d) Convulsions

(iv) Ascorbic acid ⟶ (b) Scurvy

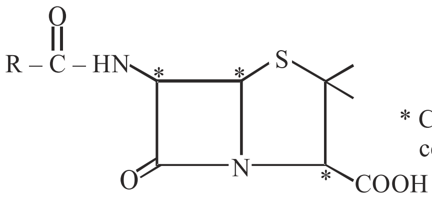
2. NTA Ans. (9.00)

Sol. 

no. of sp^2–carbon → 9

3. NTA Ans. (3.00)

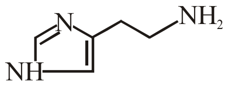
Sol. The structure of penicillin is



* Chiral center = 3

4. NTA Ans. (37.80 to 38.20)

Sol.



M.F. of Histamine is $C_5H_9N_3$

Molecular mass of Histamine is 111

Now, mass % of nitrogen = $\left(\frac{42}{111}\right)\times 100$

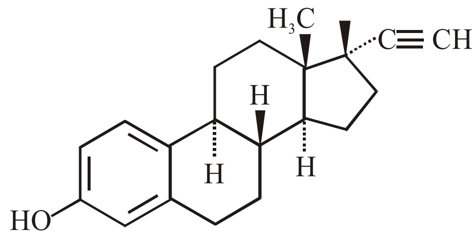
= 37.84%

5. Official Ans. by NTA (3)

Sol. Glycerol is separated by reduced pressure distillation in soap industries.

6. Official Ans. by NTA (1)

Sol.



Ethynylestradiol (novestrol)

gives (1) $Br_2 + H_2O$ test

(2) Lucas test with $ZnCl_2$ + HCl

(3) $FeCl_3$ test of phenolic group.

7. Official Ans. by NTA (3)

Sol. Ranitidine → Antacid

Nardil → Antidepressant

Chloramphenicol → Antibiotic

Dimetane → Antihistamine

8. Official Ans. by NTA (4)

Sol. Anti depressant → drug which enhance the mood. Non adrenaline is neurotransmitter and its level is low in body due to some reason then person suffers from depression and in that situation anti depressant drug is required.

9. Official Ans. by NTA (3)

JANUARY & SEPTEMBER 2020 ATTEMPT (IOC)

QUANTUM NUMBER

1. Official Ans. by NTA (2)

Sol. Electronic configuration of Gd^{3+} is

${}_{64}Gd^{3+}$ = [Xe]$4f^7$

[Xe] | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Gd^{3+} having 7 unpaired electrons.

Magnetic moment (μ) = $\sqrt{n(n+2)}$B.M.

$\mu = \sqrt{7(7+2)}$B.M.

= 7.9 B.M.

n ⇒ Number of unpaired electrons.

2. Official Ans. by NTA (4)

Sol. As per (n + ℓ) rule in 6th period, order of orbitals filling is 6s, 4f, 5d, 6p.

3. Official Ans. by NTA (1)

Official Ans. by ALLEN (2,3)

Sol. l = 0 to (n + 1)

n = 1 | n = 2

l = 0, 1, 2 | l = 0, 1, 2, 3

(n + l) ⇒ $\frac{1s}{1}\ \frac{1p}{2}\ \frac{1d}{3}$ | $\frac{2s}{2}\ \frac{2p}{3}\ \frac{2d}{4}\ \frac{2f}{5}$

n = 3

l = 0, 1, 2, 3, 4

$\frac{3s}{3}\ \frac{3p}{4}\ \frac{3d}{5}\ \frac{3f}{6}\ \frac{3g}{7}$

Now, in order to write electronic configuration, we need to apply (n + l) rule

Energy order : 1s < 1p < 2s < 1d < 2p < 3s < 2d…

Option 1) 13 : $1s^2 1p^6 2s^2 1d^3$ is not half filled

Option 2) 9 : $1s^2 1p^6 2s^1$ is the first alkali metal because after losing one electron, it will achieve first noble gas configuration

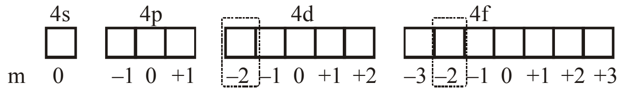
Option 3) 8 : $1s^2 1p^6$ is the first noble gas because after $1p^6$ e^- will enter 2s hence new period

Option 4) 6 : $1s^2\ 1p^4$ has 1p valence subshell.

4. Official Ans. by NTA (4)

Sol. For n = 4

ℓ = 0, 1, 2, 3



∴ 4d & 4f subshell associated with n = 4, m = –2

PERIODIC TABLE

1. NTA Ans. (1)

Sol. Order of electron gain enthalpy (magnitude) is Cl > F > Br > I

2. NTA Ans. (3)

Sol. (i) Electron affinity of second period p-block element is less than third period p-block element due to small size of second period p-block element.

E.A. order : F < Cl

(ii) Down the group electron affinity decreases due to size increases.

EA. order : S > Se

Li > Na

3. NTA Ans. (1)

Sol. Elecronic configuration of Na = [Ne] $3s^1$

Mg = [Ne] $3s^2$

Al = [Ne] $3s^2 3p^1$

Si = [Ne] $3s^2 3p^2$

So order of first ionisation energy is

$\underset{496}{Na} < \underset{737}{Mg} > \underset{577}{Al} < \underset{786}{Si}$ kj/mol

Na < Al < Mg < Si (IE_1 order)

4. NTA Ans. (4)

Sol. If the given elements are arranged according to their position in periodic table

Atomic radius

C > O > F
∧
Cl
∧
Br

Br > Cl > C > O > F

c < b < a < d < e

5. NTA Ans. (1)

Sol. Be ⇒ $1s^2\ 2s^2$

B ⇒ $1s^2\ 2s^2\ 2p^1$

B has a smaller size than Be

it is easier to remove 2p electron than 2s electron due to less pentration effect of 2p than 2s.

2p electron of Boron is more shielded from the nucleus by the inner core of electron than the 2s electron of Be

B has a smaller size than Be

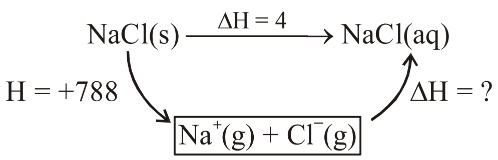
6. Official Ans. by NTA (3)

Sol. Correct order of size for isoelectronic species.

$Al^{3+} < Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

7. Official Ans. by NTA (2)

Sol.



$4 = 788 + \Delta H$

$\Delta H = -784$ kJ

8. Official Ans. by NTA (2)

Sol. $H_{(g)} + e^- \rightarrow H^-$ is exothermic

rest of all endothermic process.

9. Official Ans. by NTA (4)

Sol.

	O^{-2}	F^-	Na^+	Mg^{2+}
z	8	9	11	12
e^-	10	10	10	10
$\frac{z}{e}$	0.8	0.9	1.1	1.2

as $\frac{z}{e}$ ratio increases size decreases.

Thus correct ionic radii order is

$O^{-2} > F^- > Na^+ > Mg^{2+}$

Therefore correct option is (4)

10. Official Ans. by NTA (2)

Sol. Element with atomic no. 101 is an Actinoid element.

11. Official Ans. by NTA (2)

Sol. Let suppose element X ⇒

$$X_{(g)} \xrightarrow[800]{IE_1} \overset{+}{X}(g) \xrightarrow[2427]{IE_2} \overset{+2}{X}(g) \xrightarrow[3658]{IE_3}$$

$$\overset{+3}{X}(g) \xrightarrow[25024]{IE_4} \overset{+4}{X}(g) \xrightarrow[32824]{IE_5} \overset{+5}{X}(g)$$

X^{+3} has stable inert gas configuration as there is high jump after IE_3

So valence electrons are 3

12. Official Ans. by NTA (3)

Sol. I, A_N : Be < Mg

II IE : Be > Al

III Charge/radius ratio of Be is less than that of Al

IV Be, Al mainly form covalent compounds

13. Official Ans. by NTA (4)

Sol. 1 0 9

un nil enn

Hence correct name → unnilennium

14. Official Ans. by NTA (3)

Sol. When we are moving from left to right in a periodic table acidic character of oxides increases (as well as atomic number of atom increases)

∴ X < Y < Z (acidic character)

X < Y < Z (atomic number)

15. Official Ans. by NTA (4)

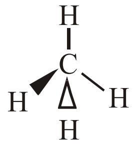
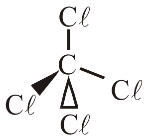
Sol. In general across a period atomic radius decreases while ionisation enthalpy, electron gain enthalpy and electronegativity increases because effective nuclear charge (Z_{eff}) increases.

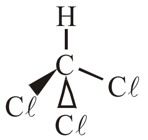
16. Official Ans. by NTA (101.00)

Sol. Unnilunium ⇒ 101

CHEMICAL BONDING

1. NTA Ans. (1)

Sol.  $\mu_{net} = 0$  $\mu_{net} = 0$

 $\mu_{net} \neq 0$

2. NTA Ans. (4)

Sol. Order is

ion – ion > ion – dipole > dipole – dipole

3. NTA Ans. (1)

Sol. According to MOT (If z is internuclear axis)

The configuration of

$$CN^- : \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2 = \pi_{2p_y}^2, \sigma_{2p_z}^2$$

Bond order $= \frac{1}{2}(10-4)$

$= 3$

CN^- is diamagnetic due to absence of unpaired electron

4. NTA Ans. (3)

Sol. Ethyl acetate $\left(H_3C\text{–}\underset{\underset{O}{\|}}{C}\text{–}O\text{–}CH_2\text{–}CH_3\right)$ is polar molecule. Hence there will be dipole-dipole attraction and london dispersion forces are present.

5. NTA Ans. (3)

Sol. Bond length order in carbon halogen bonds are in the order of C – F < C – Cl < C – Br < C – I

Hence, Bond energy order

C – F > C – Cl > C – Br > C – I

6. NTA Ans. (1)

Sol. CCl_4 is molecular solid so does not conduct electricity in liquid & solid state.

7. NTA Ans. (1)

Sol.

	number of unpaired electron	magnetic moment
$O_2^{\ominus}$	1	1.73 B.M
$O_2^{\oplus}$	1	1.73 B.M
O_2	2	2.83 BM

8. NTA Ans. (4)

Sol. 1. MgO Basic

Cl_2O Acidic

Al_2O_3 amphoteric

2. Cl_2O Acidic

CaO Basic

P_4O_{10} Acidic

3. Na_2O Basic

SO_3 Acidic

Al_2O_3 amphoteric

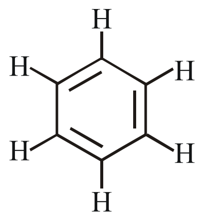
4. N_2O_3 Acidic

Li_2O Basic

Al_2O_3 amphoteric

9. NTA Ans. (4)

Sol.



Each carbon atom is sp^2 hybridized

Therefore each carbon has 3 sp^2 hybrid orbitals.

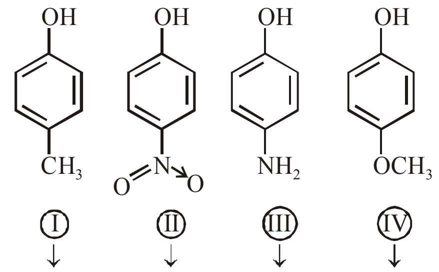
Hence total sp^2 hybrid orbitals are 18.

10. Official Ans. by NTA (1)

11. Official Ans. by NTA (3.00)

12. Official Ans. by NTA (1)

Sol.

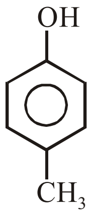


BP value from net ≃202°C ≃279° ≃284°C ≃243°C

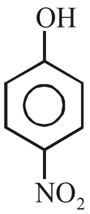
BP ∝ dipolemoment (μ)

Alter

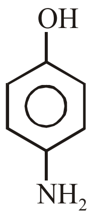
Increasing order of boiling point is :

I. 

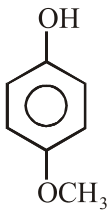
⇒ Shows hydrogen bonding from –O–H group only

II. 

⇒ Shows strongest hydrogen bonding from both sides of –OH group as well as $-NO_2$ group.

III. 

⇒ Shows stronger hydrogen from both side of –OH group as well as $-NH_2$ group.

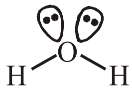
IV. 

⇒ Shows stronger hydrogen bonding from one side –OH–group and another side of $-OCH_3$ group shows only dipole-dipole interaction.

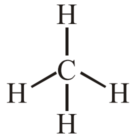
⇒ Hence correct order of boiling point is:

(I) < (IV) < (III) < (II)

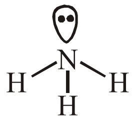
13. Official Ans. by NTA (2)

Sol. (1) 

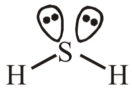
sp^3, 104°5'

(2) 

sp^3, 109°28'

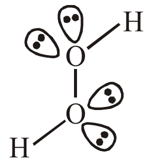
(3) 

sp^3, 107°

(4) 

92°

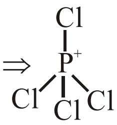
14. Official Ans. by NTA (1)

Sol. 

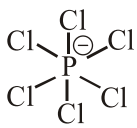
hydrogen peroxide, in the pure state, is non-planar and almost colourless (very pale blue) liquid.

15. Official Ans. by NTA (2)

Sol. $PCl_{5(s)}$ exist as $[PCl_4]^+$ and $[PCl_6]^-$

$[PCl_4]^+ \Rightarrow$  (sp^3 hybridisation)

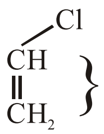
Tetrahedral

$[PCl_6]^- \Rightarrow$ 

octahedral

sp^3d^2 hybridization

16. Official Ans. by NTA (3)

Sol. 

In option (3) C—Cl bond is shortest due to resonance of lone pair of –Cl.

Due to resonance C—Cl bond acquire partial double bond character.

Hence C—Cl bond length is least.

17. Official Ans. by NTA (2)

Sol. $XeF_4 + SbF_5 \rightarrow [XeF_3]^+ [SbF_6]^-$

sp^3d^2 sp^3d sp^3d sp^3d^2

18. Official Ans. by NTA (4)

Sol. Bond order of NO^{2+} = 2.5

Bond order of NO^{+} = 3

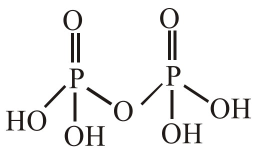
Bond order of NO = 2.5

Bond order of NO^{-} = 2

Bond order α bond strength.

19. Official Ans. by NTA (4)

Sol. Pyrophosphoric acid.



P – OH linkages = 4

P = O linkages = 2

P–O–P linkages = 1

20. Official Ans. by NTA (3)

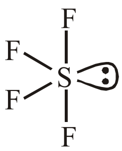
Official Ans. by ALLEN (2)

Sol.

Type of interaction	Interaction Energy(E)
ion - ion	$E \propto \frac{1}{r}$
dipole - dipole	$E \propto \frac{1}{r^3}$
London dispersion	$E \propto \frac{1}{r^6}$

21. Official Ans. by NTA (1)

Sol.

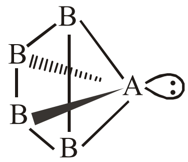


4σ bonds +1 lone pair

∴ Shape (including lone pair of electrons) is Trigonal bipyramidal

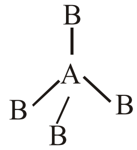
22. Official Ans. by NTA (1)

Sol. (1) If AB_4 molecule is a square pyramidal then it has one lone pair and their structure should be



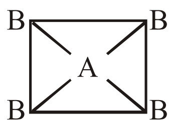
and it should be polar because dipole moment of lone pair of 'A' never be cancelled by others.

(2) If AB_4 molecule is a tetrahedral then it has no lone pair and their structure should be



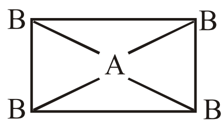
and it should be non polar due to perfect symmetry.

(3) If AB_4 molecule is a square planar then



it should be non polar because vector sum of dipole moment is zero.

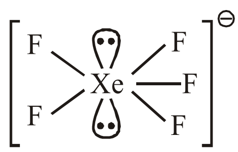
(4) If AB_4 molecule is a rectangular planar then

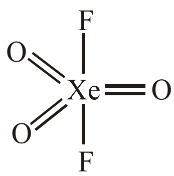


it should be non polar because vector sum of dipole moment is zero.

23. Official Ans. by NTA (1)

Sol.





XeF_5^-	XeO_3F_2
sp^3d^3	sp^3d
Pentagonal planar	Trigonal bipyramidal

COORDINATION CHEMISTRY

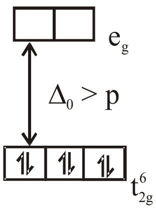
1. NTA Ans. (4)

Sol. In complex $[Ni(CO)_4]$ decrease in Ni–C bond length and increase in C–O bond length as well as it's magnetic property is explained by MOT.

2. NTA Ans. (4)

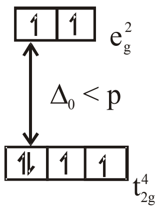
3. NTA Ans. (4)

Sol. (a) Co^{+3} (with strong field ligands)



diamagnetic

(b) If $\Delta_0 < p$;



(c) Splitting power of ethylenediamine (en) is greater than fluoride (F^-) ligand therefore more energy absorbed by $[Co(en)_3]^{3+}$ as compared to $[CoF_6]^{3-}$.

So wave length of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$

(d) $\Delta_t = \frac{4}{9}\Delta_0$

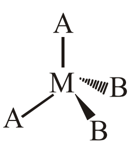
so if Δ_0 = 18,000 cm^{-1}

$$\Delta_t = \frac{4}{9} \times 18000 = 8000\ cm^{-1}$$

Statement (a) and (d) are incorrect.

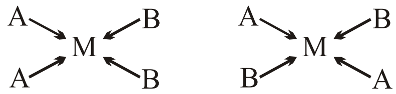
4. NTA Ans. (1)

Sol. (a) If the complex MA_2B_2 is sp^3 hybridised then the shape of this complex is tetrahedral this structure is opticaly inactive due to the presence of plane of symmetry.



Optical isomes = 0

(b) If the complex MA_2B_2 is dsp^2 hybridised then the shape of this complex is square planar.



(Cis-isomer) (Trans-isomer)

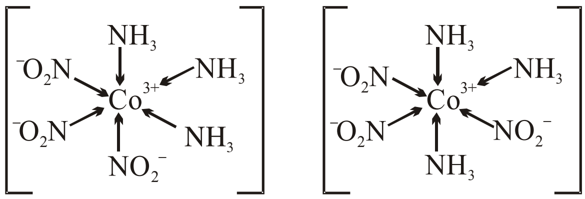
Both isomers are optically inactive due to the presence of plane of symmetry.

Optical isomers = 0

5. NTA Ans. (3)

Sol. $[Ma_3b_3]$ type complex shows fac and mer isomerism.

$[Co(NH_3)_3(NO_2)_3]$



fac-isomer mer-isomer

6. NTA Ans. (26.60 to 27.00)

Sol. Number of moles of Cl^- precipitated in $[Co(NH_3)_6]Cl_3$ is equal to number of moles of $AgNO_3$ used.

$$\frac{0.3}{267.46} \times 3 = \frac{0.125 \times V}{1000}$$

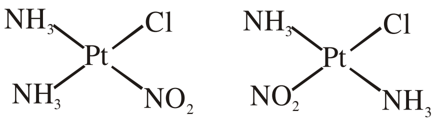
where V is volume of $AgNO_3$ (in mL)

V = 26.92 mL

7. NTA Ans. (4)

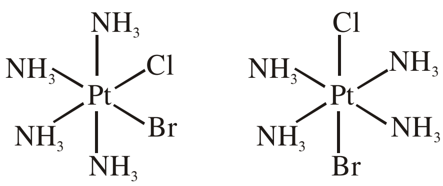
Sol. $[Pt(NH_3)_3Cl]^+$ & $[Pt(NH_3)Cl_5]^-$ does not show geometrical isomerism

(c)



cis trans

(d)



cis trans

8. NTA Ans. (1)

Sol.

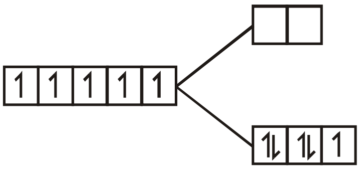
$[Ni(CO)_4]$	μ_m = 0 B.M.
$[Ni(H_2O)_6]Cl_2$	μ_m = 2.8 B.M.
$Na_2[Ni(CN)_4]$	μ_m = 0 B.M.
$[PdCl_2(PPh_3)_2]$	μ_m = 0 B.M.

A ≈ C ≈ D < B

9. NTA Ans. (20)

10. NTA Ans. (2)

Sol. $[Pb(F)(Cl)(Br)(I)]^{2-}$ have three geometrical isomer so formula for $[Fe(CN)_6]^{n-6}$ is $[Fe(CN)_6]^{3-}$ and CFSE for this complex is $Fe^{3\oplus} \Rightarrow 3d^5 4s^{\circ}$



Magnetic Moment = $\sqrt{3}$

= 1.73 B.M

CFSE = $[(-0.4\times5) + (0.6 \times 0)]\ \Delta_0$

= $-2.0\ \Delta_0$

11. NTA Ans. (1)

Sol. $Cr(H_2O)_6\ Cl_n$

if magnetic mement is 3.83 BM then it contain three unpaired electrons. It means chromium in +3 oxidation state so molecular formula is $Cr(H_2O)_6\ Cl_3$

∴ This formula have following isomers

(a) $[Cr(H_2O)_6]Cl_3$: react with $AgNO_3$ but does not show geometrical isomerism.

(b) $[Cr(H_2O)_5Cl]Cl_2.H_2O$ react with $AgNO_3$ but does not show geometrical isomerism.

(c) $[Cr(H_2O)_4Cl_2]Cl.2H_2O$ react with $AgNO_3$ & show geometrical isomerism.

(d) $[Cr(H_2O)_3Cl_3].3H_2O$ does not react with $AgNO_3$ & show geometrical isomerism.

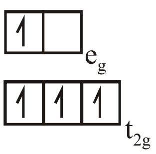
$[Cr(H_2O)_4Cl_2]Cl.2H_2O$ react with $AgNO_3$ & show geometrical isomerism and it's IUPAC nomenclature is Tetraaquadichlorido chromium (III) Chloride dihydrate.

12. NTA Ans. (2)

Sol. I $[Cr(H_2O)_6]^{2+}$

$Cr^{+2} \Rightarrow [Ar]\,3d^4$

$H_2O \rightarrow$ Weak field ligand



Unpaired e^- = 4

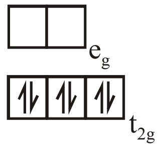
Magnetic moment = $\sqrt{24}$ BM

= 4.89 BM

II $[Fe(CN)_6]^{4-}$

$Fe^{+2} \Rightarrow [Ar]\,3d^6$

$CN^- \rightarrow$Strong field ligand



Unpaired e^- = 0

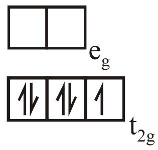
Magnetic moment = 0 BM

= 0 BM

III $[Fe(C_2O_4)_3]^{3-}$

$Fe^{+3} \Rightarrow [Ar]\,3d^5$

As $\Delta_0 > P$



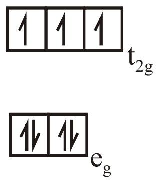
Unpaired e^- = 1

Magnetic moment = $\sqrt{3}$ BM = 1.73 BM

IV $(Et_4N)^+[CoCl_4]^{2-}$

$Co^{+2} \Rightarrow [Ar]\,3d^7$

$[CoCl_4]^{-2}$ =



Unpaired electrons = 3

Magnetic moment = $\sqrt{15}$ BM

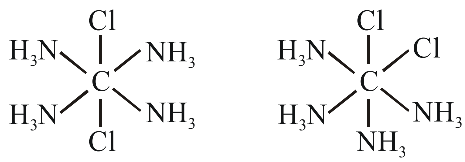
= 3.87 BM

Hence order of magnetic moment is

I > IV > III > II

13. NTA Ans. (4)

Sol. $[Co(NH_3)_4Cl_2]$ has 2 geometrical isomers



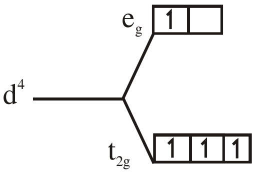
trans cis

cis isomer has Cl–Co–Cl angle of 90°

14. Official Ans. by NTA (2)

15. Official Ans. by NTA (3)

Sol.



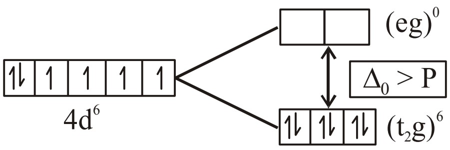
back pairing is not possible because pairing energy > Δ_O.

16. Official Ans. by NTA (00)

Sol. Magnetic moment (in B.M.) of $[Ru(H_2O)_6]^{2+}$ would be; while considering that $\Delta_0 > P$,

$Ru_{(44)}$; $[Kr]4d^7 5s^1$ (in ground state)

$\Rightarrow$ In $Ru^{2+} \Rightarrow 4d^6 \Rightarrow (t_2g)^6(eg)^0$

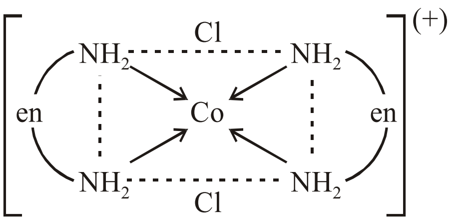


$\Rightarrow$ Here number of unpaired electrons in $Ru^{2+} = (t_2g)^6\ (eg)^0 = 0$ and Hence

$\mu_m = \sqrt{n(n+2)}\text{B.M.} = \boxed{0\ \text{B.M.}}$

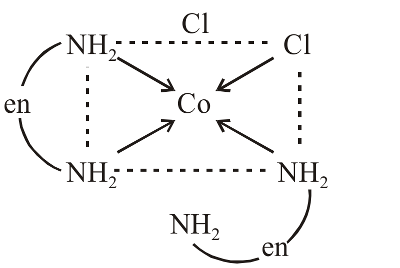
17. Official Ans. by NTA (4)

Sol. (A) *trans*-$[Co(en)_2Cl_2]^+$



$\Rightarrow$ (A) is trans form and shows plane of symmetry which is optically inactive (not optically active)

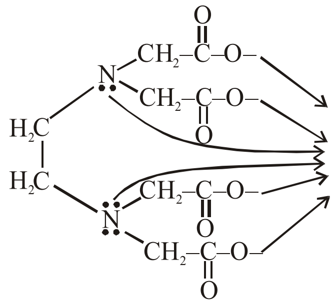
(B) *cis*-$[Co(en)_2Cl_2]^+$



$\Rightarrow$ (B) is cis form and does not shows plane of symmetry, hence it is optically active.

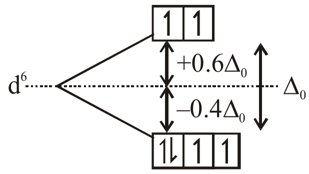
18. Official Ans. by NTA (6)

Sol. $EDTA^{4-}$ is hexadentate ligand, so its donation sites are six.



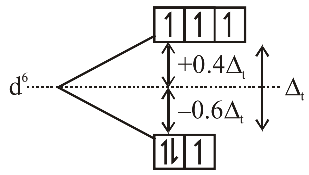
19. Official Ans. by NTA (3)

Sol. For high spin octahedral field



CFSE = (4) $(-0.4\Delta_0) + 2(0.6\ \Delta_0) = -\ 0.4\ \Delta_0$

For high spin tetrahedral field



CFSE = $3(-0.6\Delta_t) + 3(0.4\ \Delta_t) = -\ 0.6\ \Delta_t$

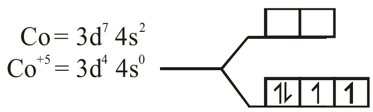
20. Official Ans. by NTA (1)

Sol. $[Ni(CN)_4]^{2-}$

dsp^2 hybridisation.

21. Official Ans. by NTA (3)

Sol. $[Co(OX)_2(OH)_2]^-$ $\Delta_0 > P$ [S.F.L]



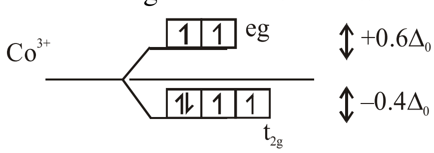
It has highest number of unpaired e^-s. so it is most paramagnetic.

22. Official Ans. by NTA (4)

Official Ans. by ALLEN (2, 4)

Sol. $[CoF_3(H_2O)_3]$ $\Delta_0 < P$

Means all ligands behaves as weak field ligands



$= \left[-0.4 \times 4 + 0.6 \times 2\right]\Delta_0$

$= \left[-1.6 + 1.2\right]\Delta_0$

$= \left[-0.4\Delta_0\right]$

23. Official Ans. by NTA (3)

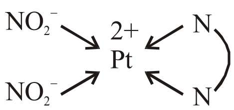
Sol.

Complex	e^- configuration	no. of unpaired e^-
$[Mn(H_2O)_6]^{2+}$ WFL	↑ ↑ eg ↑ ↑ ↑ t2g	5
$[Cr(H_2O)_6]^{2+}$ WFL	↑ ☐ eg ↑ ↑ ↑	4
$[COCl_4]^{2-}$ Tetrahedral	↑ ↑ ↑ t_2 ↑↓ ↑↓ e	3
$[Fe(H_2O)_6]^{2+}$ WFL	↑ ↑ eg ↑↓ ↑ ↑ t_2g	4
$[Co(OH)_4]^{2-}$ WFL	↑ ↑ ↑ t_2 ↑↓ ↑↓ e	3
Tetrahedral $[Fe(NH_3)_6]^{2+}$	↑ ↑ ↑↓ ↑ ↑	4

Thus complex $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ have same no. of unpaired e^- and hence same magnetic moment (spin only).

24. Official Ans. by NTA (1)

Sol. $[Pt\ (en)\ (NO_2)_2] \Rightarrow$ does not show G.I. as well as optical isomerism.



This complex will have three linkage isomers as follows :-

$[Pt\ (en)\ (NO_2)_2]$ I

$[Pt\ (en)\ (NO_2)(ONO)]$ II

$[Pt\ (en)\ (ONO)_2]$ III

25. Official Ans. by NTA (3)

Sol. % mass of water

$$= \frac{x \times 18}{(12+6\times16+35\times3+52)} \times 100 = 13.5$$

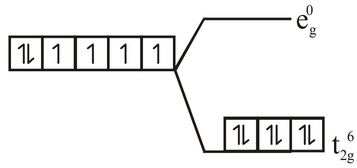
$$\Rightarrow \quad x = \frac{265\times13.5}{18\times100} \simeq 2$$

26. Official Ans. by NTA (3)

Sol. $[Ru(en)_3]Cl_2$ — Ru $\Rightarrow$ 4d series, en $\Rightarrow$ chelating ligand

CN = 6, octahedral splitting — hence large splitting of d-subshell

$Ru^{+2} \Rightarrow [Kr]4d^65s^0$

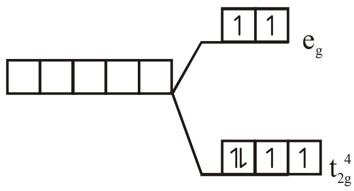


$[Fe(H_2O)_6]Cl_2 \Rightarrow H_2O \Rightarrow$ Weak filled ligand

$Fe^{+2} \Rightarrow [Ar]\ 3d^64s^0$

less splitting

CN = 6 octahedral splitting



27. Official Ans. by NTA (4)

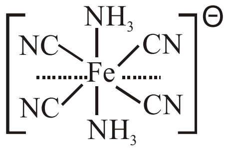
Sol. CFSE = $0.4\ \Delta_0$

$$= 0.4 \times \frac{20300}{83.7}$$

$$= 97 \text{ kJ/mol}$$

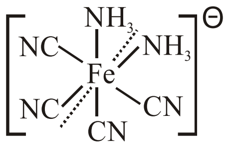
28. Official Ans. by NTA (3)

Sol. (1)



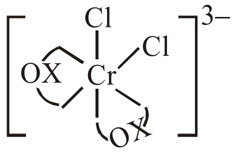
optically inactive

(2)



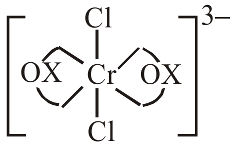
optically inactive

(3)



optically active

(4)



optically inactive

29. Official Ans. by NTA (2)

Sol. $[Ni(NH_3)_2Cl_2]$ is tetrahedral complex, therefore does not show geometrical and optical isomerism.

$[Ni(NH_3)_2Cl_2]$ does not show structural isomerism

$[Ni(NH_3)_4(H_2O)_2]^{2+}$ & $[Pt(NH_3)_2Cl_2]$ show geometrical isomerism

$[Ni(en)_3]^{2+}$ show optical isomerism

30. Official Ans. by NTA (3)

Sol. (I) Under weak field ligand, octahedral Mn(II) and tetrahedral Ni(II) both the complexes are high spin complex.

(II) Tetrahedral Ni(II) complex can very rarely be low spin because square planar (under strong ligand) complexes of Ni(II) are low spin complexes.

(III) With strong field ligands Mn (II) complexes can be low spin because they have less number of unpaired electron (unpaired electron = 1)

While with weak field ligands Mn(II) complexes can be high spin because they have more number of unpaired electron (unpaired electron = 5)

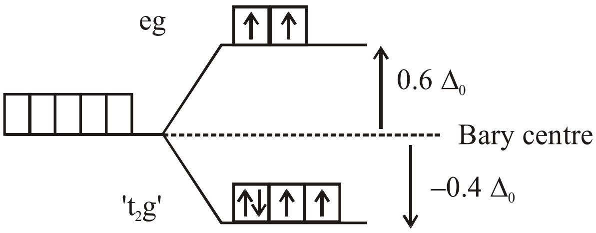
(IV) Aqueous solution of Mn(II) ions is pink in colour.

31. Official Ans. by NTA (2)

Sol. If spin only magnetic moment of the complex is 4.90 BM, it means number of unpaired electrons should be 4.

(A) In octahedral complex : $[M(H_2O)_6]^{2+}$

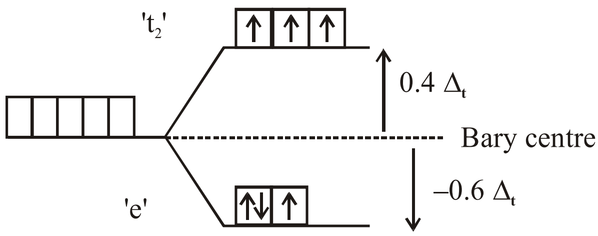
d^6



C.F.S.E. = $(-0.4\ \Delta_0) \times 4 + (+0.6\ \Delta_0) \times 2 + 0 \times P$
$= -0.4\ \Delta_0$

(B) In tetrahedral complex : $[M(H_2O)_4]^{2+}$

d^6



C.F.S.E. = $(-0.6\ \Delta_t) \times 3 + (+0.4\ \Delta_t) \times 3 + 0 \times P$
$= -0.6\ \Delta_t$

32. Official Ans. by NTA (6)

Sol. (A) $Na_4[\overset{x}{Fe}(CN)_5(NOS)]$

$(+1)4 + x + (-1)5 + (-1)\ 1 = 0$

$\boxed{x = +2}$

(B) $Na_4[\overset{y}{Fe}O_4]$

$(+1)4 + y + (-2)4 = 0$

$\boxed{y = +4}$

(C) $[\overset{z}{Fe}_2(CO)_9]$

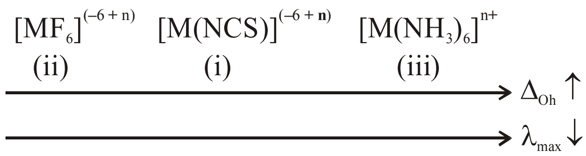
$2z + 0 \times 9 = 0$

$\boxed{z = 0}$

so $(x + y + z) = +2 + 4 + 0$
$= 6$

33. Official Ans. by NTA (2)

Sol. Strength of ligand $F^- < NCS^- < NH_3$



As given in graph : A < B < C (λ_{max})

$\therefore$ Correct matching is A–(iii), B-(i), C-(ii)

METALLURGY

1. NTA Ans. (2)

Sol. Wrought iron is purest from of commercial iron.

2. NTA Ans. (2)

Sol. Liquation method is used when the melting point of metal is less compare to the melting point of the associated impurity.

3. NTA Ans. (1)

Sol. In blast furnace (metallugy of iron) involved reactions are

(a) $CaO + SiO_2 \longrightarrow CaSiO_3$

(b) $3Fe_2O_3 + CO \longrightarrow 2Fe_3O_4 + CO_2$

4. NTA Ans. (2)

Sol. A reduces BO_2 when temperature is above 1400°C because above 1400°C A has more –ve $\Delta G°$ for AO_2 formation than B to BO_2 formation.

5. Official Ans. by NTA (2)

Sol. Impure zinc is refined by distillation method.

6. Official Ans. by NTA (4)

Sol. "Boron" and "Silicon" of very high purity can be obtained through :-
zone refining method only.
While other methods are used for other metals/elements i.e.
(i) Vapour phase refining
(ii) electrolytic refining
(iii) liquation etc.

7. Official Ans. by NTA (3)

Sol. Ellingham diagram provides information about temperature dependence of the standard gibbs energies of formation of some metal oxides.

8. Official Ans. by NTA (1)

Sol. Due to industrial process SO_2 gas is released which is responsible for acid rain & global warming.

9. Official Ans. by NTA (4)

Sol. (a) $\underset{\text{lime stone}}{CaCO_3} \xrightarrow{\Delta} CaO + CO_2$ {In Blast furnace}

(b) Ag form cyanide complex $[Ag(CN)_2]^-$ during cyaride process

$Ag / Ag_2S + CN^{\ominus} \rightarrow [Ag(CN)_2]^-$

(c) Ni is purified by mond's process

(d) Zr and Ti are purified by van arkel method

All (a), (b), (c), (d) are correct statements

Thus correct option is (4)

10. Official Ans. by NTA (2)

Sol. Cast iron is used for manufacturing of wrought iron and steel.

HYDROGEN & IT'S COMPOUND

1. Official Ans. by NTA (1)

Sol. High purity (>99.95%) dihydrogen is obtained by electrolysing warm aqueous barium hydroxide solution between nickel electrodes.

2. Official Ans. by NTA (3)

Sol. Temporary hardness of water is removed by clark method and boiling. While permanent hardness of water is removed by treatment with sodium carbonate (Na_2CO_3), calgons method and ion-exchange method

SALT ANALYSIS

1. Official Ans. by NTA (2)

Sol. $\underset{(X)}{Na_2SO_3} \xrightarrow{\text{dil. } H_2SO_4} \underset{(Y)}{SO_2} \xrightarrow{NaOH} \underset{(X)}{Na_2SO_3} \xrightarrow{SO_2 + H_2O} \underset{(Z)}{NaHSO_3}$

COMPLETE S-BLOCK

1. NTA Ans. (1)

Sol. $\underset{\text{(hot and conc.)}}{6NaOH} + 3Cl_2 \longrightarrow \underset{(A)}{NaClO_3} + \underset{\text{side product}}{5NaCl + 3H_2O}$

$\underset{\text{dry}}{2Ca(OH)_2} + 2Cl_2 \longrightarrow \underset{(B)}{Ca(OCl)_2} + \underset{\text{side product}}{CaCl_2 + 2H_2O}$

2. NTA Ans. (4)

Sol. $\underset{\text{Gypsum}}{CaSO_4.2H_2O} \xrightarrow{393K} \underset{\text{Plaster of paris}}{CaSO_4.\frac{1}{2}H_2O} + \frac{3}{2}H_2O$

3. NTA Ans. (1)

Sol. $3Mg + N_2 \xrightarrow{\Delta} Mg_3N_2$
(A) (B)
$Mg_3N_2 \xrightarrow{6H_2O} 3Mg(OH)_2 + 2NH_3$ (colourless gas)

$CuSO_4 + 4NH_3 \longrightarrow [Cu(NH_3)_4]SO_4$
deep blue solution

4. NTA Ans. (3)

Sol. Lithium has highest hydration enthalpy among alkali metals due to its small size.
LiCl is soluble in pyridine because LiCl have more covalent character.
Li does not form ethynide with ethyne.
Both Li and Mg reacts slowly with H_2O

5. Official Ans. by NTA (4)

Sol. (I) $Ca(OH)_2$ is used in white wash

(II) NaCl is used in preparation of washing soda

$2NH_3 + H_2O + CO_2 \longrightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + CO_2 \longrightarrow 2NH_4HCO_3$

$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + NaHCO_3(s)$

$2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + CO_2 + H_2O$

(III) $CaSO_4.\frac{1}{2}H_2O$ (Plaster of Paris) is used for making casts of statues

(IV) $CaCO_3$ is used as an antacid

6. Official Ans. by NTA (2)

Sol. [Be]

$BeSO_4$ is water soluble

$Be(OH)_2$ is water insoluble

BeO is stable to heat

7. Official Ans. by NTA (3)

Sol. $Li + O_2 \rightarrow Li_2O$ (Major Oxides)
excess

$Na + '' \rightarrow Na_2O_2$ ('')

$K + '' \rightarrow KO_2$ ('')

8. Official Ans. by NTA (3)

Sol. Toilet cleaning liquid has about 10.5% w/v HCl; to neutralise its affect aqueous $NaHCO_3$ is used while NaOH is avoid for this purpose because its highly corosive in nature and can burn body.

9. Official Ans. by NTA (4)

Sol. Cs used in photoelectric cell as it has least ionisation energy.

10. Official Ans. by NTA (2)

Sol. Both Li and Mg form nitride when reacts directly with nitrogen.

The hydrogen carbonate of both Li and Mg does not exist in solid state.

All alkali metal hydrogen carbonate exist in solid state except $LiHCO_3$.

COMPLETE D-BLOCK

1. NTA Ans. (3)

Sol. Atomic radius of Ag and Au is nearly same due to lanthanide contraction.

2. NTA Ans. (18.00)

Sol. $4NaCl + K_2Cr_2O_7 + 6H_2SO_4$
$\downarrow$
$2CrO_2Cl_2 + 4NaHSO_4 + 2KHSO_4 + 3H_2O$
(A)
$CrO_2Cl_2 + 4NaOH \longrightarrow Na_2CrO_4 + 2NaCl + 2H_2O$
(B)
$Na_2CrO_4 + 2H_2SO_4 + 2H_2O_2$
$\downarrow$
$CrO_5 + 2NaHSO_4 + 3H_2O$
(C)
$A = CrO_2Cl_2$
$B = Na_2CrO_4$
$C = CrO_5$
Total number of atom in A + B + C = 18

3. NTA Ans. (1)

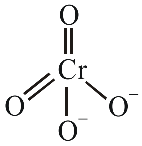
Sol. Electronic configuration of

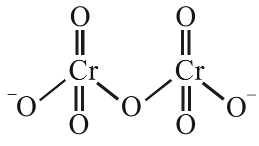
	$_{25}Mn$	$_{26}Fe$	$_{27}Co$	$_{28}Ni$
M =	$[Ar]3d^5 4s^2$	$[Ar]3d^6 4s^2$	$[Ar]3d^7 4s^2$	$[Ar]3d^8 4s^2$
M^{2+} =	$[Ar]3d^5 4s^0$	$[Ar]3d^6 4s^0$	$[Ar]3d^7 4s^0$	$[Ar]3d^8 4s^0$

So third ionisation energy is minimum for Fe.

4. NTA Ans. (12.00)
ALLEN Ans. (18.00)

Sol. Chromate CrO_4^{2-} | Dichromate $Cr_2O_7^{2-}$





Total Cr-O bonds = 6 $(4\sigma + 2\pi)$ | Total Cr-O bonds = 12 $(8\sigma + 4\pi)$

Total number of bonds between chromium and oxygen in both structures are 18.

Note :- But answer of NTA is 12. They consider only linkages between Chromium and Oxygen but in question total no. of bonds are asked so σ and π bonds must be considered separately.

5. Official Ans. by NTA (2)

6. Official Ans. by NTA (4)

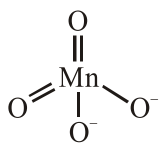
Sol. $KMnO_4$ will not give satisfactory result when it is titrated by HCl.

7. Official Ans. by NTA (2)

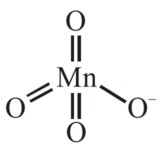
8. Official Ans. by NTA (3)

Sol. Option 1) Manganate ⇒ MnO_4^{2-},

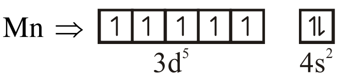
Permanganate ⇒ MnO_4^-



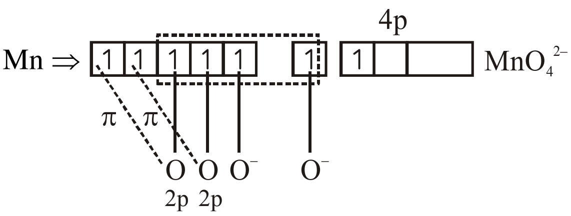
hybridisation of Mn ⇒ d^3s



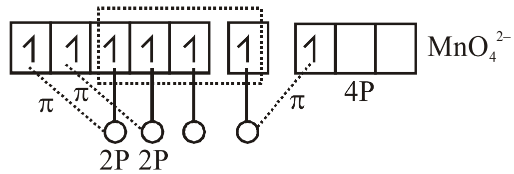
hybridisation of Mn ⇒ d^3s



After excitation



$2 \times 2p_\pi - 3d_{\pi\sigma}$



$2 \times 2P_\pi - 3d_\pi$

$1 \times 2P_\pi - 4P_\pi$

(2) MnO_4^{2-} ⇒ green

MnO_4^- ⇒ purple/violet

(3) Manganate contains 1 unpaired electron hence it is paramagnetic

where as permanganetic contains no unpaired electrons hence it is diamagnetic.

(4) Both have d^3s hybridisation hence both have tetrahedral geometry.

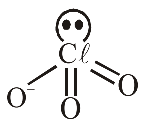
COMPLETE P-BLOCK

1. NTA Ans. (1.66 to 1.67)

Sol. $3Cl_2 + 6NaOH \rightarrow \underset{(X)}{5NaCl} + \underset{(X)}{NaClO_3} + 3H_2O$

$\underset{(X)}{NaCl} + AgNO_3 \rightarrow AgCl \downarrow + NaNO_3$

ClO_3^-



Bond order of Cl–O Bond $= 1 + \frac{2}{3} = \frac{5}{3}$

$= 1.66$ or 1.67

2. NTA Ans. (1)

Sol. (i) $N_2 + O_2 \xrightarrow{2000\text{ K}} 2NO$ (Redox reaction)

during the reaction, oxidation of nitrogen take place from 0 to 2 and reduction of oxygen take place from 0 to –2. It means this reaction is redox reaction.

(ii) $3O_2 \xrightarrow{h\nu} 2O_3$ (Non – redox reaction)

(iii) $H_2SO_4 + 2NaOH \rightarrow Na_2SO_4 + 2H_2O$ (neutralization reaction)

(iv) $[Co(H_2O)_6]Cl_3 + 3AgNO_3 \rightarrow 3AgCl\downarrow + [Co(H_2O)_6](NO_3)_3$
(White ppt.)

3. NTA Ans. (3)

Sol. (a) $H_2O_2 \rightarrow 2H_2O + O_2$

(b) $KClO_3 \xrightarrow[MnO_2]{\Delta} KCl + \frac{3}{2}O_2$

$Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2 + \frac{1}{2}O_2$

$NaNO_3 \xrightarrow{\Delta} NaNO_2 + \frac{1}{2}O_2$

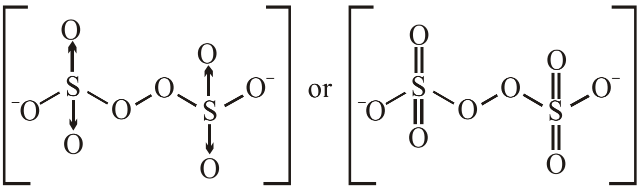
(c) 2-ethylanthraquinol $\underset{H_2/Pd}{\overset{O_2(air)}{\rightleftharpoons}}$ 2-ethylanthraquinone + H_2O_2

(d) $2H_3BO_3 + 2NaOH + 2H_2O_2 \longrightarrow Na_2[B_2(O_2)(OH)_4] + 4H_2O$

All statements are correct

4. NTA Ans. (3)

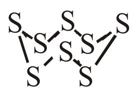
Sol. $S_2O_8^{2-}$:



8 bonds are present between sulphur and oxygen. (It is best answer in given options)

Rhombic sulphur :

(S_8)

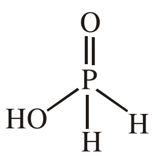


8 bonds are present between sulphur and sulphur atoms.

5. NTA Ans. (2)

Sol. $P_4 + 3NaOH + 3H_2O \longrightarrow \underset{(X)}{3NaH_2PO_2} + PH_3$

$NaH_2PO_2 + HCl \longrightarrow NaCl + \underset{(Y)}{H_3PO_2}$



(mono basic acid)

6. NTA Ans. (2)

Sol.

$\underset{(A)}{H_3N_3B_3Cl_3} + 3LiBH_4 \xrightarrow[(T.H.F.)]{\text{In tetrahydrofurane}} \underset{\substack{(B)\\ \text{Inorganic Benzene} \\ \text{(Borazene)}}}{H_3N_3B_3H_3} + 3LiCl + 3BH_3.THF$

$\underset{(A)}{H_3N_3B_3Cl_3} + \underset{(C)}{3CH_3MgBr} \longrightarrow H_3N_3B_3(CH_3)_3 + 3MgBrCl$

7. Official Ans. by NTA (4)

Sol. $2NO + N_2O_4 \xrightarrow{250K} \underset{\text{blue solid}}{2N_2O_3}$

8. Official Ans. by NTA (4)

Sol. $NH_3 + \underset{excess}{3Cl_2} \longrightarrow NCl_3 + 3HCl$

9. Official Ans. by NTA (2)

10. Official Ans. by NTA (1)

Sol. (1) Water gas shift reaction

$CO_{(g)} + H_2O_{(g)} \xrightarrow[catalyst]{673K} CO_{2(g)} + H_{2(g)}$

(2) Water gas is produced by this reaction.

$CH_{4(g)} + H_2O_{(g)} \xrightarrow[Ni]{1270K} CO_{(g)} + 3H_{2(g)}$

(3) Water gas is produced by this reaction

$C_{(s)} + H_2O_{(g)} \xrightarrow{1270K} CO_{(g)} + H_{2(g)}$

(4) producer gas is produced by this reaction.

$2C_{(s)} + O_{2(g)} + 4N_{2(g)} \xrightarrow{1270K} 2CO_{(g)} + 4N_{2(g)}$

11. Official Ans. by NTA (4)

Sol. $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + \underset{\substack{\text{Brown gas}\\(A)}}{2NO_2} + \frac{1}{2}O_2(g)$

$NO_2(g) \xrightarrow{Cooling} \underset{(B)}{N_2O_4}$

$N_2O_4 + NO \xrightarrow{\Delta} \underset{\substack{\text{Blue Solid}\\(C)}}{N_2O_3}$

O.S. of nitrogen in N_2O_3 is + 3

N_2O_3 2x + 3 (–2) = 0

x = + 3

12. Official Ans. by NTA (4)

Sol. In the stratosphere, CFCs release chlorine free radical $(\dot{Cl})$

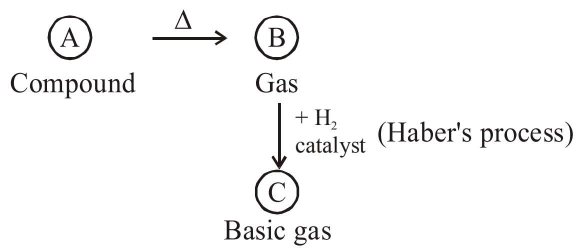
$CF_2Cl_2(g) \xrightarrow{UV} \dot{Cl}(g) + \dot{C}F_2Cl(g)$

which react with O_3 to give chlorine oxide $(Cl\dot{O})$ radical not chlorine dioxide $(\dot{Cl}O_2)$ radical.

$\dot{Cl}(g) + O_3(g) \rightarrow Cl\dot{O}(g) + O_2(g)$

13. Official Ans. by NTA (2)

Sol.



Basic gas (C) must be ammonia (NH_3).
It means (B) gas should be N_2 which is formed by heating of compound (A).

(1) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2\uparrow + Cr_2O_3 + 4H_2O\uparrow$

(2) $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2\uparrow + \frac{1}{2}O_2\uparrow$

(3) $2NaN_3 \xrightarrow{\Delta} 2Na + 3N_2\uparrow$

(4) $NH_4NO_2 \xrightarrow{\Delta} N_2\uparrow + 2H_2O\uparrow$

So, (A) should not be $Pb(NO_3)_2$

HYDROGEN AND ITS COMPOUND

1. NTA Ans. (4)

Sol. (a) Zeolite method removes only cations (Ca^{2+} and Mg^{2+} ion) present in hard water

$2NaZ + M^{2+}(aq) \rightarrow MZ_2(s) + 2Na^+(aq)$

$(M \rightarrow Mg, Ca)$

(b) Synthetic resin method removes cations (Ca^{2+} and Mg^{2+} ion) and anions (like Cl^-, HCO_3^-, SO_4^{2-} etc.)

(i) $2RNa(s) + M^{2+}(aq) \rightarrow R_2M(s) + 2Na^+(aq)$

(Cation exchange resin) $(M \rightarrow Mg, Ca)$

(ii) $RNH^+_3\,OH^-(s) + X^-(aq) \rightarrow RNH_3^+\,X^-(s) + OH^-(aq)$

(Anion exchange resin) ($X^- = Cl^-, HCO_3^-, SO_4^{2-}$ etc)

2. NTA Ans. (2)

Sol. Hydrogen has three isotopes

Isotopes	Number of neutrons
Protium (^1_1H)	0
Deutrium (^2_1H)	1
Tritium (^3_1H)	2

Hence the sum of neutrons are 3

ENVIRONMENTAL CHEMISTRY

1. NTA Ans. (4)

Sol. CO_2, H_2O, CFCs and O_3 are green house gases.

2. NTA Ans. (3)

Sol. Biochemical oxygen demand (BOD) is amount of oxygen required by bacteria to break down organic waste in a certain volume of water sample.

F-BLOCK

1. NTA Ans. (2)

Sol. $Eu_{63} \Rightarrow [Xe]\ 4f^7\ 5d^\circ\ 6s^2$

$Eu^{2\oplus} \Rightarrow [Xe]\ 4f^7$

$Ce_{58} \Rightarrow [Xe]\ 4f^1\ 5d^1\ 6s^2$

$Ce^{3\oplus} \Rightarrow [Xe]\ 4f^1$

2. Official Ans. by NTA (2)

3. Official Ans. by NTA (1)

Sol. Alloys of lanthanides with Fe are called Misch metal, which consists of a lanthanoid metal (~95%) and iron (~5%) and traces of S, C, Ca and Al.

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

Chapter Contents 03

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & SEPTEMBER 2020

MATHEMATICS

JANUARY AND SEPTEMBER 2020 ATTEMPT (MATHEMATICS)

LOGARITHM

1. Official Ans. by NTA (4)

Sol. $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+..........\text{to }\infty\right)}$

$=\left(\frac{4}{25}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}$

$=\left(\frac{1}{2}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{4}{25}\right)}=\left(\frac{1}{2}\right)^{-2}=4$

COMPOUND ANGLE

1. NTA Ans. (2)

Sol. $\tan\alpha + \tan\beta = \frac{\lambda\sqrt{2}}{k+1}$

$\tan\alpha.\ \tan\beta = \frac{k-1}{k+1}$

$\tan(\alpha+\beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1-\frac{k-1}{k+1}}=\frac{\lambda\sqrt{2}}{2}=\frac{\lambda}{\sqrt{2}}$

$\Rightarrow \frac{\lambda^2}{2}=50 \Rightarrow \lambda = 10\ \&\ -10$

2. NTA Ans. (1)

Sol. $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha}=\frac{1}{7} \Rightarrow \tan\alpha=\frac{1}{7}$

$\sin\beta=\frac{1}{\sqrt{10}} \Rightarrow \tan\beta=\frac{1}{3} \Rightarrow \tan 2\beta=\frac{3}{4}$

$\tan(\alpha+2\beta)=\frac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}=1$

Ans. 1.00

3. NTA Ans. (3)

Sol. $\cos^3\frac{\pi}{8}\cdot\sin\frac{\pi}{8}+\sin^3\frac{\pi}{8}\cdot\cos\frac{\pi}{8}$

$=\sin\frac{\pi}{8}.\cos\frac{\pi}{8}=\frac{1}{2}\sin\frac{\pi}{4}=\frac{1}{2\sqrt{2}}$

4. Official Ans. by NTA (1)

Sol. $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$\left(\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$

$\Rightarrow L = \left(\frac{1-\cos(\pi/8)}{2}\right)-\left(\frac{1-\cos(\pi/4)}{2}\right)$

$L = \frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{8}\right)\right]$

$L = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\left(\frac{\pi}{8}\right)$

$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$M = \frac{1+\cos(\pi/8)}{2}-\frac{1-\cos(\pi/4)}{2}$

$M = \frac{1}{2}\cos\left(\frac{\pi}{8}\right) + \frac{1}{2\sqrt{2}}$

QUADRATIC EQUATION

1. NTA Ans (4)

Sol. $\alpha + \beta = 1,\ \alpha\beta = -1$

$P_k = \alpha^k + \beta^k$

$\alpha^2 - \alpha - 1 = 0$

$\Rightarrow \alpha^k - \alpha^{k-1} - \alpha^{k-2} = 0$

$\&\ \beta^k - \beta^{k-1} - \beta^{k-2} = 0$

$\Rightarrow P_k = P_{k-1} + P_{k-2}$

$P_1 = \alpha + \beta = 1$

$P_2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 + 2 = 3$

$P_3 = 4$

$P_4 = 7$

$P_5 = 11$

2. NTA Ans.(4)

Sol. Let $3^x = t$; $t > 0$

$t(t-1) + 2 = |t-1| + |t-2|$

$t^2 - t + 2 = |t-1| + |t-2|$

Case-I : $t < 1$

$t^2 - t + 2 = 1 - t + 2 - t$

$t^2 + 2 = 3 - t$

$t^2 + t - 1 = 0$

$$t = \frac{-1 \pm \sqrt{5}}{2}$$

$t = \frac{\sqrt{5}-1}{2}$ is only acceptable

Case-II : $1 \le t < 2$

$t^2 - t + 2 = t - 1 + 2 - t$

$t^2 - t + 1 = 0$

$D < 0$ no real solution

Case-III : $t \ge 2$

$t^2 - t + 2 = t - 1 + t - 2$

$t^2 - 3t \;\; 5 = 0 \Rightarrow D < 0$ no real solution

(4) Option

3. NTA Ans. (8.00)

Sol. $D \ge 0 \Rightarrow (a-10)^2 - 4 \times 2 \times \left(\frac{33}{2} - 2a\right) \ge 0$

$\Rightarrow a^2 - 4a - 32 \ge 0$

$\Rightarrow a \in (-\infty, 4] \cup [8, \infty)$

4. NTA Ans. (2)

Sol. $ax^2 - 2bx + 5 = 0 <^{\alpha}_{\alpha}$

$\Rightarrow \alpha = \frac{b}{a}$; $\alpha^2 = \frac{5}{a} \Rightarrow b^2 = 5a$

$x^2 - 2bx - 10 = 0 <^{\alpha}_{\beta} \Rightarrow \alpha^2 - 2b\alpha - 10 = 0$

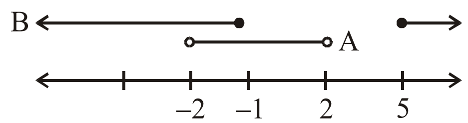
$\Rightarrow a = \frac{1}{4} \Rightarrow \alpha^2 = 20$; $\alpha\beta = -10 \Rightarrow \beta^2 = 5$

$\Rightarrow \alpha^2 + \beta^2 = 25$

5. NTA Ans. (3)

Sol. A : $x \in (-2, 2)$; B : $x \in (-\infty, -1] \cup [5, \infty)$

$\Rightarrow B - A = R - (-2, 5)$



6. NTA Ans. (4)

Sol. $e^{4x} + e^{3x} - 4e^x + e^x + 1 = 0$

Divide by e^{2x}

$$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

Let $e^x + \frac{1}{e^x} = t \Rightarrow (e^x - 1)^2 = 0 \Rightarrow x = 0.$

$\therefore$ Number of real roots = 1

7. Official Ans. by NTA (1)

Sol. α and β are roots of $5x^2 + 6x - 2 = 0$

$\Rightarrow 5\alpha^2 + 6\alpha - 2 = 0$

$\Rightarrow 5\alpha^{n+2} + 6\alpha^{n+1} - 2\alpha^n = 0$ ...(1)

(By multiplying α^n)

Similarly $5\beta^{n+2} + 6\beta^{n+1} - 2\beta^n = 0$ ...(2)

By adding (1) & (2)

$5S_{n+2} + 6S_{n+1} - 2S_n = 0$

For n = 4

$$\boxed{5S_6 + 6S_5 = 2S_4}$$

8. Official Ans. by NTA (3)

Sol. $f(x) = a(x-3)(x-\alpha)$

$f(2) = a(\alpha - 2)$

$f(-1) = 4a(1 + \alpha)$

$f(-1) + f(2) = 0 \Rightarrow a(\alpha - 2 + 4 + 4\alpha) = 0$

$a \ne 0 \Rightarrow 5\alpha = -2$

$$\alpha = -\frac{2}{5} = -0.4$$

$\alpha \in (-1, 0)$

9. Official Ans. by NTA (3)

Sol. α, β are roots of $x^2 + px + 2 = 0$

$\Rightarrow \alpha^2 + p\alpha + 2 = 0$ & $\beta^2 + p\beta + 2 = 0$

$\Rightarrow \frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + px + 1 = 0$

But $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + 2qx + 1 = 0$

$\Rightarrow p = 2q$

Also $\alpha + \beta = -p \quad \alpha\beta = 2$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \left(\frac{\alpha^2 - 1}{\alpha}\right)\left(\frac{\beta^2 - 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)$$

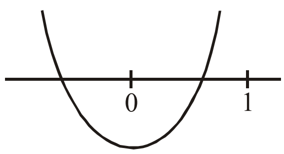
$$= \frac{(-p\alpha - 3)\,(-p\beta - 3)\,(\alpha\beta + 1)^2}{(\alpha\beta)^2}$$

$$= \frac{9}{4}(p\alpha\beta + 3p(\alpha + \beta) + 9)$$

$$= \frac{9}{4}(9 - p^2) = \frac{9}{4}(9 - 4q^2)$$

10. Official Ans. by NTA (2)

Sol. If exactly one root in (0, 1) then



$\Rightarrow f(0).f(1) < 0$

$\Rightarrow 2(\lambda^2 - 4\lambda + 3) < 0$

$\Rightarrow 1 < \lambda < 3$

Now for $\lambda = 1$, $2x^2 - 4x + 2 = 0$

$(x - 1)^2 = 0$, $x = 1, 1$

So both roots doesn't lie between (0, 1)

$\therefore \lambda \neq 1$

Again for $\lambda = 3$

$10x^2 - 12x + 2 = 0$

$\Rightarrow x = 1, \frac{1}{5}$

so if one root is 1 then second root lie between (0, 1)

so $\lambda = 3$ is correct.

$\therefore \lambda \in (1, 3]$.

11. Official Ans. by NTA (3)

Sol. $x^2 - 3x + p = 0 \begin{cases}\alpha \\ \beta\end{cases}$

$\alpha, \beta, \gamma, \delta$ in G.P.

$\alpha + \alpha r = 3$ ....(1)

$x^2 - 6x + q = 0 \begin{cases}\gamma \\ \delta\end{cases}$

$\alpha r^2 + \alpha r^3 = 6$ ...(2)

(2) ÷ (1)

$r^2 = 2$

So, $\frac{2q + p}{2q - p} = \frac{2r^5 + r}{2r^5 - r} = \frac{2r^4 + 1}{2r^4 - 1} = \frac{9}{7}$

12. Official Ans. by NTA (4)

Sol. $\alpha + \beta = 1$, $\alpha\beta = 2\lambda$

$\alpha + \beta = \frac{10}{3}$, $\alpha\gamma = \frac{27\lambda}{3} = 9\lambda$

$\gamma - \beta = \frac{7}{3}$,

$\frac{\gamma}{\beta} = \frac{9}{2} \Rightarrow \gamma = \frac{9}{2}\beta = \frac{9}{2} \times \frac{2}{3} \Rightarrow \gamma = 3$

$\frac{9}{2}\beta - \beta = \frac{7}{3}$

$\frac{9}{2}\beta = \frac{7}{3} \Rightarrow \beta = \frac{2}{3}$

$\alpha = 1 - \frac{2}{3} = \frac{1}{3}$

$2\lambda = \frac{2}{9} \Rightarrow \lambda = \frac{1}{9}$

$$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3} \times 3}{\frac{1}{9}} = 18$$

13. Official Ans. by NTA (2)

Sol. $9x^2 - 18|x| + 5 = 0$

$9|x|^2 - 15|x| - 3|x| + 5 = 0$ ($\because x^2 = |x|^2$)

$3|x|(3|x| - 5) - (3|x| - 5) = 0$

$|x| = \frac{1}{3}, \frac{5}{3}$

$x = \pm\frac{1}{3}, \pm\frac{5}{3}$

Product of roots $= \frac{25}{81}$

14. Official Ans. by NTA (1)

Sol. $7x^2 - 3x - 2 = 0$

$\alpha + \beta = \frac{3}{7} \quad \alpha\beta = \frac{-2}{7}$

$$\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2} = \frac{\alpha+\beta-\alpha\beta(\alpha+\beta)}{1-\alpha^2-\beta^2+\alpha^2\beta^2}$$

$$= \frac{\frac{3}{7}+\frac{2}{7}\left(\frac{3}{7}\right)}{1-(\alpha+\beta)^2+2\alpha\beta+\alpha^2\beta^2} = \frac{27}{16}$$

15. Official Ans. by NTA (4)

Sol. $x^2 - 64x + 256 = 0$

$\alpha + \beta = 64, \alpha\beta = 256$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8} = \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$= \frac{\alpha+\beta}{(\alpha\beta)^{5/8}} = \frac{64}{(256)^{5/8}} = 2$$

16. Official Ans. by NTA (3)

Sol. α and β are the roots of the equation

$4x^2 + 2x - 1 = 0$

$4\alpha^2 + 2\alpha = 1 \Rightarrow \frac{1}{2} = 2\alpha^2 + \alpha$ ...(1)

$\beta = \frac{-1}{2} - \alpha$

using equation (1)

$\beta = -(2\alpha^2 + \alpha) - \alpha$

$\beta = -2\alpha^2 - 2\alpha$

$\beta = -2\alpha(\alpha + 1)$

SEQUENCE & PROGRESSION

1. NTA Ans. (1)

Sol. Sum of the 40 terms of

$3 + 4 + 8 + 9 + 13 + 14 + 18 + 19\ldots$

$= (3 + 8 + 13 + \ldots$ upto 20 term)

$+ [4 + 9 + 15 + \ldots$ upto 20 terms]

$= 10 [\{6 + 19 \times 5\} + \{8 + 19 \times 5\}]$

$= 10 \times 204 = 20 \times 102$

2. NTA Ans. (1)

Sol. $a_1 + a_2 = 4$

$r^2a_1 + r^2a_2 = 16$

$\Rightarrow r^2 = 4 \Rightarrow r = -2$ as $a_1 < 0$

and $a_1 + a_2 = 4$

$a_1 + a_1(-2) = 4 \Rightarrow a_1 = -4$

$$4\lambda = (-4)\left(\frac{(-2)^9-1}{-2-1}\right) = (-4)\times\frac{513}{3}$$

$\Rightarrow \lambda = -171$

3. NTA Ans. (3)

Sol. Let the A.P is

$a - 2d, a - d, a, a + d, a + 2d$

$\because$ sum $= 25 \Rightarrow a = 5$

Product = 2520

$(25 - 4d^2)(25 - d^2) = 504$

$4d^4 - 125d^2 + 121 = 0$

$\Rightarrow d^2 = 1, \frac{121}{4}$

$\Rightarrow d = \pm 1, \pm\frac{11}{2}$

$d = \pm 1$ is rejected because none of the term can be $\frac{-1}{2}$.

$\Rightarrow d = \pm\frac{11}{2}$

$\Rightarrow$ AP will be $-6, -\frac{1}{2}, 5, \frac{21}{2}, 16$

Largest term is 16.

4. NTA Ans. (3)

Sol. $1 + 49 + 49^2 + \ldots + 49^{12}$

$= (49)^{126} - 1 = (49^{63} + 1)\dfrac{(49^{63}-1)}{(48)}$

So greatest value of k = 63

5. NTA Ans. (2)

Sol. $T_{10} = \dfrac{1}{20} = a + 9d$ ...(i)

$T_{20} = \dfrac{1}{10} = a + 19d$ ...(ii)

$a = \dfrac{1}{200} = d$

Hence, $S_{200} = \dfrac{200}{2}\left[\dfrac{2}{200} + \dfrac{199}{200}\right] = \dfrac{201}{2}$

(2) Option

6. NTA Ans. (504)

Sol. $\dfrac{1}{4}\left(\sum_{n=1}^{7} 2n^3 + \sum_{n=1}^{7} 3n^2 + \sum_{n=1}^{7} n\right)$

$= \dfrac{1}{4}\left(2\left(\dfrac{7\times 8}{2}\right)^2 + 3\left(\dfrac{7\times 8\times 15}{6}\right) + \dfrac{7\times 8}{2}\right)$

= 504

Ans. 504.00

7. NTA Ans. (1540.00)

Sol. $\sum_{k=1}^{20}\dfrac{k(k+1)}{2} = \dfrac{1}{2}\sum_{k=1}^{20}\dfrac{k(k+1)(k+2)-(k-1)k(k+1)}{3}$

$= \dfrac{1}{6}\times 20\times 21\times 22 = 1540.00$

8. NTA Ans. (3)

Sol. $x = \sum_{n=0}^{\infty}(-1)^n \tan^{2n}\theta = 1 - \tan^2\theta + \tan^4\theta + \ldots$

$\Rightarrow x = \cos^2\theta$

$y = \sum_{n=0}^{\infty}\cos^{2n}\theta \Rightarrow y = 1 + \cos^2\theta + \cos^4\theta + \ldots$

$\Rightarrow y = \dfrac{1}{\sin^2\theta} \Rightarrow y = \dfrac{1}{1-x}$

$\Rightarrow y(1 - x) = 1$

9. NTA Ans. (4)

Sol. $\sum_{n=1}^{100} a_{2n+1} = 200 \Rightarrow a_3 + a_5 + a_7 + \ldots + a_{201} = 200$

$\Rightarrow ar^2\dfrac{(r^{200}-1)}{(r^2-1)} = 200$

$\sum_{n=1}^{100} a_{2n} = 100 \Rightarrow a_2 + a_4 + a_6 + \ldots + a_{200} = 100$

$\Rightarrow \dfrac{ar(r^{200}-1)}{(r^2-1)} = 100$

On dividing r = 2

on adding $a_2 + a_3 + a_4 + a_5 + \ldots + a_{200} + a_{201} = 300$

$\Rightarrow r(a_1 + a_2 + a_3 + \ldots + a_{200}) = 300$

$\Rightarrow \sum_{n=1}^{200} a_n = 150$

10. NTA Ans. (14)

Sol. Common term are : 23, 51, 79, ... T_n

$T_n \le 407 \Rightarrow 23 + (n-1)28 \le 407$

$\Rightarrow n \le 14.71$

n = 14

11. NTA Ans. (1)

Sol. $2^{\frac{1}{4}}\cdot 4^{\frac{1}{16}}\cdot 8^{\frac{1}{48}}\cdot 16^{\frac{1}{128}}\cdot \ldots\infty$

$= 2^{\frac{1}{4}}\cdot 2^{\frac{2}{16}}\cdot 2^{\frac{3}{48}}\cdot 2^{\frac{4}{128}}\cdot \ldots\infty$

$= 2^{\frac{1}{4}}\cdot 2^{\frac{1}{8}}\cdot 2^{\frac{1}{16}}\cdot 2^{\frac{1}{32}}\cdot \ldots\infty$

$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots\infty} = (2)^{\left(\frac{1/4}{1-1/2}\right)} = 2^{1/2}$

12. Official Ans. by NTA (1)

Sol. $|x| < 1, |y| < 1, x \neq y$

$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots$

By multiplying and dividing $x - y$:

$$\frac{(x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4) + \ldots\ldots}{x - y}$$

$$= \frac{(x^2 + x^3 + x^4 + \ldots\ldots) - (y^2 + y^3 + y^4 + \ldots\ldots)}{x - y}$$

$$= \frac{\frac{x^2}{1-x} - \frac{y^2}{1-y}}{x - y}$$

$$= \frac{(x^2 - y^2) - xy(x - y)}{(1-x)(1-y)(x-y)}$$

$$= \boxed{\frac{x + y - xy}{(1-x)(1-y)}}$$

13. Official Ans. by NTA (4)

Sol. Let three terms of G.P. are $\frac{a}{r}$, a, ar

product = 27

$\Rightarrow a^3 = 27 \Rightarrow a = 3$

$$S = \frac{3}{r} + 3r + 3$$

For r > 0

$$\frac{\frac{3}{r} + 3r}{2} \geq \sqrt{3^2} \quad \text{(By AM} \geq \text{GM)}$$

$$\Rightarrow \frac{3}{r} + 3r \geq 6 \qquad \ldots(1)$$

For r < 0 $\frac{3}{r} + 3r \leq -6$ …(2)

From (1) & (2)

$S \in (-\infty - 3] \cup [9, \infty]$

14. Official Ans. by NTA (2)

Sol. $a_1 + a_2 + a_3 + \ldots\ldots + a_{11} = 0$

$\Rightarrow (a_1 + a_{11}) \times \frac{11}{2} = 0$

$\Rightarrow a_1 + a_{11} = 0$

$\Rightarrow a_1 + a_1 + 10d = 0$

where d is common difference

$\Rightarrow \boxed{a_1 = -5d}$

$a_1 + a_3 + a_5 + \ldots\ldots + a_{23}$

$= (a_1 + a_{23}) \times \frac{12}{2} = (a_1 + a_1 + 22d) \times 6$

$= \left(2a_1 + 22\left(\frac{-a_1}{5}\right)\right) \times 6$

$= -\frac{72}{5}a_1 \Rightarrow K = \frac{-72}{5}$

15. Official Ans. by NTA (3)

Sol. $S = [x + ka + 0] + [x^2 + ka + 2a] + [x^3 + ka + 4a] + [x^4 + ka + 6a] + \ldots\ldots 9$ terms

$\Rightarrow S = (x + x^2 + x^3 + x^4 + \ldots\ldots 9$ terms$) + (ka + ka + ka + ka + \ldots\ldots 9$ terms$) + (0 + 2a + 4a + 6a + \ldots\ldots 9$ terms$)$

$$\Rightarrow S = x\left[\frac{x^9 - 1}{x - 1}\right] + 9ka + 72a$$

$$\Rightarrow S = \frac{(x^{10} - x) + (9k + 72)a(x - 1)}{(x - 1)}$$

Compare with given sum, then we get, $(9k + 72) = 45$

$\Rightarrow \boxed{k = -3}$

16. Official Ans. by NTA (4)

Sol. Sum of 1st 25 terms = sum of its next 15 terms

$\Rightarrow (T_1 + \ldots\ldots + T_{25}) = (T_{26} + \ldots\ldots + T_{40})$

$\Rightarrow (T_1 + \ldots\ldots + T_{40}) = 2(T_1 + \ldots\ldots + T_{25})$

$\Rightarrow \frac{40}{2}[2 \times 3 + (39d)] = 2 \times \frac{25}{2}[2 \times 2 + 24d]$

$\Rightarrow d = \frac{1}{6}$

17. Official Ans. by NTA (3)

Sol. $S = \frac{100}{5} + \frac{98}{5} + \frac{96}{5} + \frac{94}{5} + ...n$

$S_n = \frac{n}{2}\left(2 \times \frac{100}{5} + (n-1)\left(-\frac{2}{5}\right)\right) = 188$

$n(100 - n + 1) = 488 \times 5$

$n^2 - 101n + 488 \times 5 = 0$

$n = 61, 40$

$T_n = a + (n - 1)d = \frac{100}{5} - \frac{2}{5} \times 60$

$= 20 - 24 = -4$

18. Official Ans. by NTA (39)

Sol. $3, A_1, A_2 .... A_m, 243$

$d = \frac{243-3}{m+1} = \frac{240}{m+1}$

Now $3, G_1, G_2, G_3, 243$

$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = 3$

$\therefore A_4 = G_2$

$\Rightarrow a + 4d = ar^2$

$3 + 4\left(\frac{240}{m+1}\right) = 3(3)^2$

$m = 39$

19. Official Ans. by NTA (2)

Sol. $1 + (1 - 2^2.1) + (1 - 4^2.3) + ... + (1 - 20^2.19)$

$= \alpha - 220\beta$

$= 11 - (2^2.1 + 4^2.3 + ... + 20^2.19)$

$= 11 - 2^2.\sum_{r=1}^{10} r^2(2r-1) = 11 - 4\left(\frac{110^2}{2} - 35 \times 11\right)$

$= 11 - 220(103)$

$\Rightarrow \alpha = 11, \beta = 103$

20. Official Ans. by NTA (3)

Sol. $a_n = a_1 + (n - 1)d$

$\Rightarrow 300 = 1 + (n - 1)d$

$\Rightarrow (n - 1)d = 299 = 13 \times 23$

since, $n \in [15, 50]$

$\therefore n = 24$ and $d = 13$

$a_{n-4} = a_{20} = 1 + 19 \times 13 = 248$

$\Rightarrow a_{n-4} = 248$

$S_{n-4} = \frac{20}{2}\{1 + 248\} = 2490$

21. Official Ans. by NTA (1)

Sol. Usnign AM ≥ GM

$\Rightarrow \frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x} \cdot 2^{\cos x}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1+\left(\frac{\sin x + \cos x}{2}\right)}$

$\Rightarrow \min\left(2^{\sin x} + 2^{\cos x}\right) = 2^{1-\frac{1}{\sqrt{2}}}$

22. Official Ans. by NTA (1)

Sol. Given that

$3^{4 - \sin 2\alpha} + 3^{2\sin 2\alpha - 1} = 28$

Let $3^{2\sin 2\alpha} = t$

$\frac{81}{t} + \frac{t}{3} = 28$

$t = 81, 3$

$3^{2\sin 2\alpha} = 3^1, 3^4$

$2\sin 2\alpha = 1, 4$

$\sin 2\alpha = \frac{1}{2}$, 2 (rejected)

First term $a = 3^{2\sin 2\alpha - 1}$

$a = 1$

Second term = 14

$\therefore$ common difference $d = 13$

$T_6 = a + 5d$

$T_6 = 1 + 5 \times 13$

$T_6 = 66$

23. Official Ans. by NTA (3)

Sol. $a = 2^{10}$; $r = \frac{3}{2}$; $n = 11$ (G.P.)

$$S' = (2^{10})\frac{\left(\left(\frac{3}{2}\right)^{11}-1\right)}{\frac{3}{2}-1} = 2^{11}\left(\frac{3^{11}}{2^{11}}-1\right)$$

$S' = 3^{11} - 2^{11} = S - 2^{11}$ (Given)

$\therefore S = 3^{11}$

24. Official Ans. by NTA (4)

Sol. $460 = \log_7 x \cdot (2 + 3 + 4 + ... + 20 + 21)$

$\Rightarrow 460 = \log_7 x \cdot \left(\frac{21\times 22}{2}-1\right)$

$\Rightarrow 460 = 230 \cdot \log_7 x$

$\Rightarrow \log_7 x = 2 \Rightarrow x = 49$

25. Official Ans. by NTA (2)

Sol. Let first term = a > 0

Common ratio = r > 0

$ar + ar^2 + ar^3 = 3$ ....(i)

$ar^5 + ar^6 + ar^7 = 243$ ....(ii)

$r^4(ar + ar^2 + ar^3) = 243$

$r^4(3) = 243 \Rightarrow r = 3$ as $r > 0$

from (1)

$3a + 9a + 27a = 3$

$a = \frac{1}{13}$

$S_{50} = \frac{a(r^{50}-1)}{(r-1)} = \frac{1}{26}(3^{50} - 1)$

26. Official Ans. by NTA (2)

Sol. $f(x + y) = f(x).\, f(y)$

$\sum_{x=1}^{\infty} f(x) = 2$ where $x,y \in N$

$f(1) + f(2) + f(3) +.... \infty = 2$ ...(1) (Given)

Now for $f(2)$ put $x = y = 1$

$f(2) = f(1 + 1) = f(1).\, f(1) = (f(1))^2$

$f(3) = f(2 + 1) = f(2).\, f(1) = (f(1))^3$

Now put these values in equation (1)

$f(1) + (f(1))^2 + [f(1)^2 + ...\infty = 2]$

$\frac{f(1)}{1-f(1)} = 2$

$\Rightarrow f(1) = \frac{2}{3}$

Now $f(2) = \left(\frac{2}{3}\right)^2$

$f(4) = \left(\frac{2}{3}\right)^4$

then the value of $\frac{f(4)}{f(2)} = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2} = \frac{4}{9}$

27. Official Ans. by NTA (3)

Sol. $(a^2 + b^2 + c^2)p^2 + 2(ab + bc + cd)p + b^2 + c^2 + d^2 = 0$

$\Rightarrow (a^2p^2 + 2abp + b^2) + (b^2p^2 + 2bcp + c^2) + (c^2p^2 + 2cdp + d^2) = 0$

$\Rightarrow (ab + b)^2 + (bp + c)^2 + (cp + d)^2 = 0$

This is possible only when

$ap + b = 0$ and $bp + c = 0$ and $cp + d = 0$

$p = -\frac{b}{a} = -\frac{c}{b} = -\frac{d}{c}$

or $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$\therefore$ a,b,c,d are in G.P.

28. Official Ans. by NTA (2)

Sol. $a_1, a_2, ..., a_n \rightarrow$ (CD = d)

$b_1, b_2, ..., b_m \rightarrow$ (CD = d + 2)

$a_{40} = a + 39d = -159$

...(1)

$a_{100} = a + 99d = -399$

...(2)

Subtract : $60d = -240 \Rightarrow d = -4$

using equation (1)

$a + 39(-4) = -159$

$a = 156 - 159 = -3$

$a_{70} = a + 69d = -3 + 69(-4) = -279 = b_{100}$

$b_{100} = -279$

$b_1 + 99(d + 2) = -279$

$b_1 - 198 = -279 \Rightarrow b_1 = -81$

TRIGONOMETRIC EQUATION

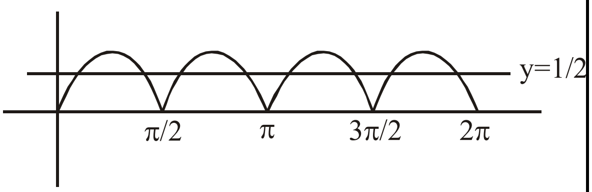
1. NTA Ans. (8.00)

Sol. $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|;\ x \in [0, 2\pi]$

$\Rightarrow \log_{1/2}|\sin x| + \log_{1/2}|\cos x| = 2$

$\Rightarrow \log_{1/2}(|\sin x\cos x|) = 2$

$\Rightarrow |\sin x\cos x| = \frac{1}{4} \Rightarrow |\sin 2x| = \frac{1}{2}$



$\Rightarrow$ 8 solutions

2. Official Ans. by NTA (4)

Sol. $\lambda = -(\sin^4\theta + \cos^4\theta)$

$\lambda = -(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$$\lambda = \frac{\sin^2 2\theta}{2} - 1$$

$$\frac{\sin^2 2\theta}{2} \in \left[0, \frac{1}{2}\right]$$

$$\lambda \in \left[-1, -\frac{1}{2}\right]$$

3. Official Ans. by NTA (1)

Sol. $$\cos\phi = \frac{\bar{p}.\bar{q}}{|\bar{p}||\bar{q}|} = \frac{ab+bc+ca}{a^2+b^2+c^2} = \frac{\Sigma ab}{1}$$

$$= abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$= \frac{abc}{\lambda}\left(\cos\theta + \cos\left(\theta+\frac{2\pi}{3}\right) + \cos\left(\theta+\frac{4\pi}{3}\right)\right)$$

$$= \frac{abc}{\lambda}\left(\cos + 2\cos(\theta+\pi)\cos\frac{\pi}{3}\right)$$

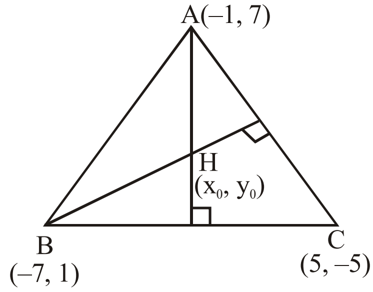
$$= \frac{abc}{\lambda}(\cos\theta - \cos\theta) = 0$$

$$\phi = \frac{\pi}{2}$$

SOLUTION OF TRIANGLE

1. Official Ans. by NTA (3)

Sol. Let orthocentre is $H(x_0, y_0)$



$m_{AH}.m_{BC} = -1$

$$\Rightarrow \left(\frac{y_0-7}{x_0+1}\right)\left(\frac{1+5}{-7-5}\right) = -1$$

$\Rightarrow 2x_0 - y_0 + 9 = 0$

.... (1)

and $m_{BH}.m_{AC} = -1$

$$\Rightarrow \left(\frac{y_0-1}{x_0+7}\right)\left(\frac{7-(-5)}{-1-5}\right) = -1$$

$\Rightarrow x_0 - 2y_0 + 9 = 0$ .... (2)

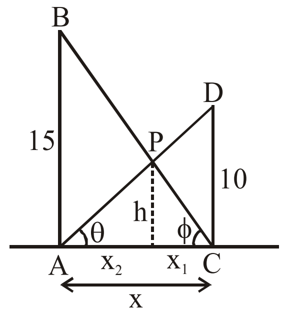
Solving equation (1) and (2) we get

$(x_0, y_0) \equiv (-3, 3)$

HEIGHT & DISTANCE

1. Official Ans. by NTA (4)

Sol.



$$\tan\theta = \frac{10}{x} = \frac{h}{x_2} \Rightarrow x_2 = \frac{hx}{10}$$

$$\tan\phi = \frac{15}{x} = \frac{h}{x_1} \Rightarrow x_1 = \frac{hx}{15}$$

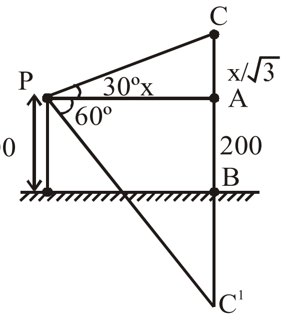
Now, $x_1 + x_2 = x = \frac{hx}{15} + \frac{hx}{10}$

$\Rightarrow 1 = \frac{h}{10} + \frac{h}{15} \Rightarrow h = 6$

2. Official Ans. by NTA (1)

Sol. Let PA = x

For ΔAPC



$AC = \frac{PA}{\sqrt{3}} = \frac{x}{\sqrt{3}}$

$AC^1 = AB + BC^1$

$AC^1 = AB + BC$

$AC^1 = 400 + \frac{x}{\sqrt{3}}$

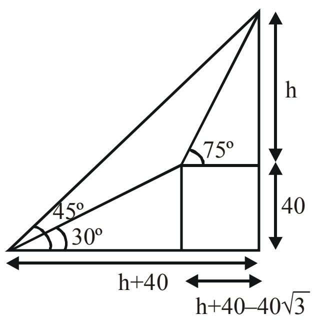
From ΔC^1PA : $AC^1 = \sqrt{3}\,PA$

$\Rightarrow \left(400 + \frac{x}{\sqrt{3}}\right) = \sqrt{3}x \Rightarrow x = (200)(\sqrt{3})$

from Δ APC : $PC = \frac{2x}{\sqrt{3}} \Rightarrow PC = 400$

3. Official Ans. by NTA (80.00)

Sol.



$\tan 75° = \frac{h}{h + 40 - 40\sqrt{3}}$

$\frac{2+\sqrt{3}}{1} = \frac{h}{h + 40 - 40\sqrt{3}}$

$\Rightarrow 2h + 80 - 80\sqrt{3} + \sqrt{3}h + 40\sqrt{3} - 120 = h$

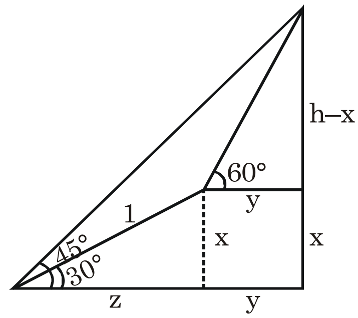
$\Rightarrow h\left(\sqrt{3}+1\right) = 40 + 40\sqrt{3}$

$\Rightarrow h = 40$

$\therefore$ Height of hill = 40 + 40 = 80m

4. Official Ans. by NTA (1)

Sol. $\sin 30° = x \Rightarrow x = \frac{1}{2}$



$\cos 30° = z \Rightarrow z = \frac{\sqrt{3}}{2}$

$\tan 45° = \frac{h}{y+z} \Rightarrow h = y + z$

$\tan 60° = \frac{h-x}{y} \Rightarrow \tan 60° = \frac{h-x}{h-z}$

$\sqrt{3}(h-z) = h - x$

$\left(\sqrt{3}-1\right)h = \sqrt{3}z - x$

$\Rightarrow \left(\sqrt{3}-1\right)h = \frac{3}{2} - \frac{1}{2}$

$\Rightarrow \left(\sqrt{3}-1\right)h = 1$

$h = \frac{1}{\sqrt{3}-1}$

DETERMINANT

1. NTA Ans. (13.00)

Sol. System has intfinitely many solution

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda = 1$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 1 \end{vmatrix} = 0$$

$\mu = 14$

$\mu - \lambda^2 = 13$

2. NTA Ans. (4)

Sol. For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0, \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$\Rightarrow (3b - 2a)(c - a) - (b - a)(4c - 2a) = 0$

$\Rightarrow 2ac = bc + ab$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ Hence $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

3. NTA Ans. (4)

Sol. $D = \begin{vmatrix} \lambda & 3 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = (\lambda+8)(2-\lambda)$

for $\lambda = 2$; $D_1 \neq 0$

Hence, no solution for $\lambda = 2$

(4) Option

4. NTA Ans. (4)

Sol. $2 \times (ii) - 2 \times (i) - (iii)$: -

$0 = 2\mu - 2 - \delta$

$\Rightarrow \delta = 2(\mu - 1)$

5. NTA Ans. (3)

Sol. $R_1 \to R_1 + R_3 - 2R_2$

$$f(x) = \begin{vmatrix} a+c-2b & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$= (a + c - 2b)((x + 3)^2 - (x + 2)(x + 4))$

$= x^2 + 6x + 9 - x^2 - 6x - 8 = 1$

$\Rightarrow f(x) = 1 \Rightarrow f(50) = 1$

6. NTA Ans. (1)

Sol. $7x + 6y - 2z = 0$ .... (1)

$3x + 4y + 2z = 0$ .... (2)

$x - 2y - 6z = 0$ .... (3)

$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0 \Rightarrow$ infinite solutions

Now (1) + (2) $\Rightarrow y = -x$ put in (1), (2) & (3) all will lead to $x = 2z$

7. Official Ans. by NTA (3)

Sol. $2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

For no solution :

$$D = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$\Rightarrow 2(-2 - \lambda^2) + 1(1 - \lambda) + 2(\lambda + 2) = 0$

$\Rightarrow -2\lambda^2 + \lambda + 1 = 0$

$\Rightarrow \lambda = 1, -\frac{1}{2}$

$$D_x = \begin{vmatrix} 2 & -1 & 2 \\ -4 & 2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -1 & 2 \\ -2 & -2 & \lambda \\ \lambda & \lambda & 1 \end{vmatrix}$$

$= 2(1 + \lambda)$

whichis not equal to zero for

$\lambda = 1, -\frac{1}{2}$

8. Official Ans. by NTA (8)

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let $x = k$

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0, \; y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k$, $y = \frac{k}{2}$, $z = 0$ put in condition

$15 \le k^2 + \left(\frac{k}{2}\right)^2 + 0 \le 150$

$12 \le k^2 \le 120$

$\Rightarrow k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

9. Official Ans. by NTA (3)

Sol. $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 +$ Cx + D.

$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_2$

$$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-1 & x-1 & x-1 \\ x-2 & 2(x-2) & 6(x-2) \end{vmatrix}$$

$$= (x-1)(x-2)\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix}$$

$= -3(x-1)^2(x-2) = -3x^3 + 12x^2 - 15x + 6$

$\therefore$ B + C = 12 – 15 = –3

10. Official Ans. by NTA (5)

Sol. $D = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0 \Rightarrow a = 8$

also, $D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0 \Rightarrow b = 3$

hence, a – b = 8 – 3 = 5

11. Official Ans. by NTA (3)

Sol. For infinite solutions

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

Now $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = \frac{9}{2}$

$\Delta_{x=0} \Rightarrow \begin{vmatrix} 2 & 1 & 1 \\ 6 & 4 & -1 \\ \mu & 2 & -\frac{9}{2} \end{vmatrix} = 0$

$\Rightarrow \mu = 5$

For $\lambda = \frac{9}{2}$ & $\mu = 5$, $\Delta_y = \Delta_z = 0$

Now check option $2\lambda + \mu = 14$

12. Official Ans. by NTA (4)

Sol. $C_3 \to C_3 - (C_1 - C_2)$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 0 \\ -\cos^2\theta & -1-\cos^2\theta & 0 \\ 12 & 10 & -4 \end{vmatrix}$$

$= -4[(1 + \cos^2\theta)\sin^2\theta - \cos^2\theta(1 + \sin^2\theta)]$

$= -4[\sin^2\theta + \sin^2\theta\cos^2\theta - \cos^2\theta - \cos^2\theta\sin^2\theta]$

$f(\theta) = 4\cos 2\theta$

$\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$2\theta \in \left[\frac{\pi}{2}, \pi\right]$

$f(\theta) \in [-4, 0]$

(m, M) = (–4, 0)

13. Official Ans. by NTA (1)

Sol. $D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix}$

$= 2(3\lambda + 2)(\lambda - 3)$

$D_1 = -2(\lambda - 3)$

$D_2 = -2(\lambda + 1)(\lambda - 3)$

$D_3 = -2(\lambda - 3)$

When $\boxed{\lambda = 3}$, then

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow$ Infinite many solution

when $\boxed{\lambda = -\frac{2}{3}}$ then D_1, D_2, D_3 none of them is zero so equations are inconsistant

$\boxed{\therefore \lambda = -\frac{2}{3}}$

14. Official Ans. by NTA (2)

Sol. $x + y + 3z = 0$ .....(i)

$x + 3y + k^2z = 0$ .....(ii)

$3x + y + 3z = 0$ .....(iii)

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 9 + 3 + 3k^2 - 27 - k^2 - 3 = 0$

$\Rightarrow k^2 = 9$

(i) – (iii) $\Rightarrow -2x = 0 \Rightarrow x = 0$

Now from (i) $\Rightarrow y + 3z = 0$

$\Rightarrow \frac{y}{z} = -3$

$x + \frac{y}{z} = -3$

15. Official Ans. by NTA (2)

Sol. $a + x = b + y = c + z + 1$

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} \quad C_3 \to C_3 - C_1$$

$$\begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} \quad C_2 \to C_2 - C_3$$

$$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \quad R_3 \to R_3 - R_1,\ R_2 \to R_2 - R_1$$

$$\begin{vmatrix} x & y & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$= (-y)[(y - x)(c - a) - (b - a)(z - x)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c - 1)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c) + b - a)$

$= -y(b - a) = y(a - b)$

16. Official Ans. by NTA (4)

Sol. For infinite many solutions

$D = D_1 = D_2 = D_3 = 0$

Now $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$

$1.(2\lambda - 9) - 1.(\lambda - 3) + 1.(3 - 2) = 0$

$\therefore \lambda = 5$

Now $D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0$

$2(10 - 9) - 1(25 - 3\mu) + 1(15 - 2\mu) = 0$

$\mu = 8$

17. Official Ans. by NTA (1)

Sol. $\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$= -1\left(\sin^2 x\right) - 1\left(1 + \sin 2x + \cos^2 x\right)$

$= -\sin 2x - 2$

$m = -3,\ M = -1$

18. Official Ans. by NTA (3.00)

Sol. $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + (3\lambda - 3)z = 0$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & 3-\lambda & \lambda-3 \\ \lambda-3 & \lambda-3 & -2(\lambda-3) \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$$(\lambda-3)^2 \begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$(\lambda - 3)^2 [(3\lambda + 1) + (3\lambda - 1)] = 0$

$6\lambda(\lambda - 3)^2 = 0 \Rightarrow \lambda = 0, 3$

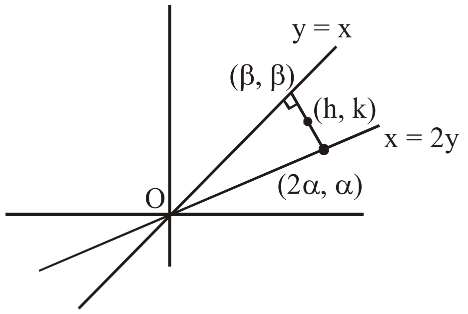
Sum = 3

STRAIGHT LINE

1. NTA Ans. (3)

Sol. $\frac{\alpha-\beta}{2\alpha-\beta}=-1$

$3\alpha = 2\beta$



$h = \frac{2\alpha+\beta}{2}$

$2h = \frac{7\alpha}{2}$

$k = \frac{\alpha+\beta}{2}$

$2k = \frac{5\alpha}{2}$

$\frac{h}{k}=\frac{7}{5}$

$5x = 7y$

2. NTA Ans. (5)

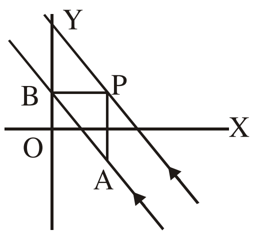
Sol. P is centroid of the triangle ABC

$\Rightarrow P \equiv \left(\frac{17}{6},\frac{8}{3}\right)$

$\Rightarrow PQ = 5$

3. NTA Ans. (3)

Sol. $\overleftrightarrow{AB} : 3x + y - 2 = 0$



Also, $\frac{1}{2}\times\sqrt{10}\times h=5$

$\Rightarrow h=\sqrt{10}$

$\Rightarrow \frac{|4\lambda-2|}{\sqrt{10}}=\sqrt{10}\Rightarrow\lambda=3,-2$

4. NTA Ans. (2)

Sol. Centroid of $\Delta = (2, 2)$

line passing through intersection of

$x + 3y - 1 = 0$ and

$3x - y + 1 = 0$, be given by

$(x + 3y - 1) + \lambda(3x - y + 1) = 0$

$\because$ It passes through (2, 2)

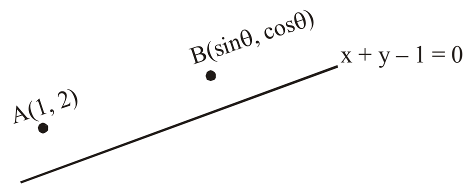
$\Rightarrow 7+5\lambda=0 \Rightarrow \lambda=-\frac{7}{5}$

$\therefore$ Required line is $8x - 11y + 6 = 0$

$\because$ (–9, –6) satisfies this equation.

5. Official Ans. by NTA (4)

Sol. Given that both points (1, 2) & (sinθ, cosθ) lie on same side of the line x + y – 1 = 0



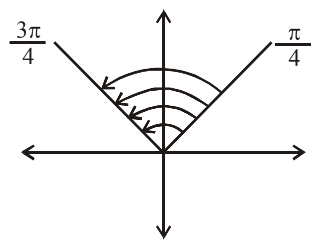
So, $\begin{pmatrix}\text{Put (1,2) in}\\\text{given line}\end{pmatrix}\begin{pmatrix}\text{Put } (\sin\theta, \cos\theta \text{ in}\\\text{given line}\end{pmatrix}>0$

$\Rightarrow (1 + 2 - 1) (\sin\theta + \cos\theta - 1) > 0$

$\Rightarrow \sin\theta + \cos\theta > 1 \ \{\div \text{by} \sqrt{2}\}$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta>\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right)>\frac{1}{\sqrt{2}}$

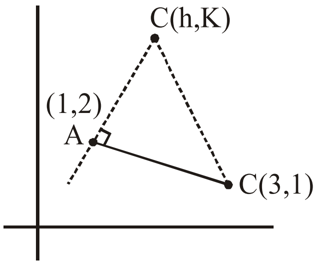


$\Rightarrow \frac{\pi}{4}<\theta+\frac{\pi}{4}<\frac{3\pi}{4}$

$\Rightarrow \boxed{0<\theta<\frac{\pi}{2}}$

6. Official Ans. by NTA (3)

Sol.



$$\left(\frac{K-2}{h-1}\right)\left(\frac{1-2}{3-1}\right)=-1 \Rightarrow K = 2h \quad ...(1)$$

$$\sqrt{5}\,|h-1|=10$$

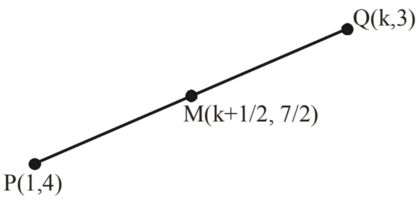
$$\because\ [\Delta ABC] = 5\sqrt{5}$$

$$\Rightarrow \frac{1}{2}\left(\sqrt{5}\right)\sqrt{(h-1)^2+(K-2)^2}=5\sqrt{5} \quad ....(2)$$

$$\Rightarrow h = 2\sqrt{5}+1 \ (h > 0)$$

7. Official Ans. by NTA (4)

Sol.



Slope = $m = \frac{1}{1-k}$

Equation of $\perp^r$ bisector is

$y + 4 = (k - 1)(x - 0)$

$\Rightarrow y + 4 = x(k - 1)$

$$\Rightarrow \frac{7}{2}+4=\frac{k+1}{2}(k-1)$$

$$\Rightarrow \frac{15}{2}=\frac{k^2-1}{2}\Rightarrow k^2=16\Rightarrow k=4,-4$$

8. Official Ans. by NTA (30)

Sol. Apply distance between parallel line formula

$4x - 2y + \alpha = 0$

$4x - 2y + 6 = 0$

$$\left|\frac{\alpha-6}{2\sqrt{5}}\right|=\frac{1}{\sqrt{5}}$$

$|\alpha - 6| = 2 \Rightarrow \alpha = 8, 4$

sum = 12

again

$6x - 3y + \beta = 0$

$6x - 3y + 9 = 0$

$$\left|\frac{\beta-9}{3\sqrt{5}}\right|=\frac{2}{\sqrt{5}}$$

$|\beta - 9| = 6 \Rightarrow \beta = 15, 3$

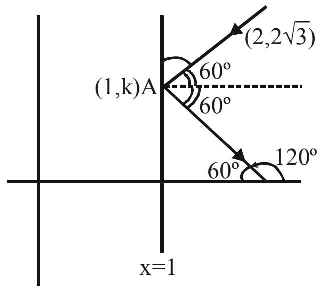
sum = 18

sum of all values of α and β is = 30

9. Official Ans. by NTA (2)

Sol. For point A

$$\tan 60° = \frac{2\sqrt{3}-k}{2-1}$$



$$\sqrt{3}=2\sqrt{3}-k$$

$$\therefore\ k=\sqrt{3}$$

so point $A\left(1,\sqrt{3}\right)$

Now slope of line AB is $m_{AB} = \tan 120°$

$m\, m_{AB} = -\sqrt{3}$

Now equation of line AB is

$$y-\sqrt{3}=-\sqrt{3}(x-1)$$

$$\sqrt{3}x+y=2\sqrt{3}$$

Now satisfy options

10. Official Ans. by NTA (3)

Sol. L : $\frac{x}{3}+\frac{y}{1}=1 \Rightarrow x + 3y - 3 = 0$

Image of point (–1, –4)

$$\frac{x+1}{1}=\frac{y+4}{3}=-2\left(\frac{-1-12-3}{10}\right)$$

$$\frac{x+1}{1}=\frac{y+4}{3}=\frac{16}{5}$$

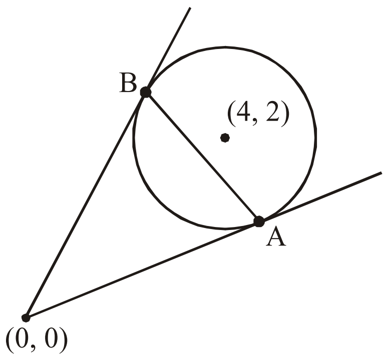
$$(x,y)\equiv\left(\frac{11}{5},\frac{28}{5}\right)$$

CIRCLE

1. NTA Ans. (4)

Sol. $R = \sqrt{16+4-16} = 2$

$L = \sqrt{S_1} = 4$



AB(Chord of contact) = $\frac{2LR}{\sqrt{L^2+R^2}} = \frac{8}{\sqrt{5}}$

$(AB)^2 = \frac{64}{5}$

2. NTA Ans. (2)

Sol. Slope of tangent to $x^2 + y^2 = 1$ at $P\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$2x + 2yy' = 0 \Rightarrow m_T\big|_P = -1$

$y = mx + c$ is tangent to $(x - 3)^2 + y^2 = 1$

$y = x + c$ is tangent to $(x - 3)^2 + y^2 = 1$

$\left|\frac{c+3}{\sqrt{2}}\right| = 1 \Rightarrow c^2 + 6c + 7 = 0$

(2) Option

3. NTA Ans. (36)

Sol. Common tangent is $S_1 - S_2 = 0$

$\Rightarrow -6x + 8y - 8 + k = 0$

Use p = r for Ist circle

$\Rightarrow \frac{|-18-8+k|}{10} = 1$

$\Rightarrow k = 36$ or $16 \Rightarrow k_{max} = 36$

4. Official Ans. by NTA (9.00)

Sol. Circle $x^2 + y^2 - 2x - 4y + 4 = 0$

$\Rightarrow (x - 1)^2 + (y - 2)^2 = 1$

Centre : (1, 2) radius = 1

line $3x + 4y - k = 0$ intersects the circle at two distinct points.

$\Rightarrow$ distance of centre from the line < radius

$\Rightarrow \left|\frac{3\times1+4\times2-k}{\sqrt{3^2+4^2}}\right| < 1$

$\Rightarrow |11 - k| < 5$

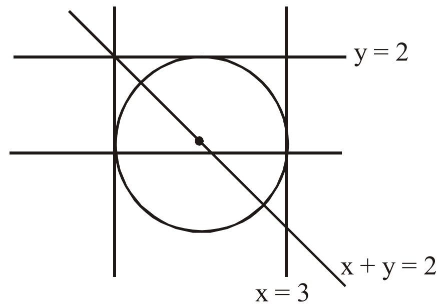
$\Rightarrow 6 < k < 16$

$\Rightarrow k \in \{7, 8, 9, ... 15\}$ since $k \in I$

Number of K is $\boxed{9}$

5. Official Ans. by NTA (3)

Sol.



$\because$ center lies on $x + y = 2$ and in 1st quadrant

center = $(\alpha, 2 - \alpha)$

where $\alpha > 0$ and $2 - \alpha > 0 \Rightarrow 0 < \alpha < 2$

$\because$ circle touches $x = 3$ and $y = 2$

$\Rightarrow |3 - \alpha| = |2 - (2 - \alpha)|$ = radius

$\Rightarrow |3 - \alpha| = |\alpha| \Rightarrow \alpha = \frac{3}{2}$

$\therefore$ radius = α

$\Rightarrow$ Diameter = $2\alpha = 3$.

6. Official Ans. by NTA (4)

Sol. Let S be the circle pasing through point of intersection of S_1 & S_2

$\therefore \; S = S_1 + \lambda S_2 = 0$

$\Rightarrow S : (x^2 + y^2 - 6x) + \lambda (x^2 + y^2 - 4y) = 0$

$\Rightarrow S : x^2 + y^2 - \left(\frac{6}{1+\lambda}\right)x - \left(\frac{4\lambda}{1+\lambda}\right)y = 0$ ...(1)

Centre $\left(\frac{3}{1+\lambda}, \frac{2\lambda}{1+\lambda}\right)$ lies on

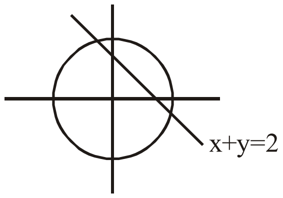
$2x - 3y + 12 = 0 \Rightarrow \lambda = -3$

put in (1) $\Rightarrow S : x^2 + y^2 + 3x - 6y = 0$

Now check options point (–3, 6) lies on S.

7. Official Ans. by NTA (7)

Sol. Let P $(3\cos\theta, 3\sin\theta)$



Q $(-3\cos\theta, -3\sin\theta)$

$$\Rightarrow \alpha\beta = \frac{|(3\cos\theta + 3\sin\theta)^2 - 4|}{2}$$

$$\Rightarrow \alpha\beta = \frac{5 + 9\sin 2\theta}{2} \le 7$$

8. Official Ans. by NTA (2)

Sol. Let chord

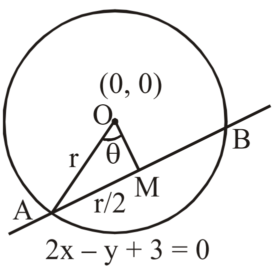
AB = r

$\because$ ΔAOM is right angled triangle

$\therefore$ $OM = \frac{r\sqrt{3}}{2}$ = perpendicular distance of line

AB from (0,0)

$$\frac{r\sqrt{3}}{2} = \left|\frac{3}{\sqrt{5}}\right|$$



$$r^2 = \frac{12}{5}$$

PERMUTATION & COMBINATION

1. NTA Ans. (1)

Sol. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 is

$$^5C_1 \times \frac{6!}{2!}$$

2. NTA Ans. (2454)

Sol. N $\to$ 2, A $\to$ 2, I $\to$ 2, E, X, M, T, O $\to$ 1

Category	Selection	Arrangement
2 alike of one kind and 2 alike of other kind	$^3C_2 = 3$	$3 \times \frac{4!}{2!\,2!} = 18$
2 alike and 2 different	$^3C_1 \times {}^7C_2$	$^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$
All 4 different	$^8C_4$	$^8C_4 \times 4! = 1680$

Total = 2454

Ans. 2454.00

3. NTA Ans. (4)

Sol. $a = {}^{19}C_{10}$, $b = {}^{20}C_{10}$ and $c = {}^{21}C_{10}$

$\Rightarrow a = {}^{19}C_9$, $b = 2({}^{19}C_9)$ and $c = \frac{21}{11}\left({}^{20}C_{10}\right)$

$\Rightarrow b = 2a$ and $c = \frac{21}{11}b = \frac{42a}{11}$

$\Rightarrow a : b : c = a : 2a : \frac{42a}{11} = 11 : 22 : 42$

4. NTA Ans. (490.00)

ALLEN Ans. (490.00 OR 13.00)

Note: If same coloured marbles are identical then, answer is 13.00. However, NTA took them as distinct and kept only one answer as 490.00

Sol. The question does not mention that whether same coloured marbles are distinct or identical. So, assuming they are distinct our required answer = $^{12}C_4 - {}^5C_4 = 490$

And, if same coloured marbles are identical then required answer = (2 + 3 + 4 + 4) = 13

5. NTA Ans. (1)

Sol. _ _ _ 2 _

No. of five digits numbers =

No. of ways of filling remaining 4 places

= 8 × 8 × 7 × 6

$$\therefore\ k = \frac{8 \times 8 \times 7 \times 6}{336} = 8$$

6. Official Ans. by NTA (309.00)

Sol. MOTHER

$1 \to E$

$2 \to H$

$3 \to M$

$4 \to O$

$5 \to R$

$6 \to T$

So position of word MOTHER in dictionary

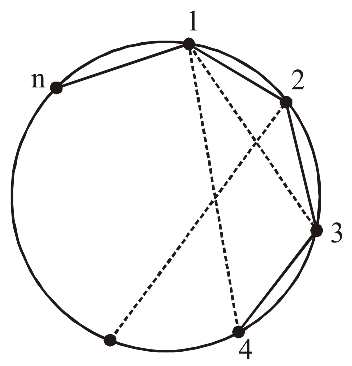
$2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1$

$= 240 + 48 + 18 + 2 + 1$

$= \boxed{309}$

7. Official Ans. by NTA (3)

Sol.



Number of blue lines = Number of sides = n

Number of red lines = number of diagonals

$= {}^nC_2 - n$

${}^nC_2 - n = 99\,n \Rightarrow \frac{n(n-1)}{2} - n = 99\,n$

$\frac{n-1}{2} - 1 = 99 \Rightarrow n = 201$

8. Official Ans. by NTA (3)

Sol. $S = (2.{}^1p_0 - 3.{}^2p_1 + 4.{}^3p_2$ .... upto 51 terms) + (1! + 2! + 3! .... upto 51 terms) $[\because {}^np_{n-1} = n!]$

$\therefore S = (2 \times 1! - 3 \times 2! + 4 \times 3! .... + 52.51!) + (1! - 2! + 3! .... (51)!)$

$= (2! - 3! + 4! .... + 52!) + (1! - 2! + 3! - 4! + ... + (51)!)$

$= 1! + 52!.$

9. Official Ans. by NTA (54)

Sol. Let three digit number is xyz

$x + y + z = 10$; $x \geq 1, y \geq 0\ z \geq 0$ ..... (1)

Let $T = x - 1 \Rightarrow x = T + 1$ where $T \geq 0$

Put in (1)

$T + y + z = 9$; $0 \leq T \leq 8, 0 \leq y, z \leq 9$

No. of non negative integral solution

$= {}^{9+3-1}C_{3-1} - 1$ (when T = 9)

$= 55 - 1 = 54$

10. Official Ans. by NTA (135)

Sol. Ways $= {}^6C_4 \cdot 1^4 \cdot 3^2$

$= 15 \times 9$

$= 135$

11. Official Ans. by NTA (240)

Sol. S_2YL_2ABU

ABCC type words

$= \underbrace{{}^2C_1}_{\text{selection of two alike letters}} \times \underbrace{{}^5C_2}_{\text{selection of of two distinct letters}} \times \underbrace{\frac{\lfloor 4}{\lfloor 2}}_{\text{arrangement of selected letters}}$

$= 240$

12. Official Ans. by NTA (4)

Sol.

A	B	C
5	5	5
1	2	2
2	1	2
2	2	1
1	1	3
1	3	1
3	1	1

Total number of selection

$= ({}^5C_1\ {}^5C_2\ {}^5C_2)\cdot 3 + ({}^5C_1\ {}^5C_1\ {}^5C_3)\cdot 3$

$= 5 \cdot 10 \cdot 10 \cdot 3 + 5 \cdot 5 \cdot 10 \cdot 3$

$= 2250$

13. Official Ans. by NTA (120.00)

Sol. LETTER

vowels = EE, consonant = LTTR

_ L _ T _ T _ R _

$\frac{4!}{2!} \times {}^5C_2 \times \frac{2!}{2!} = 12 \times 10 = 120$

BINOMIAL THEOREM

1. NTA Ans. (3)

Sol. $6\times{}^{35}C_r=(k^2-3)\,{}^{36}C_{r+1}$

$k^2-3>0 \Rightarrow k^2>3$

$k^2-3=\frac{6\times{}^{35}C_r}{{}^{36}C_{r+1}}=\frac{r+1}{6}$

Possible values of r for integral values of k, are

r = 5, 35

number of ordered pairs are 4

(5, 2), (5, –2), (35, 3), (35, –3)

2. NTA Ans. (2)

Sol. Coefficient of x^7 is

${}^{10}C_7+{}^9C_6+{}^8C_5+\ldots+{}^4C_1+{}^3C_0$

$\underbrace{{}^4C_0+{}^4C_1}_{{}^5C_1}+{}^5C_2+\ldots+{}^{10}C_7={}^{11}C_7=330$

3. NTA Ans. (30)

Sol. Let $(1+x+x^2+\ldots+x^{2n})(1-x+x^2-x^3+\ldots+x^{2n})$

$=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+\ldots+a_{4n}x^{4n}$

So,

$a_0+a_1+a_2+\ldots+a_{4n}=2n+1$ …(1)

$a_0-a_1+a_2-a_3\ldots+a_{4n}=2n+1$ …(2)

$\Rightarrow a_0+a_2+a_4+\ldots+a_{4n}=2n+1$

$\Rightarrow 2n+1=61 \qquad \Rightarrow n=30$

4. NTA Ans. (3)

Sol. $2[{}^6C_0x^6+{}^6C_2x^4(x^2-1)+{}^6C_4x^2(x^2-1)^2+{}^6C_6(x^2-1)^3]$

$\alpha=-96$ & $\beta=36$

$\therefore \alpha-\beta=-132$

(3) Option

5. NTA Ans. (4)

Sol. $T_{r+1}={}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^r$

$={}^{16}C_r(x)^{16-2r}\times\frac{1}{(\cos\theta)^{16-r}(\sin\theta)^r}$

For independent of x; $16-2r=0\Rightarrow r=8$

$\Rightarrow T_9={}^{16}C_8\frac{1}{\cos^8\theta\sin^8\theta}$

$={}^{16}C_8\frac{2^8}{(\sin2\theta)^8}$

for $\theta\in\left[\frac{\pi}{8},\frac{\pi}{4}\right]$ ℓ_1 is least for $\theta_1=\frac{\pi}{4}$

for $\theta\in\left[\frac{\pi}{16},\frac{\pi}{8}\right]$ ℓ_2 is least for $\theta_2=\frac{\pi}{8}$

$\frac{\ell_2}{\ell_1}=\frac{(\sin2\theta_1)^8}{(\sin2\theta_2)^8}=(\sqrt{2})^8=\frac{16}{1}$

6. NTA Ans. (51)

Sol. $S=1.{}^{25}C_0+5.{}^{25}C_1+9.{}^{25}C_2+\ldots+(101){}^{25}C_{25}$

$S=101\,{}^{25}C_{25}+97\,{}^{25}C_1+\ldots\ldots\ldots+1\,{}^{25}C_{25}$

$2S=(102)(2^{25})$

$S=51(2^{25})$

7. NTA Ans. (615.00)

Sol. $(1+x+x^2)^{10}$

$={}^{10}C_0+{}^{10}C_1x(1+x)+{}^{10}C_2x^2(1+x)^2$

$+{}^{10}C_3x^3(1+x)^3+{}^{10}C_4x^4(1+x)^4+\ldots\ldots.$

Coeff. of $x^4={}^{10}C_2+{}^{10}C_3\times{}^3C_1+{}^{10}C_4=615.$

8. Official Ans. by NTA (2)

Sol. Let t_{r+1} denotes

r + 1th term of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$

$t_{r+1} = {}^{10}C_r \alpha^{10-r} (x)^{\frac{10-r}{9}} . \beta^r x^{-\frac{r}{6}}$

$= {}^{10}C_r \alpha^{10-r} \beta^r (x)^{\frac{10-r}{9} - \frac{r}{6}}$

If t_{r+1} is independent of x

$\frac{10-r}{9} - \frac{r}{6} = 0 \Rightarrow r = 4$

maximum value of t_5 is 10 K (given)

$\Rightarrow {}^{10}C_4 \alpha^6 \beta^4$ is maximum

By AM ≥ GM (for positive numbers)

$$\frac{\frac{\alpha^3}{2} + \frac{\alpha^3}{2} + \frac{\beta^2}{2} + \frac{\beta^2}{2}}{4} \geq \left(\frac{\alpha^6\beta^4}{16}\right)^{\frac{1}{4}}$$

$\Rightarrow \boxed{\alpha^6\beta^4 \leq 16}$

So, 10 K = ${}^{10}C_4 16$

$\Rightarrow$ K = 336

9. Official Ans. by NTA (118)

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12$

Now $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5}$

$\Rightarrow 7r = 2n + 2$ …(1)

$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$

$\Rightarrow 17r = 5n - 12$ …(2)

On solving (1) & (2)

$\Rightarrow$ n = 118

10. Official Ans. by NTA (2)

Sol. $T_{r+1} = {}^nC_r (3)^{\frac{n-r}{2}} (5)^{\frac{r}{8}}$ $(n \geq r)$

Clearly r should be a multiple of 8.

∵ there are exactly 33 integral terms

Possible values of r can be

0, 8, 16, ... , 32 × 8

∴ least value of n = 256.

11. Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r$

$T_{r+1} = {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$

For independent of x

18 – 3r = 0, r = 6

$\therefore T_7 = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6 = \frac{21}{54} = k$

$\therefore 18k = \frac{21}{54} \times 18 = 7$

12. Official Ans. by NTA (2)

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + {}^{30}C_6$

$= {}^{50}C_6 + {}^{49}C_6 + ... + {}^{31}C_6 + \left({}^{30}C_6 + {}^{30}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{49}C_6 + ... + \left({}^{31}C_6 + {}^{31}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{50}C_7 - {}^{30}C_7$

$= {}^{51}C_7 - {}^{30}C_7$

$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$

13. Official Ans. by NTA (8)

Sol. Given $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ .... (1)

replace x by $\frac{2}{x}$ in above identity :-

$$\frac{2^{10}(2x^2 + 3x + 4)^{10}}{x^{20}} = \sum_{r=0}^{20} \frac{a_r 2^r}{x^r}$$

$$\Rightarrow 2^{10} \sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^r x^{(20-r)} \text{ (from (i))}$$

now, comparing coefficient of x^7 from both sides

(take r = 7 in L.H.S. & r = 13 in R.H.S.)

$2^{10} a_7 = a_{13} 2^{13} \Rightarrow \frac{a_7}{a_{13}} = 2^3 = 8$

14. Official Ans. by NTA (3)

Sol. Let n + 5 = N

$^{N}C_{r-1} : {}^{N}C_{r} : {}^{N}C_{r+1} = 5 : 10 : 14$

$\Rightarrow \frac{^{N}C_{r}}{^{N}C_{r-1}} = \frac{N+1-r}{r} = 2$

$\frac{^{N}C_{r+1}}{^{N}C_{r}} = \frac{N-r}{r+1} = \frac{7}{5}$

$\Rightarrow$ r = 4, N = 11

$\Rightarrow (1 + x)^{11}$

Largest coefficient = $^{11}C_6 = 462$

15. Official Ans. by NTA (13)

Sol. $T_{r+1} = {}^{22}C_r (x^m)^{22-r}\left(\frac{1}{x^2}\right)^r = {}^{22}C_r x^{22m-mr-2r}$

$= {}^{22}C_r\ x$

$\because {}^{22}C_3 = {}^{22}C_{19} = 1540$

$\therefore$ r = 3 or 19

22m – mr – 2r = 1

$m = \frac{2r+1}{22-5}$

$r = 3,\ m = \frac{7}{19} \notin N$

$r = 19,\ m = \frac{38+1}{22-19} = \frac{39}{3} = 13$

m = 13

16. Official Ans. by NTA (120.00)

Sol. $(1 + x + x^2 + x^3)^6 = \left((1+x)(1+x^2)\right)^6$

$= (1 + x)^6 (1 + x^2)^6$

$= \sum_{r=0}^{6} {}^6C_r x^r \sum_{r=0}^{6} {}^6C_t x^{2t}$

$= \sum_{r=0}^{6} \sum_{t=0}^{6} {}^6C_r {}^6C_t x^{r+2t}$

For coefficient of $x^4 \Rightarrow r + 2t = 4$

r	t
0	2
2	1
4	0

Coefficient of x^4

$= {}^6C_0 {}^6C_2 + {}^6C_2 {}^6C_1 + {}^6C_4 {}^6C_0$

= 120

17. Official Ans. by NTA (1)

Sol. $\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{(3^2)^{100}}{8}\right\} = \left\{\frac{(1+8)^{100}}{8}\right\}$

$= \left\{\frac{1 + {}^{100}C_1.8 + {}^{100}C_2.8^2 + ... + {}^{100}C_{100}8^{100}}{8}\right\}$

$= \left\{\frac{1+8m}{8}\right\} = \frac{1}{8}$

18. Official Ans. by NTA (3)

Sol. $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$

$T_{r+1} = {}^{10}C_r \left(\sqrt{x}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r$

$T_{r+1} = {}^{10}C_r.x^{\frac{10-r}{2}}.(-k)^r.x^{-2r}$

$T_{r+1} = {}^{10}C_r x^{\frac{10-5r}{2}}(-k)^r$

Constant term : $\frac{10-5r}{2} = 0 \Rightarrow r = 2$

$T_3 = {}^{10}C_2.(-k)^2 = 405$

$k^2 = \frac{405}{45} = 9$

$k = \pm 3 \Rightarrow |k| = 3$

SET

1. NTA Ans. (29.00)

Sol. n(A) = 25

n(B) = 7

$n(A \cap B) = 3$

$n(A \cup B) = 25 + 7 - 3 = 29$

2. Official Ans. by NTA (1)

Sol. A : $D \geq 0$

$\Rightarrow (m + 1)^2 - 4(m + 4) \geq 0$

$\Rightarrow m^2 + 2m + 1 - 4m - 16 \geq 0$

$\Rightarrow m^2 - 2m - 15 \geq 0$

$\Rightarrow (m - 5)(m + 3) \geq 0$

$\Rightarrow m \in (-\infty, -3] \cup [5, \infty)$

$\therefore A = (-\infty, -3] \cup [5, \infty)$

$B = [-3, 5)$

$A - B = (-\infty, -3) \cup [5, \infty)$

$A \cap B = \{-3\}$

$B - A = (-3, 5)$

$A \cup B = R$

3. Official Ans. by NTA (8)

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let $x = k$

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0, \; y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k, \; y = \frac{k}{2}, \; z = 0$ put in condition

$15 \leq k^2 + \left(\frac{k}{2}\right)^2 + 0 \leq 150$

$12 \leq k^2 \leq 120$

$\Rightarrow k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

4. Official Ans. by NTA (4)

Sol. $n(B) \leq n(A \cup B) \leq n(U)$

$\Rightarrow 76 \leq 76 + 63 - x \leq 100$

$\Rightarrow -63 \leq -x \leq -39$

$\Rightarrow 63 \geq x \geq 39$

5. Official Ans. by NTA (4)

Sol. $n(X_i) = 10. \; \bigcup_{i=1}^{50} X_i = T, \Rightarrow \; n(T) = 500$

each element of T belongs to exactly 20

elements of $X_i \Rightarrow \frac{500}{20} = 25$ distinct elements

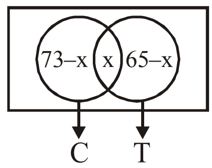
so $\frac{5n}{6} = 25 \Rightarrow n = 30$

6. Official Ans. by NTA (4)

Sol. $C \rightarrow$ person like coffee

$T \rightarrow$ person like Tea

$n(C) = 73$



$n(T) = 65$

$n(C \cup T) \leq 100$

$n(C) + n(T) - n(C \cap T) \leq 100$

$73 + 65 - x \leq 100$

$x \geq 38$

$73 - x \geq 0 \Rightarrow x \leq 73$

$65 - x \geq 0 \Rightarrow x \leq 65$

$\boxed{38 \leq x \leq 65}$

7. Official Ans. by NTA (28.00)

Sol. $2^m - 2^n = 112$

$m = 7, n = 4$

$(2^7 - 2^4 = 112)$

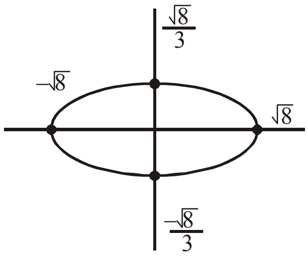
$m \times n = 7 \times 4 = 28$

RELATION

1. Official Ans. by NTA (2)

Sol. $R = \{(x, y) : x, y \in z, x^2 + 3y^2 \le 8\}$

For domain of R^{-1}



Collection of all integral of y's

For $x = 0$, $3y^2 \le 8$

$\Rightarrow y \in \{-1, 0, 1\}$

2. Official Ans. by NTA (4)

Sol. Let $a^2 + b^2 \in Q$ & $b^2 + c^2 \in Q$

eg. $a = 2 + \sqrt{3}$ & $b = 2 - \sqrt{3}$

$a^2 + b^2 = 14 \in Q$

Let $c = (1+2\sqrt{3})$

$b^2 + c^2 = 20 \in Q$

But $a^2 + c^2 = (2+\sqrt{3})^2 + (1+2\sqrt{3})^2 \notin Q$

for R_2 Let $a^2 = 1$, $b^2 = \sqrt{3}$ & $c^2 = 2$

$a^2 + b^2 \notin Q$ & $b^2 + c^2 \notin Q$

But $a^2 + c^2 \in Q$

FUNCTION

1. NTA Ans. (2)

Sol. $g(x) = x^2 + x - 1$

$g(f(x)) = 4x^2 - 10x + 5$

$= (2x - 2)^2 + (2 - 2x) - 1$

$= (2 - 2x)^2 + (2 - 2x) - 1$

$\Rightarrow f(x) = 2 - 2x$

$f\left(\frac{5}{4}\right) = \frac{-1}{2}$

2. NTA Ans. (4)

Sol. $f(x) = \begin{cases} \frac{x}{x^2+1} & ; \quad x \in (1,2) \\ \frac{2x}{x^2+1} & ; \quad x \in [2,3) \end{cases}$

$f(x)$ is decreasing function

$\therefore f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(4) Option

3. NTA Ans. (2)

Sol. $f(x) = \frac{2(2^x + 2^{-x}) + (3^x + 3^{-x})}{2} \ge 3$

(A.M ≥ G.M)

4. NTA Ans. (3)

Sol. $f(x) = y = \frac{8^{4x} - 1}{8^{4x} + 1} = 1 - \frac{2}{8^{4x} + 1}$

so, $8^{4x} + 1 = \frac{2}{1-y} \Rightarrow 8^{4x} = \frac{1+y}{1-y}$

$\Rightarrow x = \ell n\left(\frac{1+y}{1-y}\right) \times \frac{1}{4\ell n 8} = f^{-1}(y)$

Hence, $f^{-1}(x) = \frac{1}{4}\log_8 e \ell n\left(\frac{1+x}{1-x}\right)$

5. Official Ans. by NTA (1)

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow f(n) = nf(1)$

$f(n) = 2n$

$g(n) = \sum_{k=1}^{n-1} 2n = 2\left(\frac{(n-1)n}{2}\right) = n(n-1)$

$g(n) = 20 \Rightarrow n(n-1) = 20$

$n = 5$

6. Official Ans. by NTA (4)

Sol. $[x]^2 + 2[x + 2] - 7 = 0$

$\Rightarrow [x]^2 + 2[x] + 4 - 7 = 0$

$\Rightarrow [x] = 1, -3$

$\Rightarrow x \in [1, 2) \cup [-3, -2)$

7. Official Ans. by NTA (19.00)

Sol. $C = \{f : A \to B | 2 \in f(A) \text{ and f is not one-one}\}$

Case-I : If f(x) = 2 ∀ x ∈ A then number of function = 1

Case-II : If f(x) = 2 for exactly two elements then total number of many-one function = $^3C_2\ ^3C_1 = 9$

Case-III : If f(x) = 2 for exactly one element then total number of many-one functions = $^3C_1\ ^3C_1 = 9$

Total = 19

8. Official Ans. by NTA (2)

Sol. $f(x) = \frac{a-x}{a+x} \quad x \in R - \{-a\} \to R$

$$f(f(x)) = \frac{a-f(x)}{a+f(x)} = \frac{a-\left(\frac{a-x}{a+x}\right)}{a+\left(\frac{a-x}{a+x}\right)}$$

$$f(f(x)) = \frac{(a^2-a)+x(a+1)}{(a^2+a)+x(a-1)} = x$$

$\Rightarrow (a^2 - a) + x(a + 1) = (a^2 + a)x + x^2(a - 1)$

$\Rightarrow a(a - 1) + x(1 - a^2) - x^2(a - 1) = 0$

$\Rightarrow a = 1$

$$f(x) = \frac{1-x}{1+x},$$

$$f\left(\frac{-1}{2}\right) = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$$

9. Official Ans. by NTA (5.00)

Sol. $f(x + y) = f(x)\, f(y)$

put x = y = 1 $\quad f(2) = (f(1))^2 = 3^2$

put x = 2, y = 1 $f(3) = (f(1))^3 = 3^3$

⋮

Similarly $f(x) = 3^x$

$$\sum_{i=1}^{n} f(i) = 363 \Rightarrow \sum_{i=1}^{n} 3^i = 363$$

$(3 + 3^2 + ... + 3^n) = 363$

$$\frac{3(3^n-1)}{2} = 363$$

$3^n - 1 = 242 \Rightarrow 3^n = 243$

$\Rightarrow n = 5$

INVERSE TRIGONOMETRY FUNCTION

1. Official Ans. by NTA (1)

Sol. $f(x) = \sin\left(\frac{|x|+5}{x^2+1}\right)$

For domain :

$$-1 \le \frac{|x|+5}{x^2+1} \le 1$$

Since $|x| + 5$ & $x^2 + 1$ is always positive

So $\frac{|x|+5}{x^2+1} \ge 0 \ \forall x \in R$

So for domain :

$$\frac{|x|+5}{x^2+1} \le 1$$

$\Rightarrow |x| + 5 \le x^2 + 1$

$\Rightarrow 0 \le x^2 - |x| - 4$

$$\Rightarrow 0 \le \left(|x| - \frac{1+\sqrt{17}}{2}\right)\left(|x| - \frac{1-\sqrt{17}}{2}\right)$$

$$\Rightarrow |x| \ge \frac{1+\sqrt{17}}{2} \text{ or } |x| \le \frac{1-\sqrt{17}}{2} \quad \text{(Rejected)}$$

$$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$$

So, $a = \frac{1+\sqrt{17}}{2}$

2. Official Ans. by NTA (3)

Sol. $$2\pi - \left(\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right)$$

$$= 2\pi - \left(\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right)$$

$$= 2\pi - \left(\tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right)$$

$$= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

3. Official Ans. by NTA (4)

Sol. $S = \tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+...$

$S = \tan^{-1}\left(\frac{2-1}{1+1.2}\right)+\tan^{-1}\left(\frac{3-2}{1+2\times 3}\right)+\tan^{-1}$

$\left(\frac{4-3}{1+3\times 4}\right)+ ....+\tan^{-1}\left(\frac{11-10}{1+10\times 11}\right)$

$S = (\tan^{-1}2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + (\tan^{-1} 4 - \tan^{-1} 3) + ... + (\tan^{-1}(11) - \tan^{-1}(10))$

$S = \tan^{-1} 11 - \tan^{-1} 1 = \tan^{-1}\left(\frac{11-1}{1+11}\right)$

$\tan(S) = \frac{11-1}{1+11\times 1} = \frac{10}{12} = \frac{5}{6}$

LIMIT

1. NTA Ans. (36)

Sol. $\lim\limits_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}} \Rightarrow \lim\limits_{x\to 2}\frac{3^{2x}-12.3^x+27}{3^{x/2}-3}$

$= \lim\limits_{x\to 2}\frac{(3^x-9)(3^x-3)}{(3^{x/2}-3)}$

$= \lim\limits_{x\to 2}\frac{(3^{x/2}+3)(3^{x/2}-3)(3^x-3)}{(3^{x/2}-3)}$

$= 36$

2. NTA Ans. (4)

Sol. Required limit $= e^{\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}-1\right)\frac{1}{x^2}}$

$= e^{\lim\limits_{x\to 0}\left(\frac{-4}{7x^2+2}\right)} = \frac{1}{e^2}$

3. Official Ans. by NTA (40.00)

Sol. $\lim\limits_{x\to 1}\frac{x+x^2+.......+x^2-n}{x-1} = 820$

$\Rightarrow \lim\limits_{x\to 1}\left(\frac{x-1}{x-1}+\frac{x^2-1}{x-1}+......\frac{x^n-1}{x-1}\right) = 820$

$\Rightarrow 1 + 2 + ... + n = 820$

$\Rightarrow n(n + 1) = 2 \times 820$

$\Rightarrow n(n + 1) = 40 \times 41$

Since $n \in N$, so $\boxed{n = 40}$

4. Official Ans. by NTA (4)

Sol. $\lim\limits_{x\to 0}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{1/x}$

$= e^{\lim\limits_{x\to 0}\frac{1}{x}\left\{\tan\left(\frac{\pi}{4}+x\right)-1\right\}}$

$= e^{\lim\limits_{x\to 0}\left(\frac{1+\tan x-1+\tan x}{x(1-\tan x)}\right)}$

$= e^{\lim\limits_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}$

$= e^2$

5. Official Ans. by NTA (2)

Sol. LHL : $\lim\limits_{x\to 0^-}\left|\frac{1-x-x}{\lambda-x-1}\right| = \left|\frac{1}{\lambda-1}\right|$

RHL : $\lim\limits_{x\to 0^+}\left|\frac{1-x+x}{\lambda-x+1}\right| = \left|\frac{1}{\lambda}\right|$

For existence of limit

LHL = RHL

$\Rightarrow \frac{1}{|\lambda-1|} = \frac{1}{|\lambda|} \Rightarrow \lambda = \frac{1}{2}$

$\therefore L = \frac{1}{|\lambda|} = 2$

6. Official Ans. by NTA (8)

Sol. $\lim\limits_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$

$\Rightarrow \lim\limits_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)}{4\left(\frac{x^2}{2}\right)^2}\frac{\left(1-\cos\frac{x^2}{4}\right)}{16\left(\frac{x^2}{4}\right)^2} = \frac{1}{8}\times\frac{1}{32} = 2^{-k}$

$\Rightarrow 2^{-8} = 2^{-k} \Rightarrow k = 8.$

7. Official Ans. by NTA (1)

Sol. Required limit

$$L = \lim_{h\to 0}\frac{(a+2(a+h))^{1/3}-(3(a+h))^{1/3}}{(3a+a+h)^{1/3}-(4(a+h))^{1/3}}$$

$$=\lim_{h\to 0}\frac{(3a)^{1/3}\left(1+\frac{2h}{3a}\right)^{1/3}-(3a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}{(4a)^{1/3}\left(1+\frac{h}{4a}\right)^{1/3}-(4a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}$$

$$=\lim_{h\to 0}\left(\frac{3^{1/3}}{4^{1/3}}\right)\left[\frac{\left(1+\frac{2h}{9a}\right)-\left(1+\frac{h}{3a}\right)}{\left(1+\frac{h}{12a}\right)-\left(1+\frac{h}{3a}\right)}\right]$$

$$=\left(\frac{3}{4}\right)^{1/3}\frac{\left(\frac{2}{9}-\frac{1}{3}\right)}{\left(\frac{1}{12}-\frac{1}{3}\right)}=\left(\frac{3}{4}\right)^{1/3}\left(\frac{8-12}{3-12}\right)$$

$$=\left(\frac{3}{4}\right)^{1/3}\left(\frac{-4}{-9}\right)=\frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}}=\frac{4^{2/3}}{3^{5/3}}$$

$$=\frac{(8\times 2)^{1/3}}{(27\times 9)^{1/3}}=\frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

8. Official Ans. by NTA (4)

Sol. $L = \lim\limits_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}$

using L.H. rule

$L = \lim\limits_{t\to x}\frac{2tf^2(x)-x^2\cdot 2f'(t).f(t)}{1}$

$\Rightarrow$ L = 2xf(x) (f(x) – x f'(x)) = 0 (given)

$\Rightarrow$ f(x) = xf '(x) $\Rightarrow \int\frac{f'(x)dx}{f(x)}=\int\frac{dx}{x}$

$\Rightarrow \ell n\,|f(x)| = \ell n\,|x| + C$

$\because$ f(1) = e, x > 0, f(x) > 0

$\Rightarrow$ f(x) = ex, if f(x) = 1 $\Rightarrow x = \frac{1}{e}$

9. Official Ans. by NTA (1)

Sol. $x^2 - x - 2 = 0$

roots are 2 & –1

$$\Rightarrow \lim_{x\to 2^+}\frac{\sqrt{1-\cos(x^2-x-2)}}{(x-2)}$$

$$=\lim_{x\to 2^+}\frac{\sqrt{2\sin^2\frac{(x^2-x-2)}{2}}}{(x-2)}$$

$$=\lim_{x\to 2^+}\frac{\sqrt{2}\sin\left(\frac{(x-2)(x+1)}{2}\right)}{(x-2)}=\frac{3}{\sqrt{2}}$$

10. Official Ans. by NTA (4)

Sol. $\lim\limits_{x\to 0}\frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$

$\because \lim\limits_{x\to 0}\frac{\sqrt{1+x^2+x^4}-1}{x}$ ($\frac{0}{0}$ from)

$\lim\limits_{x\to 0}\frac{(1+x^2+x^4)-1}{x(\sqrt{1+x^2+x^4}+1}$

$\lim\limits_{x\to 0}\frac{x(1+x^2)}{\left(\sqrt{1+x^2+x^4}+1\right)} = 0$

So $\lim\limits_{x\to 0}\frac{x\left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)}-1\right)}{\sqrt{1+x^2+x^4}-1}$ ($\frac{0}{0}$ from)

$\lim\limits_{x\to 0}\frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}}-1}{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)} = 1$

11. Official Ans. by NTA (1)
Official Ans. by ALLEN
(Bonus-Answers musbe zero)

Sol. $\lim_{x\to 1}\frac{\int_0^{(x-1)^2} t\cos\left(t^2\right)dt}{(x-1)\sin(x-1)}\left(\frac{0}{0}\right)$

Apply L Hopital Rule

$$=\lim_{x\to 1}\frac{2(x-1).(x-1)^2\cos(x-1)^4-0}{(x-1).\cos(x-1)+\sin(x-1)}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 1}\frac{2(x-1)^3.\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\frac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to 1}\frac{2(x-1)^2\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\frac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to 1}\frac{2(x-1)^2\cos(x-1)^4}{\cos(x-1)+\frac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$=\frac{0}{1+1}=0$$

CONTINUITY

1. NTA Ans. (5.00)

Sol. $k=\lim_{x\to 0}\left(\frac{\ell n(1+3x)}{x}-\frac{\ell n(1-2x)}{x}\right)$

$k = 3 + 2 = 5$

2. NTA Ans. (2)

Sol. $A=\lim_{x\to 0}x\left[\frac{4}{x}\right]=\lim_{x\to 0}x\left(\frac{4}{x}\right)-x\left\{\frac{4}{x}\right\}=4$

$f(x) = [x^2]\sin(\pi x)$ will be discontinuous at nonintegers

$\therefore\ x=\sqrt{A+1}$ i.e. $\sqrt{5}$

3. NTA Ans. (4)

Sol. $\lim_{x\to 0^-}f(x)=\lim_{x\to 0}\left(\frac{\sin(a+2)x}{x}+\frac{\sin x}{x}\right)=a+3$

$\lim_{x\to 0^-}f(x)=\lim_{x\to 0}\frac{(x+3x^2)^{1/3}-x^{1/3}}{x^{4/3}}$

$=\lim_{x\to 0}\frac{(1+3x)^{1/3}-1}{x}=1$

$f(0) = b$

for continuity at $x = 0$

$\lim_{x\to 0^-}f(x)=f(0)=\lim_{x\to 0^+}f(x)$

$\Rightarrow a + 3 = b = 1$

$\therefore$ $a = -2$, $b = 1$

$\therefore$ $a + 2b = 0$

4. Official Ans. by NTA (8)

Sol. $x \in (-10, 10)$

$\frac{x}{2}\in(-5,\ 5)\ \rightarrow$ 9 integers

check continuity at $x = 0$

$\left.\begin{array}{l} f(0)=0 \\ f(0^+)=0 \\ f(0^-)=0 \end{array}\right\}$ continuous at $x = 0$

function will be distcontinuous when

$\frac{x}{2}=\pm 4,\ \pm 3,\ \pm 2,\ \pm 1$

8 points of discontinuity

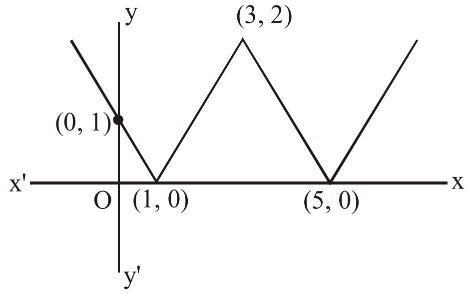
DIFFERENTIABILITY

1. NTA Ans. (3)

Sol. $f(x) = |2 - |x - 3||$

f is not differentiable at

$x = 1, 3, 5$



$\Rightarrow\ \sum_{x\in s} f(f(x))=f(f(1))+f(f(3))+f(f(5))$

$= f(0) + f(2) + f(0)$

$= 1 + 1 + 1 = 3$

2. Official Ans. by NTA (4)

Sol. $f(x)=\begin{cases} ae^{x}+be^{-x}, & -1\le x<1 \\ cx^2, & 1\le x\le 3 \\ ax^2+2cx, & 3<x\le 4 \end{cases}$

For continuity at x = 1

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)$

$\Rightarrow \boxed{ae+be^{-1}=c} \Rightarrow \boxed{b=ce-ae^2}$

...(1)

For continuity at x = 3

$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)$

$\Rightarrow$ 9c = 9a + 6c

$\Rightarrow$ c = 3a ...(2)

$f'(0) + f'(2) = e$

$(ae^x - be^x)_{x=0} + (2cx)_{x=2} = e$

$\Rightarrow \boxed{a-b+4c=e}$ ...(3)

From (1), (2) & (3)

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow a(e^2 + 13 - 3e) = e$

$\Rightarrow a=\dfrac{e}{e^2-3e+13}$

3. Official Ans. by NTA (10)

Sol. Since, $\lim_{x\to 0}\dfrac{f(x)}{x}$ exist $\Rightarrow f(0) = 0$

Now, $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\dfrac{f(h)+xh^2+x^2h}{h}$ (take y = h)

$=\lim_{h\to 0}\dfrac{f(h)}{h}+\lim_{h\to 0}(xh)+x^2$

$\Rightarrow f'(x) = 1 + 0 + x^2$

$\Rightarrow f'(3) = 10$

4. Official Ans. by NTA (1)

Sol. $f(x)=\begin{cases} \dfrac{\pi}{4}+\tan^{-1}x, & x\in(-\infty,-1]\cup[1,\infty) \\ -\dfrac{(x+1)}{2}, & x\in(-1,0] \\ \dfrac{x-1}{2}, & x\in(0,1) \end{cases}$

for continuity at x = –1

L.H.L. = $\dfrac{\pi}{4}-\dfrac{\pi}{4}=0$

R.H.L. = 0

so, continuous at x = –1

for continuity at x = 1

L.H.L. = 0

R.H.L. = $\dfrac{\pi}{4}+\dfrac{\pi}{4}=\dfrac{\pi}{2}$

so, not continuous at x = 1

For differentiability at x = –1

L.H.D. = $\dfrac{1}{1+1}=\dfrac{1}{2}$

R.H.D. = $-\dfrac{1}{2}$

so, non differentiable at x = –1

5. Official Ans. by NTA (1)

Sol. f(x) is continuous and differentiable

$f(\pi^-) = f(\pi) = f(\pi^+)$

$-1 = -k_2$

$\boxed{k_2=1}$

$f'(x)=\begin{cases} 2k_1(x-\pi)\,; & x\le\pi \\ -k_2\sin x\,; & x>\pi \end{cases}$

$f'(\pi^-) = f'(\pi^+)$

0 = 0

so, differentiable at x = 0

$f''(x)=\begin{cases} 2k_1\,; & x\le\pi \\ -k_2\cos x\,; & x>\pi \end{cases}$

$f''(\pi^-) = f''(\pi^+)$

$2k_1 = k_2$

$\boxed{k_1=\dfrac{1}{2}}$

$(k_1, k_2)=\left(\dfrac{1}{2}, 1\right)$

6. Official Ans. by NTA (5.00)

Sol. $f(x) = x^5.\sin\frac{1}{x} + 5x^2$ if $x < 0$

$f(x) = 0$ if $x = 0$

$f(x) = x^5.\cos\frac{1}{x} + \lambda x^2$ if $x > 0$

LHD of $f'(x)$ at $x = 0$ is 10

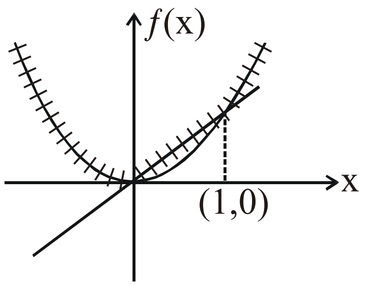
RHD of $f'(x)$ at $x = 0$ is 2λ

if $f''(0)$ exists then

$2\lambda = 10 \Rightarrow \lambda = 5$

7. Official Ans. by NTA (1)

Sol. $f(x) = \max(x, x^2)$



Non-differentiable at x = 0, 1

S = {0, 1}

METHOD OF DIFFERENTIATION

1. NTA Ans. (2)

Sol. Put $x = \sin\theta$, $y = \sin\alpha$

$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$

$\Rightarrow \sin\alpha \cdot \cos\theta + \cos\alpha \cdot \sin\theta = k$

$\Rightarrow \sin(\alpha + \theta) = k$

$\Rightarrow \alpha + \theta = \sin^{-1}k$

$\Rightarrow \sin^{-1}x + \sin^{-1}y = \sin^{-1}k$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}} \times \frac{dy}{dx} = 0$$

at $x = \frac{1}{2}$, $y = \frac{-1}{4}$

$$\frac{dy}{dx} = \frac{-\sqrt{5}}{2}$$

2. NTA Ans. (1)

Sol. $y(\alpha) = \sqrt{2\frac{(\tan\alpha + \cot\alpha)}{1+\tan^2\alpha} + \frac{1}{\sin^2\alpha}}$, $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$

$= \frac{|\sin\alpha + \cos\alpha|}{|\sin\alpha|} = \frac{-(\sin\alpha + \cos\alpha)}{\sin\alpha}$

$= -1 - \cot\alpha$

$y'(\alpha) = \text{cosec}^2\alpha$

$y'\left(\frac{5\pi}{6}\right) = 4$

3. NTA Ans. (3)

Sol. $x^k + y^k = a^k$ $(a, k > 0)$

$kx^{k-1} + ky^{k-1}\frac{dy}{dx} = 0$

$\frac{dy}{dx} + \left(\frac{x}{y}\right)^{k-1} = 0 \Rightarrow k - 1 = -\frac{1}{3} \Rightarrow k = 2/3$

4. NTA Ans. (1)

ALLEN Ans. (BONUS)

Note: The given information is insufficient to find y(x) for x < –1. So, it should be bonus, but NTA retained its answer as options.

Sol. Let $\tan^{-1}x = \theta$, $\theta \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$f(x) = (\sin\theta + \cos\theta)^2 - 1 = \sin 2\theta = \frac{2x}{1+x^2}$

Now, $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$= -\frac{1}{1+x^2}, |x| > 1$

Since, we can integrate only in the continuous interval. So we have to take integral in two cases separtely namely for x < –1 and for x > 1.

$$\Rightarrow y = \begin{cases} -\tan^{-1}x + c_1 & ; \quad x > 1 \\ -\tan^{-1}x + c_2 & ; \quad x < -1 \end{cases}$$

so, $c_1 = \frac{\pi}{2}$ as $y(\sqrt{3}) = \frac{\pi}{6}$

But we cannot find c_2 as we do not have any other additional information for x < –1. So, all of the given options may be correct as c_2 is unknown so, it should be bonus.

5. NTA Ans. (BONUS)

Note: This question has been cancelled by NTA as none option matches.

Sol. $x = 2\sin\theta - \sin 2\theta$

$\Rightarrow \frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta = 4\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)$

$y = 2\cos\theta - \cos 2\theta$

$\Rightarrow \frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta = 4\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$

$$\Rightarrow \frac{dy}{dx} = \cot\left(\frac{3\theta}{2}\right) \Rightarrow \frac{d^2y}{dx^2} = \frac{-\frac{3}{2}\operatorname{cosec}^2\left(\frac{3\theta}{2}\right)}{4\sin\left(\frac{\theta}{2}\right)\sin\frac{3\theta}{2}}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{\theta=\pi} = \frac{3}{8}$$

Alternate :-

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2\sin\theta + 2\sin 2\theta}{2\cos\theta - 2\cos 2\theta} = \frac{\sin\theta - \sin 2\theta}{-\cos\theta + \cos 2\theta}$$

$$\frac{d^2y}{dx^2}.\frac{dx}{d\theta} = \frac{(-\cos\theta + \cos 2\theta)(\cos\theta - 2\cos 2\theta) - (\sin\theta - \sin 2\theta)(\sin\theta - 2\sin 2\theta)}{(-\cos\theta + \cos 2\theta)^2}$$

$$\frac{d^2y}{dx^2}.(-2-2) = \frac{(+1+1)(-1-2)-(0)}{(1+1)^2}$$

$$\frac{d^2y}{dx^2}(-4) = \frac{2\times -3}{4} = -\frac{3}{2}$$

$$\frac{d^2y}{dx^2} = \frac{3}{8}$$

Answer should be $\frac{3}{8}$. No options is correct.

6. NTA Ans. (3)

Sol. $f(g(x)) = x$

$f'(g(x))\, g'(x) = 1$

put $x = a$

$\Rightarrow f'(b)\, g'(a) = 1$

$f'(b) = \frac{1}{5}$

7. Official Ans. by NTA (91)

Sol. Put $\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5} \quad 0 < \alpha < \frac{\pi}{2}$

Now $\frac{3}{5}\cos kx - \frac{4}{5}\sin kx$

$= \cos\alpha . \cos kx - \sin\alpha . \sin kx$

$= \cos(\alpha + kx)$

As we have to find derivate at $x = 0$

We have $\cos^{-1}(\cos(\alpha + kx))$

$= (\alpha + kx)$

$$\Rightarrow y = \sum_{k=1}^{6}(\alpha + kx)$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{at\ x=0} = \sum_{k=x}^{6} k = \frac{6\times 7\times 13}{6} = 91$$

8. Official Ans. by NTA (1)

Sol. $y^2 + \ln(\cos^2 x) = y \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

for $x = 0$ $y = 0$ or 1

Differentiating wrt x

$\Rightarrow 2yy' - 2\tan x = y'$

At $(0, 0)$ $y' = 0$

At $(0, 1)$ $y' = 0$

Differentiating wrt x

$2yy'' + 2(y')^2 - 2\sec^2 x = y''$

At $(0, 0)$ $y'' = -2$

At $(0, 1)$ $y'' = 2$

$\therefore |y''(0)| = 2$

9. Official Ans. by NTA (2)

Sol. $\left(a + \sqrt{2}b\cos x\right)\left(a - \sqrt{2}b\cos y\right) = a^2 - b^2$

$$\Rightarrow a^2 - \sqrt{2}\,ab\cos y + \sqrt{2}\,ab\cos x - 2b^2\cos x\cos y = a^2 - b^2$$

Differentiating both sides :

$$0 - \sqrt{2}\,ab\left(-\sin y\frac{dy}{dx}\right) + \sqrt{2}\,ab(-\sin x)$$

$$-2b^2\left[\cos x\left(-\sin y\frac{dy}{dx}\right) + \cos y(-\sin x)\right] = 0$$

At $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$:

$$ab\frac{dy}{dx} - ab - 2b^2\left(-\frac{1}{2}\frac{dy}{dx} - \frac{1}{2}\right) = 0$$

$$\Rightarrow \frac{dx}{dy} = \frac{ab + b^2}{ab - b^2} = \frac{a+b}{a-b} \;;\; a, b > 0$$

10. Official Ans. by NTA (2)

Sol. Let $f = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$f = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$f = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \frac{\theta}{2}$

$f = \frac{\tan^{-1}x}{2} \Rightarrow \frac{df}{dx} = \frac{1}{2(1+x^2)}$ ....(i)

Let $g = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1} x$

$g = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{1-2\sin^2\theta}\right)$

$g = \tan^{-1}(\tan 2\theta) = 2\theta$

$g = 2\sin^{-1} x$

$\frac{dg}{dx} = \frac{2}{\sqrt{1-x^2}}$ ....(ii)

$\frac{df}{dg} = \frac{1}{2(1+x^2)}\ \frac{\sqrt{1-x^2}}{2}$

at $x = \frac{1}{2}\ \left(\frac{df}{dg}\right)_{x=\frac{1}{2}} = \frac{\sqrt{3}}{10}$

INDEFINITE INTEGRATION

1. NTA Ans. (1)

Sol. $\int \frac{\cos x\,dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}} = \frac{-6}{-6}\int \frac{\cos x\,dx}{\sin^7 x\left(\frac{1}{\sin^6 x}+1\right)^{2/3}}$

$= -\frac{1}{6}\times 3\left(\frac{1}{\sin^6 x}+1\right)^{\frac{1}{3}} + c$

$= -\frac{1}{2}\frac{\left(1+\sin^6 x\right)^{\frac{1}{3}}}{\sin^2 x} + c$

Hence, $\lambda = 3$ and $f(x) = -\frac{1}{2\sin^2 x}$

so, $\lambda f\left(\frac{\pi}{3}\right) = -2$

REMARK : Technically, this question should be marked as bonus. Because $f(x)$ and λ cannot be found uniquely.

For example, another such $f(x)$ and λ can be

$-\frac{\left(1+\sin^6 x\right)^{\frac{1}{6}}}{2\sin^2 x}$ and 6 respectively.

2. NTA Ans. (1)

Sol. $I = \int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)}$

$= \int \frac{\sec^2\theta\,d\theta}{\frac{2\tan\theta}{1-\tan^2\theta}+\frac{1+\tan^2\theta}{1-\tan^2\theta}} = \int \frac{(1-\tan^2\theta)\sec^2\theta\,d\theta}{(1+\tan\theta)^2}$

$\tan\theta = t \Rightarrow \sec^2\theta\,d\theta = dt$

$I = \int \frac{1-t^2}{(1+t)^2}dt = \int \frac{(1-t)(1+t)}{(1+t)^2}dt$

$= \int \frac{1}{1+t} - \frac{t}{1+t}\,dt$

$= \ell n|1+t| - \int\left(\frac{1+t}{1+t} - \frac{1}{1+t}\right)dt$

$= \ell n|1+t| - t + \ell n|1+t| = 2\ell n|1+t| - t + C$

$= 2\ell n|1+\tan\theta| - \tan\theta + C$

$\lambda = -1,\ f(\theta) = 1 + \tan\theta$

3. NTA Ans. (1)

Sol. $I = \int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}} = \int \frac{dx}{\left(\frac{x+4}{x-3}\right)^{\frac{8}{7}}(x-3)^2}$

Let $\frac{x+4}{x-3} = t \Rightarrow \frac{dx}{(x-3)^2} = -\frac{1}{7}dt$

$\Rightarrow I = -\frac{1}{7}\int \frac{dt}{t^{8/7}} = -\frac{1}{7}\int t^{-8/7}dt$

$= t^{-1/7} + C = +\left(\frac{x+4}{x-3}\right)^{-1/7} + C = \left(\frac{x-3}{x+4}\right)^{1/7} + C$

4. Official Ans. by NTA (3)

Sol. Put $x = \tan^2\theta \Rightarrow dx = 2\tan\theta\sec^2\theta\, d\theta$

$$\int \theta.(2\tan\theta\cdot\sec^2\theta)d\theta$$

↓ ↓

I II (By parts)

$$= \theta.\tan^2\theta - \int \tan^2\theta\, d\theta$$

$$= \theta.\tan^2\theta - \int (\sec^2\theta - 1)\, d\theta$$

$$= \theta(1 + \tan^2\theta) - \tan\theta + C$$

$$= \tan^{-1}\left(\sqrt{x}\right)(1+x) - \sqrt{x} + C$$

5. Official Ans. by NTA (4)

Sol. $$\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx = \int\left(\frac{x}{\cos x}\right).\frac{x\cos x\,dx}{(x\sin x+\cos x)^2}$$

$$= \frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)$$

$$+\int\left(\frac{\cos x + x\sin x}{\cos^2 x}\right)\left(\frac{1}{x\sin x+\cos x}\right)dx =$$

$$-\frac{x\sec x}{x\sin x+\cos x} + \int \sec^2 x\, dx$$

$$= -\frac{x\sec x}{x\sin x+\cos x} + \tan x + C$$

6. Official Ans. by NTA (1)

Sol. $e^{2x} + 2e^x - e^{-x} - 1$

$= e^x(e^x + 1) - e^{-x}(e^x + 1) + e^x = [(e^x + 1)(e^x - e^{-x}) + e^x]$

so $I = \int (e^x+1)(e^x - e^{-x})e^{e^x+e^{-x}} + \int e^x\cdot e^{e^x+e^{-x}}dx =$

$(e^x+1)e^{e^x+e^{-x}} - \int e^x.e^{e^x+e^{-x}}dx + \int e^x.e^{e^x+e^{-x}}dx$

$= (e^x+1)e^{e^x+e^{-x}} + C$ $\therefore$ $g(x) = e^x + 1 \Rightarrow g(0) = 2$

7. Official Ans. by NTA (4)

Sol. $$\int\frac{\cos\theta\, d\theta}{5+7\sin\theta-2\cos^2\theta}$$

$$\int\frac{\cos\theta\, d\theta}{3+7\sin\theta+2\sin^2\theta}$$ $\boxed{\begin{array}{l}\sin\theta = t\\ \cos\theta d\theta = dt\end{array}}$

$$\int\frac{dt}{2t^2+7t+3} = \int\frac{dt}{(2t+1)(t+3)} =$$

$$\frac{1}{5}\int\left(\frac{2}{2t+1}-\frac{1}{t+3}\right)dt$$

$$= \frac{1}{5}\ln\left|\frac{2t+1}{t+3}\right| + C = \frac{1}{5}\ln\left|\frac{2\sin\theta+1}{\sin\theta+3}\right| + C$$

$$A = \frac{1}{5} \text{ and } B(\theta) = \frac{2\sin\theta+1}{\sin\theta+3}$$

DEFINITE INTEGRATION

1. NTA Ans. (4)

Sol. $2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0$

$3\sin^2\theta - 5\sin\theta + 2 = 0$

$\sin\theta = \frac{1}{2}$, 2 (Rejected)

$$\int_{\theta_1}^{\theta_2}\cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6}\frac{1+\cos 6\theta}{2}d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6}-\frac{\pi}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$$

2. NTA Ans. (3)

Sol. $$4\alpha\left[\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right] = 5$$

$$\Rightarrow 4\alpha\left(\left[\frac{e^{\alpha x}}{\alpha}\right]_{-1}^{0} + \left[\frac{e^{-\alpha x}}{-\alpha}\right]_{0}^{2}\right) = 5$$

$\Rightarrow 4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

Let $e^{-\alpha} = t$, $4t^2 + 4t - 3 = 0$, $t = \frac{1}{2}, \frac{-3}{2}$ (Rejected)

$e^{-\alpha} = \frac{1}{2}$; $\alpha = \ell n2$

3. NTA Ans. (1)

ALLEN Ans. (1 OR 3)

Note: In this Question, both options (1) as well as (3) are correct, but NTA accepte only option (1).

Sol. $f(x+1) = f(a+b-x)$

$$I = \frac{1}{(a+b)}\int_a^b x\big(f(x)+f(x+1)\big)dx \quad .....(1)$$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)\big(f(x+1)+f(x)\big)\,dx \quad ...(2)$$

from (1) and (2)

$$2I = \int_a^b \big(f(x)+f(x+1)\big)\,dx$$

$$2I = \int_a^b f\big(a+b-x\big)dx + \int_a^b f(x+1)dx$$

$$2I = 2\int_a^b f(x+1)dx \Rightarrow I = \int_a^b f(x+1)dx$$

$$= \int_{a+1}^{b+1} f(x)dx$$

OR

$$I = \frac{1}{(a+b)}\int_a^b x\big(f(x)+f(x+1)\big)dx \quad ...(1)$$

$$= \frac{1}{(a+b)}\int_a^b (a+b-x)\big(f(a+b-x)+f(a+b+1-x)\big)dx$$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)\big(f(x+1)+f(x)\big)dx \quad ...(2)$$

equation (1) + (2)

$$2I = \frac{1}{(a+b)}\int_a^b (a+b)\big(f(x+1)+f(x)\big)dx$$

$$I = \frac{1}{2}\left[\int_a^b f(x+1)dx + \int_a^b f(x)dx\right]$$

$$= \frac{1}{2}\left[\int_a^b f(a+b+1-x)dx + \int_a^b f(x)dx\right]$$

$$= \frac{1}{2}\left[\int_a^b f(x)dx + \int_a^b f(x)dx\right]$$

$$I = \int_a^b f(x)dx$$

Let $x = T + 1$

$$= \int_{a-1}^{b-1} f(T+1)dT$$

$$I = \int_{a-1}^{b-1} f(x+1)dx$$

4. NTA Ans. (1)

Sol. $$f(x) = \frac{1}{\sqrt{2x^3-9x^2+12x+4}}$$

$$f'(x) = \frac{-6(x-1)(x-2)}{2\left(2x^3-9x^2+12x+4\right)^{3/2}}$$

$\therefore f(x)$ is decreasing in (1,2)

$$f(1) = \frac{1}{3};\ f(2) = \frac{1}{\sqrt{8}}$$

$$\frac{1}{3} < I < \frac{1}{\sqrt{8}} \Rightarrow I^2 \in \left(\frac{1}{9},\frac{1}{8}\right)$$

(1) Option

5. NTA Ans. (1)

Sol. Using L.H. Rule

$$\lim_{x\to 0}\frac{x\sin(10x)}{1} = 0$$

(1) Option

6. NTA Ans. (4)

Sol. $$I = \int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx \quad ....(1)$$

$$= \left[\int_0^{\pi}\frac{x\sin^8 x}{\sin^8 x\cos^8 x}dx + \int_0^{\pi}\frac{(2\pi-x)\sin^8 x}{\sin^8 x+\cos^8 x}dx\right]$$

$$= 2\pi\int_0^{\pi}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$$

$$I = 2\pi\left[\int_0^{\pi/2}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx + \int_0^{\pi/2}\frac{\cos^8 x dx}{\sin^8 x+\cos^8 x}dx\right]$$

$$= 2\pi\int_0^{\pi/2} 1dx = 2\pi.\frac{\pi}{2} = \pi^2$$

7. NTA Ans. (3)

Sol. $f(x) = a + bx + cx^2$

$$\int_0^1 f(x)dx = \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1$$

$$= a + \frac{b}{2} + \frac{c}{3} = \frac{1}{6}[6a + 3b + c]$$

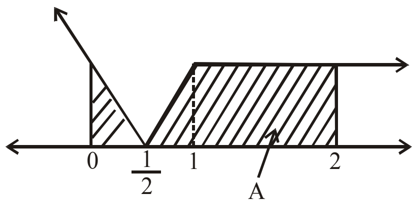
$$= \frac{1}{6}\left[f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right]$$

8. Official Ans. by NTA (1.50)

Sol. $\int_0^2 \left| |x-1| - x \right| dx$

Let f(x) $||x - 1| - x|$

$$= \begin{cases} 1, & x \geq 1 \\ |1-2x|, & x \leq 1 \end{cases}$$



$$A = \frac{1}{2} + 1 = \frac{3}{2}$$

OR

$$\int_0^{1/2} (1-2x)dx + \int_{1/2}^{1} (2x-1) + \int_0^2 1dx$$

$$= \left[x - x^2\right]_0^{\frac{1}{2}} + \left[x^2 - x\right]_{1/2}^{1} + \left[x\right]_1^2$$

$$= \boxed{3/2}$$

9. Official Ans. by NTA (1.0)

Sol. $3 < 3x < 6$

Take cases when $3 < 3x < 4$, $4 < 3x < 5$, $5 < 3x < 6$;

Now $\int_1^2 |2x - [3x]| dx$

$$= \int_1^{4/3} (3-2x)dx + \int_{4/3}^{5/3} (4-2x)\,dx + \int_{5/3}^{2} (5-2x)dx$$

$$= \frac{2}{9} + \frac{3}{9} + \frac{4}{9} = 1$$

10. Official Ans. by NTA (1)

Sol. $\int_{-\pi}^{\pi} \left|\pi - |x|\right| dx = 2\int_0^{\pi} |\pi - x| dx$

$$= 2\int_0^{\pi} (\pi - x)\,dx$$

$$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \pi^2$$

11. Official Ans. by NTA (1)

Sol. $\int_0^{1/2} \frac{((x^2-1)+1)}{(1-x^2)^{3/2}} dx$

$$\int_0^{1/2} \frac{dx}{(1-x^2)^{3/2}} - \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

$$\int_0^{1/2} \frac{x^{-3}}{(x^{-2}-1)^{3/2}} dx - (\sin^{-1} x)_0^{1/2}$$

Let $x^{-2} - 1 = t^2 \Rightarrow x^{-3}\,dx = -\,t\,dt$

$$\int_{\infty}^{\sqrt{3}} \frac{-t\,dt}{t^3} - \frac{\pi}{6} = \int_{\sqrt{3}}^{\infty} \frac{dt}{t^2} - \frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{\pi}{6} = \frac{k}{6}$$

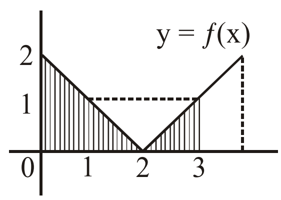
$$k = 2\sqrt{3} - \pi$$

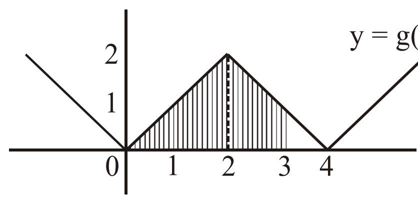
12. Official Ans. by NTA (4)

Sol. $\int_0^3 g(x) - f(x) = \int_0^3 \left| |x-2| - 2 \right| dx - \int_0^3 |x-2|\,dx$

$$= \left(\frac{1}{2} \times 2 \times 2 + 1 + \frac{1}{2} \times 1 \times 1\right) - \left(\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times 1\right)$$

$$= \left(2 + 1 + \frac{1}{2}\right) - \left(2 + \frac{1}{2}\right) = 1$$





13. Official Ans. by NTA (4)

Sol. $f(x) = \int_1^3 \frac{\sqrt{x}\,dx}{(1+x)^2} = \int_1^{\sqrt{3}} \frac{t.2t\,dt}{(1+t^2)^2}$ $\left(\text{put } \sqrt{x} = t\right)$

$= \left(-\frac{t}{1+t^2}\right)_1^{\sqrt{3}} + (\tan^{-1} t)_1^{\sqrt{3}}$ [Appling by parts]

$= -\left(\frac{\sqrt{3}}{4} - \frac{1}{2}\right) + \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{1}{2} - \frac{\sqrt{3}}{4} + \frac{\pi}{12}$

14. Official Ans. by NTA (3)

Sol. $I = \int_{\pi/6}^{\pi/3} ((2\tan^3 x \cdot \sec^2 x \cdot \sin^4 3x) + (3\tan^4 x \cdot \sin^3 3x \cdot \cos 3x))dx$

$\Rightarrow I = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} d((\sin 3x)^4 (\tan x)^4)$

$\Rightarrow I = ((\sin 3x)^4 (\tan x)^4)_{\pi/6}^{\pi/3}$

$\Rightarrow I = -\frac{1}{18}$

15. Official Ans. by NTA (21)

Sol. $\int_0^n \{x\}dx = n\int_0^1 \{x\}dx = n\int_0^1 x\,dx = \frac{n}{2}$

$\int_0^n [x]dx = \int_0^n (x - \{x\})dx = \frac{n^2}{2} - \frac{n}{2}$

$\Rightarrow \left(\frac{n^2 - n}{2}\right)^2 = \frac{n}{2} \cdot 10 \cdot n(n-1)$ (where n > 1)

$\Rightarrow \frac{n-1}{4} = 5 \Rightarrow n = 21$

16. Official Ans. by NTA (4)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx$ ....(1)

Apply King property

$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{-\sin x}}dx = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}}dx$ ...(2)

Add (1) & (2)

$2I = \int_{-\pi/2}^{\pi/2} dx = \pi$

$I = \frac{\pi}{2}$

17. Official Ans. by NTA (1)

Sol. $I_1 = \int_0^1 \left(1 - x^{50}\right)^{100} dx$ and $I_2 = \int_0^1 \left(1 - x^{50}\right)^{101} dx$

and $I_1 = \lambda I_2$

$I_2 = \int_0^1 \left(1 - x^{50}\right)^{101} dx$

$I_2 = \int_0^1 \left(1 - x^{50}\right)\left(1 - x^{50}\right)^{100} dx$

$I_2 = \int_0^1 \left(1 - x^{50}\right)dx - \int_0^1 x^{50}.\left(1 - x^{50}\right)^{100} dx$

$I_2 = I_1 - \int_0^1 \underset{I}{\underbrace{x}}.\underset{II}{\underbrace{x^{49}.\left(1 - x^{50}\right)^{100}}} dx$

Now apply IBP

$I_2 = I_1 - \left[x\int x^{49}.\left(1 - x^{50}\right)^{100} dx - \int \frac{d(x)}{dx}.\int \frac{d(x)}{dx}.\int x^{49}.\left(1 - x^{50}\right)^{100} dx\right]$

Let $(1 - x^{50}) = t$

$-50x^{49}dx = dt$

$I_2 = I_1 - \left[x.\left(-\frac{1}{50}\right)\frac{\left(1 - x^{50}\right)^{101}}{101}\Bigg|_{x=0}^{x=1} - \int_0^1 \left(-\frac{1}{50}\right)\frac{\left(1 - x^{50}\right)^{101}}{101}dx\right]$

$I_2 = I_1 - 0 - \frac{1}{50}.\frac{1}{101}.I_2 = I_1 - \frac{1}{5050}I_2$

$I_2 + \frac{1}{5050}I_2 = I_1 \Rightarrow \frac{5051}{5050}I_2 = I_1$

$\therefore \alpha = \frac{5050}{5051}$

$I_2 = \frac{5050}{5051}I_1$

$\because I_2 = \alpha.I_1$

18. Official Ans. by NTA (4)

Sol. $\int_1^2 e^x.x^x\left(2+\log_e x\right)dx$

$\int_1^2 e^x\left(2x^x+x^x\log_e x\right)dx$

$\int_1^2 e^x\left(\underbrace{x^x}_{f(x)}+\underbrace{x^x\left(1+\log_e x\right)}_{f'(x)}\right)dx$

$\left(e^x.x^x\right)_1^2=4e^2-e$

TANGENT & NORMAL

1. NTA Ans. (2)

Sol. $x^2 + 2xy - 3y^2 = 0$

m_N = slope of normal drawn to curve at (2,2) is –1

L : x + y = 4.

perpendicular distance of L from (0,0)

$=\frac{|0+0-4|}{\sqrt{2}}=2\sqrt{2}$

(2) Option

2. NTA Ans. (4.00)

Sol. Let $P(\alpha,\beta)$

so, $\beta^2 - 3\alpha^2 + \beta + 10 = 0$ ...(i)

Now, $2yy' - 6x + y' = 0$

$\Rightarrow m=\frac{6\alpha}{2\beta+1}$ ....(ii)

Also, $\frac{\beta-\frac{3}{2}}{\alpha}=-\frac{1}{m}$

$\Rightarrow \frac{2\beta-3}{2\alpha}=-\frac{(2\beta+1)}{6\alpha}$ (from (ii))

$\Rightarrow \beta = 1 \Rightarrow \alpha^2 = 4$ (from (1))

Hence, $|m|=\frac{12}{3}=4.00$

3. Official Ans. by NTA (3)

Sol. Slope of tangent to the curve $y=x+\sin y$

at (a, b) is $\frac{2-\frac{3}{2}}{\frac{1}{2}-0}=1$

$\Rightarrow \left.\frac{dy}{dx}\right]_{x=a}=1$

$\frac{dy}{dx}=1+\cos y.\frac{dy}{dx}$ (from equation of curve)

$\left.\frac{dy}{dx}\right]_{x=a}=1+\cos b.\left.\frac{dy}{dx}\right]_{x=a}$

$\Rightarrow \cos b = 0$

$\Rightarrow \sin b = \pm 1$

Now, from curve y = x + sin y

b = a + sin b

$\Rightarrow |b - a| = |\sin b| = 1$

4. Official Ans. by NTA (2)

Sol. Given equation of curve

$y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$

at $\boxed{x = 0}$

$y = (1 + 0)^{2y} + \cos^2(\sin^{-1}0)$

$y = 1 + 1$

$\boxed{y = 2}$

So we have to find the normal at (0, 2)

Now $y = e^{2y\ln(1+x)} + \cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$y = e^{2y\ln(1+x)} + \left(\sqrt{1-x^2}\right)^2$

$y = e^{2y\ln(1+x)} + (1-x^2)$ …(1)

Now differentiate w.r.t. x

$$y' = e^{2y\ln(1+x)}\left[2y\cdot\left(\frac{1}{1+x}\right) + \ln(1+x).2y'\right] - 2x$$

Put x = 0 & y = 2

$$y' = e^{2\times 2\ln 1}\left[2\times 2\left(\frac{1}{1+0}\right) + \ln(1+0).2y'\right] - 2\times 0$$

$y' = e^0[4 + 0] - 0$

$y' = 4$ = slope of tangent to the curve

so slope of normal to the curve $= -\frac{1}{4}$ $\{m_1m_2 = -1\}$

Hence equation of normal at (0, 2) is

$y - 2 = -\frac{1}{4}(x - 0)$

$\Rightarrow 4y - 8 = -x$

$\Rightarrow \boxed{x + 4y = 8}$

5. Official Ans. by NTA (1)

Sol. $\frac{d}{dt}(6a^2) = 3.6 \Rightarrow 12a\frac{da}{dt} = 3.6$

$a\frac{da}{dt} = 0.3$

$\frac{dv}{dt} = \frac{d}{dt}(a^3) = 3a\left(a\frac{da}{dt}\right)$

$= 3 \times 10 \times 0.3 = 9$

6. Official Ans. by NTA (4)

Sol. $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

$m = \left(\frac{dy}{dx}\right)_{(c, e^c)} = e^c$

$\Rightarrow$ Tangent at (c, e^c)

$y - e^c = e^c(x - c)$

it intersect x-axis

Put $y = 0 \Rightarrow x = c - 1$

......(1)

Now $y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1, 2)} = 1$

$\Rightarrow$ Slope of normal = –1

Equation of normal $y - 2 = -1(x - 1)$

$x + y = 3$ it intersect x-axis

Put $y = 0 \Rightarrow x = 3$

......(2)

Points are same

$\Rightarrow x = c - 1 = 3$

$\Rightarrow c = 4$

7. Official Ans. by NTA (0.50)

Sol. $y = x^2 - 3x + 2$

At x-axis $y = 0 = x^2 - 3x + 2$

$x = 1, 2$

$\frac{dy}{dx} = 2x - 3$

A(1, 0) B(2, 0)

$\left(\frac{dy}{dx}\right)_{x=1} = -1$ and $\left(\frac{dy}{dx}\right)_{x=2} = 1$

$x + y = a \Rightarrow \frac{dy}{dx} = -1$ So A(1, 0) lies on it

$\Rightarrow 1 + 0 = a \Rightarrow \boxed{a = 1}$

$x - y = b \Rightarrow \frac{dy}{dx} = 1$ So B(2, 0) lies on it

$2 - 0 = b \Rightarrow \boxed{b = 2}$

$\frac{a}{b} = 0.50$

8. Official Ans. by NTA (4)

Sol. $\dfrac{f(t_2)-f(t_1)}{t_2-t_1}=2at+b$

$\dfrac{a(t_2^2-t_1^2)+b(t_2-t_1)}{t_2-t_1}=2at+b$

$\Rightarrow a(t_2+t_1)+b=2at+b$

$\Rightarrow t=\dfrac{t_1+t_2}{2}$

MONOTONICITY

1. NTA Ans. (4)

Sol. f(0) = 11

f(1) = 16

$\dfrac{f(1)-f(0)}{1-0}=3c^2-8c+8$

$\Rightarrow 3c^2-8c+8=5$

$\Rightarrow 3c^2-8c+3=0$

$c \in [0, 1] \Rightarrow c=\dfrac{4-\sqrt{7}}{3}$

2. NTA Ans. (2)

Sol. Using LMVT in [–7, –1]

$\dfrac{f(-1)-f(-7)}{-1-(-7)}\le 2$

$f(-1)-f(-7)\le 12$

$\Rightarrow f(-1)\le 9$ ....(1)

Using LMVT in [–7, 0]

$\dfrac{f(0)-f(-7)}{0-(-7)}\le 2$

$f(0)-f(-7)\le 14$

$f(0)\le 11$ ....(2)

from (1) and (2)

$f(0)+f(-1)\le 20$

3. NTA Ans. (2)

ALLEN Ans. (BONUS)

Note: None of the options is correct for all *f* in S. Thus, it should be bonus, but NTA did not accept it.

Sol. Option (1), (2), (3) are incorrect for $f(x)$ = constant and option (4) is incorrect

$\dfrac{f(1)-f(c)}{1-c}=f'(a)$ where $c<a<1$ (use LMVT)

Also for $f(x)=x^2$ option (4) is incorrect.

4. NTA Ans. (2)

Sol. $\dfrac{9+\alpha}{21}=\dfrac{16+\alpha}{28}\Rightarrow \alpha=12$

Also, $f'(x)=\dfrac{7x}{x^2+12}\times\dfrac{x^2-12}{7x^2}=\dfrac{x^2-12}{x(x^2+12)}$

Hence, $c=2\sqrt{3}$

Now, $f''(c)=\dfrac{1}{12}$

5. NTA Ans. (1)

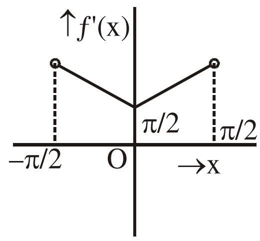
Sol. $f(x)$ is an odd function.

Now, if $x\ge 0$, then $f(x)=x\cos^{-1}(-\sin x)$

$=x\left(\dfrac{\pi}{2}-\sin^{-1}(-\sin x)\right)=x\left(\dfrac{\pi}{2}+x\right)$

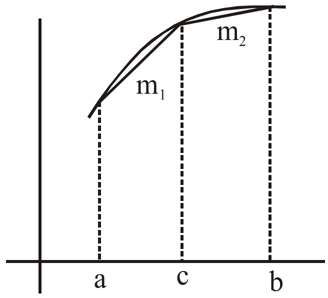
Hence, $f(x)=\begin{cases} x\left(\dfrac{\pi}{2}+x\right) & ;\ x\in\left[0,\dfrac{\pi}{2}\right] \\ x\left(\dfrac{\pi}{2}-x\right) & ;\ x\in\left[-\dfrac{\pi}{2},0\right) \end{cases}$

so, $f'(x)=\begin{cases} \dfrac{\pi}{2}+2x & ;\ x\in\left[0,\dfrac{\pi}{2}\right) \\ \dfrac{\pi}{2}-2x & ;\ x\in\left(-\dfrac{\pi}{2},0\right) \end{cases}$



6. NTA Ans. (3)

Sol.



it is clear from graph that $m_1 > m_2$

$\Rightarrow \dfrac{f(c)-f(a)}{c-a}>\dfrac{f(b)-f(c)}{b-c}$

$\Rightarrow \dfrac{f(c)-f(a)}{f(b)-f(c)}>\dfrac{c-a}{b-c}$

7. Official Ans. by NTA (1)

Sol. $$f'(x)=\frac{\frac{x}{1+x}-\ell n(1+x)}{x^2}$$

$$=\frac{x-(1+x)\,\ell n(1+x)}{x^2(1+x)}$$

Suppose $h(x) = x - (1 + x)\ \ell n(1+x)$

$\Rightarrow\ h'(x) = 1 - \ell n(1+x) - 1 = -\ell n(1+x)$

$h'(x) > 0,\ \forall\ x \in (-1, 0)$

$h'(x) < 0,\ \forall\ x \in (0, \infty)$

$h(0) = 0 \Rightarrow h'(x) < 0\ \forall\ x \in (-1, \infty)$

$\Rightarrow f'(x) < 0\ \forall\ x\in(-1, \infty)$

$\Rightarrow f(x)$ is a decreasing function for all $x\in(-1, \infty)$

8. Official Ans. by NTA (2)

Sol. $f(x) = (3x - 7)x^{2/3}$

$\Rightarrow f(x) = 3x^{5/3} - 7x^{2/3}$

$$\Rightarrow f'(x) = 5x^{2/3} - \frac{14}{3x^{1/3}}=\frac{15x-14}{3x^{1/3}}>0$$

+ – +
0 14/15

$$\therefore\ f'(x) > 0\ \forall\ x\in(-\infty,0)\cup\left(\frac{14}{15},\infty\right)$$

9. Official Ans. by NTA (3)

Sol. $f(2) = 8, f'(2) = 5, f'(x)\geq 1, f''(x) \geq 4, \forall x\in(1,6)$

$$f''(x) = \frac{f'(5)-f'(2)}{5-2}\geq 4\ \Rightarrow f'(5) \geq 17 \quad ...(1)$$

$$f'(x) = \frac{f(5)-f(2)}{5-2}\geq 1\ \Rightarrow f(5) \geq 11 \quad ...(2)$$

$$\overline{f'(5)+f(5)\geq 28}$$

10. Official Ans. by NTA (1)

Sol. $f(0) = f(1) = f'(0) = 0$

Apply Rolles theorem on $y = f(x)$ in $x \in [0, 1]$

$f(0) = f(1) = 0$

$\Rightarrow f'(\alpha) = 0$ where $\alpha \in (0, 1)$

Now apply Rolles theorem on $y = f'(x)$ in $x \in [0, \alpha]$

$f'(0) = f'(\alpha) = 0$ and $f'(x)$ is continuous and differentiable

$\Rightarrow f''(\beta) = 0$ for some , $\beta \in (0, \alpha) \in (0, 1)$

$\Rightarrow f''(x) = 0$ for some $x \in (0, 1)$

11. Official Ans. by NTA (3)

Sol. $f(x) = x\log_e x$

$$f'(x)\Big|_{(c,f(c))}=\frac{e-0}{e-1}$$

$f'(x) = 1 + \log_e x$

$$f'(x)\Big|_{(c,f(c))}=1+\log_e c=\frac{e}{e-1}$$

$$\log_e c=\frac{e-(e-1)}{e-1}=\frac{1}{e-1}\ \Rightarrow\ c=e^{\frac{1}{e-1}}$$

MAXIMA & MINIMA

1. NTA Ans. (2)

Sol. $$\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$$

$\Rightarrow f(x) = 2x^3 + ax^4 + bx^5$

$f'(x) = 6x^2 + 4ax^3 + 5bx^4$

$f'(1) = 0, f'(-1) = 0$

$$a = 0,\ b = \frac{-6}{5}\ \Rightarrow f(x) = 2x^3-\frac{6}{5}x^5$$

$f'(x) = 6x^2 - 6x^4$

$= 6x^2(1 - x)(1 + x)$

Sign scheme for $f'(x)$

–ve +ve +ve –ve
–1 0 1

Minima at $x = -1$

Maxima at $x = 1$

2. NTA Ans. (3)

Sol. $f''(x) = \lambda(x - 1)$

$$f'(x)=\frac{\lambda x^2}{2}-\lambda x+C\Rightarrow f'(-1)=0\Rightarrow c=\frac{-3\lambda}{2}$$

$$f(x)=\frac{\lambda x^3}{6}-\frac{\lambda x^2}{2}-\frac{3\lambda}{2}x+d$$

$f(1) = -6 \Rightarrow -11\lambda + 6d = -36 \quad ...(i)$

$f(-1) = 10 \Rightarrow 5\lambda + 6d = 60 \quad ...(ii)$

from (i) & (ii) $\lambda = 6$ & $d = 5$

$f(x) = x^3 - 3x^2 - 9x + 5$

Which has minima at $x = 3$

Ans. 3.00

3. NTA Ans. (1)

Sol. $F'(x) = x^2 g(x) = x^2\int_1^x f(u)\,du \Rightarrow F'(1) = 0$

$F''(x) = x^2 f(x) - 2x\int_1^x f(u)\,du$

$F''(1) = 1.f(1) - 2 \times 0$

$F''(1) = 3$

$F'(1) = 0$ and $F''(1) = 3 > 0$ So, Minima

4. NTA Ans. (3)

Sol. Let thickness of ice be 'h'.

Vol. of ice = $v = \frac{4\pi}{3}\left((10+h)^3 - 10^3\right)$

$\frac{dv}{dt} = \frac{4\pi}{3}\left(3(10+h)^2\right).\frac{dh}{dt}$

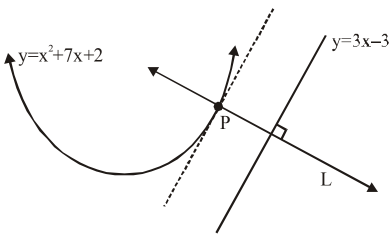
Given $\frac{dv}{dt} = 50cm^3/\min$ and h = 5cm

$\Rightarrow 50 = \frac{4\pi}{3}\left(3(10+5)^2\right)\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{50}{4\pi\times 15^2} = \frac{1}{18\pi}\ cm/\min$

5. Official Ans. by NTA (4)

Sol.



Let L be the common normal to parabola $y = x^2 + 7x + 2$ and line $y = 3x - 3$

$\Rightarrow$ slope of tangent of $y = x^2 + 7x + 2$ at P = 3

$\Rightarrow \left.\frac{dy}{dx}\right]_{For\ P} = 3$

$\Rightarrow 2x + 7 = 3 \Rightarrow x = -2 \Rightarrow y = -8$

So P(–2, –8)

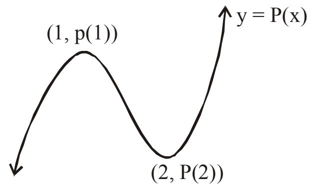
Normal at P : x + 3y + C = 0

$\Rightarrow$ C = 26 (P satisfies the line)

$\boxed{\text{Normal}: x + 3y + 26 = 0}$

6. Official Ans. by NTA (4)

Sol. Since p(x) has realtive extreme at



x = 1 & 2

so p'(x) = 0 at x = 1 & 2

$\Rightarrow p'(x) = A(x-1)(x-2)$

$\Rightarrow p(x) = \int A(x^2 - 3x + 2)dx$

$p(x) = A\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right) + C$ ...(1)

P(1) = 8

From (1)

$8 = A\left(\frac{1}{3} - \frac{3}{2} + 2\right) + C$

$\Rightarrow 8 = \frac{5A}{6} + C \Rightarrow \boxed{48 = 5A + 6C}$ ...(3)

P(2) = 4

$\Rightarrow 4 = A\left(\frac{8}{3} - 6 + 4\right) + C$

$\Rightarrow 4 = \frac{2A}{3} + C \Rightarrow \boxed{12 = 2A + 3C}$ ...(4)

From 3 & 4, C = –12

So P(0) = C = $\boxed{-12}$

7. Official Ans. by NTA (3)

Sol. $f'(x) = x(x+1)(x-1) = x^3 - x$

$\int df(x) = \int x^3 - x\,dx$

$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + C$

$f(x) = f(0)$

$\frac{x^4}{4} - \frac{x^2}{2} = 0$

$x^2(x^2 - 2) = 0$

$x = 0, 0, \sqrt{2}, -\sqrt{2}$

$x_1^2 + x_2^2 + x_3^2 = 0 + 2 + 2 = 4$

8. Official Ans. by NTA (1)

Sol. $f(x) = (3x^2 + ax - 2 - a)e^x$

$f'(x) = (3x^2 + ax - 2 - a)e^x + e^x (6x + a)$

$= e^x(3x^2 + x(6 + a) - 2)$

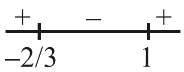
$f'(x) = 0$ at $x = 1$

$\Rightarrow 3 + (6 + a) - 2 = 0$

$a = -7$

$f'(x) = e^x(3x^2 - x - 2)$

$= e^x (x - 1) (3x + 2)$

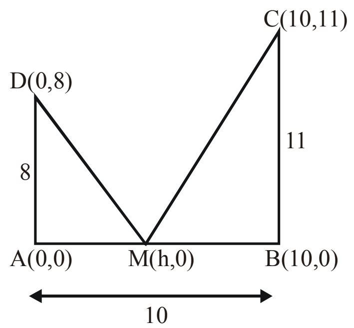


x = 1 is point of local minima

$x = \frac{-2}{3}$ is point of local maxima

9. Official Ans. by NTA (5.00)

Sol.



$(MD)^2 + (MC)^2 = h^2 + 64 + (h - 10)^2 + 121$

$= 2h^2 - 20h + 64 + 100 + 121$

$= 2(h^2 - 10h) + 285$

$= 2(h - 5)^2 + 235$

it is minimum if h = 5

10. Official Ans. by NTA (4)

Sol. $f(x) = (1 - \cos^2 x)(\lambda + \sin x) \quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$f(x) = \lambda \sin^2 x + \sin^3 x$

$f'(x) = 2\lambda \sin x \cos x + 3\sin^2 x \cos x$

$f'(x) = \sin x \cos x(2\lambda + 3\sin x)$

$\sin x = 0, \ \frac{-2\lambda}{3}, \ (\lambda \neq 0)$

for exactly one maxima & minima

$\frac{-2\lambda}{3} \in (-1, 1) \Rightarrow \lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right)$

$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

DIFFERENTIAL EQUATION

1. NTA Ans. (3)

Sol. $(y^2 - x)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dx}{dy} + x = y^2$

I.F. $= e^{\int dy} = e^y$

Solution is given by

$x e^y = \int y^2 e^y dy + C$

$\Rightarrow x e^y = (y^2 - 2y + 2)e^y + C$

$x = 0, y = 1,$ gives $C = -e$

If $y = 0$, then $x = 2 - e$

2. NTA Ans. (4)

Sol. $e^y \frac{dy}{dx} - e^y = e^x$, Let $e^y = t$

$\Rightarrow e^y \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - t = e^x$

I.F. $= e^{\int -dx} = e^{-x}$

$t e^{-x} = x + c \Rightarrow e^{y-x} = x + c$

$y(0) = 0 \Rightarrow c = 1$

$e^{y-x} = x + 1 \Rightarrow y(1) = 1 + \log_e 2$

3. NTA Ans. (1)

Sol. $2x = 4by' \Rightarrow y' = \frac{2x}{4b}$

Required D.E. is $x^2 = \frac{2x}{y'}y + \left(\frac{x}{y'}\right)^2$

$x(y')^2 = 2yy' + x$

(1) Option

4. NTA Ans. (2)

ALLEN Ans. (BONUS)

Note: As per the given informaton, x cannot be negative. So, it is invalid to ask y(x) for x < 0. Hence, it should be bonus but, NTA retained its answer as option (2).

Sol. $\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ so, $\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$

Integrating, $\sin^{-1}x + \sin^{-1}y = c$

so, $\frac{\pi}{6} + \frac{\pi}{3} = c$

Hence, $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

Put $x = -\frac{1}{\sqrt{2}}, \sin^{-1}y = \frac{3\pi}{4}$ (Not possible)

5. NTA Ans. (4)

Sol. $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

Let y = vx

$\frac{dy}{dx} = v + x.\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{xvx}{x^2+v^2x^2} = \frac{v}{1+v^2}$

$x\frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{v-v-v^3}{1+v^2} = -\frac{v^3}{1+v^2}$

$\int \frac{1+v^2}{v^3}.dv = \int -\frac{dx}{x}$

$\Rightarrow \int v^{-3}.dv + \int \frac{1}{v}dv = -\int \frac{dx}{x}$

$\Rightarrow \frac{v^{-2}}{-2} + \ell n v = -\ell n x + \lambda$

$\Rightarrow -\frac{1}{2v^2} + \ell n\left(\frac{y}{x}\right) = -\ell n x + \lambda$

$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + \ell n y - \ell n x = -\ell n x + \lambda$

$\Rightarrow -\frac{1}{2} + 0 = \lambda \Rightarrow \lambda = -\frac{1}{2}$

$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2} + \ell n y + \frac{1}{2} = 0$ at y = e

$\Rightarrow -\frac{1}{2}\frac{x^2}{e^2} + 1 + \frac{1}{2} = 0 \Rightarrow \frac{x^2}{2e^2} = \frac{3}{2} \Rightarrow x^2 = 3e^2$

$\therefore x = \sqrt{3}e$

6. NTA Ans. (3)

Sol. $f'(x) = \tan^{-1}(\sec x + \tan x)$

$f'(x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)\right)$

$\because -\frac{\pi}{2} < x < \frac{\pi}{2} \Rightarrow 0 < \frac{\pi}{4} + \frac{x}{2} < \frac{\pi}{2}$

$\Rightarrow f'(x) = \frac{\pi}{4} + \frac{x}{2}$

$\therefore f(x) = \frac{\pi}{4}.x + \frac{x^2}{4} + c$

$\because f(0) = 0 \Rightarrow c = 0$

$\Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$

$\therefore f(1) = \frac{\pi+1}{4}$

7. NTA Ans. (3.00)

Sol. $(x+1)dy - ydx = ((x+1)^2 - 3)dx$

$\Rightarrow \frac{(x+1)dy - ydx}{(x+1)^2} = \left(1 - \frac{3}{(x+1)^2}\right)dx$

$\Rightarrow d\left(\frac{y}{(x+1)}\right) = \left(1 - \frac{3}{(x+1)^2}\right)dx$

integrating both sides

$\frac{y}{x+1} = x + \frac{3}{(x+1)} + C$

Given $y(2) = 0 \Rightarrow c = -3$

$\therefore y = (x+1)\left(x + \frac{3}{(x+1)} - 3\right)$

$\therefore y(3) = 3.00$

8. Official Ans. by NTA (4)

Sol. $\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x,\ y > 0$

$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$

By integrating both sides :

$\ell n|y+1| = -\ell n|2+\sin x| + \ell nK$

$\Rightarrow y + 1 = \frac{K}{2+\sin x}$ $\quad (y + 1 > 0)$

$\Rightarrow y(x) = \frac{K}{2+\sin x} - 1$

Given y(0) = 1 $\Rightarrow$ K = 4

So, $y(x) = \frac{4}{2+\sin x} - 1$

$a = y(\pi) = 1$

$b = \left.\frac{dy}{dx}\right]_{x=\pi} = \left.\frac{-\cos x}{2+\sin x}(y(x)+1)\right]_{x=\pi} = 1$

So, (a, b) = (1, 1)

9. Official Ans. by NTA (2)

Sol. $2x^2dy = (2xy + y^2)\,dx$

$\Rightarrow \frac{dy}{dx} = \frac{2xy+y^2}{2x^2}$ {Homogeneous D.E.}

$\left\{\begin{array}{l}\text{let } y = xt \\ \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}\end{array}\right\}$

$\Rightarrow t + x\frac{dt}{dx} = \frac{2x^2t + x^2t^2}{2x^2}$

$\Rightarrow t + x\frac{dt}{dx} = t + \frac{t^2}{2}$

$\Rightarrow x\frac{dt}{dx} = \frac{t^2}{2}$

$\Rightarrow 2\int\frac{dt}{t^2} = \int\frac{dx}{x}$

$\Rightarrow 2\left(-\frac{1}{t}\right) = \ell n(x) + C$ $\left\{\text{Put } t = \frac{y}{x}\right\}$

$\Rightarrow -\frac{2x}{y} = \ell n x + C$ $\left\{\begin{array}{l}\text{Put } x = 1 \,\&\, y = 2 \\ \text{then we get } C = -1\end{array}\right\}$

$\Rightarrow \frac{-2x}{y} = \ell n(x) - 1 \Rightarrow y = \frac{2x}{1-\ell n x}$

$\Rightarrow f(x) = \frac{2x}{1-\log_e x}$

so, $\boxed{f\left(\frac{1}{2}\right) = \frac{1}{1+\log_e 2}}$

10. Official Ans. by NTA (1)

Sol. $(1 + e^{-x})(1 + y^2)\frac{dy}{dx} = y^2$

$\Rightarrow (1 + y^{-2})\,dy = \left(\frac{e^x}{1+e^x}\right)dx$

$\Rightarrow \left(y - \frac{1}{y}\right) = \ell n(1+e^x) + c$

$\therefore$ It passes through (0, 1) $\Rightarrow c = -\ell n 2$

$\Rightarrow y^2 = 1 + y\,\ell n\left(\frac{1+e^x}{2}\right)$

11. Official Ans. by NTA (2)

Sol. $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$

$\Rightarrow dy(x^3 - x^2) = dx\,(2y - xy)$

$$\Rightarrow -\int \frac{1}{y}dy = \int \frac{x-2}{x^2(x-1)}dx$$

$$\Rightarrow -\ell ny = \int\left(\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x-1)}\right)dx$$

Where A = 1, B = +2, C = –1

$$\Rightarrow -\ell ny = \ell n\,x - \frac{2}{x} - \ell n\,(x-1) + \lambda$$

$\Rightarrow y(2) = e$

$\Rightarrow -1 = \ell n\ 2 - 1 - 0 + \lambda$

$\therefore\ \lambda = -\ \ell n\,2$

$$\Rightarrow \ell n\,y = -\ \ell nx + \frac{2}{x} + \ell n(x-1) + \ell n\,2$$

Now put x = 4 in equation

$$\Rightarrow \ell n\,y = -\ell n\,4 + \frac{1}{2} + \ell n\,3 + \ell n\,2$$

$$\Rightarrow \ell n\,y = \ell n\left(\frac{3}{2}\right) + \frac{1}{2}\ell n\,e$$

$$\Rightarrow y = \frac{3}{2}\sqrt{e}$$

12. Official Ans. by NTA (1)

Sol. $x\dfrac{dy}{dx} - y = x^2(x\cos x + \sin x),\ x > 0$

$$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x)\ \Rightarrow\ \frac{dy}{dx} + Py = Q$$

so, I.F. = $e^{\int -\frac{1}{x}dx} = \dfrac{1}{|x|} = \dfrac{1}{x}$ (x > 0)

Thus, $\dfrac{y}{x} = \displaystyle\int \frac{1}{x}\big(x(x\cos x + \sin x)\big)dx$

$$\Rightarrow \frac{y}{x} = x\sin x + C$$

$\because\ y(\pi) = \pi\ \Rightarrow\ C = 1$

so, $y = x^2\sin x + x \Rightarrow (y)_{\pi/2} = \dfrac{\pi^2}{4} + \dfrac{\pi}{2}$

Also, $\dfrac{dy}{dx} = x^2\cos x + 2x\sin x + 1$

$$\Rightarrow \frac{d^2y}{dx^2} = -x^2\sin x + 4x\cos x + 2\sin x$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\frac{\pi}{2}} = -\frac{\pi^2}{4} + 2$$

Thus, $y_{\left(\frac{\pi}{2}\right)} + \dfrac{d^2y}{dx^2}_{\left(\frac{\pi}{2}\right)} = \dfrac{\pi}{2} + 2$

13. Official Ans. by NTA (3)

Sol. $\ell n(y + 3x) = z$ (let)

$$\frac{1}{y+3x}\cdot\left(\frac{dy}{dx}+3\right) = \frac{dz}{dx}$$

..(1)

$$\frac{dy}{dx} + 3 = \frac{y+3x}{\ell n(y+3x)} \qquad \text{(given)}$$

$$\frac{dz}{dx} = \frac{1}{z}$$

$$\Rightarrow z\,dz = dx \Rightarrow \frac{z^2}{2} = x + C$$

$$\Rightarrow \frac{1}{2}\ell n^2(y+3x) = x + C$$

$$\Rightarrow x - \frac{1}{2}(\ell n(y + 3x))^2 = C$$

14. Official Ans. by NTA (2)

Sol. $\dfrac{(5+e^x)}{2+y}\dfrac{dy}{dx} = -e^x$

$$\int \frac{dy}{2+y} = \int \frac{-e^x}{e^x+5}dx$$

$\ln(y + 2) = -\ln(e^x + 5) + k$

$(y + 2)(e^x + 5) = C$

$\because\ y(0) = 1$

$\Rightarrow C = 18$

$$y + 2 = \frac{18}{e^x+5}$$

at $x = \ln 13$

$$y + 2 = \frac{18}{13+5} = 1$$

$\boxed{y = -1}$

15. Official Ans. by NTA (1)

Sol. $\cos x \frac{dy}{dx} + 2y \sin x = \sin 2x$

$\frac{dy}{dx} + \frac{2\sin x}{\cos x} y = 2 \sin x$

I.F. $= e^{\int 2\frac{\sin x}{\cos x}dx}$

$= e^{2 \ln \sec x} = \sec^2 x$

$y \cdot \sec^2 x = \int 2\sin x \cdot \sec^2 x dx$

$y \sec^2 x = 2 \int \tan x \sec x \, dx$

$y \sec^2 x = 2 \sec x + c$

At $x = \frac{\pi}{3}$, $y = 0$

$\Rightarrow 0 = 2 \sec \frac{\pi}{3} + C \Rightarrow C = -4$

$\boxed{y \sec^2 x = 2\sec x - 4}$

Put $x = \frac{\pi}{4}$

$y \cdot 2 = 2\sqrt{2} - 4$

$y = \sqrt{2} - 2$

16. Official Ans. by NTA (2)

Sol. $x^4 e^y + 2\sqrt{y+1} = 3$

d.w.r. to x

$x^4 e^y y' + e^y 4x^3 + \frac{2y'}{2\sqrt{y+1}} = 0$

at P(1, 0)

$y'_P + 4 + y'_P = 0$

$\Rightarrow y'_P = -2$

Tangent at P(1, 0) is

$y - 0 = -2 (x - 1)$

$2x + y = 2$

(–2, 6) lies on it

17. Official Ans. by NTA (1)

Sol. $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$

$\Rightarrow \sqrt{(1+x)^2(1+y^2)} + xy\frac{dy}{dx} = 0$

$\Rightarrow \sqrt{1+x^2}\sqrt{1+y^2} = -xy\frac{dy}{dx}$

$\Rightarrow \int \frac{ydy}{\sqrt{1+y^2}} = -\int \frac{\sqrt{1+x^2}}{x}dx$ ...(1)

Now put $1 + x^2 = u^2$ and $1 + y^2 = v^2$

$2xdx = 2udu$ and $2ydy = 2vdv$

$\Rightarrow xdx = udu$ and $ydy = vdv$

substitude these values in equation (1)

$\int \frac{vdv}{v} = -\int \frac{u^2.du}{u^2-1}$

$\Rightarrow \int dv = -\int \frac{u^2-1+1}{u^2-1}du$

$\Rightarrow v = -\int \left(1 + \frac{1}{u^2-1}\right)du$

$\Rightarrow v = -u - \frac{1}{2}\log_e\left|\frac{u-1}{u+1}\right| + c$

$\Rightarrow \sqrt{1+y^2} = -\sqrt{1+x^2} + \frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right| + c$

$\Rightarrow \sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right| + c$

18. Official Ans. by NTA (1)

Sol. $y = \left(\frac{2x}{\pi} - 1\right)\text{cosec}\, x$ ...(1)

$\frac{dy}{dx} = \frac{2}{\pi}\text{cosec}x - \left(\frac{2x}{\pi} - 1\right)\text{cosec}x \cot x$

$\frac{dy}{dx} = \frac{2\,\text{cosec}x}{\pi} - y\cot x$

using equation (1)

$\frac{dy}{dx} + y\cot x = \frac{2\,\text{cosec}x}{\pi}$

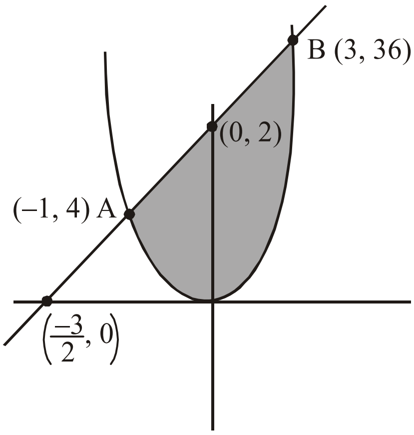
$\frac{dy}{dx} + p(x).y = \frac{2\,\text{cosec}x}{\pi} \quad x \in \left(0, \frac{\pi}{2}\right)$

Compare : p(x) = cot x

AREA UNDER THE CURVE

1. NTA Ans. (4)

Sol. $4x^2 - y \le 0$ and $8x - y + 12 \ge 0$



On solving $y = 4x^2$

and $y = 8x + 12$

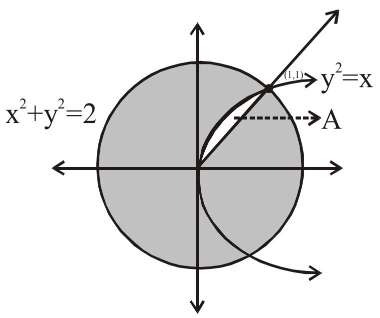
We get A (–1, 4) & B(3, 36)

Required area = area of the shaded region

$$= \int_{-1}^{3} (8x + 12 - 4x^2)dx = \frac{128}{3}$$

2. NTA Ans. (2)

Sol. $A = \int_0^1 \left(\sqrt{x} - x\right)dx$

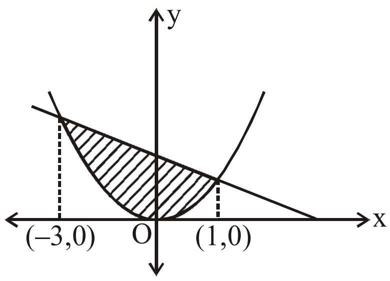


$$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{1}{6}$$

Required Area : $\pi r^2 - \frac{1}{6} = \frac{1}{6}(12\pi - 1)$

3. NTA Ans. (4)

Sol.

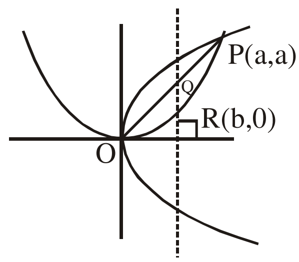


$$\text{Area} = \int_{-3}^{1} \left(3 - 2x - x^2\right)dx = \frac{32}{3}$$

(4) option

4. NTA Ans. (1)

Sol. $\int_0^b \left(\sqrt{ax} - \frac{x^2}{a}\right)dx = \frac{1}{2} \times \frac{16\left(\frac{a}{4}\right)\left(\frac{a}{4}\right)}{3}$



$$\Rightarrow \left[\frac{2\sqrt{a}}{3}x^{3/2} - \frac{x^3}{3a}\right]_0^b = \frac{a^2}{6}$$

$$\Rightarrow \frac{2\sqrt{a}}{3}b^{3/2} - \frac{b^3}{3a} = \frac{a^2}{6}$$

...(i)

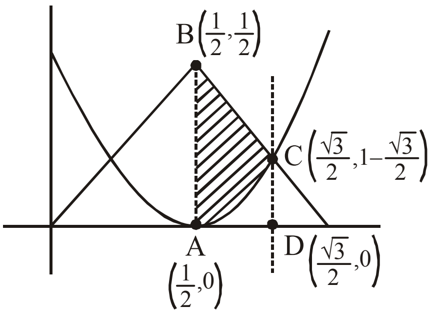
Also, $\frac{1}{2} \times b^2 = \frac{1}{2} \Rightarrow b = 1$

so, $\frac{2\sqrt{a}}{3} - \frac{1}{3a} = \frac{a^2}{6} \Rightarrow a^3 - 4a^{3/2} + 2 = 0$

$\Rightarrow a^6 + 4a^3 + 4 = 16a^3 \Rightarrow a^6 - 12a^3 + 4 = 0$

5. NTA Ans. (2)

Sol.



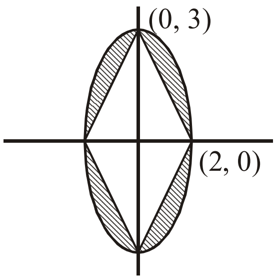
Required area = Area of trepezium ABCD – Area of parabola between $x=\frac{1}{2}$ & $x=\frac{\sqrt3}{2}$

$$A=\frac{1}{2}\left(\frac{\sqrt3}{2}-\frac{1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt3}{2}\right)-\int_{1/2}^{\sqrt3/2}\left(x-\frac{1}{2}\right)^2dx=\frac{\sqrt3}{4}-\frac{1}{3}$$

6. Official Ans. by NTA (2)

Sol. $\frac{|x|}{2}+\frac{|y|}{3}=1$

$\frac{x^2}{4}+\frac{y^2}{9}=1$



Area of Ellipse = $\pi ab = 6\pi$

Required area,

$= \pi \times 2 \times 3$ – (Area of quadrilateral)

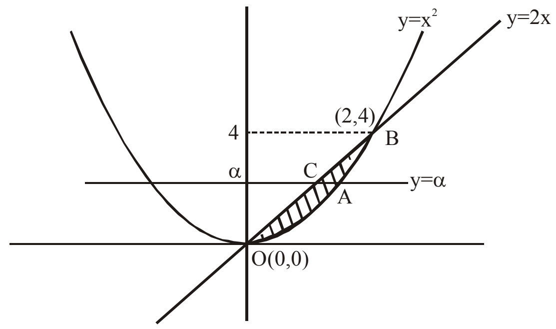
$= 6\pi-\frac{1}{2}6\times4$

$= 6\pi - 12$

$= 6(\pi - 2)$

7. Official Ans. by NTA (4)

Sol.



* $y \ge x^2 \Rightarrow$ upper region of $y = x^2$

$y \le 2x \Rightarrow$ lower region of $y = 2x$

According to ques, area of OABC = 2 area of OAC

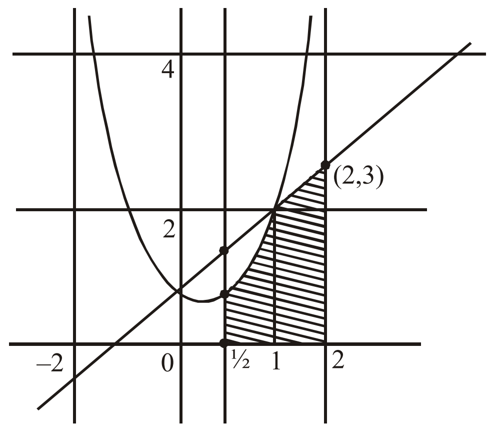
$$\Rightarrow \int_0^4\left(\sqrt{y}-\frac{y}{2}\right)dy=2\int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right).dy$$

$$\Rightarrow \frac{4}{3}=2\left[\frac{2}{3}\alpha^{3/2}-\frac{1}{4}.\alpha^2\right]$$

$$\Rightarrow \boxed{3\alpha^2-8\alpha^{3/2}+8=0}$$

8. Official Ans. by NTA (3)

Sol. $0 \le y \le x^2 + 1,\ 0 \le y \le x + 1,\ \frac{1}{2} \le x \le 2$



Required area = $\int_{1/2}^{1}(x^2+1)dx+\frac{1}{2}(2+3)\times1$

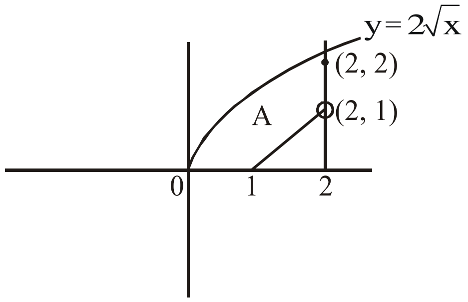
$$=\frac{19}{24}+\frac{5}{2}=\frac{79}{24}$$

9. Official Ans. by NTA (1)

Sol. $(x-1)[x] \le y \le 2\sqrt{x}$, $\boxed{0 \le x \le 2}$

Draw $y = 2\sqrt{x} \Rightarrow y^2 = 4x$ $\boxed{x \ge 0}$

$$y = (x-1)[x] = \begin{cases} 0 & , 0 \le x < 1 \\ x-1, & 1 \le x < 2 \\ 2 & , x = 2 \end{cases}$$



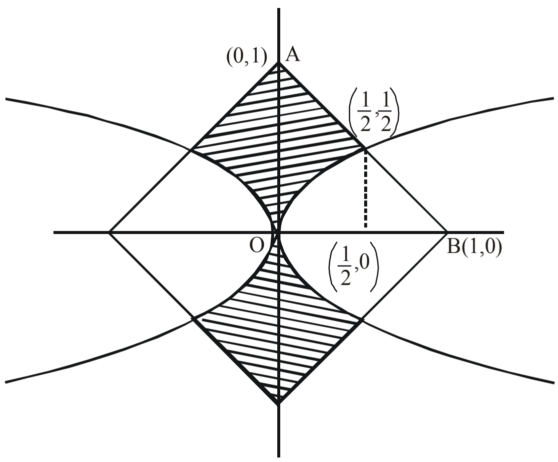
$$A = \int_0^2 2\sqrt{x}\,dx - \frac{1}{2}1\cdot 1$$

$$A = 2\cdot\left[\frac{x^{3/2}}{(3/2)}\right]_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

10. Official Ans. by NTA (4)

Sol. $|x| + |y| \le 1$

$2y^2 \ge |x|$



For point of intersection

$x + y = 1 \Rightarrow x = 1 - y$

$$y^2 = \frac{x}{2} \Rightarrow 2y^2 = x$$

$2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$

$(2y-1)(y+1) = 0$

$$y = \frac{1}{2} \text{ or } -1$$

Now Area of $\Delta OAB = \frac{1}{2}\times 1\times 1 = \frac{1}{2}$

Area of Region $R_1 = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$

Area of Region $R_2 = \frac{1}{\sqrt{2}}\int_0^{\frac{1}{2}}\sqrt{x}\,dx = \frac{1}{6}$

Now area of shaded region in first quadrant

= Area of $\Delta OAB - R_1 - R_2$

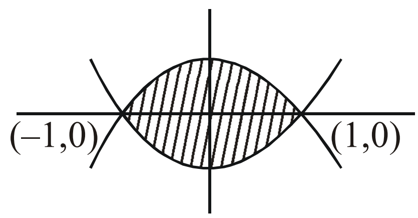
$$= \frac{1}{2} - \left(\frac{1}{6}\right) - \left(\frac{1}{8}\right) = \frac{5}{24}$$

So required area $= 4\left(\frac{5}{24}\right) = \frac{5}{6}$

so option (4) is correct.

11. Official Ans. by NTA (2)

Sol. $y = x^2 - 1$ and $y = 1 - x^2$



$$A = \int_{-1}^{1}\left((1-x^2)-(x^2-1)\right)dx$$

$$A = \int_{-1}^{1}(2-2x^2)\,dx = 4\int_0^1(1-x^2)\,dx$$

$$A = 4\left(x - \frac{x^3}{3}\right)_0^1 = 4\left(\frac{2}{3}\right) = \frac{8}{3}$$

MATRICES

1. NTA Ans. (4)

Sol. $b_{ij} = (3)^{(i+j-2)} a_{ij}$

$$B = \begin{bmatrix} a_{11} & 3a_{12} & 3^2a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2a_{31} & 3^2a_{32} & 3^2a_{33} \end{bmatrix}$$

$$\Rightarrow |B| = 3 \times 3^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2a_{31} & 3^2a_{32} & 3^2a_{33} \end{vmatrix}$$

$= 3^6|A|$

$\Rightarrow |A| = \dfrac{81}{27 \times 27} = \dfrac{1}{9}$

2. NTA Ans. (1)

Sol. $x^2 + x + 1 = 0$

$\alpha = \omega$

$\alpha^2 = \omega^2$

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow A^4 = A^2 \,.\, A^2 = I_3$

$A^{31} = A^{28}.\, A^3 = A^3.$

3. NTA Ans. (2)

Sol. $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$; $|A| = 8 - 18 = -10$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{pmatrix} 4 & -2 \\ -9 & 2 \end{pmatrix}}{-10}$$

$$10A^{-1} = \begin{pmatrix} -4 & 2 \\ 9 & -2 \end{pmatrix} = A - 6I$$

(2) Option

4. NTA Ans. (672.00)

Sol. trace $(AA^T) = \Sigma a_{ij}^2 = 3$

Hence, number of such matrices

$= {}^9C_3 \times 2^3 = 672.00$

5. NTA Ans. (3)

Sol. $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

$\Rightarrow$ $|A| = 6$

$$\frac{|\text{adj}B|}{|c|} = \frac{|\text{adj}(\text{adj}A)|}{|9A|} = \frac{|A|^4}{3^3|A|} = \frac{|A|^3}{3^3}$$

$$= \frac{(6)^3}{(3)^3} = 8$$

6. Official Ans. by NTA (4)

Sol. $|A| \neq 0$

For (P) : $A \neq I_2$

So, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$|A|$ can be -1 or 1

So (P) is false.

For (Q); $|A| = 1$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow$ tr(A) = 2

$\Rightarrow$ Q is true

7. Official Ans. by NTA (4)

Sol. $A^TA = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$

and $ab + bc + ca = 0$

Now, $(a + b + c)^2 = 1$

$\Rightarrow a + b + c = \pm 1$

So, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \pm 1 (1 - 0) = \pm 1$

$\Rightarrow 3\,abc = 2 \pm 1 = 3, 1$

$\Rightarrow abc = 1, \dfrac{1}{3}$

8. Official Ans. by NTA (2)

Sol. Given $P=\begin{bmatrix}1 & 2 & 1\\ -2 & 3 & -4\\ 1 & 9 & 1\end{bmatrix}$, Here |P| = 0 & also

given PX = 0

$$\Rightarrow \begin{bmatrix}1 & 2 & 1\\ -2 & 3 & -4\\ 1 & 9 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=0$$

$$\Rightarrow \left.\begin{matrix}x+2y+z=0\\ -2x+3y-4z=0\\ x+9y-z=0\end{matrix}\right\} \because D=0, \text{so system have}$$

infinite many solutions,

By solving these equation

we get $x=\frac{-11\lambda}{2}; y=\lambda; z=\frac{7\lambda}{2}$

Also given, $x^2 + y^2 + z^2 = 1$

$$\Rightarrow \left(\frac{-11\lambda}{2}\right)^2+(\lambda)^2+\left(\frac{7\lambda}{2}\right)^2=1$$

$$\Rightarrow \lambda=\pm\frac{1}{\sqrt{\frac{121}{4}+1+\frac{49}{4}}}$$

so, there are 2 values of λ.

$\therefore$ so, there are 2 solution set of (x,y,z).

9. Official Ans. by NTA (10)

Sol. $A=\begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}$

$$A^2=\begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}x^2+1 & x\\ x & 1\end{bmatrix}$$

$$A^4=\begin{bmatrix}x^2+1 & x\\ x & 1\end{bmatrix}\begin{bmatrix}x^2+1 & x\\ x & 1\end{bmatrix}$$

$$=\begin{bmatrix}(x^2+1)^2+x^2 & x(x^2+1)+x\\ x(x^2+1)+x & x^2+1\end{bmatrix}$$

$a_{11} = (x^2 + 1)^2 + x^2 = 109$

$\Rightarrow x = \pm 3$

$a_{22} = x^2 + 1 = 10$

10. Official Ans. by NTA (3)

Sol. $C = \text{adj } A = \begin{vmatrix}+2 & -1 & 1\\ -1 & 0 & 2\\ 1 & -2 & -1\end{vmatrix}$

$|C| = |\text{adj } A| = +2(0 + 4) + 1.(1 - 2) + 1.(2-0) = +8 - 1 + 2$

$|\text{adj } A| = |A|^2 = 9 = 9$

$\lambda = |A| = \pm 3$

$|\lambda| = 3$

$B = \text{adj } C$

$|B| = |\text{adj } C| = |C|^2 = 81$

$|(B^{-1})^T| = |B|^{-1} = \frac{1}{81}$

$(|\lambda|, \mu) = \left(3, \frac{1}{81}\right)$

11. Official Ans. by NTA (4)

Sol. $Ax_1 = b_1$

$Ax_2 = b_2$

$Ax_3 = b_3$

$$\Rightarrow |A|\begin{vmatrix}1 & 0 & 0\\ 1 & 2 & 0\\ 1 & 1 & 1\end{vmatrix}=\begin{vmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{vmatrix}$$

$\Rightarrow |A| = \frac{4}{2} = 2$

12. Official Ans. by NTA (2)

Sol. $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$A^2=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$A^2=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

$B = A + A^4$

$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\begin{bmatrix}\cos 4\theta & \sin 4\theta\\ -\sin 4\theta & \cos 4\theta\end{bmatrix}$

$B=\begin{bmatrix}(\cos\theta+\cos 4\theta) & (\sin\theta+\sin 4\theta)\\ -(\sin\theta+\sin 4\theta) & (\cos\theta+\cos 4\theta)\end{bmatrix}$

$|B| = (\cos\theta + \cos4\theta)^2 + (\sin\theta + \sin4\theta)^2$

$|B| = 2 + 2\cos3\theta$, when $\theta=\frac{\pi}{5}$

$|B|=2+2\cos\frac{3\pi}{5}=2(1-\sin 18)$

$|B|=2\left(1-\frac{\sqrt{5}-1}{4}\right)=2\left(\frac{5-\sqrt{5}}{4}\right)=\frac{5-\sqrt{5}}{2}$

VECTORS

1. NTA Ans. (1)

Sol. $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b})+2(\vec{b}\cdot\vec{c})+2(\vec{c}\cdot\vec{a})=0$

$\lambda = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\frac{-3}{2}$

$\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

$\Rightarrow \vec{d} = 3(\vec{a}\times\vec{b})$

2. NTA Ans. (4)

ALLEN Ans. (BONUS)

Note: None of the given options matches. So, it should be bonus but NTA did not accept our claim

Sol. $\vec{a}=\lambda\left(\hat{b}+\hat{c}\right)=\lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}+\frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$\vec{a}=\frac{\lambda}{3\sqrt{2}}\left(4\hat{i}+2\hat{j}+4\hat{k}\right)\Rightarrow\frac{\lambda}{3\sqrt{2}}(4\hat{i}+2\hat{j}+4\hat{k})$

$=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$\Rightarrow \alpha = 4$ and $\beta = 4$

So, $\vec{a}=4\hat{i}+2\hat{j}+4\hat{k}$

None of the given options is correct

3. NTA Ans. (3)

Sol. $\vec{b}\times\vec{c}-\vec{b}\times\vec{a}=\vec{0}$

$\vec{b}\times(\vec{c}-\vec{a})=\vec{0}$

$\vec{b}=\lambda(\vec{c}-\vec{a})$ ...(i)

$\vec{a}\cdot\vec{b}=\lambda\left(\vec{a}\cdot\vec{c}-\vec{a}^2\right)$

$4 = \lambda(0 - 6) \Rightarrow \lambda=\frac{-4}{6}=\frac{-2}{3}$

from (i) $\vec{b}=\frac{-2}{3}(\vec{c}-\vec{a})$

$\vec{c}=\frac{-3}{2}\vec{b}+\vec{a}=\frac{-1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

$\boxed{\vec{b}\cdot\vec{c}=\frac{1}{2}}$ (3) Option

4. NTA Ans. (1)

Sol. $\left\|\begin{matrix}1 & 1 & \lambda\\ 1 & 1 & 3\\ 2 & 1 & 1\end{matrix}\right\|=1\Rightarrow\lambda=2,4$

Now, $\cos\theta=\frac{\vec{u}.\vec{w}}{|\vec{u}||\vec{w}|}$

$=\frac{5}{\sqrt{6}\sqrt{6}}\text{ or }\frac{7}{\sqrt{6}\sqrt{18}}=\frac{5}{6}\text{ or }\frac{7}{6\sqrt{3}}$

5. NTA Ans. (30)

Sol. $\vec{b}.\vec{c}=10 \Rightarrow 5|\vec{c}|\cos\frac{\pi}{3}=10 \Rightarrow |\vec{c}|=4$

$\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|=|\vec{a}|\,|\vec{b}\times\vec{c}|$

$=\sqrt{3}\,.\,5\,.\,4\,.\,\sin\frac{\pi}{4}=30$

6. NTA Ans. (1.00)

Sol. $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$,

$\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and

$\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$

$\because$ $\vec{p}, \vec{q}, \vec{r}$ are coplanar

$\Rightarrow [\vec{p} \quad \vec{q} \quad \vec{r}] = 0$

$$\Rightarrow \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$\Rightarrow 3a + 1 = 0 \Rightarrow a = -\frac{1}{3}$

$\vec{p}.\vec{q} = -\frac{1}{3}$, $\vec{r}.\vec{q} = -\frac{1}{3}$

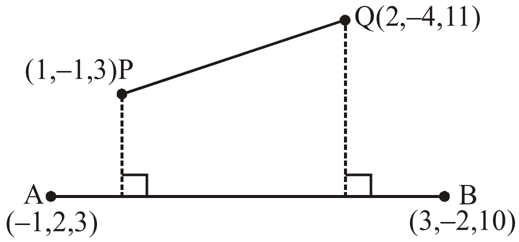
$|\vec{r}|^2 = |\vec{q}|^2 = \frac{2}{3}$

$\therefore$ $3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$

$$\Rightarrow \lambda = \frac{3(\vec{p}.\vec{q})^2}{|\vec{r} \times \vec{q}|^2} = \frac{3(\vec{p}.\vec{q})^2}{|\vec{r}|^2|\vec{q}|^2 - (\vec{r}.\vec{q})^2} = 1.00$$

7. NTA Ans. (8.00)

Sol.



Projection of $\overrightarrow{PQ}$ on $\overrightarrow{AB} = \left|\frac{\overrightarrow{PQ} \cdot \overrightarrow{AB}}{|\overrightarrow{AB}|}\right|$

$$= \left|\frac{(\hat{i} - 3\hat{j} + 8\hat{k}) \cdot (4\hat{i} - 4\hat{j} + 7\hat{k})}{9}\right| = 8$$

8. Official Ans. by NTA (2.00)

Sol. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 8$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} + |\vec{a}|^2 + |\vec{c}|^2 - 2\vec{a}.\vec{c} = 8$

$\Rightarrow 4 - 2(\vec{a}.\vec{b} + \vec{a}.\vec{c}) = 8$

$\Rightarrow \boxed{\vec{a}.\vec{b} + \vec{a}.\vec{c} = -2}$

$|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$

$= |a^2| + 4|\vec{b}|^2 + 4\vec{a}.\vec{b} + |\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a}.\vec{c}$

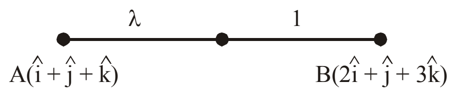
$= 10 + 4(\vec{a}.\vec{b} + \vec{a}.\vec{c})$

$= 10 - 8$

$= \boxed{2}$

9. Official Ans. by NTA (0.8)

Sol.



Using section formula we get

$$\overline{OP} = \frac{2\lambda+1}{\lambda+1}\hat{i} + \frac{\lambda+1}{\lambda+1}\hat{j} + \frac{3\lambda+1}{\lambda+1}\hat{k}$$

Now $\overline{OB} \cdot \overline{OP} = \frac{4\lambda + 2 + \lambda + 1 + 9\lambda + 3}{\lambda+1}$

$$= \frac{14\lambda + 6}{\lambda + 1}$$

$$\overline{OA} \times \overline{OP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \frac{2\lambda+1}{\lambda+1} & 1 & \frac{3\lambda+1}{\lambda+1} \end{vmatrix}$$

$$= \frac{2\lambda+1}{\lambda+1}\hat{i} + \frac{-\lambda}{\lambda+1}\hat{j} + \frac{-\lambda}{\lambda+1}\hat{k}$$

$$|\overline{OA} \times \overline{OP}|^2 = \frac{(2\lambda+1)^2 + \lambda^2 + \lambda^2}{(\lambda+1)^2}$$

$$= \frac{6\lambda^2 + 1}{(\lambda+1)^2}$$

$$\Rightarrow \frac{14\lambda+6}{\lambda+1} - 3 \times \frac{(6\lambda^2+1)}{(\lambda+1)^2} = 6$$

$\Rightarrow 10\lambda^2 - 8\lambda = 0$

$\Rightarrow \lambda = 0, \frac{8}{10} = 0.8$

$\Rightarrow \lambda = 0.8$

10. Official Ans. by NTA (3)

Sol. $\vec{r}=\hat{i}(1+2\ell)+\hat{j}(-1)+\hat{k}(\ell)$

$\vec{r}=\hat{i}(2+m)+\hat{j}(m-1)+\hat{k}(-m)$

For intersection

$1+2\ell=2+m$ ...... (i)

$-1=m-1$ ...... (ii)

$\ell=-m$ ...... (iii)

from (ii) $m=0$

from (iii) $\ell=0$

These values of m and ℓ do not satisfy equation (1).

Hence the two lines do not intersect for any values of ℓ and m.

11. Official Ans. by NTA (5)

Sol. Dr's normal to plane

$$=\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}=-\hat{i}+\hat{j}+\hat{k}$$

Equation of plane

$-1(x-1)+1(y-0)+1(z-0)=0$

$x-y-z-1=0$

......(1)

Now $\dfrac{\alpha-1}{1}=\dfrac{\beta-0}{-1}=\dfrac{\gamma-1}{-1}=-\dfrac{(1-0-1-1)}{3}$

$$\frac{\alpha-1}{1}=\frac{\beta}{-1}=\frac{\gamma-1}{-1}=\frac{1}{3}$$

$$\alpha=\frac{4}{3},\ \beta=-\frac{1}{3},\ \gamma=\frac{2}{3}$$

$$3(\alpha+\beta+\gamma)=3\left(\frac{4}{3}-\frac{1}{3}+\frac{2}{3}\right)=5$$

12. Official Ans. by NTA (4)

Sol. $f(x)=\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=\begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}=x^3-27x+26$

$f'(x)=3x^2-27=0 \Rightarrow x=\pm 3$

and $f''(-3)<0$

$\Rightarrow$ local maxima at $x=x_0=-3$

Thus, $\vec{a}=-3\hat{i}-2\hat{j}+3\hat{k}$,

$\vec{b}=-2\hat{i}-3\hat{j}-\hat{k}$,

and $\vec{c}=7\hat{i}-2\hat{j}-3\hat{k}$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=9-5-26=-22$

13. Official Ans. by NTA (18)

Sol. $\Sigma|\vec{a}-(\vec{a}\cdot i)i|^2$

$\Rightarrow \Sigma\left(|a|^2+(\vec{a}\cdot i)^2-2(\vec{a}\cdot i)^2\right)$

$\Rightarrow 3|\vec{a}|^2-\Sigma(\vec{a}\cdot i)^2$

$\Rightarrow 2|\vec{a}|^2$

$\Rightarrow 18$

14. Official Ans. by NTA (2)

Sol. $v=\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$158=\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix},\ n\ge 0$$

$158=1(12+n^2)-(6+n)+n(2n-4)$

$158=n^2+12-6-n+2n^2-4n$

$3n^2-5n-152=0$

$n=8,\ -\dfrac{38}{6}$ (rejected)

$\vec{a}\cdot\vec{c}=1+n+3n=1+4n=33$

$\vec{b}\cdot\vec{c}=2+4n-3n=2+n=10$

15. Official Ans. by NTA (6.00)

Sol. Projection of $\vec{b}$ on $\vec{a}$ = projection of $\vec{c}$ on $\vec{a}$

$$\Rightarrow \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}\Rightarrow \vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}$$

$\because \vec{b}$ is perpendicular to $\vec{c} \Rightarrow \vec{b}\cdot\vec{c}=0$

Let $|\vec{a}+\vec{b}-\vec{c}|=k$

Square both sides

$k^2=\vec{a}^2+\vec{b}^2+\vec{c}^2+2\vec{a}\cdot\vec{b}-2\vec{a}\cdot\vec{c}-2\vec{b}\cdot\vec{c}$

$k^2=\vec{a}^2+\vec{b}^2+\vec{c}^2=36$

$k=6=|\vec{a}+\vec{b}-\vec{c}|$

16. Official Ans. by NTA (4.00)

Sol. $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$

$= \sqrt{3}\left(\sqrt{2+2\cos\theta}\right)+\sqrt{2-2\cos\theta}$

$= \sqrt{6}\left(\sqrt{1+\cos\theta}\right)+\sqrt{2}\left(\sqrt{1-\cos\theta}\right)$

$= 2\sqrt{3}\left|\cos\frac{\theta}{2}\right|+2\left|\sin\frac{\theta}{2}\right|$

$\le \sqrt{\left(2\sqrt{3}\right)^2+(2)^2} = 4$

17. Official Ans. by NTA (1.00)

Sol. $|\vec{x}+\vec{y}| = |\vec{x}|$

$\sqrt{|\vec{x}|^2+|\vec{y}|^2+2\vec{x}.\vec{y}} = |\vec{x}|$

$|\vec{y}|^2+2\vec{x}.\vec{y} = 0$ .... (1)

Now $(2\vec{x}+\lambda\vec{y}).\vec{y} = 0$

$2\vec{x}.\vec{y}+\lambda|\vec{y}|^2 = 0$

from (1)

$-|\vec{y}|^2+\lambda|\vec{y}|^2 = 0$

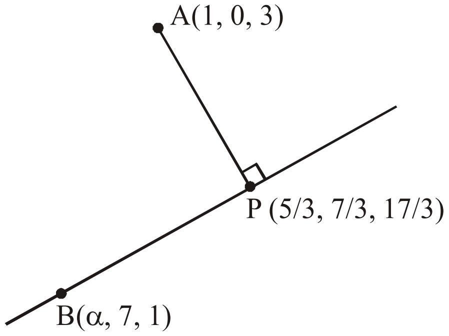
$(\lambda-1)|\vec{y}|^2 = 0$

given $|\vec{y}| \ne 0 \Rightarrow \lambda = 1$

3D

1. NTA Ans. (4.00)

Sol. D.R. of BP = $< \frac{5}{3}-\alpha, \frac{7}{3}-7, \frac{17}{3}-1 >$



D.R. of AP = $< \frac{5}{3}-1, \frac{7}{3}-0, \frac{17}{3}-3 >$

BP $\perp^r$ AP

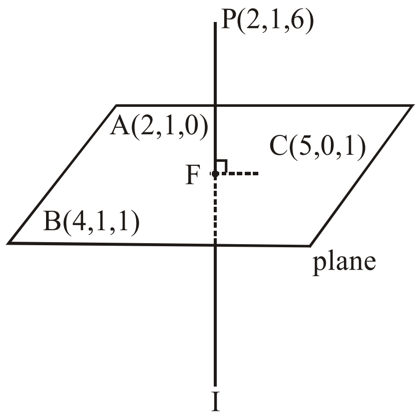
$\Rightarrow \alpha = 4$

2. NTA Ans. (1)

Sol. Plane passing through : (2, 1, 0), (4, 1, 1) and (5, 0, 1)

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow x + y - 2z = 3$



Let I and F are respectively image and foot of perpendicular of point P in the plane.

eq^n of line PI $\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \lambda(\text{say})$

Let I $(\lambda + 2, \lambda + 1, -2\lambda + 6)$

$\Rightarrow F\left(2+\frac{\lambda}{2}, 1+\frac{\lambda}{2}, -\lambda+6\right)$

F lies in the plane

$\Rightarrow 2+\frac{\lambda}{2}+1+\frac{\lambda}{2}+2\lambda-12-3 = 0$

$\Rightarrow \lambda = 4$

$\Rightarrow$ I (6, 5, –2)

3. NTA Ans. (4)

Sol. Point on plane $R\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

Normal vector of plane is $\frac{10}{3}\hat{i}+\frac{10}{3}\hat{j}+\frac{10}{3}\hat{k}$

Equation of require plane is x + y + z = 1

Hence (1, –1, 1) lies on plane

(4) Option

4. NTA Ans. (2)

Sol. Shortest distance $= \dfrac{\left\|\begin{matrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{matrix}\right\|}{\sqrt{11\times 29-49}} = \dfrac{270}{\sqrt{270}}$

$= \sqrt{270} = 3\sqrt{30}$

5. NTA Ans. (3)

Sol. If $\lambda = -7$, then planes will be parallel & distance between them will be $\dfrac{3}{\sqrt{633}} \Rightarrow k = 3$

But if $\lambda \neq -7$, then planes will be intersecting & distance between them will be 0

6. NTA Ans. (1)

Sol. For planes to intersect on a line

$\Rightarrow$ there should be infinite solution of the given system of equations

for infinite solutions

$\Delta = \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$

$\Delta_z = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 13 - \beta = 0 \Rightarrow \beta = 13$

Also for $\alpha = -3$ and $b = 13$ $\Delta_x = \Delta_y = 0$

$\therefore\ \alpha + \beta = -3 + 13 = 10$

7. Official Ans. by NTA (2)

Sol. Two points on the line (L say) $\dfrac{x}{3} = \dfrac{y}{2}, z = 1$ are (0, 0, 1) & (3, 2, 1)

So dr's of the line is < 3, 2, 0 >

Line passing through (1, 2, 1), parallel to L and coplanar with given plane is

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + t(3\hat{i} + 2\hat{j}), t \in R$ (–2, 0, 1) satisfies the line (for t = –1)

$\Rightarrow$ (–2, 0, 1) lies on given plane.

Answer of the question is (2)

We can check other options by finding eqution of plane

Equation plane : $\begin{vmatrix} x-1 & y-2 & z-1 \\ 1+2 & 2-0 & 1-1 \\ 2+2 & 1-0 & 2-1 \end{vmatrix} = 0$

$\Rightarrow 2(x - 1) - 3(y - 2) - 5(z - 1) = 0$

$\Rightarrow 2x - 3y - 5z + 9 = 0$

8. Official Ans. by NTA (2)

Sol. Hence normal is $\perp^r$ to both the lines so normal vector to the plane is

$\vec{n} = (\hat{i} - 2\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$

$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$

Now equation of plane passing through (3,1,1) is

$\Rightarrow -4(x-3) + 5(y - 1) + 7(z - 1) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0$ ...(1)

Plane is also passing through (α, –3, 5) so this point satisfies the equation of plane so put in equation (1)

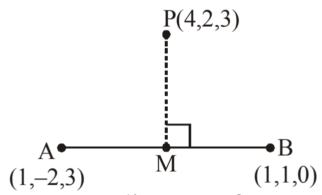
$-4\alpha + 5 \times (-3) + 7 \times (5) = 0$

$\Rightarrow -4\alpha - 15 + 35 = 0$

$\Rightarrow \boxed{\alpha = 5}$

9. Official Ans. by NTA (4)

Sol. Equation of AB = $\vec{r} = (\hat{i} + \hat{j}) + \lambda(3\hat{j} - 3\hat{k})$



Let coordinates of M = (1, (1 + 3λ), –3λ).

$\overrightarrow{PM} = -3\hat{i} + (3\lambda - 1)\hat{j} - 3(\lambda + 1)\hat{k}$

$\overrightarrow{AB} = 3\hat{j} - 3\hat{k}$

$\because\ \overrightarrow{PM} \perp \overrightarrow{AB} \Rightarrow \overrightarrow{PM} . \overrightarrow{AB} = 0$

$\Rightarrow 3(3\lambda - 1) + 9(\lambda + 1) = 0$

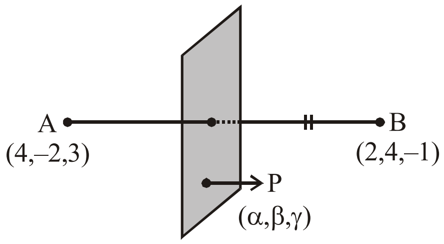
$\Rightarrow \lambda = -\dfrac{1}{3}$

$\therefore$ M = (1, 0, 1)

Clearly M lies on 2x + y – z = 1.

10. Official Ans. by NTA (1)

Sol. PA = PB



$\Rightarrow PA^2 = PB^2$

$\Rightarrow (\alpha - 4)^2 + (\beta + 2)^2 + (\gamma - 3)^2$

$= (\alpha - 2)^2 + (\beta - 4)^2 + (\gamma + 1)^2$

$\Rightarrow -4\alpha + 12\beta - 8\gamma = -8$

$\Rightarrow 2x - 6y + 4z = 4$

11. Official Ans. by NTA (3)

Sol. $D_1 = \begin{vmatrix} -7 & 4 & -1 \\ 8 & 1 & 5 \\ 15 & b & 6 \end{vmatrix} = 0 \Rightarrow b = -3$

$D = \begin{vmatrix} 1 & 4 & -1 \\ 3 & 1 & 5 \\ a & b & 6 \end{vmatrix} = 0 \Rightarrow 21a - 8b - 66 = 0$ .... (1)

P : $2x - 3y + 6z = 15$

so required distance $= \frac{21}{7} = 3$

12. Official Ans. by NTA (2)

Sol. equation of line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ passes through (1, –2, 3) is

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r$

$x = 2r + 1$

$y = 3r - 2,$

$z = -6r + 3$

So $2r + 1 - 3r + 2 - 6r + 3 = 5$

$\Rightarrow -7r + 1 = 0$

$r = \frac{1}{7}$

$x = \frac{9}{7},\ y = \frac{-11}{7},,\ z = \frac{15}{7}$

Distance is $= \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(2-\frac{11}{7}\right)^2 + \left(3-\frac{15}{7}\right)^2}$

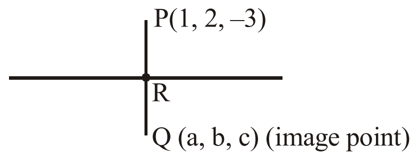
$= \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$

$= \frac{1}{7}\sqrt{4+9+36}$

$= \frac{1}{7}\sqrt{49} = 1$

13. Official Ans. by NTA (2)

Sol. Line is $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$: Let point R is $(2\lambda - 1, -2\lambda + 3, -\lambda)$



Direction ratio of PQ≡$(2\lambda - 2, -2\lambda + 1, 3 - \lambda)$

PQ is $\perp^r$ to line

$\Rightarrow 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(3 - \lambda) = 0$

$4\lambda - 4 + 4\lambda - 2 - 3 + \lambda = 0$

$9\lambda = 9 \Rightarrow \lambda = 1$

$\Rightarrow$ Point R is (1, 1, –1)

$\frac{a+1}{2} = 1$ | $\frac{b+2}{2} = 1$ | $\frac{c-3}{2} = -1$

a = 1   b = 0   c = 1

$\Rightarrow a + b + c = 2$

14. Official Ans. by NTA (4)

Sol. $L_1 \equiv \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$

$L_2 \equiv \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$

Point A(–1, 2, 1) B(–2, –1, –1)

$\because$ L_1 and L_2 are coplanar

$$\Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$$

$\alpha = -4$

$L_2 \equiv \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Check options (2, –10, –2) lies on L_2

15. Official Ans. by NTA (4)

Sol. Line of intersection of planes

$x + y + z + 1 = 0$ ...(1)

$2x - y + z + 3 = 0$ ...(2)

eliminate y

$3x + 2z + 4 = 0$

$x = \frac{-2z-4}{3}$ ...(3)

put in equaiton (1)

$z = -3y + 1$ ...(4)

from (3) and (4)

$$\frac{3x+4}{-2} = -3y + 1 = z$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\frac{1}{3}}{-\frac{1}{3}} = \frac{z-0}{1}$$

now shortest distance between skew lines

$$\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\left(\frac{1}{3}\right)}{-\frac{1}{3}} = \frac{z-0}{1}$$

$$\text{S.D.} = \left|\frac{(\vec{b}-\vec{a}).(\vec{c}\times\vec{d})}{|\vec{c}\times\vec{d}|}\right|$$

where $\vec{a} = (1, -1, 0)$

$\vec{b} = \left(-\frac{4}{3}, \frac{1}{3}, 0\right)$

$\vec{c} = (0, -1, 1)$

$\vec{d} = \left(-\frac{2}{3}, -\frac{1}{3}, 1\right)$

$\Rightarrow \text{S.D} = \frac{1}{\sqrt{3}}$

16. Official Ans. by NTA (2)

Sol. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$A \equiv (a, 0, 0)$, $B \equiv (0, b, 0)$, $C \equiv (0, 0, c)$

Centroid $\equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, 1, 2)$

$a = 3, b = 3, c = 6$

Plane : $\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$2x + 2y + z = 6$

line $\perp$ to the plane (DR of line = $2\hat{i} + 2\hat{j} + \hat{k}$)

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$$

PARABOLA

1. NTA Ans. (3)

Sol. $y = mx + 4$ is tangent to $y^2 = 4x$

$\Rightarrow m = \frac{1}{4}$

$y = \frac{1}{4}x + 4$ is tangent to $x^2 = 2by$

$\Rightarrow x^2 - \frac{b}{2}x - 8b = 0$

$\Rightarrow D = 0$

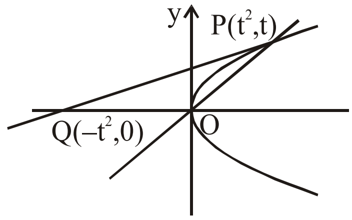
$b^2 + 128b = 0$

$\Rightarrow b = -128, 0$

$b \neq 0 \Rightarrow b = -128$

2. NTA Ans. (0.50)

Sol. $\Delta OPQ = 4$



$$\frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{matrix}\right\| = 4$$

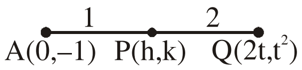
$t = 2$ ($\because$ $t > 0$)

$\therefore \ m = \frac{1}{2}$

Ans. 0.50

3. NTA Ans. (2)

Sol.

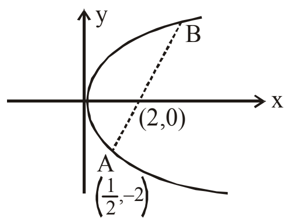


$\Rightarrow 3h = 2t$ and $3k = t^2 - 2$

$\Rightarrow \ 3y = \left(\frac{3x}{2}\right)^2 - 2 \Rightarrow 12y = 9x^2 - 8$

4. NTA Ans. (2)

Sol. $y^2 = 8x$



$4t_1 = -2 \Rightarrow t_1 = -\frac{1}{2}$,

$t_1.t_2 = -1$

$t_2 = -\frac{1}{t_1}$

$\Rightarrow t_2 = 2$

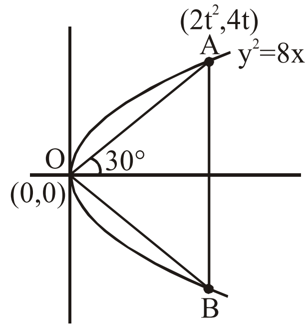
So coordinate of B is (8, 8)

$\therefore$ Equation of tangent at B is

$8y = 4(x + 8) \Rightarrow 2y = x + 8$

5. Official Ans. by NTA (3)

Sol.



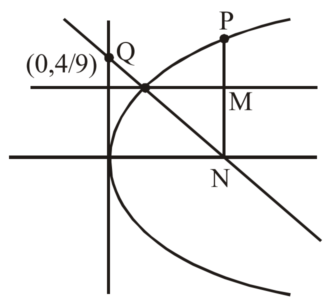
$\tan 30° = \frac{4t}{2t^2} = \frac{2}{t} \Rightarrow \ t = 2\sqrt{3}$

$AB = 8t = 16\sqrt{3}$

$\text{Area} = 256.3 \cdot \frac{\sqrt{3}}{4} = 192\sqrt{3}$

6. Official Ans. by NTA (3)

Sol. Let $P = (3t^2, 6t)$; $N = (3t^2, 0)$



$M = (3t^2, 3t)$

Equation of MQ : $y = 3t$

$\therefore \ Q = \left(\frac{3}{4}t^2, 3t\right)$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3}{4}t^2 - 3t^2\right)}(x - 3t^2)$$

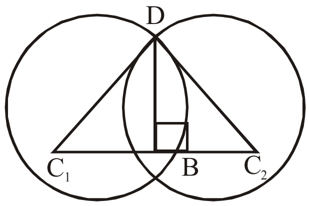
y-intercept of NQ $= 4t = \frac{4}{3} \Rightarrow \ t = \frac{1}{3}$

$\therefore \ MQ = \frac{9}{4}t^2 = \frac{1}{4}$

$PN = 6t = 2$

7. Official Ans. by NTA (1)

Sol. Length of latus rectum = 4



DB = 2

$C_1B = \sqrt{(C_1D)^2 - (DB)^2} = 4$

$C_1C_2 = 8$

8. Official Ans. by NTA (1)

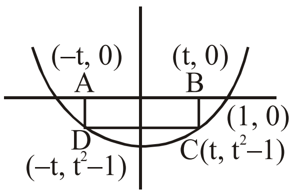
Sol. Area (A) = $2t \cdot (1 - t^2)$

$(0 < t < 1)$

$A = 2t - 2t^3$

$\frac{dA}{dt} = 2 - 6t^2$

$t = \frac{1}{\sqrt{3}}$



$\Rightarrow A_{max} = \frac{2}{\sqrt{3}}\left(1 - \frac{1}{3}\right) = \frac{4}{3\sqrt{3}}$

9. Official Ans. by NTA (3)

Sol. $y = mx + \frac{1}{m}$ (tangent at $y^2 = 4x$)

$y = mx - m^2$ (tangent at $x^2 = 4y$)

$\frac{1}{m} = -m^2$ (for common tangent)

$m^3 = -1$

$\boxed{m = -1}$

$y = -x - 1$

$x + y + 1 = 0$

This line touches circle

$\therefore$ apply p = r

$c = \left|\frac{0+0+1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

10. Official Ans. by NTA (1)

Sol. $y^2 = 4(x + 1)$

equation of tangent $y = m(x + 1) + \frac{1}{m}$

$y = mx + m + \frac{1}{m}$

$y^2 = 8(x + 2)$

equation of tangent $y = m'(x + 2) + \frac{2}{m'}$

$y = m'x + 2\left(m' + \frac{1}{m'}\right)$

since lines intersect at right angles

$\therefore mm' = -1$

Now $y = mx + m + \frac{1}{m}$ ...(1)

$y = m'x + 2\left(m' + \frac{1}{m'}\right)$

$y = -\frac{1}{m}x + 2\left(-\frac{1}{m} - m\right)$

$y = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$ ...(2)

From equation (1) and (2)

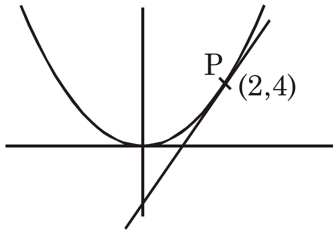
$mx + m + \frac{1}{m} = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$

$\left(m + \frac{1}{m}\right)x + 3\left(m + \frac{1}{m}\right) = 0$

$\therefore x + 3 = 0$

11. Official Ans. by NTA (2)

Sol. $y = x^2$



$$\left.\frac{dy}{dx}\right|_P = 4$$

$(y - 4) = 4(x - 2)$

$4x - y - 4 = 0$

Circle : $(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$

passes through (0, 1)

$4 + 9 + \lambda(-5) = 0 \qquad \Rightarrow \lambda = \frac{13}{5}$

Circle : $x^2 + y^2 + x(4\lambda - 4) + y(-\lambda - 8) + (20 - 4\lambda) = 0$

Centre : $\left(2 - 2\lambda, \frac{\lambda + 8}{2}\right) \equiv \left(\frac{-16}{5}, \frac{53}{10}\right)$

ELLIPSE

1. NTA Ans. (2)

Sol. $3x + 4y = 12\sqrt{12}$ is tangent to $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$

$c^2 = m^2a^2 + b^2$

$\Rightarrow a^2 = 16$

$e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

Distance between focii = 2ae = $2\sqrt{7}$

2. NTA Ans. (3)

Sol. Given 2ae = 6 $\Rightarrow \boxed{ae = 3}$ ......(1)

and $\frac{2a}{e} = 12 \Rightarrow \boxed{a = 6e}$ ....(2)

from (1) and (2)

$6e^2 = 3 \Rightarrow \boxed{e = \frac{1}{\sqrt{2}}}$

$\Rightarrow \boxed{a = 3\sqrt{2}}$

Now, $b^2 = a^2 (1 - e^2)$

$\Rightarrow b^2 = 18\left(1 - \frac{1}{2}\right) = 9$

Length of L.R = $\frac{2(9)}{3\sqrt{2}} = 3\sqrt{2}$

3. NTA Ans. (4)

Sol. Any normal to the ellipse is

$$\frac{x\sec\theta}{\sqrt{2}} - y\operatorname{cosec}\theta = -\frac{1}{2}$$

$$\Rightarrow \frac{x}{\left(\frac{-\cos\theta}{\sqrt{2}}\right)} + \frac{y}{\left(\frac{\sin\theta}{2}\right)} = 1$$

$$\Rightarrow \frac{\cos\theta}{\sqrt{2}} = \frac{1}{3\sqrt{2}} \text{ and } \frac{\sin\theta}{2} = \beta$$

$$\Rightarrow \beta = \frac{\sqrt{2}}{3}$$

4. NTA Ans. (2)

Sol. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; a > b;

$2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}} \Rightarrow b^2 = \frac{4}{3}$

tangent $y = \frac{-x}{6} + \frac{4}{3}$ compare with

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$\Rightarrow m = \frac{-1}{6} \Rightarrow \sqrt{\frac{a^2}{36} + \frac{4}{3}} = \frac{4}{3} \Rightarrow a = 4;$

$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2}\sqrt{\frac{11}{3}}$

5. Official Ans. by NTA (1)

Sol. For ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ (b < 5)

Let e_1 is eccentricity of ellipse

$\therefore\ b^2 = 25\ (1 - e_1^2)$

.... (1)

Again for hyperbola

$\frac{x^2}{16}-\frac{y^2}{b^2}=1$

Let e_2 is eccentricity of hyperbola.

$\therefore\ b^2 = 16(e_2^2 - 1)$

.... (2)

by (1) & (2)

$25(1 - e_1^2) = 16(e_2^2 - 1)$

Now $e_1.e_2 = 1$ (given)

$\therefore\ 25(1 - e_1^2) = 16\left(\frac{1-e_1^2}{e_1^2}\right)$

or $e_1 = \frac{4}{5}$ $\therefore\ e_2 = \frac{5}{4}$

Now distance between foci is 2ae

$\therefore$ distance for ellipse $= 2 \times 5 \times \frac{4}{5} = 8 = \alpha$

distance for hyperbola $= 2 \times 4 \times \frac{5}{4} = 10 = \beta$

$\therefore\ (\alpha, \beta) \equiv (8, 10)$

6. Official Ans. by NTA (1)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a > b)\ ;\ \frac{2b^2}{a}=10 \Rightarrow b^2 = 5a$ ...(i)

Now, $\phi(t)=\frac{5}{12}+t-t^2=\frac{8}{12}-\left(t-\frac{1}{2}\right)^2$

$\phi(t)_{max}=\frac{8}{12}=\frac{2}{3}=e \Rightarrow e^2=1-\frac{b^2}{a^2}=\frac{4}{9}$

... (ii)

$\Rightarrow\ a^2 = 81$ (from (i) & (ii))

So, $a^2 + b^2 = 81 + 45 = 126$

7. Official Ans. by NTA (2)

Sol. Ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

directrix : $x = \frac{a}{e}=4$ & $e = \frac{1}{2}$

$\Rightarrow a = 2$ & $b^2 = a^2\ (1-e^2) = 3$

$\Rightarrow$ Ellipse is $\frac{x^2}{4}+\frac{y^2}{3}=1$

P is $\left(1,\frac{3}{2}\right)$

Normal is : $\frac{4x}{1}-\frac{3y}{3/2}=4-3$

$\Rightarrow 4x - 2y = 1$

8. Official Ans. by NTA (2)

Sol. Given ellipse is $\frac{x^2}{5}+\frac{y^2}{4}=1$

Let point P is $(\sqrt{5}\cos\theta, 2\sin\theta)$

$(PQ)^2 = 5\cos^2\theta + 4\ (\sin\theta + 2)^2$

$(PQ)^2 = \cos^2\theta + 16\sin\theta + 20$

$(PQ)^2 = -\sin^2\theta + 16\sin\theta + 21$

$= 85 - (\sin\theta - 8)^2$

will be maximum when $\sin\theta = 1$

$\Rightarrow (PQ)^2_{max} = 85 - 49 = 36$

9. Official Ans. by NTA (1)

Sol. $\frac{x^2}{16}+\frac{y^2}{9}=1$

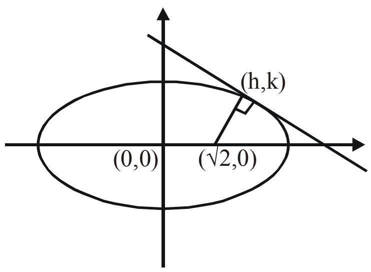
a = 4; b = 3; $e = \sqrt{\frac{16-9}{16}}=\frac{\sqrt{7}}{4}$

A and B are foci

$\Rightarrow PA + PB = 2a = 2 \times 4 = 8$

10. Official Ans. by NTA (1)

Sol. Let foot of perpendicular is (h,k)



$\frac{x^2}{4}+\frac{y^2}{2}=1$ (Given)

a =2 , $b=\sqrt{2}$, $e=\sqrt{1-\frac{2}{4}}=\frac{1}{\sqrt{2}}$

$\therefore$ Focus (ae,0) $=\left(\sqrt{2},0\right)$

Equation of tangent

$y=mx+\sqrt{a^2m^2+b^2}$

$y=mx+\sqrt{4m^2+2}$

Passes throguh (h,k)

$(k-mh)^2=4m^2+2$ ...(1)

line perpendicular to tangent will have slope

$-\frac{1}{m}$

$y-0=-\frac{1}{m}\left(x-\sqrt{2}\right)$

$my=-x+\sqrt{2}$

$(h+mk)^2=2$ ...(2)

Add equaiton (1) and (2)

$k^2(1+m^2)+h^2(1+m^2)=4(1+m^2)$

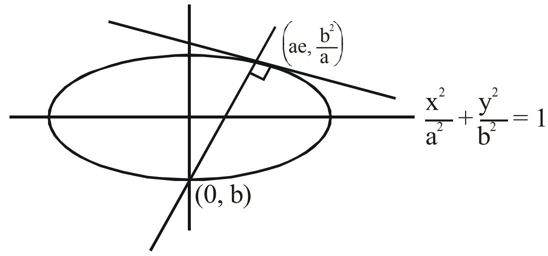
$h^2+k^2=4$

$x^2+y^2=4$ (Auxilary circle)

$\therefore\ \left(-1,\sqrt{3}\right)$ lies on the locus.

11. Official Ans. by NTA (4)

Sol. $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2e^2$



$\frac{a^2x}{ae}-\frac{b^2y}{b^2}.a=a^2e^2$

$\frac{ax}{e}-ay=a^2e^2 \Rightarrow \frac{x}{e}-y=ae^2$

passes through (0, b)

$-b=ae^2 \Rightarrow b^2=a^2e^4$

$a^2(1-e^2)=a^2e^4 \Rightarrow e^4+e^2=1$

HYPERBOLA

1. NTA Ans. (3)

Sol. $\frac{x^2}{36}-\frac{y^2}{b^2}=1$

...(i)

P(10,16) lies on (i) get $b^2=144$

$\frac{x^2}{36}-\frac{y^2}{144}=1$

Equation of normal is

$\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2e^2$

2x + 5y = 100

(3) Option

2. NTA Ans. (4)

Sol. $e_1=\sqrt{1-\frac{4}{18}}=\frac{\sqrt{7}}{3}$

$e_2=\sqrt{1+\frac{4}{9}}=\frac{\sqrt{13}}{3}$

$\because$ (e_1, e_2) lies on $15x^2+3y^2=k$

$\Rightarrow 15e_1^2+3e_2^2=k$

$\Rightarrow k=16$

3. Official Ans. by NTA (2)

Sol. Slope of tangent is 2, Tangent of hyperbola

$\frac{x^2}{4}-\frac{y^2}{2}=1$ at the point (x_1, y_1) is

$\frac{xx_1}{4}-\frac{yy_1}{2}=1$ (T = 0)

Slope : $\frac{1}{2}\frac{x_1}{y_1}=2 \Rightarrow \boxed{x_1=4y_1}$ ...(1)

(x_1, y_1) lies on hyperbola

$\Rightarrow \boxed{\frac{x_1^2}{4}-\frac{y_1^2}{2}=1}$ ...(2)

From (1) & (2)

$\frac{(4y_1)^2}{4}-\frac{y_1^2}{2}=1 \Rightarrow 4y_1^2-\frac{y_1^2}{2}=1$

$\Rightarrow 7y_1^2=2 \Rightarrow \boxed{y_1^2=2/7}$

Now $x_1^2 + 5y_1^2 = (4y_1)^2 + 5y_1^2$

$= (21)y_1^2 = 21 \times \frac{2}{7}=6$

4. Official Ans. by NTA (2)

Sol. Given $\theta \in \left(0, \frac{\pi}{2}\right)$

equation of hyperbola $\Rightarrow x^2 - y^2\sec^2\theta = 10$

$\Rightarrow \frac{x^2}{10}-\frac{y^2}{10\cos^2\theta}=1$

Hence eccentricity of hyperbola

$(e_H)=\sqrt{1+\frac{10\cos^2\theta}{10}}$ ...(1)

$\left\{e=\sqrt{1+\frac{b^2}{a^2}}\right\}$

Now equation of ellipse $\Rightarrow x^2\sec^2\theta + y^2 = 5$

$\Rightarrow \frac{x^2}{5\cos^2\theta}+\frac{y^2}{5}=1$ $\left\{e=\sqrt{1-\frac{a^2}{b^2}}\right\}$

Hence eccenticity of ellipse

$(e_E)=\sqrt{1-\frac{5\cos^2\theta}{5}}$

$(e_E)=\sqrt{1-\cos^2\theta}=|\sin\theta|=\sin\theta$ ...(2)

$\left\{\because \theta \in \left(0, \frac{\pi}{2}\right)\right\}$

given $\Rightarrow e_H = \sqrt{5}\, e_e$

Hence $1 + \cos^2\theta = 5\sin^2\theta$

$1 + \cos^2\theta = 5(1 - \cos^2\theta)$

$1 + \cos^2\theta = 5 - 5\cos^2\theta$

$6\cos^2\theta = 4$

$\cos^2\theta = \frac{2}{3}$ ...(3)

Now length of latus rectum of ellipse

$= \frac{2a^2}{b}=\frac{10\cos^2\theta}{\sqrt{5}}=\frac{20}{3\sqrt{5}}=\frac{4\sqrt{5}}{3}$

5. Official Ans. by NTA (2)

Sol. Ellipse : $\frac{x^2}{4}+\frac{y^2}{3}=1$

eccentricity $=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

$\therefore$ foci = $(\pm1, 0)$

for hyperbola, given $2a = \sqrt{2} \Rightarrow a = \frac{1}{\sqrt{2}}$

$\therefore$ hyperbola will be

$\frac{x^2}{1/2}-\frac{y^2}{b^2}=1$

eccentricity $=\sqrt{1+2b^2}$

$\therefore$ foci $=\left(\pm\sqrt{\frac{1+2b^2}{2}}, 0\right)$

$\because$ Ellipse and hyperbola have same foci

$\Rightarrow \sqrt{\frac{1+2b^2}{2}}=1$

$\Rightarrow b^2=\frac{1}{2}$

$\therefore$ Equation of hyperbola : $\frac{x^2}{1/2}-\frac{y^2}{1/2}=1$

$\Rightarrow x^2 - y^2 = \frac{1}{2}$

Clearly $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ does not lie on it.

6. Official Ans. by NTA (1)

Sol. Since, (3, 3) lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{9}{a^2}-\frac{9}{b^2}=1$

....(1)

Now, normal at (3, 3) is $y-3=-\frac{a^2}{b^2}(x-3)$,

which passes through (9, 0) $\Rightarrow b^2=2a^2$

....(2)

So, $e^2=1+\frac{b^2}{a^2}=3$

Also, $a^2=\frac{9}{2}$ (from (i) & (ii))

Thus, $(a^2, e^2)=\left(\frac{9}{2}, 3\right)$

7. Official Ans. by NTA (2)

Sol. y = mx + c is tangent to

$\frac{x^2}{100}-\frac{y^2}{64}=1$ and $x^2+y^2=36$

$c^2=100m^2-64 \mid c^2=36(1+m^2)$

$\Rightarrow 100m^2-64=36+36m^2$

$m^2=\frac{100}{64} \Rightarrow m=\pm\frac{10}{8}$

$c^2=36\left(1+\frac{100}{64}\right)=\frac{36\times164}{64}$

$4c^2=369$

COMPLEX NUMBER

1. NTA Ans. (3)

Sol. $\frac{3+i\sin\theta}{4-i\cos\theta}$ is a real number

$\Rightarrow 3\cos\theta+4\sin\theta=0$

$\Rightarrow \tan\theta=\frac{-3}{4}$

argument of $\sin\theta+i\cos\theta=\pi-\tan^{-1}\frac{4}{3}$

2. NTA Ans. (2)

Sol. $\text{Re}\left(\frac{z-1}{2z+i}\right)=1$

Put z = x + iy

$\text{Re}\left(\frac{(x+iy)-1}{2(x+iy)+i}\right)=1$

$\text{Re}\left(\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right)\left(\frac{2x-i(2y+1)}{2x-i(2y+1)}\right)\right)=1$

$\Rightarrow 2x^2+2y^2+2x+3y+1=0$

$x^2+y^2+x+\frac{3}{2}y+\frac{1}{2}=0$

$\Rightarrow$ locus is a circle whose

Centre is $\left(-\frac{1}{2},-\frac{3}{4}\right)$ and radius $\frac{\sqrt{5}}{4}$

$\Rightarrow$ diameter $=\frac{\sqrt{5}}{2}$

3. NTA Ans. (1)

Sol. $\alpha=\omega$

$a=(1+\omega)(1+\omega^2+\omega^4+.....+\omega^{200})$

$a=(1+\omega)\frac{\left(1-\left(\omega^2\right)^{101}\right)}{1-\omega^2}=1$

$b=1+\omega^3+\omega^6+.....+\omega^{300}=101$

$x^2-102x+101=0$

(1) Option

4. NTA Ans. (4)

Sol. Assuming z is a root of the given equation,

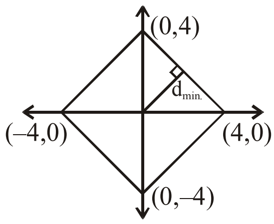
$z=\frac{-b\pm i\sqrt{180-b^2}}{2}$

so, $\left(1-\frac{b}{2}\right)^2+\frac{180-b^2}{4}=40$

$\Rightarrow -4b+184=160 \Rightarrow b=6$

5. NTA Ans. (4)

Sol. z = x + iy



|x| + |y| = 4

$|z| = \sqrt{x^2 + y^2} \Rightarrow |z|_{min} = \sqrt{8}$ & $|z|_{max} = 4 = \sqrt{16}$

So |z| cannot be $\sqrt{7}$

6. NTA Ans. (3)

Sol. $\left|\frac{z-i}{z+2i}\right| = 1$

⇒ |z – i| = |z + 2i|

⇒ z lies on perpendicular bisector of (0, 1) and (0, –2).

$\Rightarrow \text{Im} z = -\frac{1}{2}$

Let $z = x - \frac{i}{2}$

$\because |z| = \frac{5}{2} \Rightarrow x^2 = 6$

$\therefore |z + 3i| = \left|x + \frac{5i}{2}\right| = \sqrt{x^2 + \frac{25}{4}}$

$= \sqrt{6 + \frac{25}{4}} = \frac{7}{2}$

7. Official Ans. by NTA (2)

Sol. The value of $\left(\frac{1+\sin 2\pi/9 + i\cos 2\pi/9}{1+\sin\frac{2\pi}{9} - i\cos\frac{2\pi}{9}}\right)$

$= \left(\frac{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)+i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)-i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}\right)^3$

$= \left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$

$= \left(\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}\right)^3$

$= \left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3$

$= \left(\frac{e^{i5\pi/36}}{e^{-i5\pi/36}}\right)^3 = \left(e^{i5\pi/18}\right)^3$

$= \cos\frac{5\pi}{6} + i\sin 5\pi/6$

$= -\frac{\sqrt{3}}{2} + i/2$

8. Official Ans. by NTA (1)

Sol. $\left(3+2\sqrt{-54}\right) = 3 + 2\times 3\times\sqrt{6}\,i$

$= \left(3+\sqrt{6}\,i\right)^2$

$\left(3-2\sqrt{54}\right) = \left(3-\sqrt{6}\,i\right)^2$

$\left(3+2\sqrt{-54}\right)^{1/2} + \left(3-2\sqrt{-54}\right)^{1/2}$

$= \pm\left(3+\sqrt{6}\,i\right) \pm \left(3-\sqrt{6}\,i\right)$

= 6, –6, $2\sqrt{6}i, -2\sqrt{6}i,$

9. Official Ans. by NTA (4)

Sol. $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$

$\Rightarrow \left(\frac{(1+i)^2}{2}\right)^{m/2} = \left(\frac{(1+i)^2}{-2}\right)^{n/3} = 1$

$\Rightarrow (i)^{m/2} = (-i)^{n/3} = 1$

$\Rightarrow \frac{m}{2} = 4k_1$ and $\frac{n}{3} = 4k_2$

$\Rightarrow m = 8k_1$ and $n = 12k_2$

Least value of m = 8 and n = 12.

$\therefore$ GCD = 4

10. Official Ans. by NTA (4)

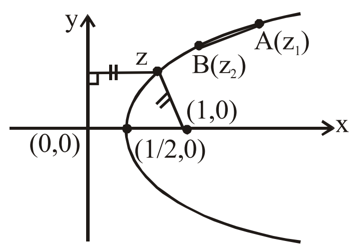
Sol. $Re(z) = |z - 1|$

$\Rightarrow x = \sqrt{(x-1)^2 + (y-0)^2}$ $(x > 0)$

$\Rightarrow y^2 = 2x - 1 = 4.\frac{1}{2}\left(x - \frac{1}{2}\right)$

$\Rightarrow$ a parabola with focus (1, 0) & directrix as imaginary axis.

$\therefore$ Vertex $= \left(\frac{1}{2}, 0\right)$



$A(z_1)$ & $B(z_2)$ are two points on it such that

slope of AB $= \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$(\arg(z_1 - z_2) = \frac{\pi}{6})$

for $y^2 = 4ax$

Let $A(at_1^2, 2at_1)$ & $B(at_2^2, 2at_2)$

$m_{AB} = \frac{2}{t_1 + t_2} = \frac{4a}{y_1 + y_2} = \frac{1}{\sqrt{3}}$

$\left(\text{Here } a = \frac{1}{2}\right)$

$\Rightarrow y_1 + y_2 = 4a\sqrt{3} = 2\sqrt{3}$

11. Official Ans. by NTA (3)

Sol. $A^2 = \begin{pmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{pmatrix}$

Similarly, $A^5 = \begin{pmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(1) $a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos 75°$

(2) $a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 0$

(3) $a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

(4) $a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

12. Official Ans. by NTA (3)

Sol. $u = \frac{2z + i}{z - ki}$

$= \frac{2x^2 + (2y+1)(y-k)}{x^2 + (y-k)^2} + i\frac{(x(2y+1) - 2x(y-k))}{x^2 + (y-k)^2}$

Since Re(u) + Im(u) = 1

$\Rightarrow 2x^2 + (2y+1)(y-k) + x(2y+1) - 2x(y-k) = x^2 + (y-k)^2$

$\left.\begin{matrix} P(0, y_1) \\ Q(0, y_2) \end{matrix}\right\rangle \Rightarrow y^2 + y - k - k^2 = 0 \left\langle\begin{matrix} y_1 + y_2 = -1 \\ y_1 y_2 = -k - k^2 \end{matrix}\right.$

$\because$ PQ = 5

$\Rightarrow |y_1 - y_2| = 5 \Rightarrow k^2 + k - 6 = 0$

$\Rightarrow k = -3, 2$

So, k = 2 (k > 0)

13. Official Ans. by NTA (4)

Sol. $\alpha = \omega \quad (\omega^3 = 1)$

$\Rightarrow (2 + \omega)^4 = a + b\omega$

$\Rightarrow 2^4 + 4.2^3\,\omega + 6.2^2\omega^3 + 4.2\,.\,\omega^3 + \omega^4 = a + b\omega$

$\Rightarrow 16 + 32\,\omega + 24\,\omega^2 + 8 + \omega = a + b\omega$

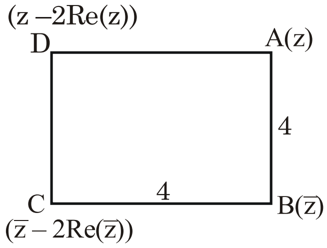
$\Rightarrow 24 + 24\,\omega^2 + 33\omega = a + b\omega$

$\Rightarrow -24\omega + 33\omega = a + b\omega$

$\Rightarrow a = 0, b = 9$

14. Official Ans. by NTA (4)

Sol. Let z = x + iy



Length of side = 4

AB = 4

$|z-\bar{z}|=4$

|2y| = 4 ; |y| = 2

BC = 4

$|\bar{z}-(\bar{z}-2\,\mathrm{Re}(\bar{z}))|=4$

|2x| = 4 ; |x| = 2

$|z| = \sqrt{x^2+y^2} = \sqrt{4+4} = 2\sqrt{2}$

15. Official Ans. by NTA (3)

Sol. $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}=\left(\frac{2\omega}{1-i}\right)^{30}$

$=\frac{2^{30}\cdot\omega^{30}}{\left((1-i)^2\right)^{30}}$

$=\frac{2^{30}\cdot 1}{(1+i^2-2i)^{15}}$

$=\frac{2^{30}}{-2^{15}\cdot i^{15}}$

$= -2^{15}i$

16. Official Ans. by NTA (4)

Sol. z = x + iy

$|z| - ke(z) \le 1$

$\Rightarrow \sqrt{x^2+y^2}-x\le 1$

$\Rightarrow \sqrt{x^2+y^2}\le 1+x$

$\Rightarrow x^2+y^2\le 1+2x+x^2$

$\Rightarrow y^2\le 2x+1$

$\Rightarrow y^2\le 2\left(x+\frac{1}{2}\right)$

17. Official Ans. by NTA (3)

Sol. z = x + iy

$z^2 = i|z|^2$

$(x + iy)^2 = i(x^2 + y^2)$

$(x^2 - y^2) - i(x^2 + y^2 - 2xy) = 0$

$(x - y)(x + y) - i(x - y)^2 = 0$

$(x - y)((x + y) - i(x - y)) = 0$

$\Rightarrow x = y$

z lies on y = x

PROBABILITY

1. NTA Ans. (3)

Sol. Probability that at most 2 machines are out of service

$= {}^5C_0\left(\frac{3}{4}\right)^5+{}^5C_1\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)+{}^5C_2\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$

$=\left(\frac{3}{4}\right)^4\times\frac{17}{8}\Rightarrow k=\frac{17}{8}$

2. NTA Ans. (3)

Sol.

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

Expected value = $\sum$ XP(k)

$-\frac{1}{32}-\frac{12}{32}-\frac{11}{32}+\frac{15}{32}+\frac{8}{32}+\frac{5}{32}$

$=\frac{28-24}{32}=\frac{4}{32}=\frac{1}{8}$

3. NTA Ans. (4)

Sol. $P(A) + P(B) - 2P(A\cap B) = \frac{2}{5}$

$P(A) + P(B) - P(A\cap B) = \frac{1}{2}$

$P(A\cap B) = \frac{1}{10}$

(4) Option

4. NTA Ans. (3)

Sol. (1) $P(A/B)=P(A)=\frac{1}{3}$

(2) $P(A/(A\cup B))=\frac{P(A\cap(A\cup B))}{P(A\cup B)}=\frac{P(A)}{P(A\cup B)}$

$$=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}-\frac{1}{18}}=\frac{3}{4}$$

(3) $P(A/B')=P(A)=\frac{1}{3}$

(4) $P(A'/B')=P(A')=\frac{2}{3}$

5. NTA Ans. (2)

Sol. $\sum P(X)=1 \Rightarrow K^2 + 2K + K + 2K + 5K^2 = 1$

$\Rightarrow 6K^2 + 5K - 1 = 0 \Rightarrow (6K - 1)(K + 1) = 0$

$\Rightarrow K = -1$ (rejected) $\Rightarrow K = \frac{1}{6}$

$P(X > 2) = K + 2K + 5K^2 = \frac{23}{36}$

6. NTA Ans. (3)

ALLEN Ans. (BONUS)

Note: Interpretating the given question, we find an answer that does not match with any of the given options. So, it should be bonus, but NTA retained the answer as option(3).

Sol. 10 different balls in 4 different boxes.

10

2,3,0,5   2,3,1,4   2,3,2,3

$$\frac{1}{4^{10}}\left(4!\times\frac{10!}{2!\times 3!\times 0!\times 5!}+4!\times\frac{10!}{2!\times 3!\times 1!\times 4!}+4!\times\frac{10!}{(2!)^2\times 2!\times(3!)^2\times 2!}\right)$$

$$=\frac{17\times 945}{2^{15}}$$

7. NTA Ans. (1)

Sol. A : Event when card A is drawn

B : Event when card B is drawn.

$P(A)=P(B)=\frac{1}{2}$

Required probability = P(AA or (AB)A or (BA)A or (ABB)A or (BAB)A or (BBA)A)

$$=\frac{1}{2}\times\frac{1}{2}+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times 2+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times 3$$

$$=\frac{1}{4}+\frac{1}{4}+\frac{3}{16}=\frac{11}{16}$$

8. Official Ans. by NTA (1)

Sol. Let B_1 be the event where Box–I is selected.

& $B_2 \rightarrow$ where box-II selected

$P(B_1) = P(B_2) = \frac{1}{2}$

Let E be the event where selected card is non prime.

For B_1 : Prime numbers :

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

For B_2 : Prime numbers :

{31, 37, 41, 43, 47}

$P(E) = P(B_1) \times P\left(\frac{E}{B_1}\right) + P(B_2)P\left(\frac{E}{B_2}\right)$

$$=\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}$$

Required probability :

$$P\left(\frac{B_1}{E}\right)=\frac{\frac{1}{2}\times\frac{20}{30}}{\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}}=\frac{\frac{2}{3}}{\frac{2}{3}+\frac{3}{4}}=\frac{8}{17}$$

9. Official Ans. by NTA (1)

Sol. Given E_1, E_2, E_3 are pairwise indepedent events so $P(E_1 \cap E_2) = P(E_1).P(E_2)$

and $P(E_2 \cap E_3) = P(E_2).P(E_3)$

and $P(E_3 \cap E_1) = P(E_3).P(E_1)$

& $P(E_1 \cap E_2 \cap E_3) = 0$

Now $P\left(\frac{\bar{E}_2 \cap \bar{E}_3}{E_1}\right) = \frac{P\left[E_1 \cap (\bar{E}_2 \cap \bar{E}_3)\right]}{P(E_1)}$

$$= \frac{P(E_1) - \left[P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)\right]}{P(E_1)}$$

$$= \frac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) - 0}{P(E_1)}$$

$= 1 - P(E_2) - P(E_3)$

$= [1 - P(E_3)] - P(E_2)$

$= P(E_3^C) - P(E_2)$

10. Official Ans. by NTA (2)

Sol. A : Sum obtained is a multiple of 4.

A = {(1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)}

B : Score of 4 has appeared at least once.

B = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}

Required probability = $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{1/36}{9/36} = \frac{1}{9}$$

11. Official Ans. by NTA (3)

Sol. First Case: Choose two non-zero digits ${}^9C_2$

Second Case : Number of 5-digit numbers containing both digits = $2^5 - 2$

Choose one non-zero & one zero as digit = ${}^9C_1$

Number of 5-digit numbers containg one non zero and one zero both = $(2^4 - 1)$

$\therefore$ Required prob.

$$= \frac{\left({}^9C_2 \times (2^5 - 2) + {}^9C_1 \times (2^4 - 1)\right)}{9 \times 10^4}$$

$$= \frac{36 \times (32 - 2) + 9 \times (16 - 1)}{9 \times 10^4}$$

$$= \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}$$

12. Official Ans. by NTA (3)

Sol. We have, 1 – (probability of all shots result in failure) > $\frac{1}{4}$

$$\Rightarrow 1 - \left(\frac{9}{10}\right)^n > \frac{1}{4}$$

$$\Rightarrow \frac{3}{4} > \left(\frac{9}{10}\right)^n \Rightarrow n \geq 3$$

13. Official Ans. by NTA (4)

Sol. $P(6) = \frac{1}{6}$, $P(7) = \frac{5}{36}$

P(A) = W + FFW + FFFFW + ...

$$= \frac{1}{6} + \frac{5}{6} \times \frac{31}{36} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \left(\frac{31}{36}\right)^2 \frac{1}{6} + \ldots$$

$$= \frac{\frac{1}{6}}{1 - \frac{155}{216}} = \frac{36}{61}$$

14. Official Ans. by NTA (11)

Sol. 4 dice are independently thrown. Each die has probability to show 3 or 5 is

$p = \frac{2}{6} = \frac{1}{3}$

$\therefore q = 1 - \frac{1}{3} = \frac{2}{3}$ (not showing 3 or 5)

Experiment is performed with 4 dices independently.

$\therefore$ Their binomial distribution is

$(q + p)^4 = (q)^4 + {}^4C_1 q^3p + {}^4C_2 q^2p^2 + {}^4C_3 qp^3 + {}^4C_4p^4$

$\therefore$ In one throw of each dice probability of showing 3 or 5 at least twice is

$= p^4 + {}^4C_3 qp^3 + {}^4C_2q^2p^2 = \frac{33}{81}$

$\therefore$ Such experiment performed 27 times

$\therefore$ so expected out comes = np

$= \frac{33}{81} \times 27$

= 11

15. Official Ans. by NTA (11.00)

Sol. $P(H) = \frac{1}{2}$

$P(\bar{H}) = \frac{1}{2}$

Let total 'n' bomb are required to destroy the target

$1 - {}^nC_n\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^n \geq \frac{99}{100}$

$1 - \frac{1}{2^n} - \frac{n}{2^n} \geq \frac{99}{100}$

$\frac{1}{100} \geq \frac{n+1}{2^n}$

Now check for value of n

$\boxed{n = 11}$

16. Official Ans. by NTA (2)

Sol. Total numbers in three familes = 3 + 3 + 4 = 10

so total arrangement = 10!

Family 1	Family 2	Family 3
3	3	4

Favourable cases

$= \underset{\text{Arrangment of 3 Families}}{3!} \quad \underset{\text{Interval Arrangment of families members}}{3! \times 3! \times 4!}$

$\therefore$ Probability of same family memebers are together $= \frac{3!\,3!\,3!\,4!}{10!} = \frac{1}{700}$

so option(2) is correct.

17. Official Ans. by NTA (3)

Sol. Out of 11 consecutive natural numbers either 6 even and 5 odd numbers or 5 even and 6 odd numbers

when 3 numbers are selected at random then total cases = ${}^{11}C_3$

Since these 3 numbers are in A.P. Let no's are a,b,c

2b $\Rightarrow$ even number

$a + c \Rightarrow \begin{pmatrix} \text{even} + \text{even} \\ \text{odd} + \text{odd} \end{pmatrix}$

so favourable cases = ${}^6C_2 + {}^5C_2$

= 15 + 10 = 25

P(3 numbers are in A.P. $= \frac{25}{{}^{11}C_3} = \frac{25}{165} = \frac{5}{33}$)

18. Official Ans. by NTA (3)

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.2$

$P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$\alpha = 1.5 - (0.2 + 0.3 + \beta) + 0.2$

$\alpha = 1.2 - \beta \in [0.85, 0.95]$

(where $\alpha \in [0.85, 0.95]$)

$\beta \in [0.25, 0.35]$

STATISTICS

1. NTA Ans. (54.00)

Sol. $\frac{3+7+9+12+13+20+x+y}{8}=10$

x + y = 16

$\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2=25$

$3^2 + 7^2 + 9^2 + 12^2 + 13^2 + 20^2 + x^2 + y^2 = 1000$

$x^2 + y^2 = 148$

xy = 54

2. NTA Ans. (18)

Sol. Variance of first 'n' natural numbers $=\frac{n^2-1}{12}=10$

$\Rightarrow$ n = 11

and variance of first 'm' even natural numbers

$= 4\left(\frac{m^2-1}{12}\right)\Rightarrow\frac{m^2-1}{3}=16\Rightarrow m=7$

m + n = 18

3. NTA Ans. (1)

Sol. $\frac{\sum x_i}{20}=10 \Rightarrow \Sigma x_i = 200$

...(i)

$\frac{\sum x_i^2}{20}-100=4 \Rightarrow \Sigma x_i^2 = 2080$

...(ii)

Actual mean = $\frac{200-9+11}{20}=\frac{202}{20}$

Variance = $\frac{2080-81+121}{20}-\left(\frac{202}{20}\right)^2=3.99$

(1) Option

4. NTA Ans. (1)

Sol. 20p – q = 10 ...(i)

and $2|p| = 1 \Rightarrow p=\pm\frac{1}{2}$ ...(ii)

so, $p=-\frac{1}{2}$ and q = –20

5. NTA Ans. (4)

Sol. $\sum_{i=1}^{10}(x_i-5)=10$

$\Rightarrow$ Mean of observation $x_i-5=\frac{1}{10}\sum_{i=1}^{3}(x_i-5)=1$

$\Rightarrow \mu$ = mean of observation $(x_i - 3)$

= (mean of observation $(x_i - 5)$) + 2

= 1 + 2 = 3

Variance of observation

$x_i-5=\frac{1}{10}\sum_{i=1}^{10}(x_i-5)^2$

– (Mean of $(x_i - 5))^2$ = 3

$\Rightarrow \lambda$ = variance of observation $(x_i - 3)$

= variance of observation $(x_i - 5)$ = 3

$\therefore (\mu, \lambda) = (3, 3)$

6. Official Ans. by NTA (1)

Sol. σ^2 = variance

μ = mean

$\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$

μ = 17

$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b)}{17}=17$

$\Rightarrow$ 9a + b = 17 ....(1)

σ^2 = 216

$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b-17)^2}{17}=216$

$\Rightarrow \frac{\sum_{x=1}^{17}a^2(x-9)^2}{17}=216$

$\Rightarrow a^2 81 - 18 \times 9a^2 + a^2\, 3 \times (35) = 216$

$\Rightarrow a^2 = \frac{216}{24} = 9 \Rightarrow a = 3\ (a > 0)$

$\Rightarrow$ From (1), b = –10

So, a + b = –7

7. Official Ans. by NTA (3.00)

Sol. Let a be the first term and d be the common difference of the given A.P. Where d > 0

$$\bar{X} = a + \frac{0 + d + 2d + ... + 10d}{11}$$

$= a + 5d$

$\Rightarrow$ varience $= \frac{\Sigma(\bar{X} - x_i)^2}{11}$

$\Rightarrow 90 \times 11 = (25d^2 + 16d^2 + 9d^2 + 4d^2) \times 2$

$\Rightarrow d = \pm 3 \Rightarrow d = 3$

8. Official Ans. by NTA (4)

Sol. $\because \sigma^2 \le \frac{1}{4}(M - m)^2$

Where M and m are upper and lower bounds of values of any random variable.

$\therefore\ \sigma^2 < \frac{1}{4}(10 - 0)^2$

$\Rightarrow 0 < \sigma < 5$

$\therefore\ \sigma \neq 6.$

9. Official Ans. by NTA (3)

Sol. $\text{Variance} = \frac{\Sigma(x_i - p)^2}{n} - \left(\frac{\Sigma(x_i - p)}{n}\right)^2$

$= \frac{9}{10} - \left(\frac{3}{10}\right)^2 = \frac{81}{100}$

S.D. $= \frac{9}{10}$

10. Official Ans. by NTA (1)

Sol. $\bar{x} = 10$

$\Rightarrow\ \bar{x} = \frac{63 + a + b}{8} = 10 \Rightarrow\ a + b = 17$ ...(1)

Since, variance is independent of origin.

So, we subtract 10 from each observation.

So, $\sigma^2 = 13.5 = \frac{79 + (a-10)^2 + (b-10)^2}{8} - (10-10)^2$

$\Rightarrow a^2 + b^2 - 20(a + b) = -171$

$\Rightarrow a^2 + b^2 = 169$ ...(2)

From (i) & (ii) ; a = 12 & b = 5

11. Official Ans. by NTA (4)

Sol. $\because$ Variance is independent of shifting of origin

$\Rightarrow$ x_i : 15 25 35 or –10 0 10

f_i : 2 x 2 2 x 2

$\Rightarrow$ Variance $(\sigma^2) = \frac{\Sigma x_i^2 f_i}{\Sigma f_i} - (\bar{x})^2$

$\Rightarrow 50 = \frac{200 + 0 + 200}{x + 4} - 0\ \{\bar{x} = 0\}$

$\Rightarrow 200 + 50x = 200 + 200$

$\Rightarrow x = 4$

12. Official Ans. by NTA (1)

Sol. $\bar{x} = \frac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8$

x + y = 14

.....(i)

$(\sigma)^2 = \frac{\sum (x_i)^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$16 = \frac{4 + 16 + 100 + 144 + 196 + x^2 + y^2}{7} - 8^2$

$16 + 64 = \frac{460 + x^2 + y^2}{7}$

$560 = 460 + x^2 + y^2$

$x^2 + y^2 = 100$ ....(ii)

Clearly by (i) and (ii), $|x - y| = 2$

Ans. 1

13. Official Ans. by NTA (2)

Sol. Mean = 5

$\frac{3 + 5 + 7 + a + b}{5} = 5$

a + b = 10 ....(i)

S.d. $= 2 \Rightarrow \sqrt{\frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{5}} = 2$

$(3-5)^2 + (5-5)^2 + (7-5)^2 + (a-5)^2 + (b-5)^2 = 20$

$\Rightarrow 4 + 0 + 4 + (a - 5)^2 + (b - 5)^2 = 20$

$a^2 + b^2 - 10(a + b) + 50 = 12$

$(a + b)^2 - 2ab - 100 + 50 = 12$

ab = 19 ....(ii)

Equation is $x^2 - 10x + 19 = 0$

14. Official Ans. by NTA (2)

Sol. S.D $=\sqrt{\dfrac{\sum_{i=1}^{n}(x_i-a)}{n}-\left(\dfrac{\sum_{i=1}^{n}(x_i-a)}{n}\right)^2}$

$=\sqrt{\dfrac{na}{n}-\left(\dfrac{n}{n}\right)^2}$

{Given $\sum_{i=1}^{n}(x_i-a)=n$ $\sum_{i=1}^{n}(x_i-a)^2=na$ }

$=\sqrt{a-1}$

15. Official Ans. by NTA (6.00)

Sol.

x	0	2	4	8		2^n
f	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$		${}^nC_n$

Mean $=\dfrac{\Sigma x_i f_i}{\Sigma f_i}=\dfrac{\sum_{r=1}^{n}2^r\ {}^nC_r}{\sum_{r=0}^{n}{}^nC_r}$

Mean $=\dfrac{(1+2)^n-{}^nC_0}{2^n}=\dfrac{728}{2^n}$

$\Rightarrow \dfrac{3^n-1}{2^n}=\dfrac{728}{2^n}$

$\Rightarrow 3^n = 729 \Rightarrow n = 6$

MATHEMATICAL REASONING

1. NTA Ans. (2)

Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$

$(A \subseteq B) \wedge (B \subseteq D) \longrightarrow (A \subseteq C)$

Contrapositive is

$\sim(A \subseteq C) \longrightarrow \sim(A \subseteq B) \vee \sim(B \subseteq D)$

$A \not\subseteq C \to (A \not\subseteq B) \vee (B \not\subseteq D)$

2. NTA Ans. (3)

Sol. $(p \to q) \wedge (q \to \sim p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$

$\equiv \sim p v(q \wedge \sim q)$

$\equiv \sim p \vee C \equiv \sim p$

3. NTA Ans. (1)

Sol. $\sim(p \vee \sim q) \to p \vee q$

$(\sim p \wedge q) \to p \vee q$

$\sim\{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$

$\sim(\sim p \wedge f)$

(1) Option

4. NTA Ans. (4)

Sol. (1) $P \wedge (P \vee Q) \equiv P$

(2) $P \vee (P \wedge Q) \equiv P$

(3) $Q \to (P \wedge (P \to Q))$

$\equiv Q \to (P \wedge (\sim P \vee Q)) \equiv Q \to (P \wedge Q)$

$\equiv (\sim Q) \vee (P \wedge Q) \equiv (P \vee (\sim Q))$

(4) $(P \wedge (P \to Q)) \to Q$

$\equiv (P \wedge (\sim P \vee Q)) \to Q \equiv (P \wedge Q) \to Q$

$\equiv ((\sim P) \vee (\sim Q)) \vee Q \equiv (\sim P) \vee t \equiv t$

5. NTA Ans. (2)

Sol. $p \to (p \wedge \sim q)$ is F $\Rightarrow$ p is T & $p \wedge \sim q$ is F $\Rightarrow$ q is T

$\therefore$ p is T, q is T

6. NTA Ans. (2)

Sol. $p=\sqrt{5}$ is an integer.

q : 5 is irrational

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$=\sqrt{5}$ is not an integer and 5 is not irrational.

7. Official Ans. by NTA (3)

Sol. Let p denotes statement

p : I reach the station in time.

q : I will catch the train.

Contrapositive of $p \to q$

is $\sim q \to \sim p$

$\sim q \to \sim p$: I will not catch the train, then I do not reach the station in time.

8. Official Ans. by NTA (1)

Sol. Option (1) is

$\sim p \wedge (p \vee q) \rightarrow q$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q) \rightarrow q$

$\equiv C \vee (\sim p \wedge q) \rightarrow q$

$\equiv (\sim p \wedge q) \rightarrow q$

$\equiv \sim(\sim p \wedge q) \vee q$

$\equiv (p \vee \sim q) \vee q$

$\equiv (p \vee q) \vee (\sim q \vee q)$

$\equiv (p \vee q) \vee t$

so $\sim p \wedge (p \vee q) \rightarrow q$ is a tautology

9. Official Ans. by NTA (1)

Sol. $p \rightarrow \sim (p \wedge \sim q)$

$= \sim p \vee \sim (p \wedge \sim q)$

$= \sim p \vee \sim p \vee q$

$= \sim (p \wedge q) \vee q$

$= \sim p \vee q$

10. Official Ans. by NTA (3)

Sol. $(p \wedge q) \rightarrow (\sim q \vee r) =$ false

when $(p \wedge q) = T$

and $(\sim q \vee r) = F$

So $(p \wedge q) = T$ is possible when $p = q =$ true

$\therefore \sim q =$ False ($q =$ true)

So $(\sim q \vee r) =$ False is possible if r is false

$\therefore p = T, q = T, r = F$

11. Official Ans. by NTA (3)

Sol. Let TV(r) denotes truth value of a statement r.

Now, if TV(p) = TV(q) = T

$\Rightarrow TV(S_1) = F$

Also, if TV(p) = T & TV(q) = F

$\Rightarrow TV(S_2) = T$

12. Official Ans. by NTA (3)

Sol. p = function is differantiable at a

q = function is continuous at a

contrapositive of statement $p \rightarrow q$ is

$\sim q \rightarrow \sim p$

13. Official Ans. by NTA (3)

Sol. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$x \leftrightarrow \sim y \equiv (x \rightarrow \sim y) \wedge (\sim y \rightarrow x)$

$\because (p \rightarrow q \equiv \sim p \vee q)$

$x \leftrightarrow \sim y \equiv (\sim x \vee \sim y) \wedge (y \vee x)$

$\sim (x \leftrightarrow \sim y) \equiv (x \wedge y) \vee (\sim x \wedge \sim y)$

14. Official Ans. by NTA (3)

Sol.

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$p \vee q$	$p \rightarrow p \vee q$	$(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

15. Official Ans. by NTA (3)

Sol. Negation of $\phi \vee (\sim p \wedge q)$

$p \vee (\sim p \wedge q) = (p \vee \sim p) \wedge (p \vee q)$

$= (T) \wedge (p \vee q)$

$= (p \vee q)$

now negation of $(p \vee q)$ is

$\sim (p \vee q) = \sim p \wedge \sim q$

16. Official Ans. by NTA (2)

Sol. Contrapositive of $(p \rightarrow q)$ is $\sim q \rightarrow \sim p$

For an integer n, if n is even then $(n^3 - 1)$ is odd